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Reaction-Diffusion Modeling in PPO Micro Planes: Analysis of Akbari Ganji and Homotopy Perturbation Methods

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Abstract This study analyzes diffusion kinetics at a polyphenol oxidase (PPO)-modified microplanar biosensor using the Akbari Ganji Method (AGM) and the Homotopy Perturbation Method (HPM) as analytical techniques. The diffusion process is governed by Fick's law, and numerical simulations using MATLAB serve as a benchmark for validation. The results indicate that both AGM and HPM provide highly accurate predictions that closely align with numerical solutions, demonstrating their effectiveness in modeling biosensor diffusion. Quantitative comparisons reveal minimal deviations, which confirms the reliability of these methods in refining the diffusion behavior. The analytical approaches not only offer precise forecasts but also provide computational efficiency over numerical simulations, making them suitable for real-time biosensor modeling. Validation against numerical results underscores their robustness in predicting sensor performance, ensuring practical applicability in biosensor design. The findings have significant implications for biomedical applications, where diffusion properties play a crucial role in sensor accuracy and response time. Improved understanding and modeling of diffusion kinetics can improve the performance of biosensors used in medical diagnostics, environmental monitoring, and glucose detection in the management of diabetes. These insights contribute to the optimization of biosensor technology, facilitating advances in real-time biochemical sensing and improving sensor reliability in healthcare applications.

1 Introduction

As valuable instruments for various biomedical applications, including disease diagnosis, medication research, and environmental monitoring, biosensors have recently attracted a lot of attention. These instruments depend on accurate target analyse detection, which is frequently accomplished through surface modification with functional materials. Poly phenol oxide (PPO) is one such substance with a high degree of stability, great sensing capabilities, and bio-compatibility. Microplanar biosensors modified with PPO have become a potentially sensitive and focused detection platform. Biosensor performance features such as response time, sensitivity, and limit of detection are heavily influenced by diffusion kinetics. To improve the design and performance of these devices, it is crucial to comprehend and forecast the diffusion behaviour within them. Diffusion mechanisms in biosensors have been extensively studied and modeled using analytical and numerical techniques.

Theory and experiment for micro-cylinder biosensors for catechol and phenol-based on layer-by-layer tyrosinase immobilization on latex particles [2].Diffusion-kinetic model analytical expressions by A. Eswari et al. concerning the concentration of catechol, o-quinone and current at PPO-modified micro cylinder biosensor [3]. A.Eswari and S.Saravanakumar's hyperbolic function method for a new mathematical study of the nonlinear simultaneous differential equation in a micro-disc biosensor [4]. The Taylor series and hyperbolic function approaches were used by Silambuselvi et al. [11] to theoretically analyse the amperometric response to PPO-based rotating disc bioelectrodes. Steady-state current mathematical model at ppo-modified micro-cylinder biosensors [13]. Voltammetry at a Rotating Disk Electrode with No Supporting Electrolyte: A Theoretical Analysis [19]. Carbon dioxide concentrations in phenyl glycidyl ether solutions at steady-state were determined using the residual method [34]. Mudassir Shams et al.[25] discussed the Embedding Family of Numerical Schemes for Solving Non-Linear Equations with Engineering Applications where they developed an advanced numerical framework that embeds iterative correction techniques to enhance the stability and convergence rate of root-finding algorithms for nonlinear equations. The physical mechanism relies on an adaptive correction strategy that refines the approximate roots iteratively, ensuring faster convergence while minimizing computational complexity. The embedding approach integrates error correction from previous iterations, making it robust for engineering applications such as structural mechanics and thermal analysis. Shams et al.[20] explained the Triangular Intuitionistic Fuzzy Linear System of Equations with Application, where they investigated the solution methodology for systems modeled using triangular intuitionistic fuzzy numbers. The physical mechanism incorporates the principles of uncertainty quantification, where intuitionistic fuzzy numbers account for hesitation degrees, enhancing the flexibility of real-world applications such as decision-making and control systems. The analytical approach utilizes an extended decomposition method to systematically extract deterministic solutions from uncertain fuzzy environments.

Shams et al.[17] explored the numerical scheme to estimate all roots of nonlinear equations with applications, in which they designed an iterative numerical technique capable of capturing multiple roots of nonlinear equations efficiently. The physical mechanism involves a systematic root-tracing algorithm that dynamically adjusts the search domain, preventing the omission of critical roots while reducing the sensitivity to initial guesses. This approach is highly applicable in computational physics, signal processing, and circuit analysis, where identifying all possible solutions is essential. Shams et al.[23] analyzed the Techniques for Finding Analytical Solutions of Generalized Fuzzy Differential Equations, where they introduced new analytical techniques to solve fuzzy differential equations under generalized conditions. The physical mechanism is based on extending classical differential calculus into the fuzzy domain, where uncertainty propagation is handled using novel differentiability concepts such as Hukuhara differentiability and generalized α-cuts. These techniques find applications in population dynamics, epidemiology, and economic modeling, where uncertainties play a significant role. Non-Michaelis-Menten Kinetics in an Amperometric Biosensor: Steady-State Substrate and Product Concentrations Using and Padé Approximants and the Hyperbolic Function method [22]. A brief overview of analytical methods for a fourth-order nonlinear integral boundary value issue with fractal derivatives [15]. Comprehensive Analysis of Electron Transfer Mediator/Heterogeneous Catalyst Composites in Polymer-Modified Electrodes Mathematically [26]. International Scholarly Research Notices, Theoretical Analysis of an Amperometric Biosensor Based on Parallel Substrates Conversion [16]. Using the homotopy perturbation method, an analytical solution of an am-perometric biosensor based on catalase-peroxidase biochemical processes was found [30]. The present study examines the analytical solution of concentrated mixtures of hydrogen sulfide and methanol in a steady state using a biofilm model [33]. Pre-Steady State Behavior of Non-Linear Double Intermediate Enzymatic Reaction: A Theoretical Analysis [21]. Nonlinear Differential Equations in Polymer Coated Microelectrodes: A Mathematical Analysis [37]

Bakhadda et al. [27] examined the existence of positive radial solutions of a nonlinear elliptic equation with a critical potential. Their study is significant in the context of reaction-diffusion systems, where the presence of a critical potential influences the spatial distribution of solutions. Physically, such equations often arise in electrostatic models, population dynamics, and quantum mechanics, where the radial symmetry represents isotropic conditions. The critical potential introduces a threshold behavior, affecting the stability and boundedness of solutions in biological and physical systems. Rasheed et al. [31] investigated the blow-up results for a reaction-diffusion equation with a Dirichlet boundary condition. Blow-up phenomena are crucial in modeling physical and biological processes where energy or concentration accumulates beyond a finite threshold, leading to singularities. In the context of chemical reactions, heat transfer, or biological populations, blow-up signifies uncontrolled growth, such as thermal runaway in combustion or tumor growth in biomedical applications. Their findings provide insights into the time scales and conditions under which such extreme behaviors occur, offering guidelines for controlling or mitigating them in practical applications. Az-Edine et al. [36] explored the weak periodic solution to nonlinear variational parabolic problems having nonlinear boundary conditions and without a sign condition. These problems arise in thermomechanical and fluid transport models where periodic forcing, such as seasonal variations in environmental conditions, affects the system's evolution. The absence of a sign condition implies that the system can exhibit both positive and negative fluxes, which is particularly relevant in multiphase transport and heat conduction in composite materials. Their study provides a mathematical framework to understand long-term oscillatory behaviors in nonlinear systems, contributing to the design of stable engineering and biomedical systems.

Shams et al.[28] investigated the Modified Block Homotopy Perturbation Method for Solving Triangular Linear Diophantine Fuzzy System of Equations, in which they extended the Homotopy Perturbation Method (HPM) to efficiently solve fuzzy Diophantine systems. The physical mechanism utilizes a block-wise decomposition approach that applies homotopy perturbation iteratively over subdomains of the problem space, leading to enhanced accuracy in solutions. This method is particularly beneficial for cryptographic applications, combinatorial optimization, and fuzzy logic-based control systems. Shams et al.[32] examined the Semi-Analytical Scheme for Solving Intuitionistic Fuzzy System of Differential Equations, where they proposed a hybrid approach combining analytical and numerical techniques to solve differential equations modeled under intuitionistic fuzzy logic. The physical mechanism leverages a decomposition-based framework that systematically reduces the complexity of fuzzy differential equations by isolating intuitionistic components. This approach is highly useful in modeling uncertainty in fluid dynamics, bioinformatics, and environmental engineering. Shams et al.[35] discussed the Highly Efficient Numerical Scheme for Solving Fuzzy Systems of Linear and Non-Linear Equations with Application in Differential Equations, where they developed an optimized numerical scheme that improves computational efficiency in solving fuzzy-based linear and nonlinear systems. The physical mechanism integrates a multi-stage refinement process, where iterative corrections are applied in an adaptive manner to balance accuracy and computational load. This scheme has wide-ranging applications in network optimization, artificial intelligence, and predictive analytics in engineering and sciences.

This research is novel since it uses my paper's modified micro-cylinder biosensor model [3]. Our AGM and HPM techniques transformed the model into a ppo-modified micro planar biosensor. The main objective of the research is to develop and analyze models of reaction-diffusion systems in the context of PPO microplanes. These systems explain the changes in chemical compounds dispersed over space due to diffusion and local chemical reactions. The Akbari Ganji method and the Homotopy Perturbation Method, assessing their accuracy, efficiency, and applicability to reaction-diffusion modeling in PPO microplanes. Validate the proposed models and solutions by comparing them with experimental data or established numerical solutions, ensuring the reliability of the methods in practical applications.

2 Mathematical formulation

An enzyme immobilized in a non-conductive substance permeable to the substrate is equally coated on a planar electrode. The electrode is utilized in a stirred solution, including additional supporting electrolytes. According to [3], the enzyme and electrode reactions are as follows:

$$O_2 + 2catechol \longrightarrow 2o - quinone + 2H_2O \tag{2.1}$$

$$o - quinone + 2H^+ + 2e^- \longrightarrow catechol \tag{2.2}$$

If the enzyme reaction follows Michaelis-Menten kinetics and the enzyme concentration is uniform, as is expected, then the response in the film is [7]. Michaelis-Menten kinetics, or M-K, can be used to model the process.

$$k_{cat} = k_1 Co_2 \; and \; M_k = k_1(k_2 + k_3)Co_2 k_2 k_3 \tag{2.3}$$

The mass balance for catechol (S) is given as follows in planar coordinates:

$$D_S \left(\frac{d^2S}{dx^2}\right) - \frac{k_{cat} C_E S}{S + M_k} = 0 \tag{2.4}$$

catechol concentration profile (S), enzyme concentration profile (C_E), diffusion coefficients (D_S and D_P), Michaelis constant (M_K), and quinone concentration profile (P) are all abbreviations for the concentration profiles of their respective compounds. Consequently, at steady-state, the equation of continuity for quinone is frequently written as

$$D_P \left(\frac{d^2P}{dx^2}\right) - \frac{k_{cat} C_E P}{P + M_k} = 0 \tag{2.5}$$

The electrode surface (x_0), as well as the film surface (x_1), have boundary conditions of [2]

$$x = x_0, s = s^*, P = 0; \tag{2.6}$$

$$x = x_1, s = s^*, P = 0; \tag{2.7}$$

where s^* is the enzyme film's partition coefficient divided by the bulk concentration of catechol. When we combine equations (4) and (5) and integrate them using the boundary conditions (6), we obtain

$$\frac{s(x)}{s^*} + \frac{D_P P(x)}{D_S S^*} = 1 \tag{2.8}$$

The steady-state current is given in [2]

$$\frac{I}{nF} = 2\pi A x_0 D_P (dP/dx)_{x=x_0} \tag{2.9}$$

Our presentation of dimensionless variables looks like this:

$$S = \frac{s}{s^*}, P = \frac{p}{s^*}, X = \frac{x}{x_0}, \gamma = \frac{s^*}{M_K}, \lambda_1 = \frac{k_{cat} C_E x_0^2}{D_S M_K}, \lambda_2 = \frac{k_{cat} C_E x_0^2}{D_P M_K}, \frac{D_P}{D_S} = \frac{\lambda_1}{\lambda_2} \tag{2.10}$$

where S and P represent the catechol and o-quinone dimensionless concentrations, X represents the parameter for distance with no dimensions. The reaction-diffusion parameters without dimensions λ_1, λ_2 and γ are the saturation parameter.

$$\frac{d^2S}{dX^2} - \frac{\lambda_1 S}{1 + \gamma S} = 0 \tag{2.11}$$

$$\frac{d^2P}{dX^2} + \frac{\lambda_2 S}{1 + \gamma S} = 0 \tag{2.12}$$

The boundary conditions are shown in the following way:

$$S = 1, P = 0, when \; X = 1 \tag{2.13}$$

$$S = 1, P = 0, when \; X = \frac{x_1}{x_0} \tag{2.14}$$

It is possible to express the dimensionless current at the micro planar as follows:

$$\psi = I/nFAD_P s^* = 2\pi(dP/dX)_{X=1} \tag{2.15}$$

3 Approximate Analytical Solution for Current and Concentrations

The HPM and AGM are mathematical approaches that provide solutions to the governing equations of diffusion phenomena. The HPM is a powerful analytical technique that solves nonlinear differential equations by introducing a small parameter and employing perturbation theory. On the other hand, the AGM is a numerical method that applies a suitable transformation to convert the governing equations into a simpler form, leading to efficient and accurate numerical solutions.

Combining these two techniques, a thorough theoretical model is created to explain the diffusion process at micro planar biosensors modified by PPO. The derived concentration profiles of analytes close to the electrode surface are compared after theoretical and numerical investigations utilizing the HPM and AGM. This comparison study aims to evaluate how well the two approaches predict diffusion kinetics in terms of precision and effectiveness.

3.1 Homotopy Perturbation Method (HPM)

The homotopy perturbation method (HPM) [5, 12, 18] is an effective mathematical technique for solving nonlinear differential equations. Ji-Huan He first presented it in 1999, and it has since become well-liked in many scientific and engineering sectors [1]. To get approximations of solutions to nonlinear problems, the method combines the ideas of homotopy and perturbation [8, 9, 10].

As part of our investigation, we focus on a non-linear partial differential equation that has been thoroughly researched in the literature [1, 8, 9, 10].
A homotopy was built to find the solution to the equations (11) & (12).

$$(1-p)\left(\frac{d^2S}{dX^2}\right)+p\left(\frac{d^2S}{dX^2}-\frac{\lambda_1 S}{1+\gamma S}\right)=0 \tag{3.1}$$

$$(1-p)\left(\frac{d^2P}{dX^2}\right)+p\left(\frac{d^2P}{dX^2}+\frac{\lambda_2 S}{1+\gamma S}\right)=0 \tag{3.2}$$

From (16) and (17), the approximate answers are

$$S=S_0+pS_1+p^2S_2+p^3S_3+...\& \tag{3.3}$$

$$P=P_0+pP_1+p^2P_2+p^3P_3+... \tag{3.4}$$

The coefficients of like powers of p are compared after substituting equations (18) and (19) into equations (16) and (17)

$$p^0:\frac{d^2S_0}{dX^2}=0 \tag{3.5}$$

$$p^1:\frac{d^2S_1}{dX^2}-\frac{\lambda_1 S_0}{1+\gamma S_0}=0 \tag{3.6}$$

$$and \tag{3.7}$$

$$p^0:\frac{d^2P_0}{dX^2}=0 \tag{3.8}$$

$$p^1:\frac{d^2P_1}{dX^2}+\frac{\lambda_1 P_0}{1+\gamma P_0}=0 \tag{3.9}$$

The following outcomes can be obtained by solving Equations (20) through (24) and applying the boundary conditions (13) and (14).

$$S_0(X)\quad=\quad 1 \tag{3.10}$$

$$S_1(X)=\frac{\lambda_1(X-1)(X-k)}{2+2\gamma} \tag{3.11}$$

and

$$P_0(X) = 0 \tag{3.12}$$

$$P_1(X) = -\frac{(-6 + (X^2 + (k+1)X + k^2 + k + 1)\lambda_2)\lambda_1(X-1)(X-k)}{24 + 24\gamma} \tag{3.13}$$

As per the HPM,

$$S(X) = S_0 + S_1 + S_2 + ... \tag{3.14}$$

$$P(X) = P_0 + P_1 + P_2 + ... \tag{3.15}$$

Using Equations (25) and (26) in Equation (29) as well as Equations (27) and (28) in (30), we obtain the final results presented below.

$$S(X) = 1 + \frac{\lambda_1(X-1)(X-k)}{2 + 2\gamma} \tag{3.16}$$

$$P(X) = 0 - \frac{(-6 + (X^2 + (k+1)X + k^2 + k + 1)\lambda_2)\lambda_1(X-1)(X-k)}{24 + 24\gamma} \tag{3.17}$$

where k = x_1/x_0

3.2 Akbari-Ganji Method (AGM)

Dr Akbari and Dr Ganji collaborated and introduced the Akbari-Ganji method, an extension of the Homotopy Perturbation Method (HPM) [6]. Their work has provided a practical method used in research to solve nonlinear differential equations and engineering, and their method has been widely adopted and applied [14, 24, 29]

We focus on a specific non-linear partial differential equation in this study that has drawn much interest from the literature [12, 6, 14, 24, 29, 4, 11]. Our investigation centers on this equation, and we explore its properties and the conclusions and theories from previous research [6, 14, 24, 29].

Let us assume that solution of the Eqn. (11),

$$S(X) = A\cosh mX + B\sinh mX \tag{3.18}$$

Here A, B and m is necessary to achieve are constants.

To resolve the equation (33) using the boundary conditions (13) & (14)

$$A = \frac{-sinh(m) - sinh(mk)}{sinh(mk)cosh(m) - sinh(m)cosh(mk)} \tag{3.19}$$

$$B = \frac{-cosh(mk) + cosh(m)}{sinh(mk)cosh(m) - sinh(m)cosh(m)} \tag{3.20}$$

Substitute Eqns. (34) and (35) in (33), we get

$$S(X) = \frac{(sinh(mk) - sinh(m))cosh(mX) + sinh(mX)(-cosh(mk) + cosh(m))}{-sinh(m)cosh(mk) + sinh(mk)cosh(m)} \tag{3.21}$$

Substitute equation (36) into (11),

$$S(X) = \frac{m^2(sinh(mk) - sinh(m))}{-sinh(m)cosh(mk) + sinh(mk)cosh(m)} + \frac{\lambda_1(-sin\lambda(mk) + sinh(m))}{(cosh(m) + \gamma)sinh(mk) - sinh(m)(\gamma + cosh(mk)} \quad (3$$

put X = 1 in Eqn. (37), we get

$$\frac{\gamma m^2 + m^2 - \lambda_1}{\gamma + 1} = 0 \tag{3.23}$$

$$m = \frac{\sqrt{(\gamma+1)\lambda_1}}{\gamma + 1} \tag{3.24}$$

After repeating the same procedure for Eqn. (12), we obtain

$$P(X) = \frac{\lambda_2(1 - S(X))}{\lambda_1} \tag{3.25}$$

The dimensionless current as follows:

$$\psi = \frac{2\pi\lambda_2(sinh(m)sinh(mk) - cosh(m)cosh(mk) + 1)}{\lambda_1(sinh(m)cosh(mk) - sinh(mk)cosh(m))} \tag{3.26}$$

$$where\ S(X) = \frac{(sinh(mk) - sinh(m))cosh(mX) + sinh(mX)(-cosh(mk) + cosh(m))}{-sinh(m)cosh(mk) + sinh(mk)cosh(m)} \tag{3.27}$$

$$k = x_1/x_0 \ \&\ m = \frac{\sqrt{(\gamma+1)\lambda_1}}{\gamma + 1} \tag{3.28}$$

4 Comparative Analysis of Analytical and Numerical Simulation

We may decide which approach is best for a given topic and develop a thorough grasp of the phenomenon being examined by comparing the findings, computational efficiency, accuracy, and simplicity. In this work, the numerical solution produced by MATLAB software was compared to the analytical expressions that were obtained.

X	λ_1 = 0.1					λ_1 = 0.5					λ_1 = 1				
	Num.	HPM	AGM	Error % of HPM	Error % of AGM	Num.	HPM	AGM	Error % of HPM	Error % of AGM	Num.	HPM	AGM	Error % of HPM	Error % of AGM
1	1	1	1	0	0	1	1	1	0	0	1	1	1	0	0
1.1	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0
1.2	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0
1.3	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0
1.4	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0
1.5	1	1	1	0	0	1	1	1	0	0	1	1	1	0	0
Average Error %				0	0				0	0				0	0

Table 1. Comparing the Substrate Concentration (S) in Equation (11) its Numerical Result for various reaction diffusion parameter (λ_1) Values with Fixed saturation parameter (γ)

5 Result and discussion

The new approximate analytical formulations of catechol and o-quinone concentrations reflect S, P, C_E, x_1/x_0, and a for all values of the parameters Eqs. (11) as well as (12). The boundary requirements of Equations (13) and (14) are met. In Fig. 1, we show a set of normalized concentration profiles for a catechol S as a function of γ saturation parameter an, film thickness x_1/x_0, and reaction-diffusion parameters. The catechol concentration S is shown as a parabola with X = 0.1, 0.2, and 0.3 as the axis. When X = 1 and X = x_1/x_0, the catechol concentration S is almost equal to 1, the attention of catechol is one for all values of x_1/x_0 when

X	γ = 0.1					γ = 0.5					γ = 1				
	Num.	**HPM**	**AGM**	**Error % of HPM**	**Error % of AGM**	**Num.**	**HPM**	**AGM**	**Error % of HPM**	**Error % of AGM**	**Num.**	**HPM**	**AGM**	**Error % of HPM**	**Error % of AGM**
1	1	1	1	0	0	1	1	1	0	0	1	1	1	0	0
1.1	0.99	0.99	0.99	0.00	0	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0
1.2	0.98	0.98	0.98	0.00	0	0.98	0.98	0.98	0	0	0.99	0.99	0.99	0	0
1.3	0.98	0.98	0.98	0.00	0	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0
1.4	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0	0.99	0.99	0.99	0	0
1.5	1	1	1	0	0	1	1	1	0	0	1	1	1	0	0
Average Error %				0	0				0	0				0	0

Table 2. Comparing the Substrate Concentration (S) in Equations (11) its Numerical Result for various saturation parameter (γ) values with fixed reaction diffusion parameter (λ_1) values.

X	λ_1 = 0.1					λ_1 = 0.5					λ_1 = 1				
	Num.	**HPM**	**AGM**	**Error % Of HPM**	**Error % of AGM**	**Num.**	**HPM**	**AGM**	**Error % of HPM**	**Error % of AGM**	**Num.**	**HPM**	**AGM**	**Error % of HPM**	**Error % of AGM**
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.1	0.19	0.19	0.19	0	0	0.19	0.19	0.19	0	0	0.19	0.19	0.19	0	0
1.2	0.27	0.27	0.27	0	0	0.27	0.27	0.27	0	0	0.27	0.27	0.27	0	0
1.3	0.26	0.26	0.26	0	0	0.26	0.26	0.26	0	0	0.26	0.26	0.26	0	0
1.4	0.16	0.16	0.16	0	0	0.16	0.16	0.16	0	0	0.16	0.16	0.16	0	0
1.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Average Error %				0	0				0	0				0	0

Table 3. Comparing the Product Concentration (P) in Equations (12) its Numerical results for saturation parameter (γ) fixed and reaction diffusion parameter (λ_1) varies.

X	γ = 0.1					γ = 0.5					γ = 1				
	Num.	**HPM**	**AGM**	**Error % Of HPM**	**Error % of AGM**	**Num.**	**HPM**	**AGM**	**Error % of HPM**	**Error % of AGM**	**Num.**	**HPM**	**AGM**	**Error % of HPM**	**Error % of AGM**
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.1	0.19	0.19	0.19	0	0	0.19	0.19	0.19	0	0	0.19	0.19	0.19	0	0
1.2	0.27	0.27	0.27	0	0	0.27	0.27	0.27	0	0	0.27	0.27	0.27	0	0
1.3	0.26	0.26	0.26	0	0	0.26	0.26	0.26	0	0	0.26	0.26	0.26	0	0
1.4	0.16	0.16	0.16	0	0	0.16	0.16	0.16	0	0	0.16	0.16	0.16	0	0
1.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Average Error %				0	0				0	0				0	0

Table 4. Comparing the Product Concentration (P) in Equations (12) its Numerical results for reaction diffusion parameter (λ_1) fixed and saturation parameter (γ) varies.

the saturation parameter (γ) is significant (or the reaction-diffusion parameter (C_E) is minimal).

We compare our analytical expressions for o-quinone (P) (Eq. (12) and catechol (S) (Eq. (11) in Figures 1-4. A satisfactory arrangement has been made. As demonstrated in Figs. 1-4, as enzyme activity rises, the catechol concentration decreases in the middle of the film. Still, it stays high at the film/solution interface because of diffusion from bulk and at the electrode/film interface because of generation at the electrode. Figures 5 depict the dimensionless current ψ vs x_1/x_0 using Eqn. (15) as an illustration. We contrast the saturated outcome in Fig. 5 with the steady-state analytical current expression (Eqn. 20). For all values, the value of current ψ increases as the film's thickness x_1/x_0 increases.

6 Conclusion

The developed diffusion-kinetic model offers a comprehensive analytical framework for understanding the steady-state behavior of PPO-modified micro planar biosensors. Using the

Homotopy perturbation method (HPM) and the Akbar Ganji method (AGM), we derived closed-form solutions that describe the spatial distribution of catechol and o-quinone concentrations within the substrate (S) and product (P) regions, as well as the ratio x_1/x_0 that characterizes the relative concentration dynamics. This model incorporates nonlinear reaction-diffusion interactions, particularly those governed by a nonlinear Michaelis-Menten kinetic scheme, providing an accurate description of the enzymatic activity at the biosensor interface. The steady-state current response of the biosensor is explicitly formulated, allowing for precise predictions of the electrochemical behavior of the system at various substrate concentrations. Furthermore, the calibration curves obtained from this study highlight the significant nonlinear contributions in catechol/phenol biosensors, emphasizing the necessity of accounting for these effects in experimental and theoretical analyses. The analytical results facilitate the estimation of catechol and o-quinone concentrations and their influence on biosensor performance, enabling a robust framework for optimizing biosensor design and improving detection accuracy in practical applications. The study shows that HPM and AGM can solve diffusion kinetics for PPO-modified micro planar biosensors, with AGM having higher accuracy and convergence. AGM concentration profiles closely match the experimental results, indicating its efficacy. This study improves our understanding of diffusion kinetics in biosensors and offers valuable insights for enhancing bio electro chemical systems.

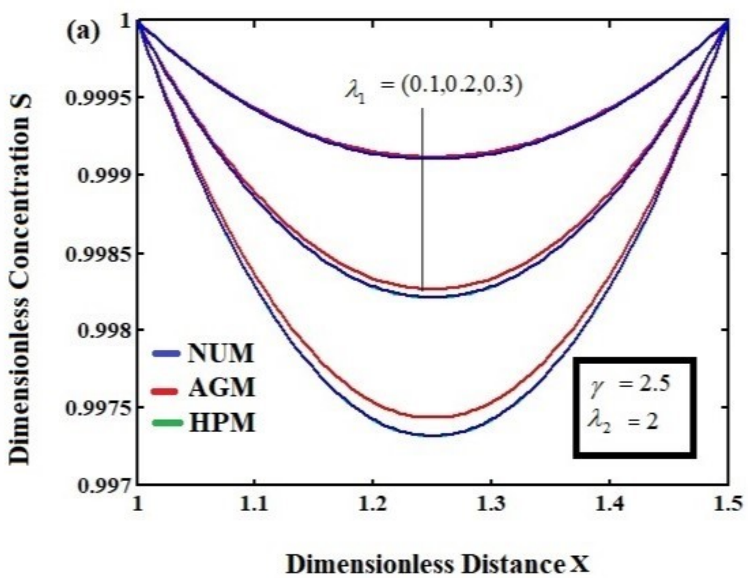

Figure 1. (a) Steady-state concentration catechol (S) for various reaction diffusion parameter (λ_1) calculated from Eqn.(16 & 18)

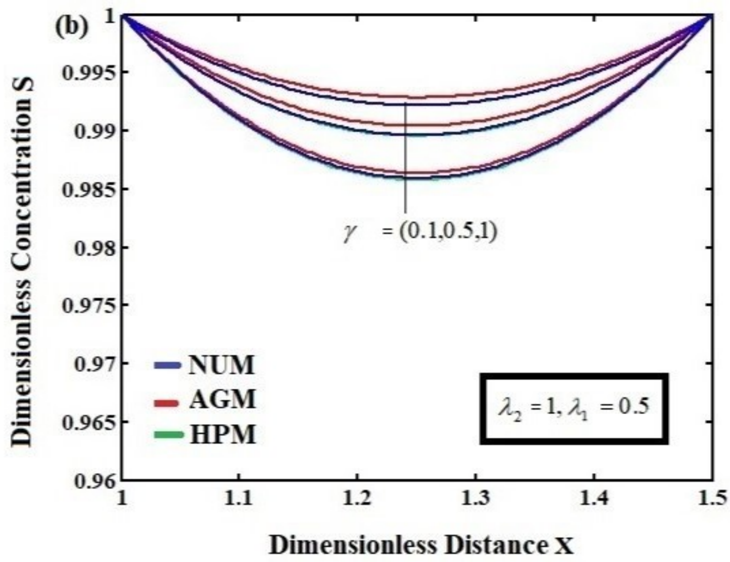

Figure 2. (b) Steady-state concentration catechol (S) for various saturation parameter value (γ) calculated from Eqn.(16 & 18)

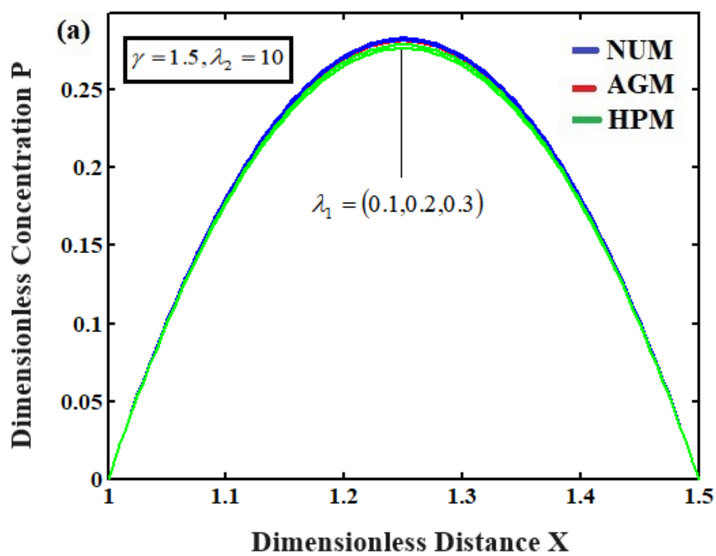

Figure 3. (a) Steady-state concentration quinone (P) for various reaction diffusion parameter (λ_1) calculated from Eqn.(17 & 19)

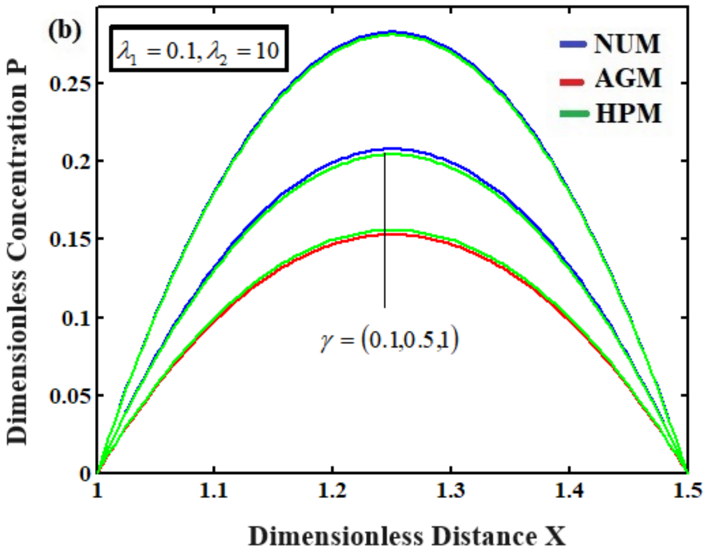

Figure 4. (b) Steady-state concentration catechol (S) for various saturation parameter value (γ) calculated from Eqn.(17 & 19)

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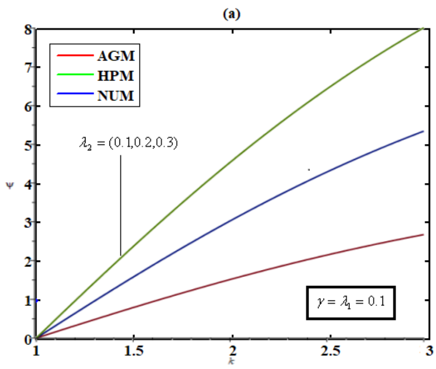

Figure 5. (a) A plot showing the dimensionless current (ψ) against x_1/x_0. In Equation (20), current is computed.

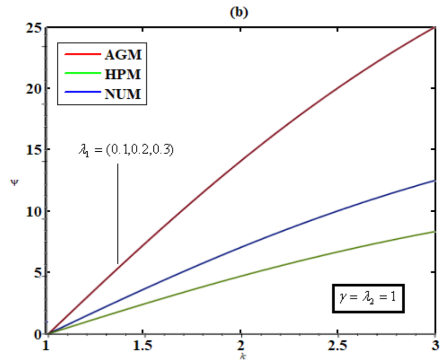

Figure 6. (b) A plot showing the dimensionless current (ψ) against x_1/x_0. In Equation (20), current is computed.

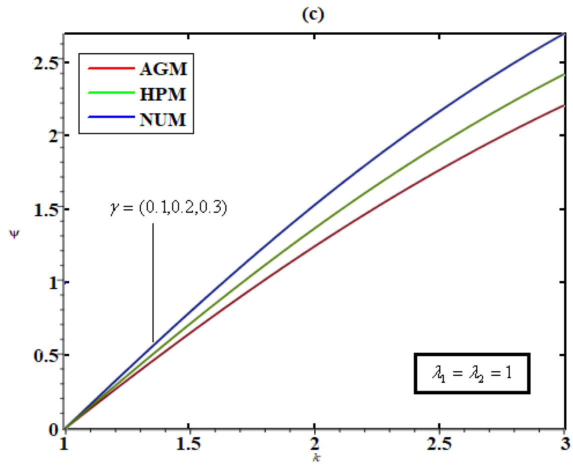

Figure 7. (c) A plot showing the dimensionless current (ψ) against x_1/x_0. In Equation (20), current is computed.

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NOMENCLATURE		
Symbol	**Definitions**	Units
D_S	Coefficient of diffusion for catechol	cm^2/s
D_P	Coefficient of diffusion for quinone	cm^2/s
s	Catechol concentration profile	$mole/cm^3$
p	Quinone concentration profile	$mole/cm^3$
C_E	Enzyme concentration profile	$mole/cm^3$
M_K	Michaelis constant	$mole/cm^3$
s*	Concentration of S in bulk	$mole/cm^3$
k_cat	Rate constant for catalysis	s^{-1}
F	Faraday constant	c $mole^{-1}$
I	Current	Ampere
x	Radius of the planar	cm
x_0	Radius of the electrode	cm
x_1	Radius of the film	cm
A	Length of the electrode	cm
x_1/x_0	Parameter without dimensions for the thickness of films	no units
χ	Parameter without dimensions for enzyme kinetic	no units
j	Sensor response without dimensions	no units
ψ	Current with no dimensions	no units
S	Concentration of catechol without dimensions	no units
P	Concentration of quinone without dimensions	no units
X	Distance with no dimensions	no units
λ_1	Reaction diffusion parameter without dimensions	no units
λ_2	Reaction diffusion parameter without dimensions	no units
γ	Saturation parameter without dimensions	no units
n	Number of electrons	no units

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