Palestine Journal of Mathematics

Vol 14(2)(2025) , 704–713

Quadripartitioned Neutrosophic Metric Spaces

Carlos Granados, Suman Das, Rakhal Das and Birojit Das

Communicated by Madeleine Al Tahan

MSC 2010 Classifications: Primary 46S40; Secondary 54A40.

Keywords and phrases: Quadripartitioned neutrosophic metric space, Baire Category Theorem, Uniform Convergence Theorem.

Corresponding Author: Carlos Granados

Abstract In this paper, the concept of new metric space with neutrosophic numbers is defined. Quadripartitioned neutrosophic metric space is a generalization of a neutrosophic metric space i.e., we divide indeterminacy into two categories. Quadripartitioned neutrosophic metric space is defined with continuous triangular norms and continuous triangular conorms. Besides, several topological and structural properties on quadripartitioned neutrosophic metric space have been shown. Additionally, the analogues of Baire Category Theorem and Uniform Convergence Theorem are proved for quadripartitioned neutrosophic metric spaces.

1 Introduction and Preliminaries

The notion of fuzzy set which was established by Zadeh [16] has influenced deeply all the scientific fields since the publication of the paper. It is seen that this notion, which is very important for real-life situations, had not enough solution to some problems in time. New quests for such problems have been coming up. Atanassov [1] initiated Intuitionistic fuzzy set for such cases. Neutrosophic set is a new version of the idea of the classical set which is defined by Smarandache [13]. Using the notions of Probabilistic metric space, the notion of fuzzy metric space was introduced in [11]. Kaleva and Seikkala [8] defined the fuzzy metric space as a distance between two points to be a non-negative fuzzy number. In [5] some basic properties of fuzzy metric space studied and the Baire Category Theorem for fuzzy metric space were proved. Moreover, some properties such as separability, countability were given and Uniform Limit Theorem was proved in [6]. Later on, neutrosophic soft normed liner space was defined by Bera and Mahapatra [2]. In [2], neutrosophic norm, Cauchy sequence in neutrosophic soft normed linear space, convexity of neutrosophic soft normed linear space, metric in neutrosophic soft normed linear space were studied. For more notion related to quadripartitioned spaces and neutrosophic metric spaces, we refer the reader to [3, 7]. In present study, from the idea of neutrosophic sets and neutrosophic metric space, new metric space was defined which is called quadripartitioned Neutrosophic metric Space. We investigate some properties of quadripartitioned Neutrosophic metric Space such as open set, Hausdorff, neutrosophic bounded, compactness, completeness, nowhere dense. Also we give Baire Category Theorem and Uniform Convergence Theorem for quadripartitioned Neutrosophic metric Spaces.

Next, some well-known definitions associated to neutrosophy are given which are useful for development of this paper.

Definition 1.1. [4] Let us consider that F is a space (objects). $A_U(a)$ denotes the truth-membership function, $B_U(a)$ is a contradiction-membership function, $C_U(a)$ is an ignorance-membership function and $C_U(a)$ is a falsity-membership function, where U is a set in F with $a \in F$. Then, if we take $I =]0^-, 1^+[$,

$$A_U(a) : F \to I,$$
$$B_U(a) : F \to I,$$
$$C_U(a) : F \to I,$$
$$D_U(a) : F \to I.$$

There is no restriction on the sum of $A_U(a), B_U(a), C_U(a)$ and $D_U(a)$. Therefore,

$$0^- \leq \sup A_U(a) + \sup B_U(a) + \sup C_U(a) + \sup D_U(a) \leq 4^+.$$

The set U which consists of with $A_U(a), B_U(a), C_U(a)$ and $D_U(a)$ in F is called a quadripartitioned neutrosophic sets and can be denoted by

$$U = \{\langle a, (A_U(a), B_U(a), C_U(a), D_U(a))\rangle : a \in F, A_U(a), B_U(a), C_U(a), D_U(a) \in I\}. \quad (1.1)$$

Definition 1.2. [4] A quadripartitioned neutrosophic set U is included in another quadripartitioned neutrosophic set V, if

$$\inf A_U(a) \leq \inf A_V(a), \sup A_U(a) \leq \sup A_V(a),$$
$$\inf B_U(a) \geq \inf B_V(a), \sup B_U(a) \geq \sup B_V(a),$$
$$\inf C_U(a) \geq \inf C_V(a), \sup C_U(a) \geq \sup C_V(a),$$
$$\inf D_U(a) \geq \inf D_V(a), \sup D_U(a) \geq \sup D_V(a).$$

For any $a \in F$.

Remark 1.3. [4] A simplification of a quadripartitioned neutrosophic set U, in (1.1), is

$$U = \{\langle a, (A_U(a), B_U(a), C_U(a), D_U(a))\rangle : a \in F\},$$

which is called a simplified quadripartitioned neutrosophic set.

Remark 1.4. [4] A simplified quadripartitioned neutrosophic set U is comprised in another simplified quadripartitioned neutrosophic set V if $A_U(a) \leq A_V(a), B_U(a) \leq B_V(a), C_U(a) \geq C_V(a)$ and $D_U(a) \geq D_V(a)$ for any $a \in F$.

Remark 1.5. [4] Let U and V be two quadripartitioned neutrosophic sets, then the following operations are given by:

(i) $U + V = \langle A_U(a) + A_V(a) - A_U(a)A_V(a), B_U(a) + B_V(a) - B_U(a)B_V(a), C_U(a) + C_V(a) - C_U(a)C_V(a), D_U(a) + D_V(a) - D_U(a)D_V(a)\rangle$,

(ii) $UV = \langle A_U(a)A_V(a), B_U(a)B_V(a), C_U(a)C_V(a), D_U(a)D_V(a)\rangle$,

(iii) $\alpha U = \langle 1 - (1 - A_U(a))^\alpha, 1 - (1 - B_U(a))^\alpha, 1 - (1 - C_U(a))^\alpha, 1 - (1 - D_U(a))^\alpha\rangle$, for $\alpha < 0$,

(iv) $U^\alpha = \langle A_U^\alpha(a), B_U^\alpha(a), C_U^\alpha(a), D_U^\alpha(a)\rangle$, for $\alpha > 0$.

Triangular norms (t-norms) (TN) were initiated by [12]. In the problem of computing the distance between two elements in space, Menger offered using probability distributions instead of using numbers for distance. TNs are used to generalize with the probability distribution of triangle inequality in metric space conditions. Triangular conorms (t-conorms) (TC) known as dual operations of TNs. TNs and TCs are very significant for fuzzy operations(intersections and unions).

Definition 1.6. [12] Given an operation $\circ : [0,1] \times [0,1] \to [0,1]$. If the operation $\circ$ is satisfies the following conditions:

(i) $s \circ 1 = s$,

(ii) If $s \leq u$ and $t \leq v$, then $s \circ t \leq u \circ v$,

(iii) $\circ$ is continuous,

(iv) $\circ$ is continuous and associative.

Then, it is called that the operation $\circ$ is continuous TN, for $s, t, u, v \in [0, 1]$.

Definition 1.7. [12] Give an operation $\bullet : [0, 1] \times [0, 1] \to [0, 1]$. If the operation $\bullet$ is satisfying the following conditions:

(i) $s \bullet 0 = s$,

(ii) If $s \leq u$ and $t \leq v$, then $s \bullet t \leq u \bullet v$,

(iii) $\bullet$ is continuous,

(iv) $\bullet$ is continuous and associative.

Then, it is called that the operation $\bullet$ is continuous TC, for $s, t, u, v \in [0, 1]$.

Remark 1.8. From the above definitions, we can see that if we take $0 < \epsilon_1, \epsilon_2 < 1$ for $\epsilon_1 > \epsilon_2$, then there exist $0 < \epsilon_3, \epsilon_4 < 0, 1$ such that $\epsilon_1 \circ \epsilon_3 \geq \epsilon_2$, $\epsilon_1 \geq \epsilon_4 \bullet \epsilon_2$. Moreover, if we take $\epsilon_5 \in (0, 1)$, then there exist $\epsilon_6, \epsilon_7 \in (0, 1)$ such that $\epsilon_6 \circ \epsilon_6 \geq \epsilon_5$ and $\epsilon_7 \bullet \epsilon_7 \leq \epsilon_5$.

2 Quadripartitioned Neutrosophic Metric Spaces

Definition 2.1. Let F be an arbitrary set, $N = \{\langle a, (A_U(a), B_U(a), C_U(a), D_U(a))\rangle : a \in F\}$ be a quadripartitioned neutrosophic set such that $N : F \times F \times F \times \mathbb{R}^+ \to [0, 1]$. Let $\circ$ and $\bullet$ show the continuous TN and continuous TC, respectively. The four-tuple $(F, N, \circ, \bullet)$ is called a quadripartitioned neutrosophic metric space (QNMS) when the following conditions are satisfied for all $a, b, c \in F$,

(i) $0 \leq A(a, b, \lambda) \leq 1$, $0 \leq B(a, b, \lambda) \leq 1$, $0 \leq C(a, b, \lambda) \leq 1$ and $0 \leq D(a, b, \lambda) \leq 1$; for all $\lambda \in \mathbb{R}^+$,

(ii) $A(a, b, \lambda) + B(a, b, \lambda) + C(a, b, \lambda) + D(a, b, \lambda) \leq 3$, for $\lambda \in \mathbb{R}^+$,

(iii) $A(a, b, \lambda) = 1$ for $\lambda > 0$ if $a = b$,

(iv) $A(a, b, c, \lambda) = A(b, a, \lambda)$, for $\lambda > 0$,

(v) $A(a, b, \lambda) \circ A(b, c, \lambda) \leq A(a, c, \lambda + \mu)$, for all $\lambda, \mu > 0$,

(vi) $A(a, b, .) : [0\infty) \to [0, 1]$ is continuous,

(vii) $\lim_{\lambda \to \infty} A(a, b, \lambda) = 1$, for all $\lambda > 0$,

(viii) $B(a, b, \lambda) = 0$ for $\lambda > 0$ if $a = b$,

(ix) $B(a, b, c, \lambda) = B(b, a, \lambda)$, for $\lambda > 0$,

(x) $B(a, b, \lambda) \bullet B(b, c, \lambda) \geq B(a, c, \lambda + \mu)$, for all $\lambda, \mu > 0$,

(xi) $B(a, b, .) : [0\infty) \to [0, 1]$ is continuous,

(xii) $\lim_{\lambda \to \infty} B(a, b, \lambda) = 0$, for all $\lambda > 0$,

(xiii) $C(a, b, \lambda) = 0$ for $\lambda > 0$ if $a = b$,

(xiv) $C(a, b, c, \lambda) = C(b, a, \lambda)$, for $\lambda > 0$,

(xv) $C(a, b, \lambda) \bullet C(b, c, \lambda) \geq C(a, c, \lambda + \mu)$, for all $\lambda, \mu > 0$,

(xvi) $C(a, b, .) : [0\infty) \to [0, 1]$ is continuous,

(xvii) $\lim_{\lambda \to \infty} C(a, b, \lambda) = 0$, for all $\lambda > 0$,

(xviii) $D(a,b,\lambda) = 0$ for $\lambda > 0$ if $a = b$,

(xix) $D(a,b,c,\lambda) = D(b,a,\lambda)$, for $\lambda > 0$,

(xx) $D(a,b,\lambda) \bullet D(b,c,\lambda) \geq D(a,c,\lambda+\mu)$, for all $\lambda, \mu > 0$,

(xxi) $D(a,b,.) : [0\infty) \to [0,1]$ is continuous,

(xxii) $\lim_{\lambda\to\infty} D(a,b,\lambda) = 0$, for all $\lambda > 0$,

(xxiii) If $\lambda < 0$, then $A(a,b,\lambda) = 0$, $B(a,b,\lambda) = 1$, $C(a,b,\lambda) = 1$ and $D(a,b,\lambda) = 0$.

Then, $N = (A,B,C,D)$ is called quadripartitioned neutrosophic metric (QNM) on F. The functions $A(a,b,\lambda)$, $B(a,b,\lambda)$, $C(a,b,\lambda)$ and $D(a,b,\lambda)$ denote the degree of degree of nearness, the degree of unknown-nearness, the degree of contradiction-nearness and the degree of non-nearness between a and b with respect to λ, respectively.

Example 2.2. Let (F,d) be a metric space. Take the operation $\circ$ and $\bullet$ as default (min) TN $a \circ b = \min\{a,b\}$ and default (max) TC $a \bullet b = \max\{a,b\}$.

$$A(a,b,\lambda)\frac{\lambda}{\lambda+d(a,b)},$$
$$B(a,b,\lambda)\frac{d(a,b)}{\lambda+d(a,b)},$$
$$C(a,b,\lambda)\frac{|\lambda-d(a,b)|}{\lambda+d(a,b)},$$
$$D(a,b,\lambda)\frac{d(a,b)}{\lambda}.$$

For all $a,b \in F$ and $\lambda > 0$. Then, $(F,N,\circ,\bullet)$ is QNMS such that $N : F\times F\times F\times \mathbb{R}^+ \to [0,1]$. This QNMS is expressed as produced by a metric d the neutrosophic metric.

Definition 2.3. Let $(F,N,\circ,\bullet)$ be a QNMS, $0 < \varepsilon < 1$, $\lambda > 0$ and $a \in F$. The set $O(a,\varepsilon,\lambda) = \{A(a,b,\lambda) > 1-\varepsilon,\ B(a,b,\lambda) < \varepsilon,\ C(a,b,\lambda) < \varepsilon,\ D(a,b,\lambda) < \varepsilon\}$ is said to be the open ball (OB) (center a and radius ε with respect to λ).

Theorem 2.4. *Every OB $O(a,\varepsilon,\lambda)$ is an open set (OS).*

Proof. Take $O(a,\varepsilon,\lambda)$ be an OB. Choose $b \in (a,\varepsilon,\lambda)$. Hence, $A(a,b,\lambda) > 1-\varepsilon$, $B(a,b,\lambda) < \varepsilon$, $C(a,b,\lambda) < \varepsilon$ and $D(a,b,\lambda)$. If we take $\varepsilon_0 = A(a,b,\lambda_0)$, then for $\varepsilon_0 > 1-\varepsilon$, $\zeta \in (0,1)$ will exist such that $\varepsilon_0 > 1-\zeta > 1-\varepsilon$. Give ε_0 and ζ such that $\varepsilon_0 > 1-\zeta$. Then, $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \in (0,1)$ will exist such that $\varepsilon_0 \circ \varepsilon_1 > 1-\zeta$, $(1-\varepsilon_0)\bullet(1-\varepsilon_2) \leq \zeta$, $(1-\varepsilon_0)\bullet(1-\varepsilon_3) \leq \zeta$ and $(1-\varepsilon_0)\bullet(1-\varepsilon_4) \leq \zeta$. Take $\varepsilon_5 = \max\{\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4\}$. Consider the OB $O(b,1-\varepsilon_4,\lambda-\lambda_0) \subset O(a,\varepsilon,\lambda)$. If we take $w \in O(b,1-\varepsilon_4,\lambda-\lambda_0)$, then $A(b,w,\lambda-\lambda_0) > \varepsilon_5$, $B(b,w,\lambda-\lambda_0) < \varepsilon_5$, $C(b,w,\lambda-\lambda_0) < \varepsilon_5$ and $D(b,w,\lambda-\lambda_0) < \varepsilon_5$. Then,

$$\begin{aligned}
&A(a,w,\lambda) \geq A(a,b,\lambda_0) \circ A(b,w,\lambda-\lambda_0) \geq \varepsilon_0 \circ \varepsilon_5 \geq \varepsilon_0 \circ \varepsilon_1 \geq 1-\zeta > 1-\varepsilon,\\
&B(a,w,\lambda) \leq B(a,b,\lambda_0) \bullet B(b,w,\lambda-\lambda_0) \leq (1-\varepsilon_0)\bullet(1-\varepsilon_5) \leq (1-\varepsilon_0)\bullet(1-\varepsilon_2)\\
&\leq \zeta < \varepsilon,\\
&C(a,w,\lambda) \leq C(a,b,\lambda_0) \bullet C(b,w,\lambda-\lambda_0) \leq (1-\varepsilon_0)\bullet(1-\varepsilon_5) \leq (1-\varepsilon_0)\bullet(1-\varepsilon_3)\\
&\leq \zeta < \varepsilon,\\
&D(a,w,\lambda) \leq D(a,b,\lambda_0) \bullet C(b,w,\lambda-\lambda_0) \leq (1-\varepsilon_0)\bullet(1-\varepsilon_5) \leq (1-\varepsilon_0)\bullet(1-\varepsilon_4)\\
&\leq \zeta < \varepsilon.
\end{aligned}$$

This shows that $w \in O(a,\varepsilon,\lambda)$ and $O(b,1-\varepsilon_5,\lambda-\lambda_0) \subset O(a,\varepsilon,\lambda)$. □

Theorem 2.5. *Every quadripartitioned neutrosophic metric space is Hausdorff.*

Proof. Let $(F, N, \circ, \bullet)$ be a QNMS. Take a and b as two distinct points in F. Then, $0 < A(a, b, \lambda) < 1$, $0 < B(a, b, \lambda) < 1$, $0 < C(a, b, \lambda) < 1$ and $0 < D(a, b, \lambda) < 1$. Take $\varepsilon_1 = A(a, b, \lambda), \varepsilon_2 = B(a, b, \lambda), \varepsilon_3 = C(a, b, \lambda), \varepsilon_4 = D(a, b, \lambda)$ and $\varepsilon = \max\{\varepsilon_1, 1-\varepsilon_2, 1-\varepsilon_3, 1-\varepsilon_4\}$. If we take $\varepsilon_0 \in (0, 1)$, then there exist $\varepsilon_5, \varepsilon_6, \varepsilon_7, \varepsilon_8$ such that $\varepsilon_5 \circ \varepsilon_5 \geq \varepsilon_0$, $(1-\varepsilon_6)\bullet(1-\varepsilon_6) \leq 1-\varepsilon_0$, $(1-\varepsilon_7)\bullet(1-\varepsilon_7) \leq 1-\varepsilon_0$ and $(1-\varepsilon_8)\bullet(1-\varepsilon_8) \leq 1-\varepsilon_0$. Let $\varepsilon_9 = \max\{\varepsilon_5, \varepsilon_6, \varepsilon_7, \varepsilon_8\}$. If we consider the open balls $O(a, 1-\varepsilon_9, \frac{\lambda}{2})$ and $O(b, 1-\varepsilon_9, \frac{\lambda}{2})$, clearly $O(a, 1-\varepsilon_9, \frac{\lambda}{2}) \cap O(b, 1-\varepsilon_9, \frac{\lambda}{2}) = \emptyset$. from here, if we take $c \in O(a, 1-\varepsilon_9, \frac{\lambda}{2}) \cap O(b, 1-\varepsilon_9, \frac{\lambda}{2})$

$$\varepsilon_1 = A(a, b, \lambda) \geq A(a, c, \frac{\lambda}{2}) \circ A(c, b, \frac{\lambda}{2}) \geq \varepsilon_9 \circ \varepsilon_9 \geq \varepsilon_5 \circ \varepsilon_5 \geq \varepsilon_0 > \varepsilon_1.$$

$$\begin{aligned} \varepsilon_2 =& B(a, b, \lambda) \leq B(a, c, \frac{\lambda}{2}) \bullet B(c, b, \frac{\lambda}{2}) \leq (1-\varepsilon_9) \bullet (1-\varepsilon_9) \\ & \leq (1-\varepsilon_6) \bullet (1-\varepsilon_6) \leq (1-\varepsilon_0) < \varepsilon_2. \end{aligned}$$

$$\begin{aligned} \varepsilon_3 =& C(a, b, \lambda) \leq C(a, c, \frac{\lambda}{2}) \bullet C(c, b, \frac{\lambda}{2}) \leq (1-\varepsilon_9) \bullet (1-\varepsilon_9) \\ & \leq (1-\varepsilon_7) \bullet (1-\varepsilon_7) \leq (1-\varepsilon_0) < \varepsilon_3. \end{aligned}$$

and

$$\begin{aligned} \varepsilon_4 =& D(a, b, \lambda) \leq D(a, c, \frac{\lambda}{2}) \bullet D(c, b, \frac{\lambda}{2}) \leq (1-\varepsilon_9) \bullet (1-\varepsilon_9) \\ & \leq (1-\varepsilon_8) \bullet (1-\varepsilon_8) \leq (1-\varepsilon_0) < \varepsilon_4. \end{aligned}$$

which is a contradiction. Therefore, we say that QNMS is Hausdorff. □

Definition 2.6. Let $(F, N, \circ, \bullet)$ be a QNMS. A subset B of F is called quadripartitioned neutrosophic-bounded (QNB), if there exists $\lambda > 0$ and $\varepsilon \in (0, 1)$ such that $A(a, b, \lambda) > 1-\varepsilon$, $B(a, b, \lambda) < \varepsilon$, $C(a, b, \lambda) < \varepsilon$ and $D(a, b, \lambda) < \varepsilon$, for all $a, b \in B$.

Definition 2.7. If $B \subseteq \bigcup_{U \in C_N} U$, a collection C_N of open sets is said to be an open cover (OC) of B.

Definition 2.8. A subspace B of a quadripartitioned neutrosophic metric space is compact if for every open cover of B has a finite subcover. If every sequence in B has a convergent subsequence to a point in B, then it is called sequential compact.

Theorem 2.9. *Every compact subset B of a quadripartitioned neutrosophic metric space is quadripartitioned neutrosophic-bounded.*

Proof. First at all, take a compact subset B of QNMS F. Now, consider the open cover $\{O(a, \varepsilon, \lambda) : a \in B\}$ for $\lambda > 0$, $\varepsilon \in (0, 1)$. Since B is compact, then there exist $a_1, a_2, ..., a_n \in B$ such that $B \subseteq \bigcup_{k=1}^{n} O(a_k, \varepsilon, \lambda)$. For some k, m and $a, b \in B$, $a \in O(a_k, \varepsilon, \lambda)$ and $b \in O(a_m, \varepsilon, \lambda)$. Then, we have $A(a, a_k, \lambda) > 1-\varepsilon$, $B(a, a_k, \lambda) < \varepsilon$, $C(a, a_k, \lambda) < \varepsilon$, $D(a, a_k, \lambda) < \varepsilon$ and $A(b, a_m, \lambda) > 1-\varepsilon$, $B(b, a_m, \lambda) < \varepsilon$, $C(b, a_m, \lambda) < \varepsilon$, $D(b, a_m, \lambda) < \varepsilon$. Let $\omega = \min\{A(a_k, a_m, \lambda) : q \leq k, m \leq n\}$, $\theta = \max\{B(a_k, a_m, \lambda) : q \leq k, m \leq n\}$, $\psi = \max\{C(a_k, a_m, \lambda) : q \leq k, m \leq n\}$ and $\phi = \max\{D(a_k, a_m, \lambda) : q \leq k, m \leq n\}$, thus $\omega, \theta, \psi, \phi > 0$. From here, for $0 < \eta_1, \eta_2, \eta_3, \eta_4 < 1$,

$$\begin{aligned} &A(a, b, 4\lambda) \geq A(a, a_k, \lambda) \circ A(a_k, a_m, \lambda) \circ A(a_m, b, \lambda) \geq (1-\varepsilon) \circ (1-\varepsilon) \circ \omega > 1-\eta_1 \\ &B(a, b, 4\lambda) \leq B(a, a_k, \lambda) \bullet B(a_k, a_m, \lambda) \bullet B(a_m, b, \lambda) \leq \varepsilon \bullet \varepsilon \bullet \theta < \eta_2 \\ &C(a, b, 4\lambda) \leq C(a, a_k, \lambda) \bullet C(a_k, a_m, \lambda) \bullet C(a_m, b, \lambda) \leq \varepsilon \bullet \varepsilon \bullet \psi < \eta_3 \\ &D(a, b, 4\lambda) \leq D(a, a_k, \lambda) \bullet D(a_k, a_m, \lambda) \bullet D(a_m, b, \lambda) \leq \varepsilon \bullet \varepsilon \bullet \phi < \eta_4. \end{aligned}$$

If we take $\eta = \max\{\eta_1, \eta_2, \eta_3, \eta_4\}$ and $\lambda_0 = 4\lambda$, we obtain $A(a, b, \lambda_0) > 1 - \eta$, $B(a, b, \lambda_0) < \eta$, $C(a, b, \lambda_0) < \eta$ and $D(a, b, \lambda_0) < \eta$ for all $a, b \in B$. Therefore, B is a QNB. □

Remark 2.10. If $(F, N, \circ, \bullet)$ is QNMS produces by a metric on F and $B \subset F$, then B is QNB if and only if it is bounded. Consequently, with Theorems 2.5 and 2.9, we have the following result.

Corollary 2.11. *In a QNMS, every compact set is closed and bounded.*

Definition 2.12. Let $(F, N, \circ, \bullet)$ be a QNMS. A sequence (a_n) in F is said to be Cauchy if for each $\varepsilon > 0$ and each $\lambda > 0$, there exists $U \in \mathbb{N}$ such that $A(a_n, a_m, \lambda) > 1-\varepsilon$, $B(a_n, a_m, \lambda) < \varepsilon$, $C(a_n, a_m, \lambda) < \varepsilon$, $D(a_n, a_m, \lambda) < \varepsilon$ for all $n, m \geq U$. $(F, N, \circ, \bullet)$ is called complete if every Cauchy sequence is convergent with respect to τ_N.

Theorem 2.13. *Let $(F, N, \circ, \bullet)$ be a QNMS. If every Cauchy sequence in F has a convergent subsequence. Then, the QNMS $(F, N, \circ, \bullet)$ is complete.*

Proof. Let the sequence (a_n) be a Cauchy and let (a_{i_n}) be a subsequence of (a_n) and $(a_n) \to a$. Let $\lambda > 0$ and $\mu \in (0, 1)$. Choose $\varepsilon \in (0, 1)$ such that $(1 - \varepsilon) \circ (1 - \varepsilon) \geq 1 - \mu$, $\varepsilon \bullet \varepsilon \leq \mu$. It is well known that the sequence (a_n) is Cauchy. Then, there is $U \in \mathbb{N}$ such that $A(a_m, a_n, \frac{\lambda}{2}) > 1 - \varepsilon$, $B(a_m, a_n, \frac{\lambda}{2}) < \varepsilon$, $C(a_m, a_n, \frac{\lambda}{2}) < \varepsilon$ and $D(a_m, a_n, \frac{\lambda}{2}) < \varepsilon$ for all $m, n \in U$. Since $a_{n_i} \to a$, there is positive integer i_p such that $i_p > U$, $A(a_{i_p}, a, \frac{\lambda}{2}) > 1 - \varepsilon$, $B(a_{i_p}, a, \frac{\lambda}{2}) < \varepsilon$, $C(a_{i_p}, a, \frac{\lambda}{2}) < \varepsilon$ and $D(a_{i_p}, a, \frac{\lambda}{2}) < \varepsilon$. Therefore, if $n \geq U$,

$$A(a_n, a, \lambda) \geq A(a_n, a_{i_p}, \frac{\lambda}{2}) \circ A(a_{i_p}, a, \frac{\lambda}{2}) > (1 - \varepsilon) \circ (1 - \varepsilon) \geq 1 - \mu,$$

$$B(a_n, a, \lambda) \leq B(a_n, a_{i_p}, \frac{\lambda}{2}) \bullet B(a_{i_p}, a, \frac{\lambda}{2}) < \varepsilon \bullet \varepsilon \leq \mu,$$

$$C(a_n, a, \lambda) \leq C(a_n, a_{i_p}, \frac{\lambda}{2}) \bullet C(a_{i_p}, a, \frac{\lambda}{2}) < \varepsilon \bullet \varepsilon \leq \mu,$$

$$D(a_n, a, \lambda) \leq D(a_n, a_{i_p}, \frac{\lambda}{2}) \bullet D(a_{i_p}, a, \frac{\lambda}{2}) < \varepsilon \bullet \varepsilon \leq \mu.$$

In consequence, $a_n \to a$. □

Theorem 2.14. *Let $(F, N, \circ, \bullet)$ be a QNMS and B be a subset of F with the subspace quadripartitioned neutrosophic metric (QNM) $(A_B, B_B, C_B, D_B) = (A|_{B^2 \times \mathbb{R}^+}, B|_{B^2 \times \mathbb{R}^+}, C|_{B^2 \times \mathbb{R}^+}, D|_{B^2 \times \mathbb{R}^+})$. Then, $(B, N_B, \circ, \bullet)$ is complete if and only if B is closed subset of F.*

Proof. Let's assume that B is a closed subset of F. Take the sequence (b_n) be a Cauchy in $(B, N_B, \circ, \bullet)$. Since (b_n) is a Cauchy in F, then there is a point $b \in F$ such that $b_n \to b$. From here, $b \in B = B$ and so (b_n) converges to B.

Contrarily, consider the $(B, N_B, \circ, \bullet)$ is complete, also assume that B is not closed. Take $a \in \overline{B} - B$. Therefore, there exist a sequence (b_n) of points in B that converges to b and so (b_n) is a Cauchy. In consequence, for $n, m \geq U$, each $0 < \mu < 1$ each $\lambda > 0$ there is $U \in \mathbb{R}$ such that $A(b_n, b_m, \lambda) > 1 - \mu$, $B(b_n, b_m, \lambda) < \mu$, $C(b_n, b_m, \lambda) < \mu$ and $D(b_n, b_m, \lambda) < \mu$. Now, we can write $A(b_n, b_m, \lambda) = A_B(b_n, b_m, \lambda)$, $B(b_n, b_m, \lambda) = B_B(b_n, b_m, \lambda)$, $C(b_n, b_m, \lambda) = C_B(b_n, b_m, \lambda)$ and $D(b_n, b_m, \lambda) = D_B(b_n, b_m, \lambda)$ because of the sequence (b_n) is in B. Therefore, (b_n) is a Cauchy in B. It is well-known that $(F, N, \circ, \bullet)$ is complete, then there is $c \in B$ such that $b_n \to c$. Hence, there is $U \in \mathbb{N}$ such that $A_B(c, b_n, \lambda) > 1 - \mu$, $B_B(c, b_n, \lambda) < \mu$, $C_B(c, b_n, \lambda) < \mu$ and $D_B(c, b_n, \lambda) < \mu$ for $n \geq U$, each $\mu \in (0, 1)$ and each $\lambda > 0$. Since the sequence (b_n) is in B and $c \in B$, we can write $A(c, b_n, \lambda) = A_B(c, b_n, \lambda)$, $B(c, b_n, \lambda) = B_B(c, b_n, \lambda)$, $C(c, b_n, \lambda) = C_B(c, b_n, \lambda)$ and $D(c, b_n, \lambda) = D_B(c, b_n, \lambda)$, and this is a contradiction. □

In proof of Lemma 2.15 and Theorem 2.16, used similar proof techniques of Propositions 4.3 and 4.4 in [9], and Lemma 3.15 and Theorem 3.16 in [10]. Therefore, their proofs are omitted.

Lemma 2.15. *Let $(F, N, \circ, \bullet)$ be QNMS. If $\lambda > 0$ and $\varepsilon_1, \varepsilon_2 \in (0, 1)$ such that $(1-\varepsilon_2)\circ(1-\varepsilon_2) \geq (1 - \varepsilon_1)$ and $\varepsilon_2 \bullet \varepsilon_2 \leq \varepsilon_1$, then $\overline{O(a, \varepsilon_2, \frac{\lambda}{2})} \subset O(a, \varepsilon_1, \lambda)$.*

Theorem 2.16. *A subset B of a QNMS $(F, N, \circ, \bullet)$ is nowhere dense if and only if every non-empty open set in F includes an open ball whose closure is disjoint from B.*

Next, we will prove Baire Category Theorem for QNMS:

Theorem 2.17. *Let $\{\omega_n : n \in \mathbb{N}\}$ be a sequence of dense open subsets of a complete QNMS $(F, N, \circ, \bullet)$. Then, $\bigcap_{n\in\mathbb{N}} \omega_n$ is also dense in F.*

Proof. Let δ be a non-empty open set of F. Since ω_1 is dense in F, $\delta \cap \omega_1 \neq \emptyset$. Let $a_1 \in \delta \cap \omega_1$. Since $\delta \cap \omega_1$ is open, then there exist $\varepsilon_1 \in (0, 1)$, $\lambda_1 > 0$ such that $O(a_1, \varepsilon_1, \lambda_1) \subset \delta \cap \omega_1$. Take $\varepsilon_1^* < \varepsilon$ and $\lambda_1^* = \min\{\lambda_1, 1\}$ such that $\overline{O(a_1, \varepsilon_1^*, \lambda_1^*)} \subset \delta \cap \omega_1$. Since ω_2 is dense in F, $O(a_1, \varepsilon_1^*, \lambda_1^*) \cap \omega_2 \neq \emptyset$. Let $a_2 \in O(a_1, \varepsilon_1^*, \lambda_1^*) \cap \omega_2$. Since $O(a_1, \varepsilon_1^*, \lambda_1^*) \cap \omega_2$ is open, then there exist $\varepsilon_2 \in (0, 1/2)$ and $\lambda_2 > 0$ such that $O(a_2, \varepsilon_2, \lambda_2) \subset O(a_1, \varepsilon_1^*, \lambda_1^*) \cap \omega_2$. Choose $\varepsilon_2^* < \varepsilon_2$ and $\varepsilon_2^* = \min\{\lambda_2, 1/2\}$ such that $\overline{O(a_2, \varepsilon_2^*, \lambda_2^*)} \subset O(a_1, \varepsilon_1^*, \lambda_1^*) \cap \omega_2$. If we continue this way, we have a sequence (b_n) in F and a sequence (λ_n^*) such that $0 < \lambda_n^* < 1/n$ and $\overline{O(a_n, \varepsilon_n^*, \lambda_n^*)} \subset O(a_{n-1}, \varepsilon_{n-1}^*, \lambda_{n-1}^*) \cap \omega_n$. Next, we will show that the sequence (a_n) is a Cauchy sequence. For $\lambda > 0$ and $\theta > 0$, choose $U \in \mathbb{N}$ such that $1/U < \lambda$ and $1/N < \theta$, Therefore, for $n \geq U$, $m \geq n$,

$$
\begin{aligned}
A(a_n, a_m, \lambda) &\geq A(a_n, a_m, 1/n) \geq 1 - 1/n > 1 - \theta \\
B(a_n, a_m, \lambda) &\leq B(a_n, a_m, 1/n) \leq 1/n \leq \theta \\
C(a_n, a_m, \lambda) &\leq C(a_n, a_m, 1/n) \leq 1/n \leq \theta \\
D(a_n, a_m, \lambda) &\leq D(a_n, a_m, 1/n) \leq 1/n \leq \theta.
\end{aligned}
$$

Hence, the sequence (a_n) is a Cauchy. It is well-known that F is complete. Then, there exists $a \in F$ such that $a_n \to a$. Since $a_i \in O(a_n, \varepsilon_n^*, \lambda_n^*)$ for $k \geq n$, then we have $a \in \overline{O(a_n, \varepsilon_n^*, \lambda_n^*)}$. Hence, $a \in \overline{O(a_n, \varepsilon_n^*, \lambda_n^*)} \subset O(a_{n-1}, \varepsilon_{n-1}^*, \lambda_{n-1}^*) \cap \omega_n$, for all $n \in \mathbb{N}$. Then, $\delta \cap \left(\bigcap_{n\in\mathbb{N}} \omega_n\right) \neq \emptyset$. Therefore, $\bigcap_{n\in\mathbb{N}} \omega_n$ is dense in F. □

Definition 2.18. Let $(F, N, \circ, \bullet)$ be a QNMS. A collection $(D_n)_{n\in\mathbb{N}}$ is said to have quadripartitioned neutrosophic diameter zero (QNDZ)if for each $\varepsilon \in (0, 1)$ and each $\lambda > 0$, then there exists $U \in \mathbb{N}$ such that $A(a, b, \lambda) > 1 - \varepsilon$, $B(a, b, \lambda) < \varepsilon$, $C(a, b, \lambda) < \varepsilon$ and $D(a, b, \lambda) < \varepsilon$, for all $a, b, \in D_U$.

Theorem 2.19. *Let QNMS $(F, N, \circ, \bullet)$ is complete if and only if very nested sequence $(D_n)_{n\in\mathbb{N}}$ of non-empty closed sets with QNDZ have non-empty intersection.*

Proof. Firstly, consider the given condition is satisfied. Then, we will show that $(F, N, \circ, \bullet)$ is complete. Take the Cauchy sequence (a_n) in F. If we define the $E_n = \{a_k : k \geq n\}$ and $D_n = \overline{E_n}$, then we can say that (D_n) has QNSZ. For given $\zeta \in (0, 1)$ and $\lambda > 0$, we take $\varepsilon \in (0, 1)$ such that $(1-\varepsilon) \bullet (1-\varepsilon) \bullet (1-\varepsilon) > 1 - \zeta$ and $\varepsilon \bullet \varepsilon \bullet \varepsilon < \zeta$. Since the sequence (a_n) is Cauchy, then there exist $U \in \mathbb{N}$ such that $A(a_n, a_m, \frac{\lambda}{3}) > 1 - \varepsilon$, $B(a_n, a_m, \frac{\lambda}{3}) < \varepsilon$, $C(a_n, a_m, \frac{\lambda}{3}) < \varepsilon$ and $D(a_n, a_m, \frac{\lambda}{3}) < \varepsilon$, for all $m, n \geq U$. Then, $A(a, b, \frac{\lambda}{3}) > 1 - \varepsilon$, $B(a, b, \frac{\lambda}{3}) < \varepsilon$, $C(a, b, \frac{\lambda}{3}) < \varepsilon$ and $D(a, b, \frac{\lambda}{3}) < \varepsilon$ for all $m, n \geq E_n$.

Take $a, b \in D_n$, there exist the sequences (a_n^*) and (b_n^*) such that $a_n^* \to a$ and $b_n^* \to b$. Thus, for sufficiently large n, $a_n^* \in O(a, \varepsilon, \frac{\lambda}{3})$ and $O(b, \varepsilon, \frac{\lambda}{3})$. Now, we obtain

$$A(a,b,\lambda) \geq A(a,b_n^*,\frac{\lambda}{3}) \circ A(a_n^*,b_n^*,\frac{\lambda}{3}) \circ A(b_n^*,b,\frac{\lambda}{3}) > (1-\varepsilon) \circ (1-\varepsilon) \circ (1-\varepsilon)$$
$$> 1-\zeta$$
$$B(a,b,\lambda) \leq A(a,b_n^*,\frac{\lambda}{3}) \bullet B(a_n^*,b_n^*,\frac{\lambda}{3}) \bullet B(b_n^*,b,\frac{\lambda}{3}) < \varepsilon \bullet \varepsilon \bullet \varepsilon < \zeta$$
$$C(a,b,\lambda) \leq C(a,b_n^*,\frac{\lambda}{3}) \bullet C(a_n^*,b_n^*,\frac{\lambda}{3}) \bullet D(b_n^*,b,\frac{\lambda}{3}) < \varepsilon \bullet \varepsilon \bullet \varepsilon < \zeta$$
$$D(a,b,\lambda) \leq D(a,b_n^*,\frac{\lambda}{3}) \bullet D(a_n^*,b_n^*,\frac{\lambda}{3}) \bullet D(b_n^*,b,\frac{\lambda}{3}) < \varepsilon \bullet \varepsilon \bullet \varepsilon < \zeta.$$

From here, $A(a,b,\lambda) > 1-\zeta$, $B(a,b,\lambda) < \zeta$, $C(a,b,\lambda) < \zeta$ and $D(a,b,\lambda) < \zeta$ for all $a,b \in D_n$. Therefore, (D_n) has QNDZ and so by hypothesis $\bigcap_{n\in\mathbb{N}} D_n$ i non-empty. Choose $a \in \bigcap_{n\in\mathbb{N}} D_n$. For $\varepsilon \in (0,1)$ and $\lambda > 0$, then there exist $U_1 \in \mathbb{N}$ such that $A(a_n,a,\lambda) > 1-\varepsilon$, $B(a_n,a,\lambda) < \varepsilon$, $C(a_n,a,\lambda) < \varepsilon$ and $D(a_n,a,\lambda) < \varepsilon$ for all $n \geq U_1$. Therefore, for each $\lambda > 0$, $A(a_n,a,\lambda) \to 1$, $B(a_n,a,\lambda) \to 0$, $C(a_n,a,\lambda) \to 0$ and $D(a_n,a,\lambda) \to 0$ as $n \to \infty$. Therefore, $a_n \to a$ that is $(F,N,\circ,\bullet)$ is complete.

Conversely, consider that $(F,N,\circ,\bullet)$ is complete. Let's $(D_n)_{n\in\mathbb{N}}$ is nested sequences of non-empty closed sets with QNDZ. For each $n \in \mathbb{N}$, take a point $a_n \in D_n$. We will show that the sequence (a_n) is Cauchy. Since (D_n) has QNDZ, for each $\lambda > 0$ and $\varepsilon \in (0,1)$, then there exist $U \in \mathbb{N}$ such that $A(a,b,\lambda) > 1-\varepsilon$, $B(a,b,\lambda) < \varepsilon$, $C(a,b,\lambda) < \varepsilon$ and $D(a,b,\lambda) < \varepsilon$ for all $a,b \in D_n$. Since the sequence (D_n) is nested, then $A(a_n,a_m,\lambda) > 1-\varepsilon$, $B(a_n,a_m,\lambda) < \varepsilon$, $C(a_n,a_m,\lambda) < \varepsilon$ and $D(a_n,a_m,\lambda) < \varepsilon$ for all $m,n \geq U$. Therefore, the sequence (a_n) is Cauchy. Since $(F,N,\circ,\bullet)$ is complete. Then, $a_n \to a$ for some $a \in F$. Hence, $a \in \overline{D_n} = D_n$ for every n, and so $a \in \bigcap_{n\in\mathbb{N}} D_n$. □

Theorem 2.20. *Every separable QNMS is second countable.*

Proof. Let $(F,N,\circ,\bullet)$ be a separable QNMS and let $A = \{a_n : n \in \mathbb{N}\}$ be a countable dense subset of F. Establish the family $O = \{O(a_k, 1/m, 1/m) : k,m \in \mathbb{N}\}$. It can be seen that O is countable. Now, we will show that O is base for the family of all open sets in F. Let μ be any open set in F, $a \in \mu$. Then, there exist $\lambda > 0$, $\varepsilon \in (0,1)$ such that $O(a,\varepsilon\lambda) \subset \mu$. Since $\varepsilon \in (0,1)$, we can take a $\zeta \in (0,1)$ such that $(1-\zeta)\circ(1-\zeta) > 1-\varepsilon$ and $\zeta \bullet \zeta < \varepsilon$. Choose $y \in \mathbb{N}$ such that $1/t = \min\{\zeta, \lambda/2\}$. Since A is dense in F, there exist $a_k \in A$ such that $a_k \in O(a, 1/y, 1/y)$. If $b \in O(a_k, 1/y, 1/y)$, we get

$$A(a,b,\lambda) \geq A(a,a_k,\frac{\lambda}{2}) \circ A(a_k,b,\frac{\lambda}{2}) \geq A(a,a_k,\frac{1}{y}) \circ A(a_k,b,\frac{1}{y})$$
$$\geq (1-\frac{1}{y}) \circ (1-\frac{1}{y}) \geq (1-\zeta)\circ(1-\zeta) > 1-\varepsilon$$
$$B(a,b,\lambda) \leq B(a,a_k,\frac{\lambda}{2}) \bullet B(a_k,b,\frac{\lambda}{2}) \leq B(a,a_k,\frac{1}{y}) \bullet B(a_k,b,\frac{1}{y}) \leq \frac{1}{y} \bullet \frac{1}{y} \leq \zeta \bullet \zeta < \varepsilon$$
$$C(a,b,\lambda) \leq C(a,a_k,\frac{\lambda}{2}) \bullet C(a_k,b,\frac{\lambda}{2}) \leq C(a,a_k,\frac{1}{y}) \bullet C(a_k,b,\frac{1}{y}) \leq \frac{1}{y} \bullet \frac{1}{y} \leq \zeta \bullet \zeta < \varepsilon$$
$$D(a,b,\lambda) \leq D(a,a_k,\frac{\lambda}{2}) \bullet D(a_k,b,\frac{\lambda}{2}) \leq D(a,a_k,\frac{1}{y}) \bullet D(a_k,b,\frac{1}{y}) \leq \frac{1}{y} \bullet \frac{1}{y} \leq \zeta \bullet \zeta < \varepsilon.$$

Then, $b \in O(a,\varepsilon,\lambda) \subset \mu$ and so O is a base. □

Remark 2.21. It can be seen that the second countability implies separability and the second countability is inheritable property. Therefore, we can imply that every subspace of a separable quadripartitioned neutrsophic metric spce is separable.

Definition 2.22. Let F be a non-empty set and $(H, N, \circ, \bullet)$ be a QNMS. The sequence of functions $(f_n) : F \to H$ is said to be converge uniformly to a function $f : F \to H$, if given $\lambda > 0$, $\varepsilon \in 0, 1)$, then ther exists $U \in \mathbb{N}$ such that $A(fn(a), f(a), \lambda) > 1 - \varepsilon$, $B(fn(a), f(a), \lambda) < \varepsilon$, $C(fn(a), f(a), \lambda) < \varepsilon$ and $D(fn(a), f(a), \lambda) < \varepsilon$ for all $n \geq U$ and $a \in F$.

Now, we will give Uniform Convergence Theorem for QNMS:

Theorem 2.23. *Let $f_n : F \to H$ be a sequence of continuous functions from a topological space F to a QNMS $(H, N, \circ, \bullet)$ If (f_n) converges uniformly to $f : F \to H$, then f is continuous.*

Proof. Choose δ be an open set of H and let $a_0 \in f^{-1}(\delta)$. Since δ is open, then there exist $\lambda > 0$, $\varepsilon \in (0, 1)$ such that $O(f(a_0), \varepsilon, \lambda) \subset \delta$. Since $\varepsilon \in (0, 1)$, we take $\zeta \in (0, 1)$ such that $(1 - \zeta) \circ (1 - \zeta) \circ (1 - \zeta) > 1 - \varepsilon$ and $\zeta \bullet \zeta \bullet \zeta < \varepsilon$. Since (f_n) converges uniformly to f, then, for $\lambda > 0$, $\zeta \in (0, 1)$, there exists $U \in \mathbb{N}$ such that $A(f_n(a), f(a), \frac{\lambda}{3}) > 1 - \zeta$, $B(f_n(a), f(a), \frac{\lambda}{3}) < \zeta$, $C(f_n(a), f(a), \frac{\lambda}{3}) < \zeta$ and $D(f_n(a), f(a), \frac{\lambda}{3}) < \zeta$ for all $n \geq U$ and for all $a \in F$. Since f_n is continuous for all $n \in \mathbb{N}$, then there exist a neighbourhood ω of a_0 such that $f_n(\omega) \subset O(f_n(a_0, \zeta, \frac{\lambda}{3})$. Therefore, $A(f_n(a), f_n(a_0), \frac{\lambda}{3}) > 1 - \zeta$, $B(f_n(a), f_n(a_0), \frac{\lambda}{3}) < \zeta$, $C(f_n(a), f_n(a_0), \frac{\lambda}{3}) < \zeta$ and $D(f_n(a), f_n(a_0), \frac{\lambda}{3}) < \zeta$ for all $a \in \omega$. Now,

$$A(f(a), f(a_0), \lambda) \geq A(f(a), f_n(a), \frac{\lambda}{3}) \circ A(f_n(a), f_n(a_0), \frac{\lambda}{3}) \circ A(f_n(a_0), f(a_0), \frac{\lambda}{3})$$
$$\geq (1 - \zeta) \circ (1 - \zeta) \circ (1 - \zeta) > 1 - \varepsilon$$
$$B(f(a), f(a_0), \lambda) \leq B(f(a), f_n(a), \frac{\lambda}{3}) \bullet B(f_n(a), f_n(a_0), \frac{\lambda}{3}) \bullet B(f_n(a_0), f(a_0), \frac{\lambda}{3})$$
$$\leq \zeta \bullet \zeta \bullet \zeta < \varepsilon$$
$$C(f(a), f(a_0), \lambda) \leq C(f(a), f_n(a), \frac{\lambda}{3}) \bullet C(f_n(a), f_n(a_0), \frac{\lambda}{3}) \bullet C(f_n(a_0), f(a_0), \frac{\lambda}{3})$$
$$\leq \zeta \bullet \zeta \bullet \zeta < \varepsilon$$
$$D(f(a), f(a_0), \lambda) \leq D(f(a), f_n(a), \frac{\lambda}{3}) \bullet D(f_n(a), f_n(a_0), \frac{\lambda}{3}) \bullet D(f_n(a_0), f(a_0), \frac{\lambda}{3})$$
$$\leq \zeta \bullet \zeta \bullet \zeta < \varepsilon.$$

Therefore, $f(a) \in O(f(a_0), \varepsilon, \lambda) \subset \delta$ for all $a \in \mu$. Hence, $f(\mu) \subset \delta$. Thus, f is continuous. □

3 Conclusion

The main of this paper is to introduce a quadripartitioned neutrosophic metric spaces and examine some properties. The structural characteristic properties of quadripartitioned neutrosophic metric spaces such as open ball, open set, Hausdorffness, compactness, completeness, nowhere dense in quadripatitioned neutrosophic metric space have been proved. Analogues of Baire Category Theorem and Uniform Convergence Theorem were given for quadripatitioned neutrosophic metric space. This new concept can also be studied to the fixed point theory, as in metric fixed metric theory and so it can constructed the quadripatitioned neutrosophic metric spac fixed point theory. As is well known, in recent years, the study of metric fixed point theory has been widely researched because the theory has a fundamental role in various areas of mathematics, science and economic studies. Moreover, this notion can be extended for pentapartitionedneutrosophic metric space or heptapartitioned neutrosophic metric space as well as notions presented in [14, 15].

Funding

This research received no external funding.

Conflicts of Interest

The authors declare no conflict of interest.

Data availability statement

This manuscript has no associated data.

References

[1] Atanassov, K.: Intuitionistic fuzzy sets. Fuzzy Sets Syst. 20, 87-96 (1986)

[2] Bera, T., Mahapatra, N.K.: Neutrosophic soft normed linear spaces. Neutrosophic Sets Syst. 23, 52-71 (2018)

[3] S. Das, R. Das, C. Granados, Topology on quadripartitioned neutrosophic sets, Neutrosophic Sets and Systems 45(2021), 54-61.

[4] Chahhtterjee, R.; Majumdar, P.; Samanta, S.K. On some similarity measures and entropy on Quadripartitioned single valued neutrosophic sets. J. Int. Fuzzy Syst. 2016, 30, 2475-2485.

[5] George, A., Veeramani, P.: On some results in fuzzy metric spaces. Fuzzy Sets Syst. 64, 395-399 (1994)

[6] George, A., Veeramani, P.: On some results of analysis for fuzzy metric spaces. Fuzzy Sets Syst. 90, 365-368 (1997)

[7] C. Granados, A. Dhital, Statistical convergence of double sequences in neutrosophic normed spaces, Neutrosophic Sets and Systems 42(2021), 334-344.

[8] Kaleva, O., Seikkala, S.: On fuzzy metric spaces. Fuzzy Sets Syst. 12, 215-229 (1984)

[9] Kirisci, M.: Multiplicative generalized metric spaces and fixed point theorems. J. Math. Anal. 8, 212-224. (2017)

[10] Kirisci, M., Simsek, N.: Neutrosophic Metric Space. Mathematical Science 14, 241-248. (2020)

[11] Kramosil, I., Michalek, J.: Fuzzy metric and statistical metric spaces. Kybernetika 11, 336-344 (1975)

[12] J. Madore, Fuzzy physics, *Ann Phys* 219 (1992), 187–98.

[13] Smarandache, F.: Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. Int. J. Pure Appl. Math. 24, 287-297 (2005)

[14] Smarandache, F, Al Tahan M.: NeutroGeometry, NeutroAlgebra, and SuperHyperAlgebra in Today's World. IGI Global. 2023.

[15] Smarandache, F, Al Tahan M.: Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras. IGI Global. 2022.

[16] Zadeh, L.A.: Fuzzy sets. Inf. Comp. 8, 338–353 (1965)

Author information

Carlos Granados, Escuela Ciencias de la Educación, Universidad Nacional Abierta y a Distancia, Barranquilla, Colombia, Colombia.
E-mail: carlosgranadosortiz@outlook.es

Suman Das, Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India.
E-mail: sumandas18843@gmail.com

Rakhal Das, Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India, India.
E-mail: rakhaldas95@gmail.com

Birojit Das, Assistant Professor, Amity Institute of Applied Sciences, Amity University Kolkata, India, India.
E-mail: dasbirojit@gmail.com

Received: 2024-06-16
Accepted: 2024-11-30