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SOME PARAMETERS IN HYPERGRAPH

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Abstract In this paper, we study isolate domination in hypergraphs and introduced the notions of an isolate set and isolate irredundant set in hypergraphs. A set I is called an isolate set if it contains at least one vertex $v \in I$ such that v is not adjacent to any vertex of I. The set I is called an isolate irredundant set if it is both isolate and irredundant. The equality among domination number, isolate domination number, isolate number and total domination number has been studied and their properties are examined. Further the integrated chain containing isolate number, isolate irredundance number along with domination related parameters has been obtained while several important results and many bounds regarding these parameters are found.

1 Introduction

In graph theory based on applications in various fields, the major focus is given to the domination and related subset problems such as independence, irredundance, vertex covering and matching. Many authors have been working on these topics and more than thousand research papers have been published during last five decades. In [6]-[7] Haynes et al. gave the detailed fundamental and advanced concepts of domination in graphs. Several variants of domination have been introduced and well-studied in the present literature and many others are being studied. In [8], [9] and [11] researchers studied isolate domination and discussed equality of some parameters in the extended domination chain in graphs. The concept of domination in hypergraphs was initiated by Acharya [1] and studied further in [2], [3]. He extended many important theorems for graphs to hypergraphs and raised several interesting open problems on hypergraphs. Many of his conjectures and open problems were solved by Jose et al. [5]. He also explored the study of domination in hypergraphs and obtained several results connecting domination number, irredundance number and independence number of a hypergraph $\mathcal{H}$ in his thesis [10]. The six parameters of domination, independence and irredundance are connected by a chain of inequalities called the domination chain of the hypergraph $\mathcal{H}$, $ir(\mathcal{H}) \leq \gamma(\mathcal{H}) \leq i(\mathcal{H}) \leq \beta_0(\mathcal{H}) \leq \Gamma(\mathcal{H}) \leq IR(\mathcal{H})$, where $ir(\mathcal{H})$ and $IR(\mathcal{H})$ denote the irredundance number and upper irredundance number of $\mathcal{H}$, $\gamma(\mathcal{H})$ and $\Gamma(\mathcal{H})$ denote the domination number and upper domination number of $\mathcal{H}$, $i(\mathcal{H})$ and $\beta_0(\mathcal{H})$ denote the independent domination number and independence number of the hypergraph $\mathcal{H}$. In [13] Jadhav and Pawar introduced the notion of edge product hypergraphs and proved some results, while in [14] the notion of an isolate domination in hypergraphs is introduced. In the present paper, we introduced some new isolate domination related parameters in hypergraphs and initiate a study on these parameters on the line of [8], [9] and [11].

2 Preliminaries

We begin with recalling some basic definitions and results required for our purpose.

Definition 2.1. [10] A hypergraph $\mathcal{H}$ is a pair $\mathcal{H}(V, E)$ where V is a finite nonempty set and E is a collection of subsets of V. The elements of V are called vertices and the elements of E are called edges or hyperedges. And $\cup_{e_i \in E} e_i = V$ and $e_i \neq \phi$ are required for all $e_i \in E$. The number of vertices in $\mathcal{H}$ is called the order of the hypergraph and is denoted by $|V|$. The number of edges in $\mathcal{H}$ is called the size of $\mathcal{H}$ and is denoted by $|E|$. A hypergraph of order n and size m is called a (n, m) hypergraph. The number $|e_i|$ is called the degree (cardinality) of the edge e_i. The rank of a hypergraph $\mathcal{H}$ is $r(\mathcal{H}) = max_{e_i \in E}|e_i|$.

Definition 2.2. [10] For any vertex v in a hypergraph $\mathcal{H}(V, E)$, the set $N[v] = \{u \in V : u \text{ is adjacent to } v\} \cup \{v\}$ is called the closed neighborhood of v in $\mathcal{H}$ and each vertex in the set $N[v] \setminus \{v\}$ is called neighbor of v. The open neighborhood of the vertex v is the set $N[v] \setminus \{v\}$. If $S \subseteq V$, then $N(S) = \cup_{v \in S} N(v)$ and $N[S] = N(S) \cup S$.

Definition 2.3. [10] A simple hypergraph (or sperner family) is a hypergraph $\mathcal{H}(V, E)$, where $E = \{e_1, e_2, \cdots, e_m\}$ such that $e_i \subset e_j$ implies $i = j$.

Definition 2.4. [10] A hypergraph is called k-uniform if $|e| = k$, for every $e \in E$.

Definition 2.5. [10] For any hypergraph $\mathcal{H}(V, E)$ two vertices v and u are said to be adjacent if there exists an edge $e \in E$ that contains both v and u and non-adjacent otherwise.

Definition 2.6. [10] For any hypergraph $\mathcal{H}(V, E)$ two edges are said to be adjacent if their intersection is nonempty. If a vertex $v_i \in V$ belongs to an edge $e_j \in E$, then we say that they are incident to each other.

Definition 2.7. [10] The vertex degree of a vertex v is the number of vertices adjacent to the vertex v in $\mathcal{H}$. It is denoted by $d(v)$.

The maximum (minimum) vertex degree of a hypergraph is denoted by $\Delta(\mathcal{H})(\delta(\mathcal{H}))$.

Definition 2.8. [10] The edge degree of a vertex v is the number of edges containing the vertex v. It is denoted by $d_E(v)$.

The maximum (minimum) edge degree of a hypergraph is denoted by $\Delta_E(\mathcal{H})(\delta_E(\mathcal{H}))$. A vertex of a hypergraph which is incident to no edge is called an isolated vertex.

Definition 2.9. [10] The hypergraph $\mathcal{H}(V, E)$ is called connected if for any pair of its vertices, there is a path connecting them. If $\mathcal{H}$ is not connected, then it consists of two or more connected components, each of which is a connected hypergraph.

Definition 2.10. [12] A complete r-partite hypergraph is an r-uniform hypergraph $\mathcal{H}(V, E)$ such that the set V can be partitioned into r non-empty parts, each edge contains precisely one vertex from each part, and all such subsets form E. It is denoted by $K^r_{n_1, n_2, \ldots, n_r}$, where n_i is the number of vertices in part V_i.

Definition 2.11. [10] For a hypergraph $\mathcal{H}(V, E)$, a subset S of V is called an independent set of $\mathcal{H}$ if no two vertices of S are adjacent in $\mathcal{H}$.

Definition 2.12. [10] For a hypergraph $\mathcal{H}(V, E)$, a set $D \subseteq V$ is called a dominating set of $\mathcal{H}$ if for every $v \in V \setminus D$ there exists $u \in D$ such that u and v are adjacent in $\mathcal{H}$.

If further, D is an independent set, then D is called an independent dominating set of $\mathcal{H}$.

Definition 2.13. [10] A dominating set D of a hypergraph $\mathcal{H}$ is called a minimal dominating set, if no proper subset of D is a dominating set of $\mathcal{H}$.

The minimum(maximum) cardinality of a minimal dominating set in a hypergraph $\mathcal{H}$ is called the domination(upper domination) number of $\mathcal{H}$ and is denoted by $\gamma(\mathcal{H})(\Gamma(\mathcal{H}))$. A dominating set of cardinality $\gamma(\Gamma)$ is called a γ-set (Γ-set).

The minimum cardinality of an independent dominating set in $\mathcal{H}$ is called the independent domination number of $\mathcal{H}$ and is denoted by $i(\mathcal{H})$. The maximum cardinality of an independent set in $\mathcal{H}$ is called the independence number of $\mathcal{H}$ and is denoted by $\beta_0(\mathcal{H})$.

Definition 2.14. [10] Let S be a set of vertices of a hypergraph $\mathcal{H}$ and let $u \in S$. Then the vertex v is said to be a private neighbor of u (with respect to S) if $N[v] \cap S = \{u\}$. The set of all private neighbors of u with respect to S is called private neighbor set of u with respect to S and is denoted by $pn[u, S] = \{v : N[v] \cap S = \{u\}\}$.

Definition 2.15. [10] A set S of vertices is irredundant if for every vertex $v \in S$, $pn[v, S] \neq \phi$. An irredundant set S is called a maximal irredundant set if no proper superset of S is irredundant. The minimum cardinality of a maximal irredundant set in a hypergraph $\mathcal{H}$ is called the irredundance number of $\mathcal{H}$ and is denoted by $ir(\mathcal{H})$. The maximum cardinality of an irredundant set in $\mathcal{H}$ is called the upper irredundance number of $\mathcal{H}$ and is denoted by $IR(\mathcal{H})$.

Definition 2.16. [14] A dominating set I of a hypergraph $\mathcal{H}$ is called an isolate dominating set of $\mathcal{H}$ if it contains at least one vertex $v \in I$ such that v is not adjacent to any vertex of I, that is $N(v) \cap I = \phi$, for at least one vertex $v \in I$.

Definition 2.17. [14] An isolate dominating set I of a hypergraph $\mathcal{H}$ is called a minimal isolate dominating set if no proper subset of I is an isolate dominating set of $\mathcal{H}$.

The minimum (maximum) cardinality of a minimal isolate dominating set in a hypergraph $\mathcal{H}$ is called the isolate (upper isolate) domination number of $\mathcal{H}$ and is denoted by $\gamma_0(\mathcal{H})(\Gamma_0(\mathcal{H}))$. An isolate dominating set of cardinality $\gamma_0(\Gamma_0)$ is called a γ_0-set (Γ_0-set).

Definition 2.18. [15] A total dominating set in $\mathcal{H}$ is a dominating set in $\mathcal{H}$ with the additional property that for every vertex v in D, there exists an edge $e \in E(\mathcal{H})$ for which $v \in e$ and $e \cap (D \setminus \{v\}) \neq \phi$. The minimum cardinality of a total dominating set in $\mathcal{H}$ is called the total domination number of hypergraph $\mathcal{H}$ and is denoted by $\gamma_t(\mathcal{H})$.

Theorem 2.19. *[1] Let $\mathcal{H} = (V, E)$ be a hypergraph and $D \subseteq V$ be a dominating set. Then D is a minimal dominating set of $\mathcal{H}$ if and only if for every $v \in D$ there exists a vertex $w \in V$ such that $N[w] \cap D = \{v\}$.*

Theorem 2.20. *[10] Every minimal dominating set S in a hypergraph $\mathcal{H}$ is a maximal irredundant set of $\mathcal{H}$.*

3 Extension of Domination Chain

In this section, we extend the domination chain of hypergraph $\mathcal{H}$ by having two more parameters namely isolate domination number and upper isolate domination number of the hypergraph $\mathcal{H}$.

Definition 3.1. [4] Let P denote an arbitrary property of sets of vertices S in a hypergraph $\mathcal{H}(V, E)$. If a set S has property P, then we say that S is a P-set, otherwise it is a $\bar{P}$-set.

Definition 3.2. [4] A P-set S is a minimal P-set if every proper subset $S^* \subset S$ is a $\bar{P}$-set. A P-set is a 1-minimal P-set if for every vertex $v \in S$, $S \setminus \{v\}$ is a $\bar{P}$-set.

As we know, minimal P-sets are always 1-minimal P-sets but the converse is not always true. The minimality and 1-minimality of sets having property P are equivalent when the property P is superhereditary. Clearly, the property of being isolate domination is neither hereditary nor superhereditary. However, we can assert the following,

Theorem 3.3. *Let $\mathcal{H}(V, E)$ be a hypergraph and let I be any isolate dominating set of $\mathcal{H}$. Then I is minimal if and only if I is 1-minimal.*

Proof. By definition, every minimal isolate dominating set is 1-minimal. For the converse, let I be a 1-minimal isolate dominating set of $\mathcal{H}$. Suppose, to the contrary, that I is not minimal. Then there exists a subset I' of I which is an isolate dominating set of $\mathcal{H}$, where $|I'| \leq |I - 2|$. Further, the set I' will contain all the vertices of I with $N(v) \cap I = \phi$. Thus for all vertices $v \in I \setminus I'$, we have $N(v) \cap I \neq \phi$. Therefore for a vertex $v \in I \setminus I'$, the set $I \setminus \{v\}$ is an isolate dominating set of $\mathcal{H}$. But this contradicts the assumption that I' is 1-miminal. $\square$

Theorem 3.4. *An isolate dominating set I of a hypergraph $\mathcal{H}$ is minimal if and only if every vertex in I has a private neighbor with respect to I.*

Proof. Let I be a minimal isolate dominating set of $\mathcal{H}$ and let $v \in I$. If $N(v) \cap I = \phi$, then v is a private neighbor of itself. Suppose $N(v) \cap I \neq \phi$. If v has no private neighbors with respect to I, then the set $I \setminus \{v\}$ is an isolate dominating set of $\mathcal{H}$, contradicting the minimality of I. Hence v must have a private neighbor with respect to I. Conversely, let I be an isolate dominating set of $\mathcal{H}$ with every vertex in I has a private neighbor with respect to I. Suppose, to the contrary, that I is not minimal. Then by Theorem 3.3, I cannot be 1-minimal. Hence there exists a vertex v in I such that $I \setminus \{v\}$ is an isolate dominating set of $\mathcal{H}$. Therefore, every vertex in $V \setminus (I \setminus \{v\})$ is adjacent to at least one vertex of $I \setminus \{v\}$, which implies that the vertex v can have no private neighbor with respect to I. This contradicts the assumption. Hence the proof. □

Corollary 3.5. *Every minimal isolate dominating set of a hypergraph $\mathcal{H}$ is a minimal dominating set of $\mathcal{H}$.*

Proof. Let I be a minimal isolate dominating set of $\mathcal{H}$. Then by Theorem 3.4, every vertex of I has a private neighbor with respect to I. Consequently, I is a minimal dominating set of $\mathcal{H}$, by Theorem 2.19. □

Corollary 3.6. *For any hypergraph $\mathcal{H}$, $\gamma(\mathcal{H}) \leq \gamma_0(\mathcal{H}) \leq \Gamma_0(\mathcal{H}) \leq \Gamma(\mathcal{H})$.*

Proof. Follows from Corollary 3.5. □

Theorem 3.7. *Every maximal independent set of $\mathcal{H}$ is a minimal isolate dominating set.*

Proof. Let I be a maximal independent set and $u \in V \setminus I$. Then $I \cup \{u\}$ is not an independent set and hence there exists a vertex v in I such that u and v are adjacent. Thus I is a dominating set of the hypergraph $\mathcal{H}$. Since I is an independent set, it is an isolate dominating set of $\mathcal{H}$ and every vertex of I has a private neighbor namely itself. Hence by Theorem 3.4, the result follows. □

Corollary 3.8. *For any hypergraph $\mathcal{H}$, $\gamma_0(\mathcal{H}) \leq i(\mathcal{H}) \leq \beta_0(\mathcal{H}) \leq \Gamma_0(\mathcal{H})$.*

Proof. Follows from Theorem 3.7. □

The Corollaries 3.6 and 3.8 extend the existing domination chain of the hypergraph $\mathcal{H}$ as follows,

$$ir(\mathcal{H}) \leq \gamma(\mathcal{H}) \leq \gamma_0(\mathcal{H}) \leq i(\mathcal{H}) \leq \beta_0(\mathcal{H}) \leq \Gamma_0(\mathcal{H}) \leq \Gamma(\mathcal{H}) \leq IR(\mathcal{H}). \tag{3.1}$$

4 Isolate Variants

In this section, the notions of an isolate set and isolate irredundant set of a hypergraph $\mathcal{H}$ are introduced. Also their corresponding parameters are defined with suitable examples and many important results are explored. Later, we obtain the integrated chain containing these parameters along with the domination related parameters.

Definition 4.1. For a hypergraph $\mathcal{H}(V, E)$, a set $I \subseteq V$ is called an isolate set if it contains at least one vertex $v \in I$ such that v is not adjacent to any vertex of I.

Definition 4.2. The minimum(maximum) cardinality of a maximal isolate set in a hypergraph $\mathcal{H}$ is called the isolate(upper isolate) number of a hypergraph $\mathcal{H}$ and is denoted by $i_0(\mathcal{H})(I_0(\mathcal{H}))$.

Definition 4.3. An isolate set I of a hypergraph $\mathcal{H}$ is called a maximal isolate set if no proper superset of I is an isolate set of $\mathcal{H}$. An isolate set of cardinality $i_0(I_0)$ is called a i_0-set (I_0-set).

Example 4.4. Consider the hypergraph $\mathcal{H}(V, E)$, where $V = \{v_1, v_2, \ldots, v_{10}\}$ and $E = \{e_1, e_2, e_3, e_4\}$. In which the edges of $\mathcal{H}$ are defined as follows:

$$e_1 = \{v_1, v_2, v_3, v_4\}, \qquad e_2 = \{v_5, v_6, v_7, v_8\},$$
$$e_3 = \{v_1, v_2, v_5, v_6, v_9\}, \qquad e_4 = \{v_5, v_9, v_{10}\}.$$

Then the sets $I_1 = \{v_8, v_9\}$, $I_2 = \{v_3, v_4, v_5\}$, $I_3 = \{v_3, v_4, v_7, v_8, v_9\}$, and $I_4 = \{v_1, v_2, v_3, v_4, v_6, v_7, v_8, v_{10}\}$ are isolate sets of $\mathcal{H}$. But among these all are maximal isolate sets except I_1. In fact, I_2 is a maximal isolate set of $\mathcal{H}$ with minimum cardinality and I_4 is that of maximum cardinality. Hence $i_0(\mathcal{H}) = 3$ and $I_0(\mathcal{H}) = 8$.

Definition 4.5. A subset $I \subseteq V$ is called an isolate irredundant set of a hypergraph $\mathcal{H}$ if it is both isolate and irredundant. The minimum(maximum) cardinality of a maximal isolate irredundant set in a hypergraph $\mathcal{H}$ is called the isolate(upper isolate) irredundance number of $\mathcal{H}$ and is denoted by $ir_0(\mathcal{H})(IR_0(\mathcal{H}))$.

Example 4.6. Consider the hypergraph $\mathcal{H}(V, E)$, where $V = \{v_1, v_2, \dots, v_{10}\}$ and $E = \{e_1, e_2, e_3, e_4\}$. In which the edges of $\mathcal{H}$ are defined as follows:

$$e_1 = \{v_1, v_2, v_3, v_4\}, \qquad e_2 = \{v_4, v_5, v_6\},$$
$$e_3 = \{v_2, v_3, v_7, v_8, v_9\}, \qquad e_4 = \{v_7, v_{10}\}.$$

Here the sets $I_1 = \{v_1, v_7\}$, $I_2 = \{v_4, v_7\}$, $I_3 = \{v_2, v_4, v_{10}\}$ and $I_4 = \{v_1, v_5, v_8, v_{10}\}$ are all isolate irredundant sets in which except I_1, all of them are maximal. Also, I_2 is a maximal isolate irredundant set of minimum cardinality, whereas, I_4 is that of maximum cardinality. Hence $ir_0(\mathcal{H}) = 2$ and $IR_0(\mathcal{H}) = 4$.

Theorem 4.7. *Every minimal isolate dominating set is a maximal isolate irredundant set.*

Proof. Let I be a minimal isolate dominating set of $\mathcal{H}$. Since I is a minimal isolate dominating set in $\mathcal{H}$, it follows from Corollary 3.5 that I is a minimal dominating set in $\mathcal{H}$. Again by Theorem 2.20, I is a maximal irredundant set in $\mathcal{H}$. Consequently, I is a maximal isolate irredundant set. □

Corollary 4.8. *For any hypergraph $\mathcal{H}$, $ir_0(\mathcal{H}) \leq \gamma_0(\mathcal{H}) \leq \Gamma_0(\mathcal{H}) \leq IR_0(\mathcal{H})$.*

Proof. Immediate from the above Theorem 4.7. □

Theorem 4.9. *Let $\mathcal{H}(V, E)$ be a hypergraph and let I be an isolate set of $\mathcal{H}$. Then I is a maximal isolate set if and only if every vertex in $V \setminus I$ is adjacent to all the vertices $v \in I$ with $N(v) \cap I = \phi$.*

Proof. Let I be a maximal isolate set of $\mathcal{H}$. Suppose there exists a vertex $u \in V \setminus I$ such that u is not adjacent to all $v \in I$ with $N(v) \cap I = \phi$. Then $I \cup \{u\}$ will be an isolate set of $\mathcal{H}$, contradicting the assumption that I is a maximal isolate set. Hence the result. The converse is obvious. □

From the above theorem, it is clear that every maximal isolate set is an isolate dominating set. However, a maximal isolate set need not be a minimal isolate dominating set. For example, the set I_4 in the Example 4.6 is a maximal isolate set but not a minimal isolate dominating set of $\mathcal{H}$. Hence $\gamma_0(\mathcal{H}) \leq i_0(\mathcal{H})$.
Further, $I_0(\mathcal{H})$ is the maximum cardinality of an isolate set in $\mathcal{H}$ and hence $IR_0(\mathcal{H}) \leq I_0(\mathcal{H})$.
Also, since $ir(\mathcal{H}) \leq ir_0(\mathcal{H})$, we have the following inequality chains,

$$ir(\mathcal{H}) \leq ir_0(\mathcal{H}) \leq \gamma_0(\mathcal{H}) \leq i_0(\mathcal{H}) \leq I_0(\mathcal{H}). \tag{4.1}$$

$$\Gamma_0(\mathcal{H}) \leq IR_0(\mathcal{H}) \leq I_0(\mathcal{H}). \tag{4.2}$$

Theorem 4.10. *For any hypergraph $\mathcal{H}$, $i(\mathcal{H}) \leq i_0(\mathcal{H})$.*

Proof. Let I be an i_0-set of $\mathcal{H}$ and let S be the set of vertices $v \in I$ with $N(v) \cap I = \phi$. Then the set $I_1 \cup S$ is an independent set of $\mathcal{H}$, where I_1 is an independent dominating set of $I \setminus S$. Further, I_1 dominates all the vertices of $I \setminus S$ and the single vertex of S dominates all vertices of $V \setminus I$, by Theorem 4.9. Hence the set $I_1 \cup S$ is an independent dominating set of $\mathcal{H}$. □

Remark 4.11. The bound given in Theorem 4.10 is sharp and attained for complete k-partite hypergraphs.

Further, since $\gamma_0(\mathcal{H}) \leq i(\mathcal{H})$ and $i(\mathcal{H}) \leq i_0(\mathcal{H})$, we have the extended chain as follows:

$$ir(\mathcal{H}) \leq ir_0(\mathcal{H}) \leq \gamma_0(\mathcal{H}) \leq i(\mathcal{H}) \leq i_0(\mathcal{H}) \leq I_0(\mathcal{H}). \tag{4.3}$$

It can be noticed from Equation 3.1 and 4.3 that each of the parameters γ and ir_0 lies between ir and γ_0. Note, however, there is no relation between ir_0 and γ.

The following examples illustrate that there is no relation between domination number and isolate irredundance number of a hypergraph $\mathcal{H}$.

Example 4.12. Consider the hypergraph $\mathcal{H}(V, E)$, where $V = \{v_1, v_2, \ldots, v_{12}\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$. In which the edges of $\mathcal{H}$ are defined as follows:

$$e_1 = \{v_1, v_2, v_3, v_4\}, \qquad e_2 = \{v_3, v_4, v_5, v_6\},$$
$$e_3 = \{v_3, v_4, v_7, v_8, v_9\}, \qquad e_4 = \{v_8, v_9, v_{10}, v_{11}\},$$
$$e_5 = \{v_8, v_9, v_{12}\}.$$

Here $\gamma(\mathcal{H}) = 2 < ir_0(\mathcal{H}) = 3$.

Example 4.13. Consider the hypergraph $\mathcal{H}(V, E)$, where $V = \{v_1, v_2, \ldots, v_{11}\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$. In which the edges of $\mathcal{H}$ are defined as follows:

$$e_1 = \{v_1, v_2, v_3, v_4\}, \qquad e_2 = \{v_3, v_4, v_5, v_6, v_7, v_8\},$$
$$e_3 = \{v_7, v_8, v_9\}, \qquad e_4 = \{v_9, v_{10}\},$$
$$e_5 = \{v_{10}, v_{11}\}.$$

Here $ir_0(\mathcal{H}) = 2 < \gamma(\mathcal{H}) = 3$.

Now we focus our attention on the relationships among the maximum parameters,

Theorem 4.14. *For any hypergraph $\mathcal{H}$, $\Gamma(\mathcal{H}) = \Gamma_0(\mathcal{H})$ and $IR_0(\mathcal{H}) = IR(\mathcal{H})$.*

Proof. Since every minimal isolate dominating set is a minimal dominating set, it follows that $\Gamma_0(\mathcal{H}) \leq \Gamma(\mathcal{H})$, for any hypergraph $\mathcal{H}$. Now we go for another part of the equality. Let D be a Γ-set of $\mathcal{H}$. Then D need not be an isolate dominating set of $\mathcal{H}$. However, the set $I^* = (D \backslash \{v\}) \cup I_1$, where I_1 is a minimal isolate dominating set of $pn[v, D]$ is a minimal isolate dominating set of $\mathcal{H}$ with cardinality more than or equal to D. And the minimality of I^* follows from the minimality of I_1 and minimality of D. Hence $\Gamma(\mathcal{H}) = \Gamma_0(\mathcal{H})$. Now let I be an IR-set of $\mathcal{H}$. If for any $v \in I$, we have $N(v) \cap I = \phi$, then I itself is a maximal isolate irredundant set of $\mathcal{H}$ and the result follows. Otherwise every vertex in I has a private neighbor in $v \setminus I$. Therefore the set $(I \setminus \{v\}) \cup I'$, where I' is an IR_0-set of $pn[v, I]$ forms an isolate irredundant set of $\mathcal{H}$. Further, for every vertex $v \in I$, $IR_0(pn[v, I]) \geq 1$, which implies that $|I| \leq |(I \setminus \{v\}) \cup I'|$. Hence $IR(\mathcal{H}) \leq IR_0(\mathcal{H})$. Also, since $IR(\mathcal{H})$ is the maximum cardinality of an irredundant set of $\mathcal{H}$, it follows that $IR_0(\mathcal{H}) \leq IR(\mathcal{H})$. Hence $IR_0(\mathcal{H}) = IR(\mathcal{H})$. □

Theorem 4.15. *For any hypergraph $\mathcal{H}$, $I_0(\mathcal{H}) = n - \delta(\mathcal{H})$.*

Proof. Let $\mathcal{H}$ be a hypergraph. Since for any vertex v of $\mathcal{H}$, $V(\mathcal{H}) \setminus N(v)$ is an isolate set of $\mathcal{H}$, it follows that $I_0(\mathcal{H}) \geq max_{v \in V(\mathcal{H})} |V(\mathcal{H}) \setminus N(v)| = n - \delta(\mathcal{H})$. Further, let I be an isolate set of $\mathcal{H}$ and $u \in I$ with $N(u) \cap I = \phi$. Then all neighbors of u must lie outside the set I. Consequently, $I_0(\mathcal{H}) \leq n - \delta(\mathcal{H})$. Hence for any hypergraph $\mathcal{H}$, we have $I_0(\mathcal{H}) = n - \delta(\mathcal{H})$. □

Now all these results and inequities can be summarized as,

Theorem 4.16. *For any hypergraph $\mathcal{H}$, $ir(\mathcal{H}) \leq ir_0(\mathcal{H}) \leq \gamma_0(\mathcal{H}) \leq i(\mathcal{H}) \leq \beta_0(\mathcal{H}) \leq \Gamma_0(\mathcal{H}) = \Gamma(\mathcal{H}) \leq IR_0(\mathcal{H}) = IR(\mathcal{H}) \leq I_0(\mathcal{H})$.*

It should be noted that we have excluded two parameters namely γ and i_0 from the chain as there is no relation among ir_0, γ and i_0. However we also have, $ir(\mathcal{H}) \leq \gamma(\mathcal{H}) \leq \gamma_0(\mathcal{H}) \leq i(\mathcal{H}) \leq i_0(\mathcal{H}) \leq I_0(\mathcal{H})$.

5 More on Isolate Parameters

In this section, we study more about isolate number and isolate irredundance number of a hypergraph $\mathcal{H}$. We also determine the values of these parameters for some hypergraphs such as disconnected hypergraphs, complete n-partite hypergraphs etc.

Theorem 5.1. *Let $\mathcal{H}$ be a disconnected hypergraph with $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3, \ldots, \mathcal{H}_k$ as its components. Then*

(1) $i_0(\mathcal{H}) = i_0(\mathcal{H}_1) + |V(\mathcal{H})| - |V(\mathcal{H}_1)|$, *where* $i_0(\mathcal{H}_1) = min\ \{i_0(\mathcal{H}_i), i = 1, 2, \dots, k\}$.

(2) $ir_0(\mathcal{H}) = min_{1\leq i\leq k}\{s_i\}$, *where* $s_i = ir_0(\mathcal{H}_i) + \sum_{j=1,j\neq i}^{k} ir(\mathcal{H}_j)$.

Proof. (1) Let I_1 be an i_0-set of $\mathcal{H}_1$. Then the set I_1 together with all the vertices of each of the remaining components $\mathcal{H}_2, \mathcal{H}_3, \dots, \mathcal{H}_k$ is a maximal isolate set of $\mathcal{H}$. Hence $i_0(\mathcal{H}) \leq i_0(\mathcal{H}_1) + |V(\mathcal{H})| - |V(\mathcal{H}_1)|$. Now let I be any maximal isolate set of $\mathcal{H}$. Then $I \cap V(\mathcal{H}_j)$ is an i_0-set of $\mathcal{H}_j$, for some j and $I \cap V(\mathcal{H}_i) = V(\mathcal{H}_i)$ for all $i \neq j$. Hence $i_0(\mathcal{H}_1) + |V(\mathcal{H})| - |V(\mathcal{H}_1)| \leq i_0(\mathcal{H})$.

(2) Every ir_0-set of $\mathcal{H}_i$ together with the set $\cup_{j=1,j\neq i}^{k} D_j$ forms a maximal isolate irredundant set of $\mathcal{H}$, where D_j is a ir-set of $\mathcal{H}_j$ and $1 \leq i \leq k$. Thus, $ir_0(\mathcal{H}) \leq \min_{1\leq i\leq k}\{s_i\}$.
Now, let I be any maximal isolate irredundant set of $\mathcal{H}$. Then $I \cap V(\mathcal{H}_i)$ is a maximal irredundant set of $\mathcal{H}$, for every $i = 1, 2, \dots, k$. Further, for at least one i, say j we have $I \cap V(\mathcal{H}_j)$ is a maximal isolate irredundant set. Therefore $|I| \geq ir_0(\mathcal{H}_j) + \sum_{i=1,i\neq j}^{k} ir(\mathcal{H}_i) = s_j \geq \min_{1\leq i\leq k}\{s_i\}$. Hence $ir_0(\mathcal{H})$=$\min_{1\leq i\leq k}\{s_i\}$.

□

Corollary 5.2. *Let $\mathcal{H}$ be a hypergraph. Then $i_0(\mathcal{H}) = n$ if and only if $\mathcal{H}$ has an isolated vertex.*

Proof. Follows from first part of Theorem 5.1. □

Proposition 5.3. *If $\mathcal{H}$ is a hypergraph with an isolated vertex, then $V(\mathcal{H})$ is the only maximal isolate set of $\mathcal{H}$.*

Theorem 5.4. *For a complete r-partite hypergraph $\mathcal{H} = K^r_{n_1,n_2,\dots,n_r}$, $i_0(\mathcal{H})$= min $\{n_1, n_2, \dots, n_r\}$.*

Proof. Let $\mathcal{H}$ be a hypergraph. Let I be any maximal isolate set of $\mathcal{H}$. Then I will not intersect more than one part of $\mathcal{H}$. Also, being a maximal isolate set, I cannot be a proper subset of any part of $\mathcal{H}$. Hence the parts of $\mathcal{H}$ are the only maximal isolate sets of $\mathcal{H}$, which completes the proof. □

Now we see some bounds for the isolate number i_0 of $\mathcal{H}$. It is obvious that for any hypergraph $\mathcal{H}$ of order n, we have $1 \leq i_0(\mathcal{H}) \leq n$. Corollary 5.2 tells us that $i_0(\mathcal{H}) = n$ only when the hypergraph $\mathcal{H}$ has an isolated vertex. And, if $i_0(\mathcal{H}) = 1$, then $\gamma_0(\mathcal{H}) = 1$ as $\gamma_0(\mathcal{H}) \leq i_0(\mathcal{H})$ and consequently $\Delta(\mathcal{H}) = n - 1$. Further, if $\mathcal{H}$ is a hypergraph with $\Delta(\mathcal{H}) = n - 1$, then we have $i_0(\mathcal{H}) = 1$. Hence for any hypergraph $\mathcal{H}$, $i_0(\mathcal{H}) = 1$ if and only if $\Delta(\mathcal{H}) = n - 1$.

Now following is the theorem that characterizes hypergraphs for which we have, $i_0(\mathcal{H}) = 2$.

Theorem 5.5. *Let $\mathcal{H}(V, E)$ be a hypergraph with $i_0(\mathcal{H}) = 2$. Then $i_0(\mathcal{H}) = \gamma_0(\mathcal{H}) = \gamma(\mathcal{H}) = i(\mathcal{H}) = ir_0(\mathcal{H}) = ir(\mathcal{H})$.*

Proof. Let $\mathcal{H}$ be a hypergraph with $i_0(\mathcal{H}) = 2$ and let I be an i_0-set of $\mathcal{H}$. Then by Theorem 4.9, every vertex of $V \setminus I$ is adjacent to both vertices of I. Hence I is a dominating set of $\mathcal{H}$ and $\gamma(\mathcal{H}) \leq 2$. Also $\Delta(\mathcal{H}) < V(\mathcal{H}) - 1$, otherwise $i_0(\mathcal{H})$ would be less than 2, which is absurd. Hence $\gamma(\mathcal{H}) = 2$. Now, as I is an independent set, it follows that $\gamma_0(\mathcal{H}) = 2$. Further, since $\Delta(\mathcal{H}) \leq V(\mathcal{H}) - 2$, and every vertex of $V \setminus I$ is adjacent to two vertices of I, we get $ir(\mathcal{H}) = 2$. Hence $ir(\mathcal{H}) = ir_0(\mathcal{H}) = \gamma(\mathcal{H}) = \gamma_0(\mathcal{H}) = i(\mathcal{H}) = i_0(\mathcal{H})$. □

6 Equality of Parameters

In this section, we study the equality among domination number, isolate domination number, isolate number and total domination number of a hypergraph $\mathcal{H}$ and examine their properties. We have determined the properties of hypergraphs for which some of the above said parameters equals or have some relation between them. We also study the conditions when $\gamma(\mathcal{H}) \equiv \gamma_t(\mathcal{H})$ in a hypergraph $\mathcal{H}$.

For complete n-partite hypergraphs, we have $\gamma(\mathcal{H}) = \gamma_0(\mathcal{H})$. In the following theorem, we determine one more class of hypergraphs for which we have $\gamma(\mathcal{H}) = \gamma_0(\mathcal{H})$.

Theorem 6.1. *Let $\mathcal{H}$ be a hypergraph with either $\delta(\mathcal{H}) = 0$ or $\Delta(\mathcal{H}) = n - 1$. Then $\gamma(\mathcal{H}) = \gamma_0(\mathcal{H})$.*

Proof. Let $\mathcal{H}$ be a hypergraph and let v be a vertex in $\mathcal{H}$ such that $\delta(v) = 0$. Then v must belong to every γ-set of $\mathcal{H}$. Thus, $\gamma(\mathcal{H}) = \gamma_0(\mathcal{H})$. Further when $\Delta(\mathcal{H}) = n - 1$, $\gamma(\mathcal{H}) = 1$. □

In the next few theorems, we derived some properties of the hypergraphs with $\gamma(\mathcal{H}) = \gamma_0(\mathcal{H})$.

Proposition 6.2. *For a hypergraph $\mathcal{H}$, $\gamma_0(\mathcal{H}) = \gamma(\mathcal{H})$ if and only if there is a γ-set D with $v \in D$ such that $N(v) \cap D = \phi$.*

Proof. Let $\mathcal{H}$ be a hypergraph with $\gamma_0(\mathcal{H}) = \gamma(\mathcal{H})$. Then clearly there is a γ-set D with $v \in D$ such that $N(v) \cap D = \phi$. Conversely, suppose that there is a γ-set D with $v \in D$ such that $N(v) \cap D = \phi$. Then D is an isolate dominating set as well. Hence $\gamma_0(\mathcal{H}) = \gamma(\mathcal{H})$. □

Corollary 6.3. *Let $\mathcal{H}(V, E)$ be a hypergraph. Then $\gamma_0(\mathcal{H}) \neq \gamma(\mathcal{H})$ if and only if every γ-set is a total dominating set.*

Proof. Let $\mathcal{H}$ be a hypergraph with $\gamma_0(\mathcal{H}) \neq \gamma(\mathcal{H})$. Then by Proposition 6.2, there is no γ-set D with $v \in D$ such that $N(v) \cap D = \phi$. Therefore, every minimum dominating set of $\mathcal{H}$ is a total dominating set of $\mathcal{H}$. Conversely, suppose that every minimum dominating set is a total dominating set. Then no γ-set D with $v \in D$ such that $N(v) \cap D = \phi$. Hence by Proposition 6.2, $\gamma_0(\mathcal{H}) \neq \gamma(\mathcal{H})$. □

Theorem 6.4. *For any hypergraph $\mathcal{H}$, $\gamma_0(\mathcal{H}) = \gamma(\mathcal{H})$ if and only if $\gamma(\mathcal{H}) \not\equiv \gamma_t(\mathcal{H})$.*

Proof. Let $\mathcal{H}$ be a hypergraph with $\gamma_0(\mathcal{H}) = \gamma(\mathcal{H})$. Then by Corollary 6.3, every minimum dominating set is not a total dominating set. Hence $\gamma(\mathcal{H}) \not\equiv \gamma_t(\mathcal{H})$. Conversely, suppose that $\gamma(\mathcal{H}) \not\equiv \gamma_t(\mathcal{H})$. Then every minimum dominating set is not a total dominating set. Hence by Corollary 6.3, $\gamma(\mathcal{H}) = \gamma_0(\mathcal{H})$. □

Example 6.5. Consider the hypergraph $\mathcal{H}(V, E)$, where $V = \{v_1, v_2, \ldots, v_6\}$ and $E = \{e_1, e_2, e_3\}$. In which the edges of $\mathcal{H}$ are defined as follows:

$$e_1 = \{v_1, v_2, v_3, v_4\}, \qquad e_2 = \{v_2, v_5\},$$
$$e_3 = \{v_4, v_6\}.$$

Here the sets $D_1 = \{v_2, v_4\}$, $D_2 = \{v_2, v_6\}$ and $D_3 = \{v_4, v_5\}$ are the minimum dominating sets of $\mathcal{H}$. Hence $\gamma(\mathcal{H}) = 2$. And among these D_1 is a γ-set and D_2, D_3 are the γ_0-sets of $\mathcal{H}$. Clearly, every minimum dominating set is not total dominating. Hence $\gamma(\mathcal{H}) \not\equiv \gamma_t(\mathcal{H})$.

Corollary 6.6. *Let $\mathcal{H}$ be a hypergraph with $\gamma(\mathcal{H}) \neq \gamma_t(\mathcal{H})$. Then $\gamma_0(\mathcal{H}) = \gamma(\mathcal{H})$.*

Proof. Let $\mathcal{H}$ be a hypergraph with $\gamma(\mathcal{H}) \neq \gamma_t(\mathcal{H})$. Let D be a minimum dominating set of $\mathcal{H}$. Then $|D| < \gamma_t(\mathcal{H})$. Further, there must exist a vertex $v \in D$ such that $N(v) \cap D = \phi$, otherwise D would become a total dominating set $\mathcal{H}$ with $|D| < \gamma_t(\mathcal{H})$. Thus, the set D is an isolate dominating set as well. Therefore $\gamma_0(\mathcal{H}) = \gamma(\mathcal{H})$. □

The following Lemma provides a necessary condition for $\gamma(\mathcal{H}) \equiv \gamma_t(\mathcal{H})$ in a hypergraph $\mathcal{H}$:

Lemma 6.7. *If $\mathcal{H}$ is a hypergraph with $\gamma(\mathcal{H}) \equiv \gamma_t(\mathcal{H})$, then every vertex in γ-set D has at least two private neighbors in $V \setminus D$.*

Proof. Let $\mathcal{H}$ be a hypergraph with $\gamma(\mathcal{H}) \equiv \gamma_t(\mathcal{H})$. Let D be a γ-set of $\mathcal{H}$. Since $\gamma(\mathcal{H}) \equiv \gamma_t(\mathcal{H})$, it follows that D is also a γ_t-set of $\mathcal{H}$ which implies for every $v \in D$, we have $N(v) \cap D \neq \phi$. Hence each vertex in D has at least one private neighbor in $V \setminus D$ with respect to D. Suppose $v \in D$ and $pn[v, D] = \{u\}$. Then $(D \setminus \{v\}) \cup \{u\}$ is a γ-set of $\mathcal{H}$ which is not a γ_t-set, contradicting the fact that $\gamma(\mathcal{H}) \equiv \gamma_t(\mathcal{H})$. Hence every vertex in D has at least two private neighbors in $V \setminus D$. □

Now we study the properties of hypergraphs with $\gamma_0(\mathcal{H}) = i_0(\mathcal{H})$.

Theorem 6.8. *Let $\mathcal{H}$ be a hypergraph with $\gamma_0(\mathcal{H}) = i_0(\mathcal{H})$. Then every γ_0-set of $\mathcal{H}$ is independent.*

Proof. Let $\mathcal{H}$ be a hypergraph with $\gamma_0(\mathcal{H}) = i_0(\mathcal{H})$. Let I' be an i_0-set of $\mathcal{H}$ and $v \in I'$ with $N(v) \cap I' = \phi$. Then $|I'| = i_0(\mathcal{H}) = \gamma_0(\mathcal{H})$. Since I' is a maximal isolate set, it follows from Theorem 4.9, that every vertex of $V \setminus I'$ is adjacent to v. Suppose there exist two vertices u and w in I' such that u and w are adjacent. Then $u, w \neq v$ and $I' \setminus \{u\}$ is an isolate dominating set of $\mathcal{H}$ with $\gamma_0(\mathcal{H}) \leq |I'| - 1 = \gamma_0(\mathcal{H}) - 1$, a contradiction. Hence the set I' is independent. In fact, I' is a maximal isolate set of $\mathcal{H}$ in which for every vertex $v \in I'$, we have $N(v) \cap I' = \phi$. Hence, again by Theorem 4.9, we have every vertex of $V \setminus I'$ is adjacent to all vertices of I'. Therefore every isolate set I of $\mathcal{H}$ is either a subset of I' or subset of $V \setminus I'$. In particular, if I is a γ_0-set of $\mathcal{H}$, then either $I \subseteq I'$ or $I \subseteq V \setminus I'$. If $I \subseteq I'$, then the result follows. Suppose that $I \subseteq V \setminus I'$. Now, if there exist two vertices p and q in I such that p and q are adjacent, then $I \setminus \{p\}$ is an isolate dominating set of cardinality less than $\gamma_0(\mathcal{H})$, a contradiction. Hence the set I is independent. □

Corollary 6.9. *If $\mathcal{H}$ is a hypergraph with $\gamma_0(\mathcal{H}) = i_0(\mathcal{H})$, then $\gamma_0(\mathcal{H}) = i(\mathcal{H})$.*

Proof. Let $\mathcal{H}$ be a given hypergraph. Since an i-set is a minimal isolate dominating set of $\mathcal{H}$, $\gamma_0(\mathcal{H}) \leq i(\mathcal{H})$. Further, by Theorem 6.8, it follows that every γ_0-set of $\mathcal{H}$ is independent. Hence $i(\mathcal{H}) \leq \gamma_0(\mathcal{H})$. □

Remark 6.10. The converse is not true. For this, consider the following example,

Example 6.11. Consider the hypergraph $\mathcal{H}(V, E)$, where $V = \{v_1, v_2, \ldots, v_9\}$ and $E = \{e_1, e_2, e_3, e_4\}$. In which the edges of $\mathcal{H}$ are defined as follows:

$$e_1 = \{v_1, v_2, v_3, v_4\}, \qquad e_2 = \{v_3, v_4, v_5, v_6\},$$
$$e_3 = \{v_6, v_7, v_8\}, \qquad e_4 = \{v_8, v_9\}.$$

Clearly, $\{v_4, v_8\}$ is a γ_0-set of $\mathcal{H}$ which is independent. Hence $\gamma_0(\mathcal{H}) = i(\mathcal{H})$ whereas $i_0(\mathcal{H}) = 4$.

Theorem 6.12. *If $\mathcal{H}(V, E)$ is a hypergraph with $\gamma_0(\mathcal{H}) = i_0(\mathcal{H})$, then $i_0(\mathcal{H}) = n - \Delta(\mathcal{H})$.*

Proof. Let $\mathcal{H}$ be a hypergraph with $\gamma_0(\mathcal{H}) = i_0(\mathcal{H})$. Since $\gamma_0(\mathcal{H}) \leq n - \Delta(\mathcal{H})$, it follows that $i_0(\mathcal{H}) \leq n - \Delta(\mathcal{H})$. Further, it can be observed from Theorem 4.9 that $i_0(\mathcal{H}) \geq n - \Delta(\mathcal{H})$. Hence $i_0(\mathcal{H}) = n - \Delta(\mathcal{H})$. □

Remark 6.13. The converse part of the above theorem is not true. For this, consider the hypergraph $\mathcal{H}(V, E)$ of order 9, where $V = \{v_1, v_2, \ldots, v_9\}$ and $E = \{e_1, e_2, e_3, e_4\}$. In which the edges of $\mathcal{H}$ are defined as follows:

$$e_1 = \{v_1, v_2, v_3, v_4\}, \qquad e_2 = \{v_4, v_5, v_6\},$$
$$e_3 = \{v_6, v_7, v_8\}, \qquad e_4 = \{v_5, v_6, v_9\}.$$

The isolate number of the given hypergraph $\mathcal{H}$ is 4 while the isolate domination number of $\mathcal{H}$ is 2 and $\Delta(\mathcal{H}) = 5$.

Theorem 6.14. *For a complete k-partite hypergraph $\mathcal{H}$, $\gamma_0(\mathcal{H}) = i(\mathcal{H}) = i_0(\mathcal{H}) = ir_0(\mathcal{H})$.*

Proof. Let I be a γ_0-set of $\mathcal{H}$. Since k-parts of $\mathcal{H}$ are the only minimal isolate dominating set of $\mathcal{H}$, it follows that I must be among k-parts of $\mathcal{H}$ with minimum cardinality. Also each vertex of I is the only private neighbor of itself and every vertex in $V \setminus I$ is adjacent to all the vertices of I. Therefore, by Theorem 4.9, I is a maximal isolate set of $\mathcal{H}$. Thus $i_0(\mathcal{H}) \leq \gamma_0(\mathcal{H})$. Hence by chains 4.1 and 4.3, we have $i_0(\mathcal{H}) = \gamma_0(\mathcal{H}) = i(\mathcal{H})$. Clearly, $ir_0(\mathcal{H}) = \gamma_0(\mathcal{H})$. □

Theorem 6.15. *For any hypergraph $\mathcal{H}$, $\gamma_t(\mathcal{H}) \leq i_0(\mathcal{H}) + 1$.*

Proof. Let $\mathcal{H}$ be a hypergraph and let I be an i_0-set of $\mathcal{H}$. Then by Theorem 4.9, every vertex in $V \setminus I$ is adjacent to all vertices $v \in I$ with $N(v) \cap I = \phi$. This implies that I is a dominating set $\mathcal{H}$. Now choose any vertex $v \in V \setminus I$. Then the set $I \cup \{v\}$ is not an isolate set. Consequently, $I \cup \{v\}$ is a total dominating set of $\mathcal{H}$. Hence $\gamma_t(\mathcal{H}) \leq |I \cup \{v\}| \leq i_0(\mathcal{H}) + 1$. □

References

[1] B.D. Acharya, Domination in Hypergraphs, AKCE Int. J. Graphs Combin., 4(2) (2007), 117-126.

[2] B.D. Acharya, Domination in hypergraphs: II-New Directions, Proc. Int. Conf. – ICDM, (2008), 1-16.

[3] B.D. Acharya, Weak Edge-Degree Domination in Hypergraphs, Czechoslovak Mathematical Journal, Vol. 56 (2006), No. 1, 99–108.

[4] C. Berge, *Hypergraphs, Combinatorics of Finite Sets*, North-Holland, Amsterdam, (1989).

[5] Bibin K. Jose, Zs. Tuza, Hypergraph Domination and Strong Independence, Appl. Anal. Discrete Math., **3** (2009), 347—358.

[6] Haynes T.W., Hedetniemi S.T. and Slater P.J. *Fundamentals of Domination in Graphs*, New York: Dekker (1998).

[7] Haynes T.W., Hedetniemi S.T. and Slater P.J. *Domination in Graphs-Advanced Topics*, New York : Dekker, (1998)

[8] I. Sahul Hamid and S. Balamurugan, Extended Chain of Domination Parameters in Graphs, ISRN Combinatorics, Volume 2013, Article ID 792743, 4 pages.

[9] I. Sahul Hamid and S. Balamurugan, Isolate Domination in Graphs, Arab Journal of Mathematical Sciences, 22(2016), 232-241.

[10] Jose B. K., *Domination in Hypergraphs*, Ph.D. Dissertation, Mahatma Gandhi University, Kottayam, 2011.

[11] Lay Lay Nwe and Min Min Maung, Equality of Domination Number and Isolate Domination Number in Some Graphs, University of Mandalay, Research Journal, Vol.11, (2020), 292-298.

[12] Vitaly I. Voloshin *Introduction to Graph and Hypergraph Theory*, Nova Science Publishers, Inc. New York , 2009.

[13] Megha M. Jadhav and Kishor F. Pawar, On Edge Product Hypergraphs, Journal of Hyperstructures, 10(1) (2021), 1-12.

[14] Megha M. Jadhav and Kishor F. Pawar, On Isolate Domination in Hypergraphs, Malaya Journal of Matematik, 10(01)(2022), 55-62.

[15] Michael A. Henning, Stephen T. Hedetniemi, Teresa W. Haynes, *Structures of Domination in Graphs*, Springer International Publishing, 2021.

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