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WEAK FILTERS AND MULTIPLIERS IN SHEFFER STROKE HILBERT ALGEBRAS

Y. Bae Jun and T. Oner

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Abstract With the aim of discussing the weak filters and multipliers of the Sheffer stroke Hilbert algebra, the concept of weak filters that weakened the filter conditions in the Sheffer stroke Hilbert algebra is first introduced and their properties are investigated. A method of making a weak filter using the notion of ideals is presented, and the shape of the weak filter is investigated in the Cartesian product of the Sheffer stroke Hilbert algebra. Second, the concept of multipliers in Sheffer stroke Hilbert algebras is introduced, and the various properties involved are examined. The image and pre-image of weak filters by multipliers are discussed. The kernel and a fixed set of multipliers are found to be weak filters. The composition of multipliers is studied, and the conditions under which the two multipliers are equal are explored. The conditions under which the two multipliers are equal are explored. By assigning a Sheffer stroke to the set of multipliers, a new Sheffer stroke Hilbert algebra is derived.

1 Introduction

The concept of Sheffer operation (the so-called Sheffer stroke in [1]) was first introduced by Sheffer [2] in 1913, and it was used in Boolean algebras as a very successful tool because Boolean operations can be replaced by Sheffer stroke. Sheffer stroke has been applied to several algebraic structures, for example, Boolean algebra, basic algebras, MV-algebra, BL-algebra, BCK-algebra, BE-algebra, ortholattices, and Hilbert algebra, etc. (see [3, 4, 5, 6, 7, 8, 9]). Oner et al. [10] discussed fuzzy filters of Sheffer stroke Hilbert algebras. Moreover, there are many applications of Sheffer stroke reduction different algebraic structures, for instance, Łukasiewicz fuzzy filters of Sheffer stroke Hilbert algebras [11], state operators in Sheffer stroke basic algebras [12], Riečan and Bosbach states notions and very true operator concept on Sheffer stroke MTL-algebras [13, 14] and also congruences of Sheffer stroke basic algebras [15]. In accordance with these, the concept of reduction Sheffer stroke can be used different algebraic structures such as [16, 17, 18, 19].

The aim of this paper is to discuss weak filters and multipliers in Sheffer stroke Hilbert algebras. We first introduce the concept of weak filters that have weakened the filter conditions in the Sheffer stroke Hilbert algebra and investigate several properties. We present how to make weak filters using ideals. We examine the shape of the weak filter in the Cartesian product of Sheffer stroke Hilbert algebras. Second, we introduce the concept of multipliers in Sheffer stroke Hilbert algebras and examine the various properties involved. We investigate the image and pre-image of weak filters by multipliers. We show that the kernel and fixed set of multiplier become weak filters. We study the composition of multipliers. We explore the conditions under which two multipliers are equal. We assign a Sheffer Stroke to a set of multipliers to derive a new Sheffer Stroke Hilbert algebra.

2 Preliminaries

Definition 2.1 ([2]). Let $\mathcal{A} := (A, |)$ be a groupoid. Then the operation "$|$" is said to be *Sheffer stroke* or *Sheffer operation* if it satisfies:

(s1) $(\forall a, b \in A)\ (a|b = b|a)$,

(s2) $(\forall a, b \in A)\ ((a|a)|(a|b) = a)$,

(s3) $(\forall a, b, c \in A)\ (a|((b|c)|(b|c)) = ((a|b)|(a|b))|c)$,

(s4) $(\forall a, b, c \in A)\ ((a|((a|a)|(b|b)))|(a|((a|a)|(b|b))) = a)$.

Definition 2.2 ([20]). A *Sheffer stroke Hilbert algebra* is a groupoid $\mathcal{H} := (H, |)$ with a Sheffer stroke "$|$" that satisfies:

(sH1) $(x|((A)|(A)))|(((B)|((C)|(C)))|((B)|((C)|(C)))) = x|(x|x)$,
where $A := y|(z|z)$, $B := x|(y|y)$ and $C := x|(z|z)$,

(sH2) $x|(y|y) = y|(x|x) = x|(x|x) \ \Rightarrow\ x = y$

for all $x, y, z \in H$.

Let $\mathcal{H} := (H, |)$ be a Sheffer stroke Hilbert algebra. Then the order relation "$\preceq$" on H is defined as follows:

$$(\forall a, b \in H)(a \preceq b \ \Leftrightarrow\ a|(b|b) = 1). \tag{2.1}$$

We observe that the relation "$\preceq$" is a partial order in a Sheffer stroke Hilbert algebra $\mathcal{H} := (H, |)$ (see [20]). Recall that the Sheffer stroke Hilbert algebra $\mathcal{H} := (H, |)$ satisfies the identity $a|(a|a) = b|(b|b)$, which is denoted by 1, for all $a, b \in H$ (see [20]).

Proposition 2.3 ([20]). *Every Sheffer stroke Hilbert algebra $\mathcal{H} := (H, |)$ satisfies:*

$$(\forall a \in H)(a|(a|a) = 1), \tag{2.2}$$
$$(\forall a \in H)(a|(1|1) = 1), \tag{2.3}$$
$$(\forall a \in H)(1|(a|a) = a), \tag{2.4}$$
$$(\forall a, b \in H)(a \preceq b|(a|a)), \tag{2.5}$$
$$(\forall a, b \in H)((a|(b|b))|(b|b) = (b|(a|a))|(a|a)), \tag{2.6}$$
$$(\forall a, b \in H)\ (((a|(b|b))|(b|b))|(b|b) = a|(b|b))\,, \tag{2.7}$$
$$(\forall a, b, c \in H)\ (a|((b|(c|c))|(b|(c|c))) = b|((a|(c|c))|(a|(c|c))))\,, \tag{2.8}$$
$$(\forall a, b, c \in H)(a \preceq b \ \Rightarrow\ c|(a|a) \preceq c|(b|b),\ b|(c|c) \preceq a|(c|c)), \tag{2.9}$$
$$(\forall a, b, c \in H)(a|((b|(c|c))|(b|(c|c))) = (a|(b|b))|((a|(c|c))|(a|(c|c)))). \tag{2.10}$$

By (2.3), we know that the element 1 is the greatest element in $\mathcal{H} := (H, |)$ with respect to the order $\preceq$.

Proposition 2.4. *Let $(H, |)$ be a Sheffer stroke Hilbert algebra with the smallest element* 0. *Then*

$$0|0 = 1,\ 1|1 = 0, \tag{2.11}$$
$$1|(0|0) = 0,\ 0|(0|0) = 1. \tag{2.12}$$

Definition 2.5 ([10]). Let $(H, |)$ be a Sheffer stroke Hilbert algebra. A subset F of H is called a *filter* of $(H, |)$ if it satisfies:

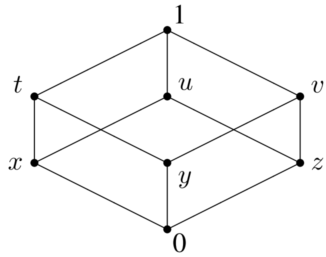
$$1 \in F, \tag{2.13}$$
$$(\forall a, b \in H)(b \in F \ \Rightarrow\ a|(b|b) \in F), \tag{2.14}$$
$$(\forall a, b, c \in H)(b, c \in F \ \Rightarrow\ (a|(b|c))|(b|c) \in F). \tag{2.15}$$

3 Weak filters

This section deals with the so-called weak filter, which has weakened the conditions of the filter. In what follows, $\mathcal{H} := (H, |)$ stands for a Sheffer stroke Hilbert algebra, unless otherwise stated.

Definition 3.1. A subset F of H is called a *weak filter* of $\mathcal{H} := (H, |)$ if it satisfies (2.13) and (2.14).

Example 3.2. Let $H = \{0, 1, x, y, z, t, u, v\}$ be a set with the following Hasse diagram:



Define a Sheffer stroke "|" on H by Table 1.

Table 1. Cayley table for the Sheffer stroke "|"

$\mid$	0	x	y	z	t	u	v	1
0	1	1	1	1	1	1	1	1
x	1	v	1	1	v	v	1	v
y	1	1	u	1	u	1	u	u
z	1	1	1	t	1	t	t	t
t	1	v	u	1	z	v	u	z
u	1	v	1	t	v	y	t	y
v	1	1	u	t	u	t	x	x
1	1	v	u	t	z	y	x	0

Then $\mathcal{H} := (H, |)$ is a Sheffer stroke Hilbert algebra (see [20]). It is routine to verify that $F_1 := \{1, t, u\}$, $F_2 := \{1, u, v\}$ and $F_3 := \{1, t, u, v\}$ are weak filters of $\mathcal{H} := (H, |)$.

It is obvious that every filter is a weak filter, but the converse may not be true. In fact, the weak filter $F_2 := \{1, u, v\}$ in Example 3.2 is not a filter of $\mathcal{H} := (H, |)$ since $(z|(u|v))|(u|v) = z \notin F$.

Theorem 3.3. *If F and G are weak filters of $\mathcal{H} := (H, |)$, then so are the intersection $F \cap G$ and the union $F \cup G$.*

Proof. It is clear that $1 \in F \cap G$ and $1 \in F \cup G$. Let $x \in H$ and $y \in F \cap G$. Then $y \in F$ and $y \in G$, and thus $x|(y|y) \in F$ and $x|(y|y) \in G$. Hence $x|(y|y) \in F \cap G$, and therefore $F \cap G$ is a weak filter of $\mathcal{H} := (H, |)$. Let $x \in H$ and $y \in F \cup G$. Then $y \in F$ or $y \in G$. If $y \in F$, then $x|(y|y) \in F \subseteq F \cup G$. If $y \in G$, then $x|(y|y) \in G \subseteq F \cup G$. Thus $F \cup G$ is a weak filter of $\mathcal{H} := (H, |)$. $\square$

Theorem 3.4. *For every $a \in H$, the set $\vec{a} := \{x \in H \mid a|(x|x) = 1\}$ is a weak filter of $\mathcal{H} := (H, |)$.*

Proof. Let a be an arbitrary element of H. Clearly, $1 \in \vec{a}$ by (2.3). If $y \in \vec{a}$, then $a|(y|y) = 1$, and so

$$a|((x|(y|y))|(x|(y|y))) = x|((a|(y|y))|(a|(y|y))) = x|(1|1) = 1$$

for all $x \in H$ by (2.8). Hence $x|(y|y) \in \vec{a}$, and therefore $\vec{a}$ is a weak filter of $\mathcal{H} := (H, |)$. $\square$

Theorem 3.5. *For every $a \in H$, the set $H^a := \{x \in H \mid a|(x|x) = x\}$ is a weak filter of $\mathcal{H} := (H, |)$.*

Proof. Let a be an arbitrary element of H. It is clear that $1 \in H^a$ by (2.3). Let $x \in H$ and $y \in H^a$. Then $a|(y|y) = y$, and so

$$a|((x|(y|y))|(x|(y|y))) = x|((a|(y|y))|(a|(y|y))) = x|(y|y)$$

by (2.8). Hence $x|(y|y) \in H^a$, and consequently H^a is a weak filter of $\mathcal{H} := (H, |)$. □

Theorem 3.6. *If F is a weak filter of $\mathcal{H} := (H, |)$, then the set*

$$F_a := \{z \in H \mid z = a|(x|x),\, x \in F\}$$

is a weak filter of $\mathcal{H} := (H, |)$.

Proof. Let F be a weak filter of $\mathcal{H} := (H, |)$. Since $1 = a|(1|1)$ by (2.3) and $1 \in F$, we get $1 \in F_a$. Let $x \in H$ and $y \in F_a$. Then $y = a|(z|z)$ for some $z \in F$. Since F is a weak filter of $\mathcal{H} := (H, |)$, we get $x|(z|z) \in F$. It follows from (2.8) that

$$x|(y|y) = x|((a|(z|z))|(a|(z|z))) = a|((x|(z|z))|(x|(z|z))).$$

Hence $x|(y|y) \in F_a$, and thus F_a is a weak filter of $\mathcal{H} := (H, |)$. □

Corollary 3.7. *For every $a \in H$, the set*

$$H_a := \{a|(x|x) \mid x \in H\}$$

is a weak filter of $\mathcal{H} := (H, |)$.

Theorem 3.8. *If F is a weak filter of $\mathcal{H} := (H, |)$, then the set*

$$F^* := \{x \in H \mid (\forall y \in F)(x|(y|y) = 1 \ \Rightarrow \ y = 1)\}$$

is a weak filter of $\mathcal{H} := (H, |)$.

Proof. Let a be an arbitrary element of H. It is clear that $1 \in F^*$ by (2.4). Let $z \in H$ and $x \in F^*$. Assume that $(z|(x|x))|(y|y) = 1$ for every $y \in F$. Then $x \preceq z|(x|x) \preceq y$ by (2.5) and (2.1), and so $y = 1$ since $x \in F^*$. Hence $z|(x|x) \in F^*$, and therefore F^* is a weak filter of $\mathcal{H} := (H, |)$. □

Proposition 3.9. *If F is a weak filters of $\mathcal{H} := (H, |)$, then*

$$F \cap F^* = \vec{a} \cap H^a = \{1\}$$

for all $a \in H$,

Proof. By the combination of Theorems 3.4 and 3.5, we know that $\vec{a} \cap H^a = \{1\}$ for all $a \in H$. It is obvious that $F \cap F^* = \vec{a} \cap H^a$. □

Definition 3.10 ([20]). Let $\mathcal{H} := (H, |)$ be a Sheffer stroke Hilbert algebra with the smallest element 0. A subset F of H is called an *ideal* of $\mathcal{H} := (H, |)$ if it satisfies:

$$0 \in F, \tag{3.1}$$

$$(\forall x, y \in H)\,((x|(y|y))|(x|(y|y)) \in F,\, y \in F \ \Rightarrow \ x \in F). \tag{3.2}$$

Given a nonempty subset F of H, consider the set:

$$F^{\&} := \{y \in H \mid y = x|x \text{ for } x \in F\}.$$

Proposition 3.11. *Let $\mathcal{H} := (H, |)$ be a Sheffer stroke Hilbert algebra with the smallest element 0. Then every ideal F of $\mathcal{H} := (H, |)$ satisfies:*

$$1 \in F^{\&}, \tag{3.3}$$

$$(\forall x, y \in H)\left(x \in F^{\&},\, x|(y|y) \in F^{\&} \ \Rightarrow \ y \in F^{\&}\right). \tag{3.4}$$

Proof. Let F be an ideal of $\mathcal{H} := (H, |)$. Then $0 \in F$, and so $1 = 0|0 \in F^{\&}$ by (2.11). Let $x, y \in H$ be such that $x \in F^{\&}$ and $x|(y|y) \in F^{\&}$. Then $x = a|a$ and $x|(y|y) = b|b$ for some $a, b \in F$. It follows from (s1) and (s2) that

$$\begin{aligned}((y|y)|(a|a))|((y|y)|(a|a)) &= ((y|y)|x)|((y|y)|x)\\ &= (x|(y|y))|(x|(y|y))\\ &= (b|b)|(b|b) = b \in F.\end{aligned}$$

Hence $y|y \in F$ by (3.2), and so $y = (y|y)|(y|y) \in F^{\&}$ by (s2). □

(i) Is the set $F^{\&}$ a weak filter or an ideal of $\mathcal{H} := (H, |)$?

(ii) If F is a weak filter of $\mathcal{H} := (H, |)$, then is the set $F^{\&}$ a weak filter or an ideal of $\mathcal{H} := (H, |)$?

(iii) If F is an ideal of $\mathcal{H} := (H, |)$, then is the set $F^{\&}$ a weak filter or an ideal of $\mathcal{H} := (H, |)$?

In Example 3.2, if we take $F := \{y, v\}$, then $F^{\&} = \{x, u\}$ and it is neither a weak filter nor an ideal of $\mathcal{H} := (H, |)$. If we take the weak filter $F_3 := \{1, t, u, v\}$, then $F_3^{\&} = \{0, x, y, z\}$ is not a weak filter of $\mathcal{H} := (H, |)$. Since $(v|(y|y))|(v|(y|y)) = z \in F_3^{\&}$ and $y \in F_3^{\&}$, nut $v \notin F_3^{\&}$. Hence $F_3^{\&}$ is not an ideal of $\mathcal{H} := (H, |)$. Therefore, the answer to (i) and (ii) in Question 3 is negative. To answer Question 3(iii), let's look at the following example.

Example 3.12. Consider a set $H = \{1, x, y, 0\}$, and define a Sheffer stroke "|" on H by Table 2.

Table 2. Cayley table for the Sheffer stroke "|"

$\vert$	1	x	y	0
1	0	y	x	1
x	y	y	1	1
y	x	1	x	1
0	1	1	1	1

Then $\mathcal{H} := (H, |)$ is a Sheffer stroke Hilbert algebra with the smallest element 0 (see [20]). Take a subset $F := \{0, y\}$ of H. Then it is an ideal of $\mathcal{H} := (H, |)$ and $F^{\&} = \{1, x\}$ is a weak filter of $\mathcal{H} := (H, |)$. But it is not an ideal of $\mathcal{H} := (H, |)$.

Based on Example 3.12, we induces the following theorem.

Theorem 3.13. *Let $\mathcal{H} := (H, |)$ be a Sheffer stroke Hilbert algebra with the smallest element* 0. *If F is an ideal of $\mathcal{H} := (H, |)$, then the set $F^{\&}$ is a weak filter of $\mathcal{H} := (H, |)$.*

Proof. Let F be an ideal of $\mathcal{H} := (H, |)$. Then $1 \in F^{\&}$ by (3.3). Let $x \in H$ and $y \in F^{\&}$. Since $y \preceq x|(y|y)$ by (2.5), we get

$$y|((x|(y|y))|(x|(y|y))) = 1 \in F^{\&}.$$

It follows from (3.4) that $x|(y|y) \in F^{\&}$. Therefore, $F^{\&}$ is a weak filter of $\mathcal{H} := (H, |)$. □

The following example shows that the converse of Theorem 3.13 may not be true.

Example 3.14. In Example 3.2, let's take $F := \{0, x, y\}$. Then $F^{\&} = \{1, u, v\}$ and it is a weak filter of $\mathcal{H} := (H, |)$. But F is not an ideal of $\mathcal{H} := (H, |)$ since $x \in F$ and $(t|(x|x))|(t|(x|x)) = y \in F$, but $t \notin F$.

Let $\mathcal{H} := (H, |)$ and $\mathcal{K} := (K, \uparrow)$ be Sheffer stroke Hilbert algebras. Then $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$ is also a Sheffer stroke Hilbert algebra where the Sheffer stroke " $\Uparrow$ " on $H \times K$ is given as follows:

$$(x, a) \Uparrow (y, b) = (x|y, a \uparrow b) \tag{3.5}$$

for all $x, y \in H$ and $a, b \in K$.

Theorem 3.15. *If F_H and F_K are weak filters of Sheffer stroke Hilbert algebras $\mathcal{H} := (H, |)$ and $\mathcal{K} := (K, \uparrow)$, respectively, then $F_H \times F_K$ is a weak filter of $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$.*

Proof. Assume that F_H and F_K are weak filters of Sheffer stroke Hilbert algebras $\mathcal{H} := (H, |)$ and $\mathcal{K} := (K, \uparrow)$, respectively. It is clear that $(1, 1) \in F_H \times F_K$. Let $(x, a) \in \mathcal{H} \times \mathcal{K}$ and $(y, b) \in F_H \times F_K$. Then

$$(x, a) \Uparrow ((y, b) \Uparrow (y, b)) = (x, a) \Uparrow (y|y, b \uparrow b) = (x|(y|y), a \uparrow (b \uparrow b)) \in F_H \times F_K.$$

Therefore $F_H \times F_K$ is a weak filter of $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$. □

Theorem 3.16. *If F_H and F_K are filters of Sheffer stroke Hilbert algebras $\mathcal{H} := (H, |)$ and $\mathcal{K} := (K, \uparrow)$, respectively, then $F_H \times F_K$ is a filter of $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$.*

Proof. Assume that F_H and F_K are filters of Sheffer stroke Hilbert algebras $\mathcal{H} := (H, |)$ and $\mathcal{K} := (K, \uparrow)$, respectively. Then they are weak filters, and so $F_H \times F_K$ is a weak filter of $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$ by Theorem 3.15. Let $(x, a), (y, b), (z, c) \in \mathcal{H} \times \mathcal{K}$ be such that $(y, b) \in F_H \times F_K$ and $(z, c) \in F_H \times F_K$. Then

$$\begin{aligned}
&((x, a) \Uparrow ((y, b) \Uparrow (z, c))) \Uparrow ((y, b) \Uparrow (z, c)) = ((x, a) \Uparrow (y|z, b \uparrow c)) \Uparrow (y|z, b \uparrow c) \\
&= (x|(y|z), a \uparrow (b \uparrow c)) \Uparrow (y|z, b \uparrow c) = ((x|(y|z))|(y|z), (a \uparrow (b \uparrow c)) \uparrow (b \uparrow c)) \\
&\in F_H \times F_K.
\end{aligned}$$

Therefore $F_H \times F_K$ is a filter of $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$. □

Theorem 3.17. *Every weak filter of $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$ can be expressed as the Cartesian product of weak filters of $\mathcal{H} := (H, |)$ and $\mathcal{K} := (K, \uparrow)$.*

Proof. Let F be a weak filter of $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$. Consider projections $\pi_H : \mathcal{H} \times \mathcal{K} \to \mathcal{H}$ and $\pi_K : \mathcal{H} \times \mathcal{K} \to \mathcal{K}$. Let F_H and F_K be the projections of F on $\mathcal{H}$ and $\mathcal{K}$, respectively, that is, $\pi_H(F) = F_H$ and $\pi_K(F) = F_K$. It is clear that $1 \in F_H \cap F_K$. Let $x \in H$ and $y \in F_H$. Then $(x, 1) \in \mathcal{H} \times \mathcal{K}$ and $(y, 1) \in F$. Since F is a weak filter, we have

$$\begin{aligned}
(x|(y|y), 1) &= (x|(y|y), 1 \uparrow (1 \uparrow 1)) \\
&= (x, 1) \Uparrow (y|y, 1 \uparrow 1) \\
&= (x, 1) \Uparrow ((y, 1) \Uparrow (y, 1)) \in F
\end{aligned}$$

and so $x|(y|y) = \pi_H(x|(y|y), 1) \in \pi_H(F) = F_H$. Hence F_H is a weak filter of $\mathcal{H} := (H, |)$. By the similar way, we can check that F_K is a weak filter of $\mathcal{K} := (K, \uparrow)$. □

Theorem 3.18. *Every filter of $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$ can be expressed as the Cartesian product of filters of $\mathcal{H} := (H, |)$ and $\mathcal{K} := (K, \uparrow)$.*

Proof. Let F be a filter of $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$. Then it is weak filter, and so $F = F_H \times F_K$ where F_H and F_K are weak filters of $\mathcal{H} := (H, |)$ and $\mathcal{K} := (K, \uparrow)$, respectively (see Theorem 3.17). Let $x, y, z \in H$ be such that $y, z \in F_H$. Then $(y, 1), (z, 1) \in F$, and so

$$\begin{aligned}
((x|(y|z))|(y|z), 1) &= ((x|(y|z))|(y|z), (1 \uparrow (1 \uparrow 1)) \uparrow (1 \uparrow 1)) \\
&= ((x, 1) \Uparrow ((y, 1) \Uparrow (z|1))) \Uparrow ((y, 1) \Uparrow (z|1)) \in F.
\end{aligned}$$

Hence $(x|(y|z))|(y|z) \in \pi_H(F) = F_H$ Similarly, we can verify that if $b, c \in F_K$, then $(a \uparrow (b \uparrow c)) \uparrow (b \uparrow c) \in F_K$. Therefore F_H and F_K are filters of $\mathcal{H} := (H, |)$ and $\mathcal{K} := (K, \uparrow)$, respectively. This completes the proof. □

4 Multipliers

Definition 4.1. A self-map $f : H \to H$ is called a *multiplier* of $\mathcal{H} := (H, |)$ if it satisfies:

$$(\forall x, y \in H)(f(x|(y|y)) = x|(f(y)|f(y))). \tag{4.1}$$

It is clear that the identity function $f_1 : H \to H,\ x \mapsto x$, is a multiplier of $\mathcal{H} := (H, |)$. Also a constant function $\gamma : H \to H,\ x \mapsto 1$, is a multiplier of $\mathcal{H} := (H, |)$.

Example 4.2. Consider the Sheffer stroke Hilbert algebra $\mathcal{H} := (H, |)$ in Example 3.2 and define a self-map f as follows:

$$f : H \to H,\ b \mapsto \begin{cases} 1 & \text{if } b \in \{x, t, u, 1\}, \\ v & \text{otherwise.} \end{cases}$$

It is routine to verify that f is a multiplier of $\mathcal{H} := (H, |)$. Also, if we consider a self-map g given by

$$g : H \to H,\ c \mapsto \begin{cases} 1 & \text{if } c \in \{1, u\}, \\ y & \text{if } c \in \{0, y\}, \\ t & \text{if } c \in \{x, t\}, \\ v & \text{if } c \in \{z, v\}, \end{cases}$$

then g is a multiplier of $\mathcal{H} := (H, |)$.

Proposition 4.3. *The multiplier f of $\mathcal{H} := (H, |)$ satisfies:*

$$f(1) = 1, \tag{4.2}$$
$$(\forall x \in H)(x \preceq f(x)), \tag{4.3}$$
$$(\forall x \in H)(f(f(x)|(x|x)) = 1), \tag{4.4}$$
$$(\forall x, y \in H)(f(x)|(f(y)|f(y)) \preceq f(x|(y|y))), \tag{4.5}$$
$$(\forall x, y \in H)(f(x)|(y|y) \preceq f(x|(y|y))). \tag{4.6}$$

Proof. Using (2.3), (4.1) and (2.2), we have

$$f(1) = f(f(1)|(1|1)) = f(1)|(f(1)|f(1)) = 1.$$

Using (4.2), we get $1 = f(1) = f(x|(x|x)) = x|(f(x)|f(x))$ and so $x \preceq f(x)$ for all $x \in H$. The combination of (2.2) and (4.1) induces (4.4). If we use (4.3), (2.9) and (4.1), then

$$f(x)|(f(y)|f(y)) \preceq x|(f(y)|f(y)) = f(x|(y|y))$$

and $f(x)|(y|y) \preceq x|(y|y) \preceq x|(f(y)|f(y)) = f(x|(y|y))$ for all $x, y \in H$. □

Corollary 4.4. *The multiplier f of $\mathcal{H} := (H, |)$ satisfies:*

$$(\forall x\, y \in H)(x \preceq y \ \Rightarrow\ x \preceq f(y)). \tag{4.7}$$

Proposition 4.5. *If a multiplier f of $\mathcal{H} := (H, |)$ satisfies:*

$$(\forall x, y \in H)(f(x)|(y|y) = x|(f(y)|f(y))), \tag{4.8}$$

then it is an identity map.

Proof. Let f be a multiplier of $\mathcal{H} := (H, |)$ satisfying the condition (4.8). Then

$$f(y) = f(1|(y|y)) = 1|(f(y)|f(y)) = f(1)|(y|y) = 1|(y|y) = y$$

for all $y \in H$ by (2.4), (4.1), (4.8) and (4.2). Hence f is an identity map. □

Theorem 4.6. *Let $\mathcal{H} := (H, |)$ and $\mathcal{K} := (K, \uparrow)$ be Sheffer stroke Hilbert algebras. Define a map $f : H \times K \to H \times K$ by $f(x, a) = (x, 1)$ for all $(x, a) \in H \times K$. Then f is a multiplier of $\mathcal{H} \times \mathcal{K} := (H \times K, \Uparrow)$.*

Proof. For every $(x,a),(y,b)\in H\times K$, we have

$$\begin{aligned} f((x,a)\Uparrow((y,b)\Uparrow(y,b)) &= f((x,a)\Uparrow(y|y,b\uparrow b)) \\ &= f(x|(y|y),a\uparrow(b\uparrow b)) \\ &= f(x|(y|y),1)=f(x|(y|y),a\uparrow(1\uparrow 1)) \\ &= f((x,a)\Uparrow((y|y),1\uparrow 1)) \\ &= f((x,a)\Uparrow((y,1)\Uparrow(y,1))) \\ &= (x,a)\Uparrow(f(y,b)\Uparrow f(y,b)). \end{aligned}$$

Hence f is a multiplier of $\mathcal{H}\times\mathcal{K}:=(H\times K,\Uparrow)$. □

Proposition 4.7. *Given an element $a\in H$, define a self-map f_a as follows:*

$$f_a: H\to H,\ x\mapsto a|(x|x). \tag{4.9}$$

Then f_a is a multiplier of $\mathcal{H}:=(H,|)$ which is called the a-simple multiplier. Moreover,

$$(\forall a,b\in H)(a\preceq b\ \Rightarrow\ f_b\subseteq f_a,\ i.e., f_b(x)\preceq f_a(x),\ \forall x\in H), \tag{4.10}$$

$$(\forall a,b\in H)(a\neq b\ \Rightarrow\ f_a\neq f_b). \tag{4.11}$$

Proof. For every $x,y\in H$, we have

$$f_a(x|(y|y))=a|((x|(y|y))|(x|(y|y)))=x|((a|(y|y))|(a|(y|y)))=x|(f_a(y)|f_a(y))$$

by (2.8). Hence f_a is a multiplier of $\mathcal{H}:=(H,|)$. The condition (4.10) is induced by (2.9). Let $a,b\in H$ be such that $a\neq b$. Assume that $f_a=f_b$. Then $a|(x|x)=b|(x|x)$ for all $x\in H$. If we replace x with b (resp., a), then $a|(b|b)=b|(b|b)=1$ (resp., $b|(a|a)=a|(a|a)=1$) by (2.2). Hence $a\preceq b$ and $b\preceq a$, and so $a=b$, a contrsdiction. Thus $f_a\neq f_b$. □

Proposition 4.8. *The* 1*-simple multiplier f_1 of $\mathcal{H}:=(H,|)$ is the identity map on H where* 1 *is the greatest element in $\mathcal{H}:=(H,|)$ with respect to the order $\preceq$. Also, if $\mathcal{H}:=(H,|)$ has the least element* 0*, then the* 0*-simple multiplier f_0 of $\mathcal{H}:=(H,|)$ is the constant map with value* 1*.*

Proof. For every $x\in H$, we have $f_1(x)=1|(x|x)=x$. If 0 is the least element of $\mathcal{H}:=(H,|)$, then $0\preceq x$ for all $x\in H$. Hence $f_0(x)=0|(x|x)=1$. □

In the example below, we give a Sheffer stroke Hilbert algebra, which lists all simple multipliers.

Example 4.9. Consider the Sheffer stroke Hilbert algebra $\mathcal{H}:=(H,|)$ in Example 3.2. At this time, all simple multipliers are f_0, f_1, f_x, f_y, f_z, f_t, f_u, and f_v where $f_0(x)=1$ and $f_1(x)=x$ for all $x\in H$, and

$$f_x: H\to H,\ x\mapsto\begin{cases} 1 & \text{if } x\in\{0,x,t,u\} \\ v & \text{if } x\in\{1,y,z,v\}. \end{cases}$$

$$f_y: H\to H,\ x\mapsto\begin{cases} u & \text{if } x\in\{0,x,z,u\}, \\ 1 & \text{if } x\in\{1,y,t,v\}. \end{cases}$$

$$f_z: H\to H,\ x\mapsto\begin{cases} t & \text{if } x\in\{0,x,y,t\}, \\ 1 & \text{if } x\in\{1,z,u,v\}. \end{cases}$$

$$f_t: H\to H,\ x\mapsto\begin{cases} z & \text{if } x\in\{0,z\}, \\ 1 & \text{if } x\in\{1,t\}, \\ u & \text{if } x\in\{x,u\}, \\ v & \text{if } x\in\{y,v\}. \end{cases}$$

$$f_u : H \to H,\ x \mapsto \begin{cases} y & \text{if } x \in \{0, y\}, \\ 1 & \text{if } x \in \{1, u\}, \\ t & \text{if } x \in \{x, t\}, \\ v & \text{if } x \in \{z, v\}. \end{cases}$$

$$f_v : H \to H,\ x \mapsto \begin{cases} x & \text{if } x \in \{0, x\}, \\ 1 & \text{if } x \in \{1, v\}, \\ t & \text{if } x \in \{y, t\}, \\ u & \text{if } x \in \{z, u\}. \end{cases}$$

It can be checked by routinely that they satisfy (4.10) and (4.11).

Theorem 4.10. *If $\mathcal{H} := (H, |)$ is a finite Sheffer stroke Hilbert algebra with n elements, then more than n multipliers of $\mathcal{H} := (H, |)$ are formed.*

Proof. Assume that $|H| = n$. Since f_a is a multiplier of $\mathcal{H} := (H, |)$ for every $a \in H$ (see Example 4.2(i)), we know that there exists at least n multipliers of $\mathcal{H} := (H, |)$. Therefore, we can form more than n multipliers of $\mathcal{H} := (H, |)$ by (4.11). □

Theorem 4.11. *Let f be the multiplier of $\mathcal{H} := (H, |)$. If F is a weak filter of $\mathcal{H} := (H, |)$, then so are $f(F)$ and $f^{-1}(F)$.*

Proof. Let F be a weak filter of $\mathcal{H} := (H, |)$. Then $1 = f(1) \in f(F)$ by (4.2). Let $x, y \in H$ be such that $y \in f(F)$. Then $f(b) = y$ for some $b \in F$, and so $x|(y|y) = x|(f(b)|f(b)) = f(x|(b|b)) \in f(F)$ by (2.14) and (4.1). Thus $f(F)$ is a weak filter of $\mathcal{H} := (H, |)$. Using (2.13) and (4.2) leads to $1 \in f^{-1}(F)$. Let $x, y \in H$ be such that $y \in f^{-1}(F)$. Then $f(y) \in F$, and so $f(x|(y|y)) = x|(f(y)|f(y)) \in F$ by (4.1) and (2.14). Hence $x|(y|y) \in f^{-1}(F)$. Therefore $f^{-1}(F)$ is a weak filter of $\mathcal{H} := (H, |)$. □

Theorem 4.12. *If f is the multiplier of $\mathcal{H} := (H, |)$, then the set*

$$\ker(f) := \{x \in H \mid f(x) = 1\}, \tag{4.12}$$

which is called the kernel of f, is a weak filter of $\mathcal{H} := (H, |)$.

Proof. Since $f(1) = 1$ by (4.2), we have $1 \in \ker(f)$. Let $x, y \in H$ be such that $y \in \ker(f)$. Then $f(x|(y|y)) = x|(f(y)|f(y)) = x|(1|1) = 1$ by (4.1) and (2.3), and thus $x|(y|y) \in \ker(f)$. Hence $\ker(f)$ is a weak filter of $\mathcal{H} := (H, |)$. □

Theorem 4.13. *If f is the multiplier of $\mathcal{H} := (H, |)$, then the set*

$$Fix_f(H) := \{x \in H \mid f(x) = x\} \tag{4.13}$$

which is called the fixed set of H by f, is a weak filter of $\mathcal{H} := (H, |)$.

Proof. Since $f(1) = 1$ by (4.2), we have $1 \in Fix_f(H)$. Let $x, y \in H$ be such that $y \in Fix_f(H)$. Then $f(y) = y$, and so $f(x|(y|y)) = x|(f(y)|f(y)) = x|(y|y)$, that is, $x|(y|y) \in Fix_f(H)$. Hence $Fix_f(H)$ is a weak filter of $\mathcal{H} := (H, |)$. □

Denote by $M(H)$ the collection of all multipliers of $\mathcal{H} := (H, |)$. We define a function composition "$\circ$" on $M(H)$ as follows:

$$\circ : M(H) \times M(H) \to M(H), (f, g) \mapsto f \circ g$$

where $f \circ g : H \to H,\ x \mapsto f(g(x))$.

Lemma 4.14. *If f and g are multipliers of $\mathcal{H} := (H, |)$, then so is their composition $f \circ g$, that is, $f, g \in M(H) \Rightarrow f \circ g \in M(H)$.*

Proof. Let f and g be multipliers of $\mathcal{H} := (H, |)$. For every $x, y \in H$, we have

$$\begin{aligned}(f \circ g)(x|(y|y)) &= f(g(x|(y|y))) = f(x|(g(y)|g(y))) \\ &= x|(f(g(y))|f(g(y))) \\ &= x|((f \circ g)(y)|(f \circ g)(y)).\end{aligned}$$

Hence $f \circ g$ is a multiplier of $\mathcal{H} := (H, |)$. □

Theorem 4.15. *The collection of all multipliers of $\mathcal{H} := (H, |)$ forms a monoid under the function composition "$\circ$" with the identity element f_1.*

Proof. Straightforward. □

Proposition 4.16. *If f is a multiplier of $\mathcal{H} := (H, |)$, then $(f \circ f)(x) = x$ for all $x \in Fix_f(H)$.*

Proof. Let $x \in Fix_f(H)$. Then $(f \circ f)(x) = f(f(x)) = f(x) = x$. □

Proposition 4.17. *For every element $x \in H$, the x-simple multiplier f_x of $\mathcal{H} := (H, |)$ is idempotent, that is, $(f_x \circ f_x)(a) = f_x(a)$ for all $a \in H$.*

Proof. For every $a \in H$, we have

$$(f_x \circ f_x)(a) = f_x(f_x(a)) = f_x(x|(a|a)) = x|((x|(a|a))|(x|(a|a))) = x|(a|a) = f_x(a),$$

and so f_x is idempotent. □

Denote by $M_s(H)$ the set of all simple multipliers of $\mathcal{H} := (H, |)$.

Lemma 4.18. *If $f_a, f_b \in M_s(H)$ for $a, b \in H$, then $f_a \circ f_b \in M_s(H)$ and $f_a \circ f_b = f_b \circ f_a$.*

Proof. Using (2.8) induces

$$\begin{aligned}(f_a \circ f_b)(x) &= f_a(f_b(x)) = f_a(b|(x|x)) \\ &= a|((b|(x|x))|(b|(x|x))) \\ &= b|((a|(x|x))|(a|(x|x))) \\ &= f_b(a|(x|x)) = f_b(f_a(x)) \\ &= (f_b \circ f_a)(x)\end{aligned}$$

and

$$\begin{aligned}(f_a \circ f_b)(x|(y|y)) &= f_a(f_b(x|(y|y))) \\ &= f_a(b|((x|(y|y))|(x|(y|y)))) \\ &= a|((b|((x|(y|y))|(x|(y|y))))|(b|((x|(y|y))|(x|(y|y))))) \\ &= x|((a|((b|(y|y))|(b|(y|y))))|(a|((b|(y|y))|(b|(y|y))))) \\ &= x|((a|(f_b(y)|f_b(y)))|(a|(f_b(y)|f_b(y)))) \\ &= x|(f_a(f_b(y))|f_a(f_b(y))) \\ &= x|((f_a \circ f_b)(y)|(f_a \circ f_b)(y))\end{aligned}$$

for all $x, y \in H$. Hence $f_a \circ f_b = f_b \circ f_a$ and $f_a \circ f_b \in M_s(H)$. □

Theorem 4.19. *$(M_s(H), \circ)$ is a subsemigroup of $(M(H), \circ)$.*

Proof. Straightforward. □

We explore the conditions under which two multipliers are equal.

Proposition 4.20. *Let $f, g \in M(H)$ such that f and g are idempotent and $f \circ g = g \circ f$. If $Fix_f(H) = Fix_g(H)$, then $f = g$. Also, if $f(H) = g(H)$, then $Fix_f(H) = Fix_g(H)$.*

Proof. Assume that $Fix_f(H) = Fix_g(H)$. Let $x \in H$. Since f and g are idempotent, we have $f(x) \in Fix_f(H) = Fix_g(H)$ and $g(x) \in Fix_g(H) = Fix_f(H)$. Hence $g(f(x)) = f(x)$ and $f(g(x)) = g(x)$. Since $f \circ g = g \circ f$, it follows that

$$f(x) = g(f(x)) = (g \circ f)(x) = (f \circ g)(x) = f(g(x)) = g(x).$$

Hence $f = g$. Suppose that $f(H) = g(H)$. If $x \in Fix_f(H)$, then $x = f(x) \in f(H) = g(H)$ and thus $x = g(a)$ for some $a \in H$. Since g is idempotent, it follows that $g(x) = g(g(a)) = g(a) = x$. Hence $x \in Fix_g(H)$, and so $Fix_f(H) \subseteq Fix_g(H)$. The similar way induces $Fix_g(H) \subseteq Fix_f(H)$. □

Theorem 4.21. *If we define a binary operation "$\uparrow$" on $M(H)$ as follows:*

$$\uparrow : M(H) \times M(H) \to M(H),\ (f, g) \mapsto f \uparrow g$$

where $(f \uparrow g)(x) = f(x)|g(x)$ for all $x \in H$, then "$\uparrow$" is a Sheffer stroke on $M(H)$ and $(M(H), \uparrow)$ forms a Sheffer stroke Hilbert algebra.

Proof. The calculation for proof is routine, so we omit it. □

We deine an order relation "$\precapprox$" on $M(H)$ as follows:

$$(\forall f, g \in M(H))(f \precapprox g \;\Leftrightarrow\; f(x) \preceq g(x) \text{ for all } x \in H).$$

It is obvious that the relation "$\precapprox$" is a partial order in $(M(H), \uparrow)$.

Theorem 4.22. *The set $M_s(H)$ is a weak filter of $(M(H), \uparrow)$.*

Proof. Since $f_a \precapprox f_0$ for all $f_a \in M_s(H)$, it is obtained that f_0 is the greatest element of $M_s(H)$, and so, $f_0 \in M_s(H)$. Let $g_a \in M_s(H)$ and $f \in M(H)$. Using (2.8) and (2.10), we have

$$\begin{aligned}
&(f \uparrow (g_a \uparrow g_a))(x|(y|y)) = f(x|(y|y))|(g_a(x|(y|y))|g_a(x|(y|y)))\\
&= (x|(f(y)|f(y)))|((a|((x|(y|y))|(x|(y|y))))|(a|((x|(y|y))|(x|(y|y)))))\\
&= (x|(f(y)|f(y)))|((x|((a|(y|y))|(a|(y|y))))|(x|((a|(y|y))|(a|(y|y)))))\\
&= x|((f(y)|((a|(y|y))|(a|(y|y))))|(f(y)|((a|(y|y))|(a|(y|y)))))\\
&= x|((f(y)|(g_a(y)|g_a(y)))|(f(y)|(g_a(y)|g_a(y))))\\
&= x|((f \uparrow (g_a \uparrow g_a))(y)|(f \uparrow (g_a \uparrow g_a))(y))
\end{aligned}$$

for all $x, y \in H$. Hence $f \uparrow (g_a \uparrow g_a) \in M_s(H)$. This completes the proof. □

Definition 4.23. A relation θ on $\mathcal{H} := (H, |)$ is called a *Sheffer congruence relation* if it is an equivalence relation satisfying the condition below:

$$(\forall a, x, y \in H)((x, y) \in \theta \;\Rightarrow\; (a|(x|x), a|(y|y)) \in \theta). \tag{4.14}$$

Example 4.24. Consider the Sheffer stroke Hilbert algebra $\mathcal{H} := (H, |)$ in Example 3.12. Then a relation

$$\theta = \{(0,0), (1,1), (x,x), (y,y), (1,x), (x,1), (1,y), (y,1), (x,y), (y,x)\}$$

is a Sheffer congruence relation on $\mathcal{H} := (H, |)$.

Theorem 4.25. *Let f be a multiplier of $\mathcal{H} := (H, |)$. If we define a relation θ_f on $\mathcal{H} := (H, |)$ as follows:*

$$(\forall x, y \in H)((x, y) \in \theta_f \;\Leftrightarrow\; f(x) = f(y)), \tag{4.15}$$

then θ_f is a Sheffer congruence relation on $\mathcal{H} := (H, |)$.

Proof. It is clear that θ_f is an equivalence relation on $\mathcal{H} := (H, |)$. Let $(x, y) \in \theta_f$ for $x, y \in H$. Then $f(x) = f(y)$, and so

$$f(a|(x|x)) = a|(f(x)|f(x)) = a|(f(y)|f(y)) = f(a|(y|y))$$

for all $a \in H$. Hence $(a|(x|x), a|(y|y)) \in \theta_f$, and therefore θ_f is a Sheffer congruence relation on $\mathcal{H} := (H, |)$. $\square$

Lemma 4.26. *If a multiplier f of $\mathcal{H} := (H, |)$ is idempotent, then $f(x) = x$ for all $x \in \mathrm{Im}(f)$ and*

$$(\forall x, y \in \mathrm{Im}(f))((x, y) \in \theta_f \Rightarrow x = y). \tag{4.16}$$

Proof. If $x \in \mathrm{Im}(f)$, then $x = f(a)$ for some $a \in H$ and so $f(x) = (f \circ f)(x) = f(f(x)) = f(a) = x$. If $(x, y) \in \theta_f$ for all $x, y \in \mathrm{Im}(f)$, then $f(x) = f(y)$ and thus $x = f(x) = f(y) = y$. $\square$

Theorem 4.27. *If f is an idempotent multiplier of $\mathcal{H} := (H, |)$, then $|f(H) \cap \theta_f[x]| = 1$ for all $x \in H$, where $\theta_f[x]$ is the Sheffer congruence class of x with respect to θ_f.*

Proof. Let f be an idempotent multiplier f of $\mathcal{H} := (H, |)$. Then θ_f is a Sheffer congruence relation on $\mathcal{H} := (H, |)$. Let $x \in H$. Then $(x, f(x)) \in \theta_f$ since f is idempotent, and so $f(x) \in \theta_f[x]$. Since $f(x) \in f(H)$, it follows that $f(x) \in f(H) \cap \theta_f[x]$, that is, $f(H) \cap \theta_f[x] \neq \emptyset$. If $a, b \in f(H) \cap \theta_f[x]$, then $a = b$ by Lemma 4.26, and so $|\theta_f[x] \cap F| = 1$. $\square$

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Author information

Y. Bae Jun, Department of Mathematics Education, Gyeongsang National University, Jinju 52828, Korea.
E-mail: skywine@gmail.com

T. Oner, Department of Mathematics, Faculty of Science, Ege University, Izmir,, Türkiye.
E-mail: tahsin.oner@ege.edu.tr

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