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DETOUR PEBBLING NUMBER FOR CYCLE AND WHEEL RELATED GRAPHS

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Abstract A detour pebbling number of a vertex v in a graph G is defined as least positive number $f^*(v,G)$ which allows a pebble to be put on v via a detour path by repeatedly performing pebbling moves (A pebbling move is to remove two pebbles from a vertex and move a pebble to an adjacent vertex) with any configuration of $f^*(v,G)$ pebbles on G. Then $f^*(G)$, the detour pebbling number of G, is the maximum $f^*(v,G)$ where $v \in G$. Here, we study the detour pebbling number for cycle and wheel related graphs.

1 Introduction

One recent development in graph theory suggested by, Lagarias and Saks and called pebbling, has been the subject of much research. It was first introduced into the literature by Chung [1], and has been developed by many others including Hulbert, who published a survey of graph pebbling [3]. Lourdusamy et al. [9] defined the detour pebbling number as follows: "A detour pebbling number of a vertex v of a graph G is the smallest number $f^*(G,v)$ that allows a pebble to be moved to v via a detour path by a sequence of pebbling moves for any placement of $f^*(G,v)$ pebbles on the vertices of G. The detour pebbling number of a graph G, denoted by $f^*(G)$, is the maximum $f^*(G,v)$ over all the vertices of G". The readers can refer to [4, 6, 8, 10] for further information about detour pebbling number.

Throughout this article, we denote $p(z)$ for the number of pebbles initially placed on the vertex z and $p(S)$ denote the number of pebbles on the vertices of S. P_i means detour path and $\tilde{P_i}$ means the set of vertices which are not on the detour path.

The readers can get the information about the graphs duplicating an arbitrary vertex of a cycle C_n, duplicating an arbitrary edge of a cycle C_n, Q_n, $P_2 \odot C_n$, $C_m *_e C_n$, H_n, CH_n, Wb_n and G_n in [2, 5, 7, 11]. Here, we study the detour pebbling number of above mentioned graphs.

Remark 1.1. ω is assumed to be target. For the initial distribution we take $p(\omega) = 0$.

2 Main Results

Theorem 2.1. *If G is a graph obtained by duplicating a vertex of C_n, then $f^*(G) = 2^n$.*

Proof. Let $V(C_n) = \{a_1, a_2 \cdots a_n\}$. Then the graph $G = D(C_n, a^{'})$ is obtained by duplicating a_1 in C_n. Let $V(G) = \{a_r, a_1^{'} : 1 \le r \le n\}$ and $E(G) = \{a_r a_{r+1} : 1 \le r \le n-1\} \bigcup \{a_n a_1, a_2 a_1^{'}, a_n a_1^{'}\}$. The detour path P_1 be $a_1^{'} a_2 a_3 \cdots a_n a_1$ is of length n. Placing $2^n - 1$ pebbles on the vertex $a_1^{'}$, we cannot reach a_1. Thus $f^*(G) \ge 2^n$. Now we prove the sufficient condition. Let D be any configuration of 2^n pebbles on the vertices of G.
Case 1. Let $\omega = a_1$.
Through the detour path $P_2 : a_1^{'} a_2 \cdots a_n a_1$ we can move a pebble to a_1. By symmetry the result

is true for the target vertices a_3 and a_{n-1}.
Case 2. Let $\omega = a_2$.
The detour path $P_3 : a_1^{'}a_na_{n-1}\cdots a_3a_2$ has 2^{n-1} pebbles. Since the length of P_3 is $n-1$. We can move a pebble to a_2 using fewer than 2^n pebbles. By symmetry we can prove for the vertices $a_4, a_5 \cdots a_{n-2}$ and a_n.
Case 3. Let $\omega = a_1^{'}$.
The detour path $P_4 : a_1a_n\cdots a_3a_2a_1$ has 2^n pebbles. It is easy to move a pebble to $a_1^{'}$ through P_4. Hence $f^*(G) = 2^n$. □

Theorem 2.2. *If G is a graph obtained by duplicating an edge in cycle C_n, then $f^*(G) = 2^{n+1}$.*

Proof. Let $V(C_n) = \{a_1, a_2\cdots a_n\}$. Let $e = a_1a_2$ and $e^{'} = a_1^{'}a_2^{'}$. Consider the graph $G = D(C_n, e^{'})$ which is obtained by duplicating an arbitrary edge e in C_n. Then $V(G) = \{a_r, a_1^{'}, a_2^{'} : 1 \le r \le n\}$ and $E(G) = \{a_ra_{r+1} : 1 \le r \le n-1\}\bigcup\{a_na_1, a_na_1^{'}, a_3a_2^{'}, a_1^{'}a_2^{'}\}$. Consider the detour path $P_1 : a_1^{'}a_2^{'}a_3a_4\cdots a_na_1a_2$ is of length $n+1$. If we place $2^{n+1}-1$ pebbles on $a_1^{'}$, we cannot move a pebble to a_2. So $f^*(G) \ge 2^{n+1}$.
Now we prove the sufficient condition. Let D be any configuration of 2^{n+1} pebbles on G.
Case 1. Let $\omega = a_1$.
We distribute 2^{n+1} pebbles path $P_2 : a_2^{'}a_1^{'}a_na_{n-1}\cdots a_3a_2a_1$. This is also a detour path from $a_2^{'}$ to a_1 of length $n-1$. So we can put a pebble on a_1. By symmetry the proof follows for $\omega = a_2, a_4$ and a_{n-1}.
Case 2. Let $\omega = a_3$.
We distribute 2^{n-1} pebbles on the detour path $P_3 : a_2^{'}a_1^{'}a_na_{n-1}\cdots a_4a_3$ which is of length $n-1$. It is easy to see that a pebble can be moved to a_3 using fewer than 2^{n-1} pebbles. By symmetry we can proof follows for $\omega = a_n$.
Case 3. Let $\omega = a_{n-2}$.
We distribute 2^n pebbles on the detour path $P_4 : a_2^{'}a_1^{'}a_na_1a_2\cdots a_{n-3}a_{n-2}$ which is of length n. So we can move a pebble to a_{n-2} using P_4. By symmetry the follows for $\omega = a_r$, $r \ne 1, 2, 3, 4, n-1, n$.
Case 4. Let $\omega = a_1^{'}$.
We distribute 2^{n+1} pebbles on the path $P_5 : a_2a_1a_na_{n-1}\cdots a_3a_2^{'}a_1^{'}$ which is a detour path of length $n+1$. Obviously we can put a pebble on $a_1^{'}$ using the path P_5. By symmetry the proof follows for $\omega = a_2^{'}$.
Hence $f^*(G) = 2^{n+1}$. □

Theorem 2.3. *For quadrilateral snake Q_n, $f^*(Q_n) = 2^{3n}$.*

Proof. Let $V(Q_n) = \{u_r : 1 \le r \le n+1\} \cup \{v_r, w_r : 1 \le r \le n\}$ and $E(Q_n) = \{u_ru_{r+1}, v_rw_r, w_ru_{r+1}, u_rv_r : 1 \le r \le n\}$. Put $2^{3n}-1$ pebbles on v_1. Then we cannot move a pebble to u_{n+1}. Since the length of the detour path from v_1 to u_{n+1} is $3n$. Thus $f^*(Q_n) \ge 2^{3n}$.
Let us now prove the sufficiency part, let D be any configuration of 2^{3n} pebbles on $V(Q_n)$.
Case 1. Let $\omega = u_{n+1}$.
Consider the detour path $P : u_1v_1w_1u_2v_2w_2\cdots u_nv_nw_nu_{n+1}$. Through the path P we can put a pebble on ω. Through the path P we can put a pebble on ω. By symmetry the proof follows for $\omega = u_1$.
Case 2. Let $\omega = u_r, 2 \le r \le n$.
Consider the path $P_1 : u_1v_1w_1u_2v_2w_2\cdots u_r$ of length $3r-3$ and the path $P_2 : u_rv_rw_ru_{r+1}\cdots u_{n+1}$ of length $3n-3r+3$. Either P_1 contains at least 2^{3r-3} pebbles or P_2 contains at least $2^{3n-3r+3}$ pebbles. Using the pebbles in either P_1 or P_2 we can move a pebble to ω using fewer than 2^{3n} pebbles.
Case 3. Let $\omega = v_r, 1 \le r \le n$.
We consider the paths $P_3 : u_1v_1w_1u_2v_2w_2\cdots u_ru_{r+1}w_rv_r$ of length $3r$ and the paths $P_4 : v_rw_ru_{r+1}v_{r+1}w_{r+1}\cdots u_{n+1}$ of length $3n-3r+2$. Then either P_3 contains 2^{3r} pebbles or P_4 contains atleast $2^{3n-3r+2}$ pebbles. Using the pebbles in either P_3 or P_4 we can move a pebble to ω using fewer than 2^{3n} pebbles.
Case 4. Let $\omega = w_r, 1 \le r \le n$.
The paths $P_5 : u_1v_1w_1u_2v_2w_2\cdots u_rv_rw_r$ is of length $3r-1$ and the paths $P_4 : w_rv_ru_ru_{r+1}v_{r+1}w_{r+1}\cdots u_nv_n$ is of length $3r$. Then either P_3 contains 2^{3r-1} pebbles or P_4 contains at least 2^{3r}

pebbles. Then we can move a pebble to ω through the path P_5 or P_6 using fewer than 2^{3n}. Hence, $f^*(Q_n) = 2^{3n}$. □

Theorem 2.4. *Let $G = C_m *_e C_n$. Then G is obtained by identifying of an edge of C_m with an edge of C_n. Then $f^*(G) = 2^{m+n-3}$.*

Proof. Let $V(C_m) = \{a_1, a_2 \cdots a_m\}$ and $V(C_n) = \{b_1, b_2, \cdots b_n\}$. Let as identify $a_1 a_m$ in C_m with $b_n b_1$ in C_n. The detour path $P_1 : b_{n-1} b_{n-2} \cdots b_2 a_m a_1 a_2 \cdots a_{m-1}$ is of length $m + n - 3$. By placing $2^{m+n-3} - 1$ pebbles that it is not possible to move a pebble to a_{m-1}. This leads to $f^*(G) \geq 2^{m+n-3}$.
Let us now we prove the sufficiency part. Let D be any configuration of 2^{m+n-3} pebbles on $V(G)$.
Case 1. Let $\omega = a_1$.
The detour path $P_2 : b_{n-1} b_{n-2} \cdots b_2 a_m a_{m-1} \cdots a_2 a_1$ is of length $m+n-3$. By placing 2^{m+n-3} pebbles on P_2 we can put a pebble on a_1. By symmetry the proof follows for $\omega = a_r, 2 \leq r \leq m$.
Case 2. Let $\omega = b_{n-1}$.
The detour path $P_3 : a_1 a_2 \cdots a_{m-1} a_m b_2 b_3 \cdots b_{n-1}$ is of length $m + n - 3$. So we reach ω. By symmetry the proof follows for $\omega = b_r, 3 \leq r \leq n - 1$.
Hence, $f^*(G) = 2^{m+n-3}$ □

Theorem 2.5. *For $P_2 \odot C_n$, $f^*(P_2 \odot C_n) = 2^{2n+1}$.*

Proof. Let $V(P_2 \odot C_n) = \{a_s, a_r^s : 1 \leq r \leq n, 1 \leq s \leq 2\}$ and $E(P_2 \odot C_n) = \{a_r^s a_{r+1}^s : 1 \leq r \leq n-1, 1 \leq s \leq 2\} \cup \{a_s a_r^s : 1 \leq r \leq n, 1 \leq s \leq 2\} \cup \{a_1^s a_n^s : 1 \leq s \leq 2\} \cup \{a_1 a_2\}$. The detour path from a_n^1 to a_n^2 is of length $2n + 1$. So if we place $2^{2n+1} - 1$ pebbles on a_n^1 it is not possible to move a pebble to a_n^2. Hence $f^*(P_2 \odot C_n) \geq 2^{2n+1}$.
Let us now prove the sufficient condition.
Case 1. Let $\omega = a_1^1$.
Consider the detour path $P_1 : a_n^2 a_{n-1}^2 \cdots a_1^2 a_2 a_1 a_n^1 a_{n-1}^1 \cdots a_1^1$. Using 2^{2n+1} pebbles through the path P_1, we can move a pebble to a_1^1. By symmetry the proof follows for $\omega = a_r^s : 1 \leq r \leq n, 1 \leq s \leq 2$.
Case 2. Let $\omega = a_1$.
Consider the detour path $P_2 : a_n^2 a_{n-1}^2 \cdots a_1^2 b_2 b_1$. Using 2^{n+1} pebbles on P_2 and n pebbles on $\tilde{P}_2$ we can put a pebble on a_1. By symmetry the proof follows for $\omega = a_2$.
Hence, $f^*(P_2 \odot C_n) = 2^{2n+1}$. □

Theorem 2.6. *For H_n, $f^*(H_n) = 2^{n+1} + (n - 1)$.*

Proof. Let $V(H_n) = \{a, a_r, b_r : 1 \leq r \leq n\}$ and $E(H_n) = \{a_r a_{r+1}, a_n a_1 : 1 \leq r \leq n - 1\} \bigcup \{a_r b_r, a a_r : 1 \leq r \leq n\}$. Consider the detour path $P_1 : b_1 a_1 a_2 \cdots a_n a$. Let $\omega = a$. Placing $2^{n+1} - 1$ pebbles on b_1 and 1 pebble each on b_k where $k \neq 1$ we cannot reach ω. Hence, $f^*(H_n) \geq 2^{n+1} + (n - 1)$.
Now we prove the sufficient condition. Let D be a configuration having $2^{n+1} + (n - 1)$ pebbles on $V(H_n)$.
Case 1. Let $\omega = b_1$.
Consider the detour path $P_2 : a a_n a_{n-1} \cdots a_1 b_1$. The distance between a and b_1 is $n + 1$. Using 2^{n+1} pebbles on P_2 we can transfer a pebble to b_1, without using $n - 1$ pebbles.
Subcase 1.1 $p(V(P_2)) < 2^{n+1}$ and $p(V(\tilde{P}_2)) \geq n - 1$.
If there exist more than one pebble on a vertex in $V(\tilde{P}_2)$ we can shift at least 1 pebble to the detour path P_2. Then using $2^{n+1} - 1$ pebbles on the detour path P_2 we can move a pebble to ω. If the vertices of $\tilde{P}_2$ has at least two pebbles each then using $2^n + (2^{n-2})$ pebbles on the detour path P_2 we can put a pebble to ω. If the detour path has zero pebbles then using at least $2^n + (n - 2)$ pebbles on $V(\tilde{P}_2)$ we can move a pebble to b_1. By symmetry the proof follows for $\omega = b_r, 2 \leq r \leq n$ and $\omega = a$.
Case 2. Let $\omega = a_1$.
In the detour path $P_3 : a a_n a_{n-1} \cdots a_1$ the distance from a to a_1 is n. Then distributing 2^n pebbles on the detour path P_3 we can reach a_1. If the number of pebbles on $V(P_3)$ is less than 2^n then we need to transfer pebbles from $\tilde{P}_3$. Now we consider the following subcase.
Subcase 2.1 $p(V(P_3)) < 2^n$ and $p(V(\tilde{P}_3)) \geq n - 1$.

If there exist more than one pebble on a vertex in $V(\tilde{P}_3)$ we can shift at least 1 pebble to the detour path P_3. If $p(V(P_3)) = 2^n - 1$ we can reach the destination. If the vertices of $\tilde{P}_3$ contain at least two pebbles each then using 2 pebbles on the detour path we can move a pebble to a_1. If the detour path has zero pebbles then using at least $2^{n+1} + (n-2)$ pebbles on $V(\tilde{P}_3)$ we reach a_1. By symmetry proof follows for $a_r, 2 \leq r \leq n$. Hence, $f^*(H_n) = 2^{n+1} + (n-1)$. □

Theorem 2.7. *For CH_n, $f^*(CH_n) = 2^{2n}$.*

Proof. Let $V(CH_n) = \{a, a_r, b_r; 1 \leq r \leq n\}$ and $E(CH_n) = \{aa_r, a_rb_r : 1 \leq r \leq n\} \cup \{a_ra_{r+1}, b_rb_{r+1} : 1 \leq r \leq n-1\}$. Since the detour distance from a to b_1 is $2n$. By placing $2^{2n} - 1$ pebbles on a, we cannot transfer a pebble to b_1. Thus $f^*(CH_n) \geq 2^{2n}$.
We now prove the sufficiency part. Let us consider any distribution of 2^{2n} pebbles on CH_n.
Case 1. Let $\omega = b_n$.
The Path $P_1 : aa_na_{n-1}\cdots a_1b_1b_2\cdots b_n$ is the detour path of length $2n$. Using 2^{2n} pebbles on the spanning path P_1 we can reach b_n. By symmetry the proof follows for $\omega = b_r, 1 \leq r \leq n-1$ and $\omega = a$.
Case 2. Let $\omega = a_1$.
Consider the detour path $P_2 : b_1b_2\cdots b_na_na_{n-1}\cdots a_1$. Note that the detour path between b_1 and a_1 is $2n-1$. Using 2^{2n-1} pebbles on the detour path P_2, we can reach a_1. By symmetry the proof follows for $\omega = a_r, 2 \leq r \leq n$.
Hence, $f^*(CH_n) = 2^{2n}$. □

Theorem 2.8. *For Wb_n, $f^*(Wb_n) = 2^{2n+1} + (n-1)$.*

Proof. Let $V(Wb_n) = \{a, a_r, b_r, u_r; 1 \leq r \leq n\}$ and $E(CH_n) = \{aa_r, a_rb_r, b_ru_r : 1 \leq r \leq n\} \cup \{a_ra_{r+1}, b_rb_{r+1} : 1 \leq r \leq n-1\}$. Consider the detour path $P_1 : u_1b_1b_2\cdots b_na_na_{n-1}\cdots a_2a_1a$. Let $\omega = a$. Placing $2^{n+1} - 1$ pebbles on u_1 and 1 pebble each on u_k where $k \neq 1$ we cannot reach ω. Thus $f^*(Wb_n) \geq 2^{2n+1} + (n-1)$.
Now for proving the sufficient condition. Cnsider a configuration of $2^{2n+1} + (n-1)$ pebbles on $V(Wb_n)$.
Case 1. Let $\omega = u_1$.
In the detour path $P_2 : aa_1a_2\cdots a_nb_nb_{n-1}\cdots b_1u_1$ the distance between a and u_1 is $2n+1$. Let $p(V(P_2)) = 2^{2n+1}$. Then we can transfer a pebble to u_1, without using $n-1$ pebbles.
Subcase 1.1 $p(V(P_2)) < 2^{2n+1}$ and $p(V(\tilde{P}_2)) \geq n-1$.
If there exist more than one pebble on a vertex in $V(\tilde{P}_2)$ we can shift at least 1 pebble to the detour path P_2. Then using $2^{2n+1} - 1$ pebbles on P_2 we can put a pebble on ω. If the vertices of $\tilde{P}_2$ have at least two pebbles each then using $2^{2n} + (2^{n+1})$ pebbles on P_2 we can reach ω. If the detour path has zero pebbles then using at least $2^{n+3} + (n-2)$ pebbles on $V(\tilde{P}_2)$ we put a pebble on u_1. By symmetry the proof follows for $\omega = u_r, 2 \leq r \leq n$.
Case 2. Let $\omega = b_1$.
Consider the detour path $P_3 : aa_1a_2\cdots a_nb_nb_{n-1}\cdots b_1$. The detour distance from a to b_1 is $2n$. Then distributing 2^{2n} pebbles on P_3 we can reach b_1. If the number of pebbles on the vertices of P_3 is less than 2^n pebbles then we need to transfer pebbles from $\tilde{P}_3$. Now we consider the following subcase.
Subcase 2.1 $p(V(P_3)) < 2^{2n}$ and $p(V(\tilde{P}_3)) \geq n-1$.
If there exist more than one pebble on a vertex in $V(\tilde{P}_3)$ we can shift at least 1 pebble to the detour path P_3. If $p(V(P_3)) = 2^{2n} - 1$ we can reach the destination. If the vertices of $\tilde{P}_3$ contain at least two pebbles each, we can move a pebble on b_1. If the detour path has zero pebbles then using at least $2^{n+2} + (n-2)$ pebbles on $V(\tilde{P}_3)$ we can reach b_1. By symmetry the proof follows for $\omega = b_r, 2 \leq r \leq n$.
Case 3. Let $\omega = a_1$.
Consider the detour path $P_4 : u_1b_1b_2\cdots b_na_na_{n-1}\cdots a_1$. Note that the distance from u_1 to a_1 is $2n$. Hence we are done by case 2.
Case 4. Let a be a target.
In the detour path $P_5 : u_1b_1b_2\cdots b_na_na_{n-1}\cdots a_1a$ the distance from u_1 to a is $2n+1$. Then by case 1, we can reach a.
Hence, $f^*(Wb_n) = 2^{2n+1} + (n-1)$. □

Theorem 2.9. *For G_n, $f^*(G_n) = 2^{2n}$.*

Proof. Let $V(G_n) = \{a, a_r, b_r : 1 \leq r \leq n\}$. Let $E(G_n) = \{aa_r, a_rb_r : 1 \leq r \leq n\} \cup \{b_ra_{r+1}, b_na_1 : 1 \leq r \leq n-1\}$. Note that the detour path from a to b_1 has length $2n$. Therefore by placing $2^{2n} - 1$ pebbles on a we cannot put a pebble on b_1. Therefore, $f^*(G_n) \geq 2^{2n}$.
For proving the sufficient condition, let D be a configuration having 2^{2n} pebbles on G_n.
Case 1. Let $\omega = b_n$.
The detour path $P_1 : aa_1b_1a_2b_2 \cdots a_nb_n$ is of length $2n$. So it is sufficient to have 2^{2n} pebbles on P_1 to reach ω. By symmetry the proof follows for $\omega = b_r, 2 \leq r \leq n$.
Case 2. Let $\omega = a_1$.
The detour path $P_2 : b_na_nb_{n-1}a_{n-1} \cdots b_2a_2b_1a_1$ is of length $2n-1$. So we can reach ω using 2^{2n-1} pebbles. By symmetry the proof follows for $\omega = a_r, 2 \leq r \leq n$.
Case 3. Let $\omega = a$.
The detour path $P_3 : b_na_nb_{n-1}a_{n-1} \cdots b_2a_2b_1a_1a$ is of length $2n$. So we can reach ω using 2^{2n} pebbles.
Hence, $f^*(G_n) = 2^{2n}$. □

3 Conclusion

In this paper, we have computed detour pebbling number of cycle related graphs and wheel related graphs. Therefore, the results of this work are variant, significant and so it is interesting and capable to develop its study in the future.

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