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Heron triangles with polynomial value sides. II

Yong Zhang and Mei Jiang

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Corresponding Author: Yong Zhang

Abstract Using the method of undetermined coefficients, we prove that there are infinitely many isosceles Heron triangles whose sides are $(x(x-B), y(Cy+D), y(Cy+D))$, where B, C, D are positive integers. By the theory of Pellian equation, we show that there exist an infinity of isosceles Heron triangles whose sides are $(Bx^2+2CDEx+4CD^2-2B, Cy^2-CEy+2CD^2-B, Cy^2-CEy+2CD^2-B)$, where B, C, D, E are positive integers such that $4C^2D^4-B^2>0$ is not a perfect square, and $(f(x), g(y), g(y))$, where $f(x)$ and $g(y)$ are three classes of polynomials with degree $n \geq 3$. These results give a partial answer to Question 3.3 of Jiang and Zhang [Acta Math. Hungar., **165(2)**, 275–286, (2021)].

1 Introduction

A Pythagorean triangle is a right triangle with integer side lengths. A Heron triangle is a triangle having integer side lengths and integer area. Therefore, a Pythagorean triangle is a special Heron triangle. There are many results concerning Pythagorean (Heron) triangles whose two or three sides are special numbers or polynomial values. We can refer to [1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29] and the references given there.

In 2019, Peng and Zhang [15, 16] showed that there are infinitely many classes of isosceles Heron triangles whose sides are triangular numbers, and infinitely many isosceles Heron triangles whose sides are polygonal numbers $\big(P_n^x, P_n^y, P_n^y\big)$ for $n \geq 5$, and binomial coefficients $\big(\binom{x}{n}, \binom{y}{n}, \binom{y}{n}\big)$ for $n \geq 3$, where

$$P_n^x = \frac{x\big((n-2)(x-1)+2\big)}{2}$$

and

$$\binom{x}{n} = \frac{x(x-1)\cdots(x-n+1)}{n!}.$$

In 2021, we [10] continued the study of [15] and discussed the isosceles Heron triangles with certain polynomial value sides. In fact, we proved that there exist an infinite number of isosceles Heron triangles whose sides are

$$(f_i(x), f_i(y), f_i(y)),\ i = 1, 2, 3, 4,$$

where

$$f_1(x)=\frac{x(Bx-C)}{2},\ B,C\in\mathbb{Z}^+,$$

$$f_2(x)=x\prod_{i=0}^{n-2}(x-k^i),\ n\geq 3,\ k\geq 2,$$

$$f_3(x)=x\prod_{i=0}^{n-2}\left(x-\sum_{j=0}^{i}k^j\right),\ n\geq 3,\ k\geq 2,$$

$$f_4(x)=x\prod_{i=0}^{n-2}(x+ak^i),\ n\geq 3,\ k\geq 2,\ a=4k^{2n}-1,\ \text{or}\ 2k^n-1.$$

In this paper, we consider the isosceles Heron triangles whose sides are of the form

$$\big(f(x),g(y),g(y)\big),$$

where $f(x)$ and $g(y)$ are different polynomials. By the Heron formula, we need to study the positive integer solutions of the Diophantine equation

$$g(y)^2-\left(\frac{f(x)}{2}\right)^2=v^2.$$

Noting that the three sides are positive and satisfy the triangle inequality, and we obtain the condition

$$f(x)>0,\ g(y)>0,\ 2g(y)>f(x). \tag{1.1}$$

Using the method of undetermined coefficients, we get the following theorem.

Theorem 1.1. *Let*

$$f_1(x)=x(x-B),\ g_1(y)=y(Cy+D),$$

where B,C,D are positive integers, then there are infinitely many isosceles Heron triangles whose sides are $(f_1(x),g_1(y),g_1(y))$.

By the theory of the Pellian equation, we obtain the following theorems.

Theorem 1.2. *Let B,C,D,E be positive integers such that $4C^2D^4-B^2>0$ is not a perfect square, and*

$$f_2(x)=Bx^2+2CDEx+4CD^2-2B,\ g_2(y)=Cy^2-CEy+2CD^2-B,$$

then there are an infinite number of isosceles Heron triangles whose sides are of the form $(f_2(x),g_2(y),g_2(y))$.

Theorem 1.3. *Let $n\geq 3$ and $k\geq 2$ be positive integers, if*

$$f_3(x)=(x-B)\prod_{i=0}^{n-2}(x+i),\ g_3(y)=(Cy+D)\prod_{i=-1}^{n-3}(y+i),$$

$$f_4(x)=x(x-B)\prod_{i=0}^{n-3}(x+k^i),\ g_4(y)=y(Cy+D)\prod_{i=1}^{n-2}(y+k^i),$$

$$f_5(x)=x(x-B)\prod_{i=0}^{n-3}\left(x+\sum_{j=0}^{i}k^j\right),\ g_5(y)=(Cy+D)\prod_{i=0}^{n-2}\left(y+\sum_{j=0}^{i}k^j\right),$$

where B,C,D are positive integers, then there exist an infinity of isosceles Heron triangles whose sides are $(f_i(x),g_i(y),g_i(y)),\ i=3,4,5$.

These three Theorems give a partial answer to Question 3.3 in [10].

2 Proofs of the theorems

Proof of Theorem 1.1. Assume that the isosceles Heron triangle has sides

$$(s_1, s_2, s_3) = (f_1(x), g_1(y), g_1(y)),$$

and its area is A, where $f_1(x) = x(x-B),\ g_1(y) = y(Cy+D)$, and B, C, D are positive integers.

By the Heron formula, we get

$$A^2 = \frac{f_1(x)^2}{16} F(x, y, B, C, D),$$

where

$$F(x, y, B, C, D) = 4g_1(y)^2 - f_1(x)^2 = 4y^2(Cy+D)^2 - x^2(x-B)^2.$$

Let $x = at^4 + bt^3 + ct^2 + dt + e,\ y = t(at^4 + bt^3 + ct^2 + dt + e)$, then

$$F(x, y, B, C, D) = (at^4 + bt^3 + ct^2 + dt + e)^2 \left(\sum_{i=0}^{12} a_i t^i\right), \tag{2.1}$$

where

$$\begin{cases} a_0 = -(B-e)^2,\ a_1 = 2(B-e)d,\ a_2 = 2cB + 4D^2 - 2ce - d^2, \\ a_3 = 8eCD + 2bB - 2be - 2cd,\ a_4 = 4e^2C^2 + 8dCD + 2aB - 2ae - 2bd - c^2, \\ a_5 = 8deC^2 + 8cCD - 2ad - 2bc,\ a_6 = 8ceC^2 + 4d^2C^2 + 8bCD - 2ac - b^2, \\ a_7 = 8beC^2 + 8cdC^2 + 8aCD - 2ab,\ a_8 = aeC^2 + 8bdC^2 + 4c^2C^2 - a^2, \\ a_9 = 8(ad+bc)C^2,\ a_{10} = 4(2ac+b^2)C^2,\ a_{11} = 8abC^2,\ a_{12} = 4a^2C^2. \end{cases}$$

According to (2.1), we consider the decomposition

$$\begin{aligned} F(x, y, B, C, D) &= (at^4 + bt^3 + ct^2 + dt + e)^2 \\ &\quad \times \left(2aCt^6 + b_5t^5 + b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0\right)^2. \end{aligned}$$

By the method of undetermined coefficients, we obtain

$$b_5 = 2bC,\ b_4 = 2cC,\ b_3 = 2dC,\ b_2 = \frac{8eC^2 - a}{4C},\ b_1 = \frac{8CD - b}{4C},\ b_0 = \frac{-c}{4C},$$

and

$$a = 16BC^2,\ b = 16CD,\ c = 0,\ d = 0,\ e = B.$$

Then

$$\begin{aligned} x &= 16BC^2t^4 + 16CDt^3 + B, \\ y &= t(16BC^2t^4 + 16CDt^3 + B), \\ A &= 8Ct^4(4Ct^2 - 1)(4Ct^2 + 1)(BCt + D)^2(16BC^2t^4 + 16CDt^3 + B)^2. \end{aligned}$$

In view of B, C, D are positive integers, it is easy to verify that the condition (1.1) is satisfied.

Hence, there are infinitely many isosceles Heron triangles with sides

$$(s_1, s_2, s_3) = (f_1(x), g_1(y), g_1(y)),$$

where x and y are given in above and t is a positive integer parameter. □

Example 2.1. When $B = C = 1, D = 2,\ f_1(x) = x(x-1),\ g_1(y) = y(y+2)$, there exist an infinity of isosceles Heron triangles with sides

$$(s_1, s_2, s_3) = (x(x-1), y(y+2), y(y+2)),$$

where

$$x = 16t^4 + 32t^3 + 1,\ y = t(16t^4 + 32t^3 + 1),\ t \geq 1.$$

Proof of Theorem 1.2. For

$$f_2(x) = Bx^2 + 2CDEx + 4CD^2 - 2B,\ g_2(y) = Cy^2 - CEy + 2CD^2 - B,$$

let $y = Dx + E$, by the Heron formula, we have

$$A^2 = \frac{x^2 f_2(x)^2}{16}\left((4C^2D^4 - B^2)x^2 + 4(2CD^2 - B)CDEx + 4(2CD^2 - B)^2\right). \tag{2.2}$$

To get integral values of x and A, it needs to study the positive integer solutions (x, v) of the quadratic equation

$$(4C^2D^4 - B^2)x^2 + 4(2CD^2 - B)CDEx + 4(2CD^2 - B)^2 = v^2.$$

Put $U = (2CD^2 - B)((2CD^2 + B)x + 2CDE),\ V = v$, we get the Pellian equation

$$U^2 - (4C^2D^4 - B^2)V^2 = 4(2CD^2 - B)^2(B^2 - 4C^2D^4 + C^2D^2E^2). \tag{2.3}$$

If $4C^2D^4 - B^2 > 0$ is not a perfect square, then the Pellian equation $U^2 - (4C^2D^4 - B^2)V^2 = 1$ has infinitely many positive integer solutions. It is easy to verify that

$$(U_0, V_0) = (2CDE(2CD^2 - B), 2(2CD^2 - B))$$

is a positive integer solution of (2.3). Suppose that $(U, V) = (s, t)$ is a positive integer solution of the Pellian equation $U^2 - (4C^2D^4 - B^2)V^2 = 1$. Thus, an infinity of positive integer solutions of (2.3) are given by

$$\begin{aligned} U_m + V_m\sqrt{4C^2D^4 - B^2} &= 2(2CD^2 - B)\left(CDE + \sqrt{4C^2D^4 - B^2}\right) \\ &\quad \times \left(s + t\sqrt{4C^2D^4 - B^2}\right)^m,\ m \geq 0, \end{aligned}$$

which leads to

$$U_m = sU_{m-1} + t(4C^2D^4 - B^2)V_{m-1}, \quad V_m = tU_{m-1} + sV_{m-1}.$$

Then

$$\begin{cases} U_m = 2sU_{m-1} - U_{m-2}, & U_0 = 2(2CD^2 - B)CDE, \\ & U_1 = 2(2CD^2 - B)\left((4C^2D^4 - B^2)t + CDEs\right); \\ V_m = 2sV_{m-1} - V_{m-2}, & V_0 = 2(2CD^2 - B), \\ & V_1 = 2(2CD^2 - B)(CDEt + s). \end{cases}$$

From

$$x = \frac{U - 2CDE(2CD^2 - B)}{4C^2D^4 - B^2}, \quad v = V,$$

we have

$$\begin{cases} x_m = 2sx_{m-1} - x_{m-2} + \dfrac{4(s-1)CDE}{2CD^2 + B}, & x_0 = 0, \\ & x_1 = \dfrac{2\left((4C^2D^4 - B^2)t + CDE(s-1)\right)}{2CD^2 + B}; \\ v_m = 2sv_{m-1} - v_{m-2}, & v_0 = 2(2CD^2 - B), \\ & v_1 = 2(2CD^2 - B)(CDEt + s). \end{cases}$$

Using the recurrence relation of x_m twice, we get

$$x_{m+2} = 2(2s^2 - 1)x_m - x_{m-2} + \frac{8(s^2 - 1)CDE}{2CD^2 + B}.$$

Noting that $s^2 - (4C^2D^4 - B^2)t^2 = 1$, then $s^2 - 1 = (2CD^2 + B)(2CD^2 - B)t^2$. Replacing m by $2m$, we have

$$x_{2m+2} = 2(2s^2 - 1)x_{2m} - x_{2m-2} + 8CDE(2CD^2 - B),$$

where

$$x_0 = 0, \quad x_2 = 4t(CDEt + s)(2CD^2 - B).$$

In view of $4C^2D^4 - B^2 > 0$ and the recurrence relation of x_m, we get

$$x_{2m} \in \mathbb{Z}^+,\ m \geq 1.$$

It can easily be shown that

$$x_{2m} \equiv 0 \pmod 2, \quad v_{2m} \in \mathbb{Z}^+, \quad \text{and} \quad v_{2m} \equiv 0 \pmod 2,\ m \geq 1.$$

Then (2.2) has infinitely many positive integer solutions

$$(x, A) = \left(x_{2m},\ \frac{v_{2m}x_{2m}f_2(x_{2m})}{4}\right).$$

Due to B, C, D, E are positive integers and $4C^2D^4 - B^2 > 0$, we can check that the condition (1.1) is satisfied.

Therefore, there exist an infinite number of isosceles Heron triangles with sides

$$(s_1, s_2, s_3) = \left(f_2(x_{2m}), g_2(Dx_{2m} + E), g_2(Dx_{2m} + E)\right),$$

where $m \geq 1$. □

Example 2.2. When $B = C = D = E = 1$, we have

$$f_2(x) = x^2 + 2x + 2,\ g_2(y) = y^2 - y + 1.$$

By Theorem 1.2, there are infinitely many isosceles Heron triangles with sides

$$(s_1, s_2, s_3) = (x_{2m}^2 + 2x_{2m} + 2,\ x_{2m}^2 + x_{2m} + 1,\ x_{2m}^2 + x_{2m} + 1),$$

where

$$x_{2m+2} = 14x_{2m} - x_{2m-2} + 8, \quad x_0 = 0,\ x_2 = 12,\ m \geq 1.$$

Remark 2.3. In Example 2.2, the polynomials $f_2(x) = x^2 + 2x + 2,\ g_2(y) = y^2 - y + 1$ are irreducible. In fact, if $4C^2D^2E^2 - 16BCD^2 + 8B^2 \neq 0$ and $C(-8CD^2 + CE^2 + 4B) \neq 0$ are not perfect squares, then the polynomials

$$f_2(x) = Bx^2 + 2CDEx + 4CD^2 - 2B \text{ and } g_2(y) = Cy^2 - CEy + 2CD^2 - B$$

are irreducible.

Proof of Theorem 1.3. 1) Consider the isosceles Heron triangle with sides

$$(s_1, s_2, s_3) = (f_3(x), g_3(y), g_3(y)),$$

and its area A, where

$$f_3(x) = (x - B)\prod_{i=0}^{n-2}(x + i),\ g_3(y) = (Cy + D)\prod_{i=-1}^{n-3}(y + i),\ n \geq 3,$$

and B, C, D are positive integers.

By the Heron formula, we have

$$A^2 = \frac{f_3(x)^2}{16}\left(4g_3(y)^2 - f_3(x)^2\right).$$

Let $y = x + 1$, then

$$A^2 = \frac{f_3(x)^2}{16}\left(\prod_{i=0}^{n-2}(x+i)\right)^2 \left(4(C(x+1)+D)^2 - (x-B)^2\right). \tag{2.4}$$

To obtain integral values of x and A, let us investigate the positive integer solutions (x, v) of the quadratic equation

$$4(C(x+1)+D)^2 - (x-B)^2 = v^2.$$

Put $U = (4C^2-1)x + 4C^2 + 4CD + B,\ V = v$, we get the Pellian equation

$$U^2 - (4C^2-1)V^2 = 4(BC+C+D)^2. \tag{2.5}$$

If C is a positive integer, then $4C^2 - 1$ is not a perfect square. It is easy to check that

$$(U_0, V_0) = (4C(BC+C+D), 2(BC+C+D))$$

is a positive integer solution of (2.5), and

$$(U, V) = (2C, 1)$$

is a positive integer solution of $U^2 - (4C^2-1)V^2 = 1$. Thus, an infinity of positive integer solutions of (2.5) are given by

$$U_m + V_m\sqrt{4C^2-1} = \left(4C(BC+C+D) + 2(BC+C+D)\sqrt{4C^2-1}\right) \times \left(2C + \sqrt{4C^2-1}\right)^m,\ m \geq 0.$$

We can complete the proof by the same method of Theorem 1.2.

2) Assume that the isosceles Heron triangle has sides

$$(s_1, s_2, s_3) = (f_4(x), g_4(y), g_4(y)),$$

and its area is A, where

$$f_4(x) = x(x-B)\prod_{i=0}^{n-3}(x+k^i),\ g_4(y) = y(Cy+D)\prod_{i=1}^{n-2}\left(y+k^i\right),\ n \geq 3,\ k \geq 2,$$

and B, C, D are positive integers.

By the Heron formula, we obtain

$$A^2 = \frac{f_4(x)^2}{16}\left(4g_4(y)^2 - f_4(x)^2\right).$$

Let $y = kx$, then

$$A^2 = \frac{f_4(x)^2}{16}\left(x\prod_{i=0}^{n-3}(x+k^i)\right)^2 \left(4k^{2n-2}(Ckx+D)^2 - (x-B)^2\right). \tag{2.6}$$

To get integral values of x and A, it needs to study the positive integer solutions (x, v) of the quadratic equation

$$4k^{2n-2}(Ckx+D)^2 - (x-B)^2 = v^2.$$

Take $U = (4C^2k^{2n} - 1)x + 4CDk^{2n-1} + B,\ V = v$, we have the Pellian equation

$$U^2 - (4C^2k^{2n} - 1)V^2 = 4k^{2n-2}(BCk + D)^2. \tag{2.7}$$

For any positive integer $k \geq 2$, if C is a positive integer, then $4C^2k^{2n} - 1$ is not a perfect square. It can easily be verified that

$$(U_0, V_0) = (4Ck^{2n-1}(BCk + D), 2k^{n-1}(BCk + D))$$

is a positive integer solution of (2.7), and

$$(U, V) = (2Ck^n, 1)$$

is a positive integer solution of $U^2 - (4C^2k^{2n} - 1)V^2 = 1$. The rest of the proof is similar as 1).

3) Consider the isosceles Heron triangle with sides

$$(s_1, s_2, s_3) = (f_5(x), g_5(y), g_5(y)),$$

and its area A, where

$$f_5(x) = x(x - B)\prod_{i=0}^{n-3}\left(x + \sum_{j=0}^{i} k^j\right),$$

$$g_5(y) = (Cy + D)\prod_{i=0}^{n-2}\left(y + \sum_{j=0}^{i} k^j\right),\ n \geq 3,\ k \geq 2,$$

and B, C, D are positive integers.

By the Heron formula, we have

$$A^2 = \frac{f_5(x)^2}{16}\left(4g_5(y)^2 - f_5(x)^2\right).$$

Let $y = kx - 1$, then

$$A^2 = \frac{f_5(x)^2}{16}\left(x\prod_{i=0}^{n-3}(x + \sum_{j=0}^{i} k^j)\right)^2\left(4k^{2n-2}(C(kx-1) + D)^2 - \left(x - B\right)^2\right). \tag{2.8}$$

To obtain integral values of x and A, let us study the positive integer solutions (x, v) of the quadratic equation

$$4k^{2n-2}(C(kx-1) + D)^2 - \left(x - B\right)^2 = v^2.$$

Let $U = (4C^2k^{2n} - 1)x - 4C(C - D)k^{2n-1} + B,\ \ V = v$, we get the Pellian equation

$$U^2 - (4C^2k^{2n} - 1)V^2 = 4k^{2n-2}(BCk - C + D)^2. \tag{2.9}$$

Note that

$$(U_0, V_0) = \left(4Ck^{2n-1}(BCk - C + D), 2k^{n-1}(BCk - C + D)\right)$$

is a positive integer solution of (2.9). We can complete the proof as 1). □

Example 2.4. When $B = 1, C = 2, D = 3, n = 3$, we have

$$f_3(x) = (x - 1)x(x + 1),\ g_3(y) = (y - 1)y(2y + 3).$$

By Theorem 1.3, there are infinitely many isosceles Heron triangles with sides

$$\begin{aligned}(s_1, s_2, s_3) = ((x_{2m} - 1)x_{2m}(x_{2m} + 1),\ & x_{2m}(x_{2m} + 1)(2x_{2m} + 5),\\ & x_{2m}(x_{2m} + 1)(2x_{2m} + 5)),\end{aligned}$$

where

$$x_{2m+2} = 62x_{2m} - x_{2m-2} + 164, \quad x_0 = 1,\ x_2 = 225,\ m \geq 1.$$

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Author information

Yong Zhang, School of Mathematics and Statistics, Changsha University of Science and Technology, China.
E-mail: zhangyongzju@163.com

Mei Jiang, School of Mathematics and Statistics, Changsha University of Science and Technology, China.
E-mail: jiangmeicsust@163.com

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