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QUASI-$*$-$A(n,k)$ COMPOSITION OPERATORS

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Abstract In this paper, we characterize quasi-$*$-A (n, k) composition operators on L^2 space. We give an example for quasi-$*$-A (n, k) composition operator. Also we obtained a necessary and sufficient condition for quasi-$*$-A (n, k) weigted composition operators on L^2 space. More over, we characterize quasi-$*$-A(n, k) composition operators on Fock space.

1 Introduction

Let $B(\mathcal{H})$ denote the algebra of all bounded linear operators on an infinite dimensional complex Hilbert space $\mathcal{H}$. An operator $T \in B(\mathcal{H})$ is said to be hyponormal if $T^*T \geq TT^*$ where T^* is the adjoint of T. $T \in B(\mathcal{H})$ is said to be p-hyponormal if $(T^*T)^p \geq (TT^*)^p$ $(0 < p \leq 1)$, class A if $|T^2| \geq |T|^2$ [7] and $*$-class A if $|T^2| \geq |T^*|^2$ [5] where $|T| = (T^*T)^{\frac{1}{2}}$. An operator $T \in B(\mathcal{H})$ is said to be paranormal if $||Tx||^2 \leq ||T^2x||$ and n-paranormal if $||Tx||^{n+1} \leq ||T^{n+1}x||||x||^n$ for every $x \in \mathcal{H}$ and for $n \in \mathbb{N}$. One of the recent interest in operator theory is to study the extension of hyponormal operators (see for instance [16, 18, 19, 20]). An operator For $n \geq 2$, $k \geq 0$, $T \in B(\mathcal{H})$ is called quasi-$*$-A(n, k) if $T^{*k}|T^n|T^k \geq T^{*k}|T^*|^nT^k$ [19].

Let $(X, \mathscr{F}, \mu)$ be a σ-finite measure space and the map $T : X \to X$ be such that $T^{-1}(S) \in \mathscr{F}$ for all $S \in \mathscr{F}$. A measurable transformation T is said to be nonsingular if $\mu(T^{-1}(S)) = 0$ whenever $\mu(S) = 0$ for all $S \in \mathscr{F}$. If T is a nonsingular measurable transformation on $(X, \mathscr{F}, \mu)$ and if the Radon-Nikodym derivative $\frac{d\mu T^{-1}}{d\mu}$, denoted by h, is essentially bounded, then the composition operator C on $L^2(\mu)$ induced by T is given by $Cf = f \circ T$ for every $f \in L^2(\mu)$. For $k \in \mathbb{N}$ we have $C^k f = f \circ T^k$ for $f \in L^2(\mu)$. We denote h_k as the Radom-Nikodym derivative of $\mu \circ (T^{-1})^k$.

If $T^{-1}(\mathscr{F}) \subset \mathscr{F}$, then there exists an operator $E : L^p(\mathscr{F}) \to L^p(T^{-1}\mathscr{F})$ called conditional expectation operator which is uniquely determined by the following conditions : $E(f)$ is $T^{-1}(\mathscr{F})$ measurable and if S is any $T^{-1}(\mathscr{F})$ measurable set for which $\int_S f d\mu$ converges, then $\int_S f d\mu = \int_S E(f) d\mu$. It is known that for $f, g \in L^2(\mu)$, $E(g) = g$ if and only if g is $T^{-1}(\mathscr{F})$ measurable, if g is $T^{-1}(\mathscr{F})$ measurable, then $E(fg) = E(f)g$, $E(fg \circ T) = (E(f))(g \circ T)$ and $E(E(f)g) = E(f)E(g)$, $E(1) = 1$ hold for each $\mathscr{F}$-measurable g. E is the identity operator in $L^2(\mu)$ if and only if $T^{-1}(\mathscr{F}) = \mathscr{F}$, and E is the projection operator from $L^2(\mu)$ onto $\overline{C(L^2(\mu))}$. For more details see [10, 12, 17]. The adjoint C^* of the composition operator C is given by $C^*f = hE(f) \circ T^{-1}$ and $C^{*k}f = h_k E(f) \circ T^{-k}$ for $k \in \mathbb{N}$.

A detailed study of weighted composition operators on $L^2(\mu)$ for quasi-class A has been done by Emamalipour, Jabbarzadeh and Moayyerizadeh [6]. Jabbarzadeh and Azimi characterized weighted composition operators for some weaker p-hyponormal and paranormal classes [11] . Composition operators for (n, k) quasi class Q on Fock space was studied by Braha, Ilmi Hoxha, and Estaremi [1]. The class-p-$A(s, t)$ composition operators has been studied by Prasad [14]. The characterization of (m, n) class Q and (m, n) class Q^* composition operators on L^2 space can be found in [16].

In this paper we study the measure-theoretic characterization of quasi-$*$-$A(n,k)$ composition operators and weighted composition operators on L^2 space and discuss quasi-$*$-$A(n,k)$ composition operator on Fock space.

2 Quasi-$*$-$A(n,k)$ composition operators

In this section, we characterize quasi-$*$-$A(n,k)$ composition operators on $L^2(\mu)$. We begin with the following lemma.

Lemma 2.1. *[21] Let C be the composition induced by T on $L^2(\mu)$ and P be the projection from $L^2(X,\mathscr{F},\mu)$ onto $\overline{R(C)}$. Then for every $f \in L^2(\mu)$, the following holds:*

(i) $CC^*f = (h\circ T)Pf, C^*Cf = hf$.

(ii) $(CC^*)^k f = (h\circ T)^k Pf$ for each $k \in \mathbb{N}$.

(iii) $(C^*C)^k f = h^k f$ for each $k \in \mathbb{N}$.

Theorem 2.2. *Let C be the composition operator on $L^2(\mu)$. Then C is a quasi-$*$-A(n, k) operator if and only if $h_k E((h_n)^{\frac{1}{2}}) \geq h_k E((h\circ T)^{\frac{n}{2}})\circ T^{-k}$, where E is the conditional expectation operator from $L_p(\mathscr{F})$ to $L_p(T^{-1}(\mathscr{F}))$.*

Proof. C is quasi-$*$-A(n,k) if and only if

$$\langle C^{*k}|C^n|C^k - C^{*k}|C^*|^nC^k f, f\rangle = \langle C^{*k}(C^{*n}C^n)^{\frac{1}{2}}C^k - C^{*k}((CC^*)^{\frac{1}{2}})^nC^k f, f\rangle \geq 0, \quad (2.1)$$

for $f \in L^2(\mu)$. Now,

$$\begin{aligned} C^{*k}(C^{*n}C^n)^{\frac{1}{2}}C^k f &= C^{*k}(C^{*n}C^n)^{\frac{1}{2}}(f\circ T^k) \\ &= C^{*k}h_n^{\frac{1}{2}}(f\circ T^k) \\ &= h_k E(h_n^{\frac{1}{2}}(f\circ T^k))\circ T^{-k} \\ &= h_k E(h_n^{\frac{1}{2}})f. \end{aligned}$$

Also,

$$\begin{aligned} C^{*k}((CC^*)^{\frac{1}{2}})^nC^k f &= C^{*k}((CC^*)^{\frac{1}{2}})^n(f\circ T^k) \\ &= C^{*k}(h\circ T)^{\frac{n}{2}}E(f\circ T^k) \\ &= h_k E((h\circ T)^{\frac{n}{2}}E(f\circ T^k))\circ T^{-k} \\ &= h_k E((h\circ T)^{\frac{n}{2}})E(f\circ T^k)\circ T^{-k} \\ &= h_k E((h\circ T)^{\frac{n}{2}})\circ T^{-k} f. \end{aligned}$$

Hence,

$$C^{*k}(C^{*n}C^n)^{\frac{1}{2}}C^k - C^{*k}((CC^*)^{\frac{1}{2}})^nC^k \geq 0$$

if and only if

$$h_k E(h_n^{\frac{1}{2}}) - h_k E((h\circ T)^{\frac{n}{2}})\circ T^{-k} \geq 0$$

.
Thus C is quasi-$*$-A(n,k) if and only if $h_k E(h_n^{\frac{1}{2}}) \geq h_k E((h\circ T)^{\frac{n}{2}} \circ T^{-k}$. □

Example 2.3. Let $X = \mathbb{N}$ with counting measure and $T : X \to X$ be defined by

$$T(n) = \begin{cases} 1 \text{ for } 1 \leq n \leq 9 \\ k+1 \text{ for } n = 10k+m,\ k \geq 1 \text{ and } 0 \leq m \leq 9. \end{cases}$$

.
Then

$$h(k) = \begin{cases} 9 \text{ for } k = 1 \\ 10 \text{ for } k > 1 \end{cases} \quad \text{and}$$
$$h_2(k) = \begin{cases} 89 \text{ for } k = 1 \\ 100 \text{ for } k > 1. \end{cases}$$

Hence $h(h_2)^{\frac{1}{2}} \geq h^2$. Thus C is a quasi-$*$-$A(2,1)$ operator.

Theorem 2.4. *Let C be the composition operator on $L^2(\mu)$. Then C^* is a quasi-$*$-A(n, k) operator if and only if $(h_n \circ T^n)^{\frac{1}{2}} E(h_k) \circ T^k \geq (h^{\frac{n}{2}} h_k) \circ T^k$.*

Proof. By definition, C^* is quasi-$*$-A(n,k) if and only if

$$\langle C^k (C^n C^{*n})^{\frac{1}{2}} C^{*k} - C^k ((C^* C)^{\frac{1}{2}})^n C^{*k} f, f \rangle \geq 0 \text{ for all } f \in L^2(\mu). \tag{2.2}$$

For $f \in L^2(\mu)$, we have

$$\begin{aligned} C^k (C^n C^{*n})^{\frac{1}{2}} C^{*k} f &= C^k (C^n C^{*n})^{\frac{1}{2}} h_k E(f) \circ T^{-k} \\ &= C^k (h_n \circ T^n)^{\frac{1}{2}} E(h_k E(f) \circ T^{-k}) \\ &= [(h_n \circ T^n)^{\frac{1}{2}} E(h_k E(f) \circ T^{-k})] \circ T^k \\ &= (h_n \circ T^n)^{\frac{1}{2}} E(h_k) \circ T^k f. \end{aligned}$$

Also,

$$\begin{aligned} C^k ((C^* C)^{\frac{1}{2}})^n C^{*k} f &= C^k ((C^* C)^{\frac{1}{2}})^n h_k E(f) \circ T^{-k} \\ &= C^k h^{\frac{n}{2}} h_k E(f) \circ T^{-k} \\ &= h^{\frac{n}{2}} h_k \circ T^k f. \end{aligned}$$

Thus,

$$C^k (C^n C^{*n})^{\frac{1}{2}} C^{*k} - C^k ((C^* C)^{\frac{1}{2}})^n C^{*k} \geq 0$$

if and only if

$$(h_n \circ T^n)^{\frac{1}{2}} E(h_k) \circ T^k \geq h^{\frac{n}{2}} h_k \circ T^k.$$

□

3 Quasi-$*$-$A(n,k)$ Weighted composition operators on $L^2(\mu)$

In this section, we study the weighted composition operators on L^2 space for quasi-$*$-$A(n,k)$ operators. Let $(X, \mathscr{F}, \mu)$ be a σ-finite measure space. The weighted composition operator $W_{\pi,T}$ induced by a measurable transformation π and T is given by $W_{\pi,T} f = Wf = \pi \cdot f \circ T$ for $f \in L^2(\mu)$. We have, $W^k f = \pi_k \cdot f \circ T^k$ for $k \in \mathbb{N}$, where $\pi_k = \pi \cdot \pi \circ T \cdots \pi \circ T^{k-1}$. The adjoint W^* of W is given by $W^* f = h \cdot E(\bar{\pi} \cdot f) \circ T^{-1}$. For $f \in L^2(\mu)$, we have $W^{*k} f = h_k E(\bar{\pi_k} f) \circ T^{-k}$, $(W^* W) f = hE(|\pi|^2) \circ T^{-1} f = Jf$, $(WW^* f) = \pi(h \circ T) E(\bar{\pi} f)$, $W^{*k} W^k f = h_k E(|\pi_k|^2) \circ T^{-k} f = J_k f$, and $(W^k W^{*k} f) = \pi_k (h_k \circ T^k) E(\bar{\pi_k} f)$ ([3, 6, 11]).

Theorem 3.1. *The weighted composition operator on $L^2(\mu)$, W is quasi-$*$-A(n,k) if and only if $h_k E[\bar{\pi_k} (J_n)^{\frac{1}{2}} \pi_k] \circ T^{-k} \geq h_k E[\bar{\pi_k} (\pi (h \circ T) E(\bar{\pi})^{\frac{n}{2}}) \pi_k] \circ T^{-k}$.*

Proof. By definition, W is quasi-$*$-A(n,k) if and only if

$$\langle W^{*k}(W^{*n}W^n)^{\frac{1}{2}}W^k - W^{*k}((WW^*)^{\frac{1}{2}})^n W^k f, f\rangle \geq 0\,\forall\, f \in L^2(\mu)$$

For f in $L^2(\mu)$, we have

$$\begin{aligned} W^{*k}(W^{*n}W^n)^{\frac{1}{2}}W^k f &= W^{*k}(W^{*n}W^n)^{\frac{1}{2}}\pi_k(f \circ T^k) \\ &= W^{*k}(J_n)^{\frac{1}{2}}\pi_k(f \circ T^k) \\ &= h_k E[\bar{\pi_k}(J_n)^{\frac{1}{2}}\pi_k(f \circ T^k)] \circ T^{-k} \\ &= h_k E[\bar{\pi_k}(J_n)^{\frac{1}{2}}\pi_k] \circ T^{-k} f. \end{aligned}$$

Now,

$$\begin{aligned} W^{*k}((WW^*)^{\frac{1}{2}})^n W^k f &= W^{*k}((WW^*)^{\frac{1}{2}})^n \pi_k(f \circ T^k) \\ &= W^{*k}[\pi(h \circ T)E(\bar{\pi})]^{\frac{n}{2}}\pi_k(f \circ T^k) \\ &= h_k E[\bar{\pi_k}(\pi(h \circ T)E(\bar{\pi}))^{\frac{n}{2}}\pi_k(f \circ T^k)] \circ T^{-k} \\ &= h_k E[\bar{\pi_k}(\pi(h \circ T)E(\bar{\pi})^{\frac{n}{2}})\pi_k] \circ T^{-k} f. \end{aligned}$$

Thus W is quasi-$*$-A(n,k) if and only if

$$h_k E[\bar{\pi_k}(J_n)^{\frac{1}{2}}\pi_k] \circ T^{-k} \geq h_k E[\bar{\pi_k}(\pi(h \circ T)E(\bar{\pi})^{\frac{n}{2}})\pi_k] \circ T^{-k}.$$

□

Theorem 3.2. *W^* is quasi-$*$-A(n,k) if and only if*

$$\langle \pi_k \circ T^k[\pi_n(h_n \circ T^n)E(\bar{\pi_n})]^{\frac{1}{2}}(h_k E(\bar{\pi_k} f) - \pi_k \circ T^k (J)^{\frac{n}{2}} h_k E(\bar{\pi_k} f), f\rangle \geq 0$$

for every $f \in L^2(\mu)$.

Proof. By definition, W^* is quasi-$*$-A(n,k) if and only if

$$\langle (W^k(W^nW^{*n})^{\frac{1}{2}}W^{*k} - W^k(W^*W)^{\frac{n}{2}}W^{*k})f, f\rangle \geq 0 \text{ forall } f \in L^2(\mu).$$

We have,

$$\begin{aligned} W^k(W^nW^{*n})^{\frac{1}{2}}W^{*k} f &= W^k(W^nW^{*n})^{\frac{1}{2}}(h_k E(\bar{\pi_k} f) \circ T^{-k}) \\ &= W^k[\pi_n(h_n \circ T^n)E(\bar{\pi_n})]^{\frac{1}{2}}(h_k E(\bar{\pi_k} f) \circ T^{-k}) \\ &= \pi_k\{[\pi_n(h_n \circ T^n)E(\bar{\pi_n})]^{\frac{1}{2}}(h_k E(\bar{\pi_k} f) \circ T^{-k})\} \circ T^k \\ &= \pi_k \circ T^k.[\pi_n(h_n \circ T^n)E(\bar{\pi_n})]^{\frac{1}{2}}(h_k E(\bar{\pi_k} f). \end{aligned}$$

Also,

$$\begin{aligned} W^k(W^*W)^{\frac{n}{2}}W^{*k} f &= W^k(W^*W)^{\frac{n}{2}}(h_k E(\bar{\pi_k} f) \circ T^{-k}) \\ &= W^k(J)^{\frac{n}{2}}(h_k E(\bar{\pi_k} f) \circ T^{-k}) \\ &= \pi_k[(J)^{\frac{n}{2}}(h_k E(\bar{\pi_k} f) \circ T^{-k})] \circ T^k \\ &= \pi_k \circ T^k (J)^{\frac{n}{2}} h_k E(\bar{\pi_k} f). \end{aligned}$$

Thus W^* is quasi-$*$-A(n,k) if and only if

$$\langle \pi_k \circ T^k.[\pi_n(h_n \circ T^n)E(\bar{\pi_n})]^{\frac{1}{2}}(h_k E(\bar{\pi_k} f) - \pi_k \circ T^k (J)^{\frac{n}{2}} h_k E(\bar{\pi_k} f), f\rangle \geq 0$$

for every $f \in L^2(\mu)$. □

4 Quasi-$*$-$A(n,k)$ composition operators on Fock-space

In this section, we characterize quasi-$*$-$A(n,k)$ composition operators on Fock-space with $n = 2l$ and $k = 1$.

The Hilbert space of all holomorphic functions on $\mathbb{C}^m$ with inner product

$$\langle f, g\rangle = \frac{1}{(2\pi)^m}\int_{\mathbb{C}^m} f(z)\overline{g(z)}e^{\frac{-1}{2}|z|^2}d\nu(z),$$

where ν denote the Lebesgue measure on $\mathbb{C}^m$ is called Fock space and is denoted by $\mathscr{F}_m^2$. The functions $k_w(z) = e^{\frac{\langle z,w\rangle}{2}}$, with $\langle z, w\rangle = \sum_1^m z_j\overline{w_j}$ are the reproducing kernel function for the Fock space $\mathscr{F}_m^2$. The composition operator $C_\phi : \mathscr{F}_m^2 \to \mathscr{F}_m^2$ is defined by $C_\phi(f) = f\circ\phi$, $f \in \mathscr{F}_m^2$, where $\phi : \mathbb{C}^m \to \mathbb{C}^m$ is a given holomorphic mapping . For an entire function u on $\mathscr{F}_m^2$, the multiplication operator M_u induced by u is defined as $M_u f(z) = u(z)f(z)$ for an entire function f.

Lemma 4.1. *[4] If $\phi(z) = Az + B$ where A is an $m\times m$ matrix having $||A|| \le 1$ and B is an $m\times 1$ vector, and if $|A\zeta| = |\zeta|$ for some $\zeta\in\mathbb{C}^m$, then $\langle A\zeta, B\rangle = 0$. So that C_ϕ is bounded and $C_\phi^* = M_{k_B}C_\tau$ where $\tau(z) = A^*z$ and M_{k_B} is the multiplication by the kernel function k_B.*

Theorem 4.2. *For $n = 2l$, the composition operator C_ϕ on Fock space is quasi-$*$-$A(2l,1)$ if and only if*

$$\langle M_{u_b}C_\tau(M_{u_b}M_{u_b\circ\tau}\cdots M_{U_b\circ\tau^{2l-1}}C_{\phi^{2l}\circ\tau^{2l}})(f\circ\phi) - (M_{u_b}C_\tau)^{l+1}f, f\rangle \ge 0$$

for every $f\in\mathscr{F}_m^2$.

Proof. The composition operator C_ϕ on Fock space is quasi-$*$-$A(2l,1)$ if and only if

$$\langle (C_\phi^*(C_\phi^{*2l}C_\phi^2 l)^{\frac{1}{2}}C_\phi - C_\phi^*(C_\phi C_\phi^*)^{\frac{2l}{2}}C_\phi)f, f\rangle \ge 0.$$

For $f\in\mathscr{F}_m^2$ we have

$$\begin{aligned} C_\phi^*(C_\phi^{*2l}C_\phi^{2l})^{\frac{1}{2}}C_\phi f &= C_\phi^*(C_\phi^{*2l}C_\phi^{2l})^{\frac{1}{2}}(f\circ\phi) \\ &= C_\phi^*(M_{u_b}M_{u_b\circ\tau}\cdots M_{U_b\circ\tau^{2l-1}}C_{\phi^{2l}\circ\tau^{2l}})(f\circ\phi) \\ &= M_{u_b}C_\tau(M_{u_b}M_{u_b\circ\tau}\cdots M_{U_b\circ\tau^{2l-1}}C_{\phi^{2l}\circ\tau^{2l}})(f\circ\phi). \end{aligned}$$

Also,

$$\begin{aligned} C_\phi^*(C_\phi C_\phi^*)^{\frac{n}{2}}C_\phi f &= C_\phi^*C_\phi(C_\phi^*C_\phi)^{l-1}C_\phi^*C_\phi f \\ &= (C_\phi^*C_\phi)^{l+1}f \\ &= (M_{u_b}C_\tau)^{l+1}f. \end{aligned}$$

This completes the proof. □

5 Conclusion remarks

In this paper, we studied measure-theoretic characterization of quasi-$*$-$A(n,k)$ composition operators and weighted composition operators on L^2 space. Also we obtained characterization of quasi-$*$-$A(n,k)$ composition operator on Fock space.

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