

Static Deformation in an Initially Stressed Magneto-Elastic Transversely Isotropic Half-Space affected by Imperfect Interface

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Abstract. In this research paper, we have investigated the deformation behavior of a "magneto elastic transversely isotropic" half-space under the influence of imperfect bonding among the interface, normal loading and the presence of hydrostatic initial stress. The obtained results highlight the deviation from idealized models by incorporating the effect of an imperfect interface causing variation and discontinuity in the displacements and stresses. For the methodology, we have used the Fourier transformation technique, matrix method and Eigenvalue approach. The influence of the imperfect interface on the behavior of the bi-materials is illustrated using graphical representations, mesh grids, and contour maps of the analytical solutions, which assist experimental validation. This analysis employs advanced numerical techniques and mathematical models to state the deformation results, giving insights into the stress distribution and patterns of displacement within the materials.

1 Introduction

Due to its wide range of applications in Seismology, Geology and engineering and technology, the study of deformation in magneto-elastic materials has gained significant attention. Magneto-elastic materials that exhibit coupling behavior between elastic and magnetic fields are pivotal in the design of sensors, activators and other smart devices. In the case of transversely isotropic elastic medium, these materials exhibit unique directional properties complicating their mechanical behavior. For optimizing the reliability and performance of the materials, it is important to understand their characteristics. In governing the coupling between magnetic and elastic field a magneto elastic parameter plays crucial role. Therefore, this study primarily focus on examining a magneto elastic half-space with transverse isotropy under the influence of normal loading and hydrostatic initial stress present on the surface. Additionally, we have introduced a layer of complexity by considering the presence of an imperfect interface due to its wide range of applications in real world as perfect bonding between two mediums is rarely achieved. Leading to deviations from idealized theoretical predictions, the stresses and displacement fields can be significantly influenced by the Imperfect Interfaces.

Seismic deformation is affected by many factors i.e. loading, presence of initial stress, variations in elastic half-spaces, bonding conditions between the interfaces, presence of irregularity in the medium etc. The study of imperfect interfaces began with examining the influence of interfacial imperfections on the overall properties of materials. Primary models and research were based on the impact of voids, cracks, faults and enclosures with in the interface. Eshelby's inclusion problems [17] gave notable contribution which helps for understanding the stress fields around imperfections. "The composite sphere assemblage model" given by Hashin's [28], gave insights into the elastic performance of the composite system. Boutaghou [1] studies the non homogeneous way axial symmetry and continuity of an orthogonal pseudospectral collocation. Mansouri [26] studied the weakly coupled and boundary partially damped plate equation for an indirect stabilization an a system.

Many researchers and seismologist, Head [7], Steketee [16], Maruyama [27], Singh and Garg [24], Madan et al. [12], priyadarshan and poonia [20] etc. gave significant contribution in the study of dislocation theory for distinct interfaces. Yu [15] proposed a new dislocation-like model by introducing a thin layer of interface material near the interface and gave the boundary conditions for the phenomena. Fan and wang [14] showed that the forces acting on a screw dislocation an imperfect interface resides between those associated with a free surface and a perfect interface. They claim that their study is the initial step of the research in "dislocation or imperfect interfaces". Madan et. al [11] studied the impact of imperfect interface and obtained analytical solutions for displacement and stresses affected by various imperfect interface condones and distinct loading. Later Madan and Kumari [3] studied a dislocation-like model caused by a vertical tensile fault and observed the significant difference in deformation graphically. Recently Kumari and Madan [4] has investigated the strike-slip dislocations for two homogeneous orthotropic elastic half-space having linear spring like interface. They derived analytic solutions for displacements and stresses in two homogeneous orthotropic elastic half-spaces connected by a linear spring-like interface, originated by a line source, by including vertical strike-slip fault cases.

The influence of irregularity on deformation in a transversely isotropic elastic media was examined by Madan et al. [10]. Kumar and Saini [22][21] studied the propagation of love waves affecting by various parameter i.e. anisotropy, homogeneity, porosity, irregularity for a fluid saturated porous layer. They [5][6] also showed the variation in propagation of love waves due to rigidity and irregularity for transversely isotropic elastic medium. Chugh et al. [23] investigated how the presence of tangential and normal loads affected the displacement and stresses in an unstressed elastic medium. considering hydrostatic initial stress conditions, Singh et al. [8] examined the effects of "initial stress, irregularity depth, irregularity factor, and magneto-elastic coupling parameter" on the dynamic response resulting from a normal moving load traveling at a constant speed on the force surface of an irregular magneto-elastic half-space with transverse isotropy. Gaba et al. [2] have the results for altered deformation due to irregularity present in isotropic elastic medium with cubic symmetry. Using a basic dislocation model, Pan [19] developed a solution for the static deformation problem for a transversely isotropic and layered

half-space.

Most of the recent research has focused on the advanced models that exhibit the effect of magnetic, thermal and electrical interfaces. For instance, with the help of the phase-field technique, micro structures with imperfect interfaces have been developed considering various external fields. Also these days, the multi-scale modeling techniques connecting continuous and atomistic scales are fetching more attention. These models give more insights into the relationship between various scales and imperfect interfaces.

In this paper, we examine a two-dimensional plane strain problem in a transversely isotropic elastic medium with magnetic elasticity. Our study demonstrates how imperfect bonding within the medium influences deformation and stress distribution. Considering the resisting forces on Earth, we incorporate hydrostatic initial compressive stress into the model. The problem is formulated with a normal line load. We present the effects of magneto-elasticity, initial stress, and imperfect bonding on deformation through detailed graphs and 3-D plots.

2 Construction of the Problem

In this study, we examine an infinite magneto-elastic transversely isotropic half-space influenced by hydrostatic initial stress and imperfect interface, with the x-axis oriented vertically downwards. This problem involves plane strain deformation parallel to the xy-plane, where the displacement components are independent of the z-axis and can be expressed accordingly:

$$u = u(x, y), v = v(x, y), w = 0 \quad (2.1)$$

2.1 Theory

At $x = 0$, assuming a vertical line-load V_N per unit length (Figure 1) is acting vertically downwards on a line parallel to z-axis. Neglecting the mechanical body force, the required equation of equilibrium for transversely isotropic half-space under the influence of hydrostatic stress at initial state and in presence of constant magnetic field can be written as [18]:

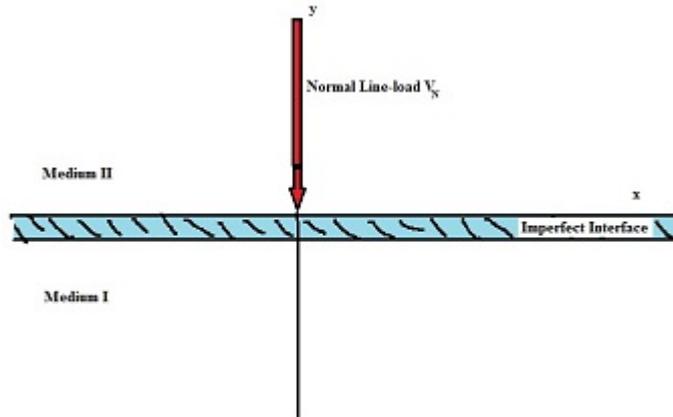


Figure 1. Geometric Framework

$$(A - \rho_o + \lambda) \frac{\partial^2 u}{\partial x^2} + (F + L + \lambda) \frac{\partial^2 v}{\partial x \partial y} + (L - \rho_o) \frac{\partial^2 u}{\partial y^2} = 0 \quad (2.2)$$

$$(L - \rho_o) \frac{\partial^2 v}{\partial x^2} + (F + L + \lambda) \frac{\partial^2 u}{\partial x \partial y} + (A - \rho_o + \lambda) \frac{\partial^2 v}{\partial y^2} = 0 \quad (2.3)$$

Here A , F and L are elastic constant for initially stressed transversely isotropic elastic half-space, ρ_o is the parameter for hydrostatic initial stress and $\frac{\lambda}{L}$ is used as magneto elastic coupling parameter. The non-zero stresses components for the plane strain problem can be expressed as [9]:

$$s_{11} = A \frac{\partial u}{\partial x} + F \frac{\partial v}{\partial y} \quad (2.4)$$

$$s_{12} = L \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (2.5)$$

Taking Fourier transform of the equation (2) and (3) we get,

$$a \frac{d^2 \bar{u}}{dx^2} - ck^2 \bar{u} - ikb \frac{d\bar{v}}{dx} = 0 \quad (2.6)$$

$$b \frac{d^2 \bar{v}}{dx^2} - ak^2 \bar{v} - ikc \frac{d\bar{u}}{dx} = 0 \quad (2.7)$$

where the bar denotes the Fourier Transform with the fourier parameter k and $a = A - \rho_o + \lambda$, $b = L - \rho_o$, $c = F + L + \lambda$. Expressing in the form of vector matrix differential equation, the equation (6) and (7) can be written as

$$P_1 \frac{d^2 U}{dx^2} - ikP_2 \frac{dU}{dx} - k^2 P_3 U = 0 \quad (2.8)$$

here

$$P_1 = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, P_2 = \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix}, P_3 = \begin{pmatrix} b & 0 \\ 0 & a \end{pmatrix}, U = \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix} \quad (2.9)$$

Assuming the solution of the matrix equation (8) can be written as

$$U(x, k) = E(k)e^{mx} \quad (2.10)$$

where m is a parameter and $E(k)$ is a matrix of the type 2×1 .

Using equation (10) in to equation (9), we obtained the following characteristics equation

$$m^4 - 2k^2 m^2 + k^4 = 0 \quad (2.11)$$

The solution of the characteristics equation (11) is:

$$m = m_1 = m_2 = m_3 = m_4 = |k| \quad (2.12)$$

which are the required eigen values.

Ross [25] has given a method to solve the problem of repeated eigen values. Using this method we have,

$$\frac{dU_1}{dx} = A_1 U_1 \quad (2.13)$$

where

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{b}{a}k^2 & 0 & 0 & \frac{c}{a}ik \\ 0 & \frac{a}{b}k^2 & \frac{c}{a}ik & 0 \end{pmatrix}, \quad (2.14)$$

after solving for eigen values the required corresponding eigen vectors are:

$$V_1 = \begin{pmatrix} i|k| \\ k \\ ik^2 \\ |k| \end{pmatrix}, V_2 = \begin{pmatrix} ix|k| - \frac{2a}{c} \\ k(x - \frac{1}{|k|}) \\ i|k|(x|k| - \frac{a+b}{c}) \\ xk|k| \end{pmatrix}, V_3 = \begin{pmatrix} -i|k| \\ k \\ ik^2 \\ -k|k| \end{pmatrix}, V_4 = \begin{pmatrix} -ix|k| + \frac{2a}{c} \\ k(x + \frac{1}{|k|}) \\ i|k|(x|k| + \frac{a+b}{c}) \\ -xk|k| \end{pmatrix} \quad (2.15)$$

The general solution of equation (13) can be written as

$$U_1 = (BV_1 + CV_2)e^{|k|x} + (DV_3 + GV_4)e^{-|k|x} \quad (2.16)$$

the constants used in the above equation will be determined using boundary conditions. The required analytical expressions for displacement and stresses can be determined by solving equation (10) and using equation (15)-(16).

$$\begin{aligned} u &= i[B|k| + C(x|k| - \frac{2a}{c})e^{|k|x} - D|k| + G(x|k| + \frac{2a}{c})e^{-|k|x}] \\ v &= k[B + C(x - \frac{1}{|k|})e^{|k|x} + D + G(x + \frac{1}{|k|})e^{-|k|x}] \end{aligned} \quad (2.17)$$

With the help of equation (17) in (2.4) and (2.5), we get the required equation for normal and tangential stress.

$$\begin{aligned} s_{11} &= i[Bk^2(A - F) + C((x|k|^2 - \frac{a+b}{c}|k|)A + (xk^2 - k)F)e^{|k|x} + \\ &\quad Dk^2(A + F) + G((x|k|^2 + \frac{a+b}{c}|k|)A + (xk^2 + k)F)e^{-|k|x}] \\ s_{12} &= L[2Bk|k| + C(2xk|k| - \frac{2a}{c})e^{|k|x} - 2Dk|k| + G(2xk|k| - \frac{2a}{c})e^{-|k|x}] \end{aligned} \quad (2.18)$$

By applying inverse Fourier transformation on equation (17) and (18), the required expressions for displacement and stresses are as follows:

$$\begin{aligned} u(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} i[B|k| + C(x|k| - \frac{2a}{c})e^{|k|x} - D|k| + G(x|k| + \frac{2a}{c})e^{-|k|x}]e^{-iky} dk \\ v(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} k[B + C(x - \frac{1}{|k|})e^{|k|x} + D + G(x + \frac{1}{|k|})e^{-|k|x}]e^{-iky} dk \\ s_{11}(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} i[Bk^2(A - F) + C((x|k|^2 - \frac{a+b}{c}|k|)A + (xk^2 - k)F)e^{|k|x} + \\ &\quad Dk^2(A + F) + G((x|k|^2 + \frac{a+b}{c}|k|)A + (xk^2 + k)F)e^{-|k|x}]e^{-iky} dy \\ s_{12}(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} L[2Bk|k| + C(2xk|k| - \frac{2a}{c})e^{|k|x} - 2Dk|k| + G(2xk|k| - \frac{2a}{c})e^{-|k|x}]e^{-iky} dy \end{aligned} \quad (2.19)$$

2.2 Deformation due to line source

For finding the displacements and stresses at arbitrary point of an initially stressed magneto elastic transversely isotropic elastic half-space due to Normal Line Load V_N , the infinite medium is considered to be composed of Medium I and Medium II defined with distinct material properties and can be written as:

For Medium I ($x < 0$)

$$\begin{aligned}
 u^I(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} i[B|k| + C(x|k| - \frac{2a}{c})e^{|k|x}/e^{-iky} dy \\
 v^I(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} k[B + C(x - \frac{1}{|k|})e^{|k|x}]e^{-iky} dy \\
 s_{11}^I(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} i[Bk^2(A - F) + C((x|k|^2 - \frac{a+b}{c}|k|)A + (xk^2 - k)F)e^{|k|x}]e^{-iky} dy \quad (2.20) \\
 s_{12}^I(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} L[2Bk|k| + C(2xk|k| - \frac{2a}{c})e^{|k|x}]e^{-iky} dy
 \end{aligned}$$

and for medium II ($x > 0$)

$$\begin{aligned}
 u^{II}(x, y) &= -\frac{1}{2\pi} \int_{-\infty}^{\infty} i[D|k| + G(x|k| + \frac{2a}{c})e^{-|k|x}]e^{-iky} dy \\
 v^{II}(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} k[D + G(x + \frac{1}{|k|})e^{-|k|x}]e^{-iky} dy \\
 s_{11}^{II}(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} i[Dk^2(A + F) + G((x|k|^2 + \frac{a+b}{c}|k|)A + (xk^2 + k)F)e^{-|k|x}]e^{|k|x}]e^{-iky} dy \quad (2.21) \\
 s_{12}^{II}(x, y) &= -\frac{1}{2\pi} \int_{-\infty}^{\infty} L[2Dk|k| + G(2xk|k| - \frac{2a}{c})e^{-|k|x}]e^{-iky} dy
 \end{aligned}$$

2.3 Boundary Conditions for Imperfect Interface

To simulate a concentrated mechanical disturbance and analyze material response on an interface, we have applied a Normal line-load V_N acting vertically downwards and along the positive x -direction on the interface at $x=0$ along z -axis on the elastic half-space and the bonding between the interface is imperfect (figure 1), thus the required boundary conditions for the formulated problem at $x=0$ are [11]:

$$\begin{aligned}
 u^I(x, y) &= k_{11}u^{II}(x, y) + k_{12}v^{II}(x, y) \\
 u^I(x, y) &= k_{21}u^{II}(x, y) + k_{22}v^{II}(x, y) \\
 s_{11}^I(x, y) - s_{11}^{II}(x, y) &= -Q_0\delta(y), \\
 s_{12}^I(x, y) - s_{12}^{II}(x, y) &= 0
 \end{aligned} \quad (2.22)$$

here δ is direc-delta satisfying the following properties:

$$\int_{-\infty}^{\infty} \delta(y) dy = 1, \delta(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iky} dk$$

and k_{ij} ($i, j=1, 2$) is a constant matrix describing the bonding conditions along the interface.

By applying boundary conditions (22) in equation (20) and (21) we get the following value of

elastic constants.

$$\begin{aligned}
 B &= F_0 \frac{\delta(y)}{2\delta} \left[\frac{T_1}{k|k|} + \frac{T_2}{k^2} \right], \\
 C &= F_0 \frac{\delta(y)}{2\delta} \left[\frac{T_3}{k} + \frac{T_4}{|k|} \right], \\
 D &= -F_0 \frac{\delta(y)}{2\delta} \left[\frac{T_5}{k^2} + \frac{T_6}{k|k|} \right], \\
 G &= F_0 \frac{\delta(y)}{2\delta} \left[\frac{T_7}{|k|} + \frac{T_8}{k} \right]
 \end{aligned} \tag{2.23}$$

$$\begin{aligned}
 T_1 &= -i \frac{8a^2}{c^2} k_{21} - \frac{2a}{c} (1 + k_{22}) - k_{11} k_{22} \frac{4a^2}{c^2} k|k| - k_{11} k_{22} \frac{4a^2}{c^2} + \\
 &\quad \frac{4a}{c} \left(1 + \frac{a}{c} k_{22} \right) \\
 T_2 &= \frac{4a k_{22} (a - c)}{c^2} - i k_{11} \frac{2a}{c} (1 + k_{22}) - k_{21} k_{12} \frac{4a^2}{c^2} + i k_{11} \frac{4a}{c} + \\
 &\quad \left(1 + \frac{1}{c} k_{22} \right) + i k_{12} k_{21} \frac{2a}{c} \\
 T_3 &= -14 k_{21} \frac{a}{c} + 2 k_{22} \left(\frac{a - c}{c} \right) + 2 k_{12} \left(\frac{2a}{c} + k_{22} \right) + i k_{22} k_{11} \frac{4a}{c} - k_{21} k_{11} \frac{2a}{c} + \\
 &\quad 2 k_{12} (1 + k_{22}) \\
 T_3 &= -4 i k_{21} \frac{a}{c} + 2 k_{22} \left[\frac{a - c}{c} \right] + 2 k_{12} \left(\frac{2a}{c} + k_{22} \right) + i k_{11} k_{21} \frac{4a}{c} - k_{21} k_{11} \frac{2a}{c} + 2 k_{12} (1 + k_{12}) \\
 T_4 &= -i 2 k_{11} \left(\frac{2a}{c} + k_{22} \right) + i 4 k_{12} k_{21} \frac{a}{c} + \left(2(1 + k_{22}) i k_{11} \frac{2a}{c} + k_{21} k_{12} \right) \\
 Y_5 &= \frac{2a}{c} (1 + k_{22}) + 8 \frac{a^2}{c^2} + 2 k_{22} \frac{a}{c} + i 2 k_{11} \frac{a(a - c)}{c^2} \\
 T_6 &= i 4 k_{21} \frac{a^2}{c^2} - i 8 k_{21} \frac{a^2}{c^2} + 2 k_{12} \frac{c - a}{c} \\
 T_7 &= 2 \left(1 + k_{22} \frac{a}{c} - 2(1 + k_{22}) \frac{a}{c} + i k_{11} \frac{c - a}{c} \right) \\
 T_8 &= k_{12} \frac{c - a}{c}
 \end{aligned} \tag{2.24}$$

and

$\delta = 2k^4 D_r$, where

$$\begin{aligned}
 D_r &= \left[\left(\frac{b - c}{a} A + F \right) \frac{c - 2a}{a} \right] + \frac{2a}{c} \left(\frac{2(a + b)}{c} A \right) (i k_{21} + k_{22} - i k_{11} - k_{12}) + \\
 &\quad \left(\frac{b - c}{a} A - F \right) \left(i k_{21} \frac{2a}{c} + k_{22} \right) + \left(\frac{b - c}{a} A - F \right) \left(i k_{11} + k_{12} - \frac{2a}{c} (i k_{21} + k_{22}) \right) \\
 &\quad (2A - F) \left(\frac{2a}{c} (i k_{21} + k_{22}) - i k_{11} \frac{2a}{c} + k_{12} \right) + \left(\frac{3a + b}{c} A - F \right) \left(i \frac{2a - c}{c} (k_{12} k_{21} - k_{11} k_{22}) \right)
 \end{aligned}$$

Using equation (23) and (24) in to the equation (20) and (21) and after integration using wolfram Mathematica (version 11) we derived the following closed from analytical solutions for

displacements and stresses.

$$u = i\delta(y)\frac{F_0}{2\delta}\left[M_7\left(\frac{2x^2}{(y^2+x^2)}\right) - M_8\left(\frac{i2xy}{(y^2+x^2)}\right) - \left(M_5 + M_7\frac{2a}{c}\right)\log(y^2+x^2) - 2\left(M_6 + M_8\frac{2a}{c}\right)\tan^{-1}\frac{y}{x}\right] \quad (2.25)$$

$$v = \delta(y)\frac{F_0}{2\delta}\left[-T_52\tan^{-1}\frac{y}{x} - (T_6 + T_7)\log(y^2+x^2) - iT_7\frac{2xy}{y^2+x^2} + T_8\left(\frac{2x^2}{y^2+x^2} + \frac{2(x^2-y^2)}{y^2+x^2}\right)\right] \quad (2.26)$$

$$s_{11} = -\delta(y)\frac{F_0}{2\delta}\left[\left(AT_5 + T_7 + xT_8A\frac{a+b}{c}\right)\frac{2x}{y^2+x^2} - T_8(xA+1)\frac{4ixy}{(y^2+x^2)^2} + x(T_7A + T_8)\frac{2(x^2-y^2)}{(y^2+x^2)^2} - \left(AT_8\frac{a+b}{c} + AT_6\right)\frac{-2iy}{y^2+x^2}\right] \quad (2.27)$$

$$s_{12} = -\frac{LF_0\delta y}{\delta}\left[-2\left(T_5 - \frac{2a}{c}T_7\right)\frac{2iy}{y^2+x^2} + \left(T_6 - T_8\frac{2a}{c}\right)\frac{2x}{y^2+x^2} + 8iT_8\frac{x^2y}{(y^2+x^2)^2} - 4T_8x\frac{(x^2-y^2)}{(y^2+x^2)^2}\right] \quad (2.28)$$

3 Numerical Analysis and Discussion

This section presents 2D and 3D graphs to study the variation in deformation due to the influence of Imperfect bounding between the interfaces, presence of hydro static initial stress and magnetic presence in the medium. For the numerical analysis, we have considered the value of elastic constants given by Haojiang et. al [13].

For medium I, the values of elastic constants in units of GPa are taken for Aluminum Oxide, i.e. $c_{11} = 1060, c_{12} = 180, c_{13} = 15, c_{33} = 0.37, c_{44} = 0.35$ For medium II, the considered material is Graphite and the elastic constants for Graphite are: $c_{11} = 460.2, c_{12} = 174.7, c_{13} = 127.4, c_{33} = 509.5, c_{44} = 136.9$ ($A = c_{11}, L = \frac{c_{11}-c_{12}}{2}, F + L = \frac{c_{11}+c_{12}}{2}$) The Imperfection of the bonding has been showed by three constants matrix (K1,K2 and K3) and perfect bonding is shown by Matrix K4:

$$K1 = \begin{bmatrix} 0.25 & 0.15 \\ 0.15 & 0.35 \end{bmatrix} K2 = \begin{bmatrix} 0.45 & 0.25 \\ 0.25 & 0.65 \end{bmatrix} K3 = \begin{bmatrix} 0.75 & 0.25 \\ 0.35 & 0.75 \end{bmatrix} K4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ The Aluminum}$$

oxide and Graphite have contrasting yet complimentary properties , as Aluminum oxide is a stiff, electrically insulating ceramic with good transverse isotropic, while graphite is a softer, electrically conductive and weakly diamagnetic material with strong anisotropy. Therefore this combination of materials enhance the sensitivity of the study. In these figures we have taken the value of at fixed initial stress $\frac{\rho_0}{L}=0.2$ and fixed magneto elastic parameter $\frac{\lambda}{L}=0.5$.

For more effective results, we have obtained dimensionless displacements and stresses and further depicted the results graphically.

Figure (2) and (3) shows the 2D plots for the comparison in variation of normal displacement and tangential displacements . These variations are due to the presence of various imperfect interface (K1, K2 and K3) at fixed initial stress $\frac{\rho_0}{L}=0.2$ and fixed magneto elastic parameter $\frac{\lambda}{L}=0.5$. From figure (4) and (5), it has been found that variation in imperfect interface and presence of initial compressive stress affect the normal and tangential stresses significantly. It has been observed that when the initial stress is compressive, it may reduce the normal displacement by increasing the material stiffness and when it exceeds a critical threshold, it might lead to buckling and drastically increases the normal and tangential displacement. As 3D figure are important to understand the significance of various parameter on deformation, therefore for more accurate display we have plotted mesh-grids in figure (6)-(9).

Mesh-grid (6) and (7) shows the comparison in Normal displacement due to imperfect and perfect interfaces respectively. Similarly mesh-grid (8) and (9) shows the comparison in tangential

displacement due to imperfect and perfect bonding between the interface respectively. In figure (10) and (11), a 3D contour maps has been plotted using K1 matrix for imperfection in the medium to show the variation in Normal and tangential Stress.

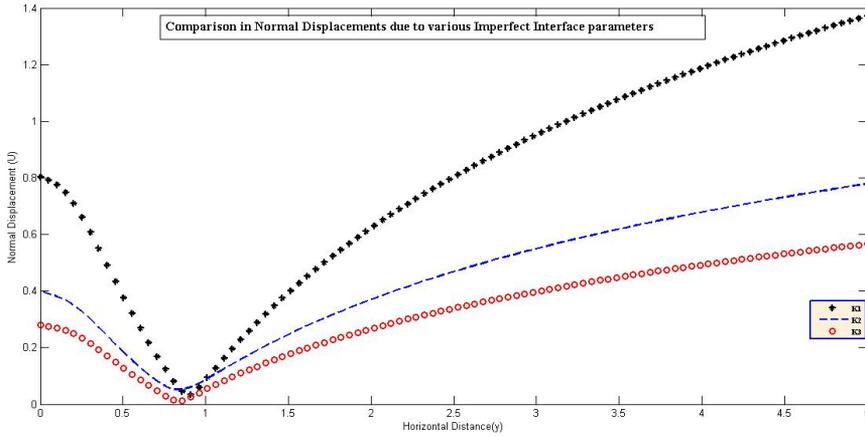


Figure 2. Comparison of Normal Displacement due to the effect of various Imperfect Interface.

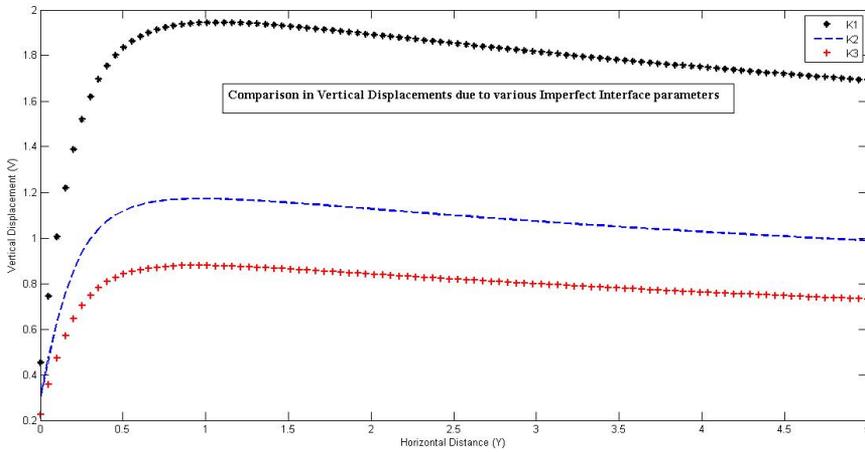


Figure 3. Comparison of Tangential Displacement due to the effect of various Imperfect Interface.

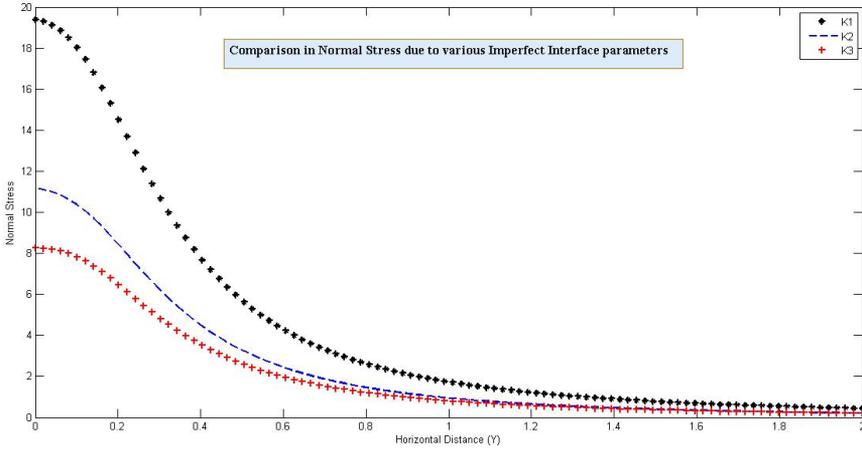


Figure 4. Comparison of Normal Stress due to the effect of various Imperfect Interface.

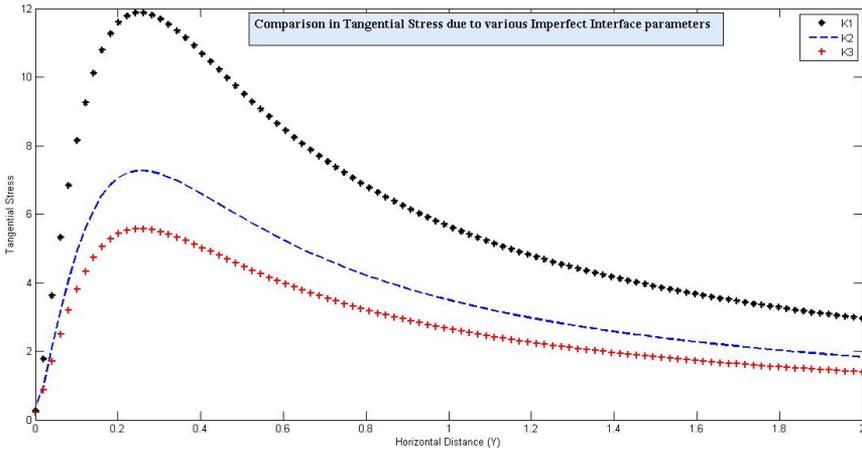


Figure 5. Comparison of Tangential Stress due to the effect of various Imperfect Interface.

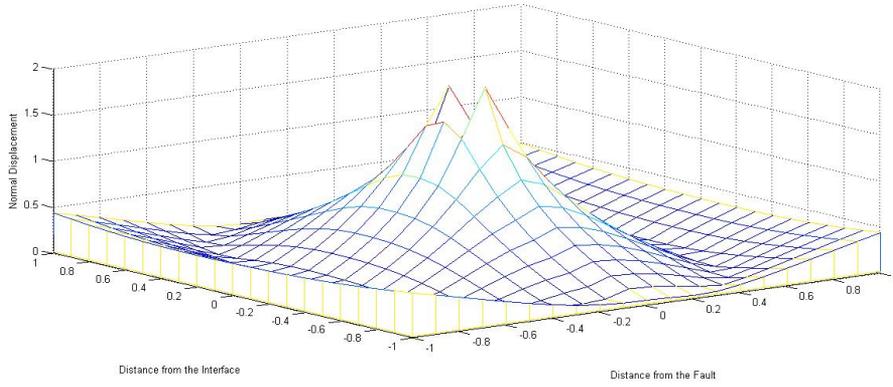


Figure 6. A 3D mesh-grid depicting normal displacement due to imperfect interface

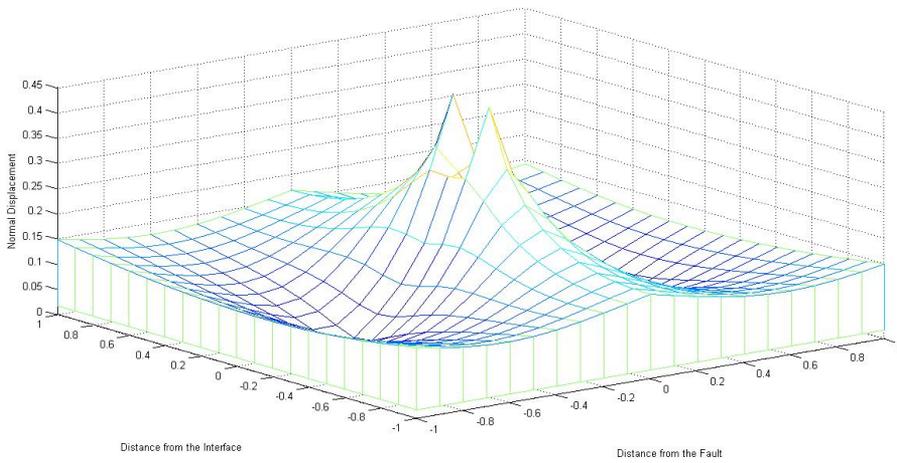


Figure 7. A 3D mesh-grid depicting normal displacement due to Perfect interface

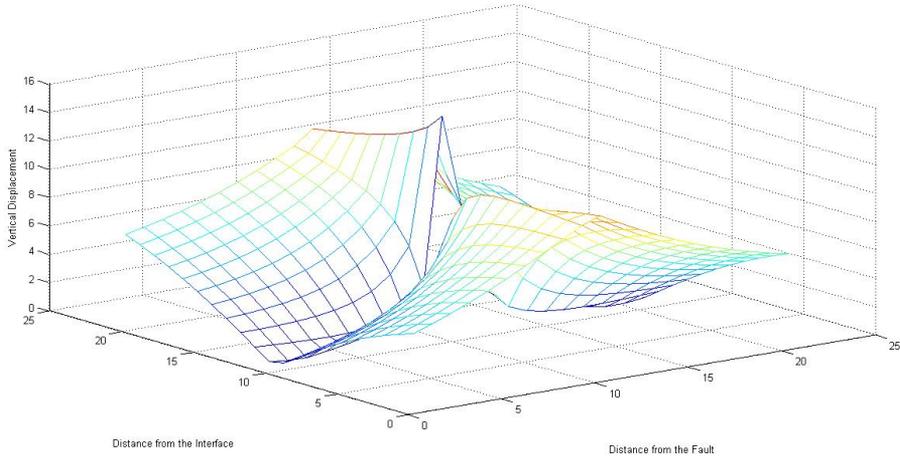


Figure 8. A 3D mesh-grid depicting Tangential displacement due to imperfect interface

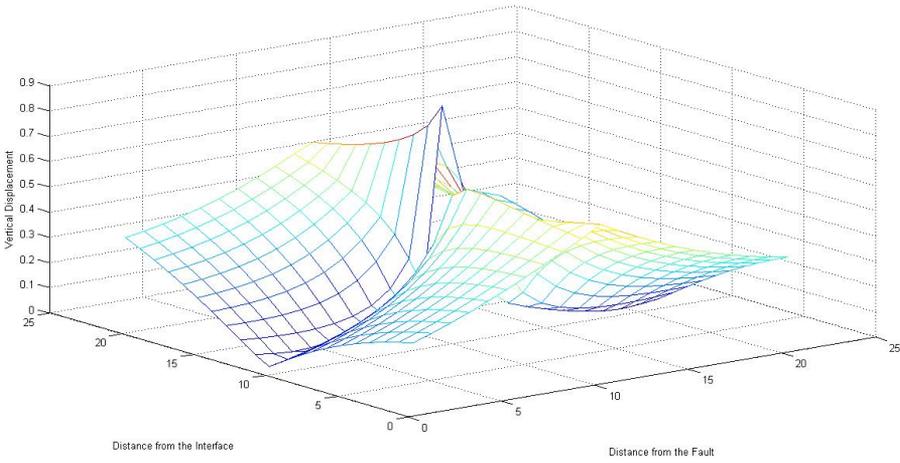
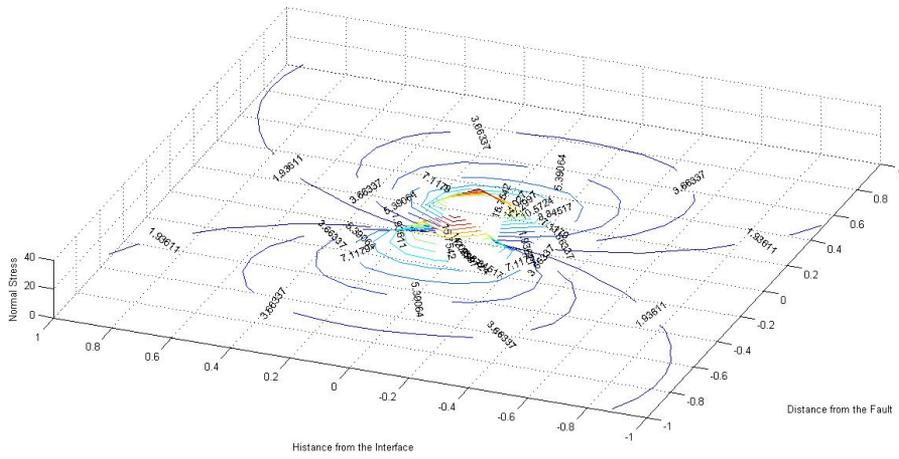


Figure 9. A 3D mesh-grid depicting Tangential displacement due to perfect interface



The degree of coupling between the magnetic and mechanical field has been represented by magnetic elastic parameter (λ), this coupling becomes significant in modifying the stress displacement field near the interface because in plane strain problem deformation is restricted in one direction. After analyzing these graphs, mesh grids and contour maps, our observations indicates that the deformation in the medium is significantly influenced by the bonding conditions between the interfaces, the strength of the magnetic field present in the medium and also by the initial stress residing on the surface. The non-uniform initial stress can introduce a shear strain that can affect the displacements in an anisotropic and pre-stressed material. These results suggest a primarily linear relationship between interface bonding and initial stress whereas the presence of magneto elastic parameter introduces a nonlinear behavior.

4 Conclusion

In this work, we have examined how the displacement and stress distribution in a transversely isotropic elastic material are affected by an imperfect interface, hydrostatic initial stress, and magneto-elasticity. Our results show that these parameters have a major impact on the medium's mechanical behavior. While hydrostatic initial stress impacts the overall stress distribution and deformation characteristics, the existence of an imperfect contact changes the patterns of stress displacement and concentration. Further complexity is added by including magneto-elastic effects, which modify the direction and amplitude of stresses and displacements. These findings help in the experimental study of material behavior under the combined physical influence and understanding how the deformation is scattered due to imperfect interfaces, and fault mechanisms, helps in monitoring volcanic and earthquake activity etc. The results can also be used for designing magneto-elastic sensors, geophysical models and advanced composite materials. It can also help in predicting failure mechanisms in layered magnetic elastic structures.

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