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A NEW EXTENSION OF HESITANT BIPOLAR FUZZY SET WITH MULTI ATTRIBUTE DECISION MAKING PROBLEM USE OF ENTROPY- VIKOR METHOD

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Abstract In this paper, we introduce a novel approach combining interval-valued hesitant bipolar fuzzy entropy with an interval-valued hesitant bipolar fuzzy VIKOR (Vise Kriterijumska Optimizacija I Kompromisno Resenje) method to evaluate criteria importance and rank alternatives. With the increasing adoption of battery-powered electric vehicles over fuel-based ones, the development of high-performance batteries remains a challenge. Selecting the optimal Li-ion battery, considering various cathode and anode materials, involves significant complexity. Interval-valued hesitant bipolar fuzzy sets (IVHBFs) provide an effective mathematical framework to capture individual hesitant judgments, accommodating multiple possible interval-valued bipolar fuzzy positive and negative membership values. In this study, we utilize the proposed interval-valued hesitant bipolar fuzzy entropy and VIKOR methods to facilitate the selection of Li-ion battery materials, thereby addressing the complexities in decision-making for battery optimization.

1 Introduction

In this paper, we examine and introduce the interval-valued hesitant bipolar fuzzy VIKOR method for selecting the material for lithium batteries. In real life, many characters make decisions about a variety of issues, and more and more people struggle with decision-making in both their personal and professional lives. Examples include deciding where to locate a new restaurant, deciding whether to test a new crop, and other issues.Given these findings, it is crucial to help decision-makers articulate their evaluation information based on certain inherently conflicting criteria and to consolidate all the information for the final ranking of alternatives. As a result, numerous multi-attribute decision making (MADM) models have been developed and widely implemented in practice. Zadeh [1] was the first to introduce the concept of fuzzy sets to address ambiguity.

However after that, we obtained a number of fuzzy set extensions, Zhang [2] developed the idea of a bipolar fuzzy set and proposed two membership functions, which relate to membership in a set and membership in a complimentary set. According to Torra [3], the embedding of HFs in them is free to deal with uncertainties and allows DMs to express their assessment information with a set of values that belong within a interval [0,1]. However, the major concern is how to calculate the smoothness value based on a certain criterion to reflect the uncertainty and ambiguity of the DMs in a complex real MCDM. As an illustration, suppose the DMs are unable to agree on which exact number should be assigned to the alternative under a certain attribute. Instead of a single number, he / she may provide a range of numbers to indicate the results of the assessment. Therefore, HFs have a wider range of applications and are more important practically than other expanded forms of FSs. According to different application contexts, these different forms of FSs have been extensively organised in academic research over the last few decades and have produced some outstanding research results. Although using bipolar fuzzy logic offers advantages, it is used less frequently than other fuzzy logic extensions to solve MCDM problems. The examples that follow may be considered some of the more rare uses of BFs for MCDM problems. Alghamdi et al. [4] have invented a new method for MCDM problems that combines bipolar fuzzy information with subjective data provided by the decision maker. Muhammad Akram et al. [5] used bipolar fuzzy PROMETHEE for the selection of green suppliers. Han et al. [6] present a comprehensive TOPSIS-based mathematical approach to improve the accuracy of the clinical diagnosis of bipolar disorder.

Although this is occurring at a much slower rate due to the continued development of better performing batteries, battery-powered electric vehicles are gradually displacing the fuel-based vehicles. The demanding requirements to be met while updating battery technology are fast charging, long distance driving, long battery life and low cost. Electric Vehicles (EVs) using batteries were created and produced in the early 18s. The key benefits of electric cars include no carbon dioxide emissions, low operating costs, quiet operation and minimal maintenance costs. A types of renewable energy sources are used to power EVs. The Li-ion battery is one example of such a power source. Due to its superior energy efficiency compared to conventional batteries, long lifespan and rapid charging, Li-ion batteries dominate in electric vehicles. The cathode/anode materials have the most impact on the performance of Li-ion batteries. The materials chosen for electrodes are important in determining battery performance, especially in terms of cost and life. The practical problem is that it is challenging for manufacturers of electric vehicles to select a better Li-ion battery in order to overcome the trade-off between performance, price and durability. Based on the selection of materials that contain high-quality cathode and anode to get the finest battery performance characteristics, there have been considerable advancements in the production of batteries. Experts in multi-criteria decision-making challenges often have different backgrounds and come from a diverse range of specific professions. Decision-makers have varying degrees of knowledge. They differ in their viewpoints. Zadeh introduces the fuzzy set to clarify frequently recurring misunderstandings in the provided information. Wei et al. [7] introduced the concept of interval-valued bipolar fuzzy sets, incorporating various operators based on the interval-valued bipolar fuzzy weighted geometric operator and the interval-valued bipolar fuzzy weighted averaging operator. Jie Lan et al. [8] developed several interval-valued bipolar uncertain linguistic aggregating operators, extending traditional information aggregation operators into interval-valued bipolar uncertain linguistic sets.

We look at some of the applications for the hesitant fuzzy set theory related to the theme of this research. Loganathan et al. [9] used the MCDM method to select a better battery technology for electric vehicles. However, the results for the best selection of lithium batteries have certain limitations in terms of various factors such as performance factors, safety, cost and reliability. To choose the best among them and compare several alternatives based on many end criteria, here a simple and effective MCDM method is used for analyzing. In general, fuzzy entropy is an essential tool for determining the degree of fuzzy in uncertain information. In 2009, Zhang et al.[10] investigated the relationship between entropy and similarity measures, defining the entropy measure for interval-valued fuzzy sets based on distance measures. Joshi and Kumar [11] improved the Hamming distance for IVIHFs and employed the TOPSIS method to rank the options.

Opricovic [12] introduced the VIKOR technique to describe discrete decision-making problems including inconsistent and conflicting criteria. The VIKOR technique is a tool used in MCDM to identify a compromise solution that meets the requirements of maximizing group utility for the majority while minimizing individual regret for the opposition. A context based on connection numbers obtained from interval-valued bipolar fuzzy numbers, the linguistic VIKOR approach was introduced by Riaz and Tehrim in 2021 [13]. Rajalakshmi and Julia Rose Mary [14] developed the hesitant bipolar fuzzy CRITIC based VIKOR approach for alternative fuel selection and in that paper, the CRITIC method is used to discover the criteria weights and the VIKOR method is utilized to sort the alternatives.

In this paper, the interval-valued hesitant bipolar fuzzy set is an extension of hesitant bipolar fuzzy set. An effective mathematical tool for expressing individual hesitant thoughts is the Interval-Valued Hesitant Bipolar Fuzzy set (IVHBFs). An IVHBFs agrees with a range of possible positive and negative interval-valued bipolar fuzzy membership values. Here, we also proposed various operations, a scoring function and weighted aggregated operators for IVHBFs using the VIKOR approach.

2 Mathematical Preliminaries

Definition 2.1. Let U be a fixed set. An Interval-Valued Bipolar Fuzzy set (IVBFs) is a set having the form. [7]

$$\tilde{B}_{IVBF} = \{< u, \tilde{p}(u), \tilde{q}(u) > /u \in U\} \tag{2.1}$$

where $\tilde{p}(u) \subset [0,1]$ and $\tilde{q}(u) \subset [-1,0]$ are positive and negative membership function of an element $u \in U$. An Interval-Valued Bipolar Fuzzy Numbers (IVBFNs) can be represented by,

$$\tilde{B}_{IVBF} = (\tilde{p}, \tilde{q}) = ([\tilde{p}^L, \tilde{p}^U], [\tilde{q}^L, \tilde{q}^U])$$

where $\tilde{p}^L, \tilde{p}^U, \tilde{q}^L$ and $\tilde{q}^U$ are associated lower and upper limits of the IVBFs $\tilde{p}$, $\tilde{q}$ respectively.

3 Interval-Valued Hesitant Bipolar Fuzzy Sets (IVHBFs)- The New Extension of the Hesitant Bipolar Fuzzy Set

In this section, we provide an IVHBF-set and its properties. Also, we give the distance measure, IVHBF weighted average operator and IVHBF weighted geometric operator. After setting the background, we introduce a novel multi-criteria cecision making (mcdm) approach aided by Interval-Valued Hesitant Bipolar Fuzzy (IVHBF) guidance. Additionally, we demonstrate the effectiveness of this new method through a numerical example focused on selecting Li-ion battery materials for electric vehicles.

Definition 3.1. Assume U is a finite set. An interval-valued hesitant bipolar fuzzy set on U defined as,

$$B^*_{IVHBFS} = \left\{\langle u, h^+_{B^*}(u), h^-_{B^*}(u)\rangle/u \in U\right\} = \left\{\langle u, (\tilde{p}(u), \tilde{q}(u))\rangle/u \in U\right\} \tag{3.1}$$

In above equation $h^+_{B^*}, h^-_{B^*}$ represent the positive and negative membership degrees, respectively. The positive membership functions are represented by several closed intervals within the real unit interval [0,1], and the positive membership degree is denoted by $\tilde{p}$. Conversely, the negative membership functions are represented by several closed intervals within the real unit interval [-1,0], and the negative membership degree is denoted by $\tilde{q}$.

$$B^*_{IVHBFS} = \left\{\langle u, [\tilde{p}^L, \tilde{p}^U], [\tilde{q}^L, \tilde{q}^U]\rangle/u \in U\right\} \tag{3.2}$$

Each element of membership degree $h_{B^*} \in U$ and also the positive and negative membership degree is $\tilde{p} : U \to [0,1]$ and $\tilde{q} : U \to [-1,0]$
The interval-valued hesitant bipolar fuzzy set meets the following conditions, namely:

$$B^*_{IVHBFS} = (\tilde{p}, \tilde{q}) = \left([\tilde{p}^L, \tilde{p}^U], [\tilde{q}^L, \tilde{q}^U]\right). \tag{3.3}$$

Operations and Properties

In this context, we have introduced several new operations on interval-valued hesitant bipolar fuzzy numbers.

Definition 3.2. Let us consider three IVHBFNs

$$\check{h} = (\tilde{p}, \tilde{q}) = \left([\tilde{p}^L, \tilde{p}^U], [\tilde{q}^L, \tilde{q}^U]\right),$$

$$\check{h}_1 = (\tilde{p}_1, \tilde{q}_1) = \left([\tilde{p}^L_1, \tilde{p}^U_1], [\tilde{q}^L_1, \tilde{q}^U_1]\right)$$

and

$$\check{h}_2 = (\tilde{p}_2, \tilde{q}_2) = \left([\tilde{p}^L_2, \tilde{p}^U_2], [\tilde{q}^L_2, \tilde{q}^U_2]\right)$$

and the basic operations are defined as,

1) $\check{h}_1 \oplus \check{h}_2$

$$= \bigcup_{\substack{([\tilde{p}_1^L,\tilde{p}_1^U],[\tilde{q}_1^L,\tilde{q}_1^U])\in(\tilde{p}_1,\tilde{q}_1)\\ ([\tilde{p}_2^L,\tilde{p}_2^U],[\tilde{q}_2^L,\tilde{q}_2^U])\in(\tilde{p}_2,\tilde{q}_2)}} \begin{pmatrix} \left[\tilde{p}_1^L+\tilde{p}_2^L-\tilde{p}_1^L\tilde{p}_2^L,\ \tilde{p}_1^U+\tilde{p}_2^U-\tilde{p}_1^U\tilde{p}_2^U\ \right], \\ \left[-|\tilde{q}_1^L||\tilde{q}_2^L|, -|\tilde{q}_1^U||\tilde{q}_2^U|\ \right] \end{pmatrix}$$

2) $\check{h}_1 \otimes \check{h}_2$

$$= \bigcup_{\substack{([\tilde{p}_1^L,\tilde{p}_1^U],[\tilde{q}_1^L,\tilde{q}_1^U])\in(\tilde{p}_1,\tilde{q}_1)\\ ([\tilde{p}_2^L,\tilde{p}_2^U],[\tilde{q}_2^L,\tilde{q}_2^U])\in(\tilde{p}_2,\tilde{q}_2)}} \begin{pmatrix} \left[\tilde{p}_1^L\tilde{p}_2^L,\tilde{p}_1^U\tilde{p}_2^U\ \right], \left[-\left(|\tilde{q}_1^L|+|\tilde{q}_2^L|-|\tilde{q}_1^L||\tilde{q}_2^L|\right), \right. \\ \left. -(|\tilde{q}_1^U|+|\tilde{q}_2^U|-|\tilde{q}_1^U||\tilde{q}_2^U|)\right] \end{pmatrix}$$

3)

$$\lambda\check{h} = \bigcup_{([\tilde{p}^L,\tilde{p}^U],[\tilde{q}^L,\tilde{q}^U])\in(\tilde{p},\tilde{q})} \begin{pmatrix} \left[1-(1-\tilde{p}^L)^\lambda, 1-(1-\tilde{p}^U)^\lambda\right], \\ \left[-|\tilde{q}^L|, -|\tilde{q}^U|\right] \end{pmatrix}; \quad \lambda > 0$$

4)

$$(\check{h})^\lambda = \bigcup_{([\tilde{p}^L,\tilde{p}^U],[\tilde{q}^L,\tilde{q}^U])\in(\tilde{p},\tilde{q})} \begin{pmatrix} \left[(\tilde{p}^L)^\lambda, (\tilde{p}^U)^\lambda\right], \\ \left[-1+|1+\tilde{q}^L|^\lambda, -1+|1+\tilde{q}^U|^\lambda\right] \end{pmatrix}; \quad \lambda > 0$$

5)

$$(\check{h})^c = \bigcup_{([\tilde{p}^L,\tilde{p}^U],[\tilde{q}^L,\tilde{q}^U])\in(\tilde{p},\tilde{q})} \begin{pmatrix} \left[1-\tilde{p}^U, 1-\tilde{p}^L\right], \\ \left[|\tilde{q}^U|-1, |\tilde{q}^L|-1\right] \end{pmatrix}$$

Definition 3.3. The score value $S(\check{h})$ and accuracy value $H(\check{h})$ for $\check{h} = (\tilde{p},\tilde{q}) = ([\tilde{p}^L,\tilde{p}^U],[\tilde{q}^L,\tilde{q}^U])$ are computed as,

$$S(\check{h}) = \frac{1}{\#l_h}\frac{\left[(1+\tilde{p}^L+\tilde{q}^L)+(1+\tilde{p}^U+\tilde{q}^U)\right]}{4}, S(\check{h}) \in [0,1] \tag{3.4}$$

The accuracy function for IVHBF H of $\check{h}$ is computed as follows,

$$H(\check{h}) = \frac{1}{\#l_h}\frac{\left[(\tilde{p}^L-\tilde{q}^L)+(\tilde{p}^U-\tilde{q}^U)\right]}{4}, H(\check{h}) \in [0,1] \tag{3.5}$$

where the number of hesitant fuzzy element is represented as $\#l_h$. Next, we define an ordered relation between two IVHBFNs as $\check{h}_1 = (\tilde{p}_1,\tilde{q}_1) = ([\tilde{p}_1^L,\tilde{p}_1^U],[\tilde{q}_1^L,\tilde{q}_1^U])$ and $\check{h}_2 = (\tilde{p}_2,\tilde{q}_2) = ([\tilde{p}_2^L,\tilde{p}_2^U],[\tilde{q}_2^L,\tilde{q}_2^U])$.

Definition 3.4. Suppose two IVHBFNs of $\check{h}_1$ & $\check{h}_2$ are defined as above, then the score function satisfies the following conditions
(i.e) (1) If $S(\check{h}_1) > S(\check{h}_2)$, then $\check{h}_1 > \check{h}_2$.
(2) If $S(\check{h}_1) = S(\check{h}_2)$ then there is a requirement to compare accuracy functions
(3) If $H(\check{h}_1) > H(\check{h}_2)$, then $\check{h}_1$ is superior to $\check{h}_2$ denoted by $\check{h}_1 > \check{h}_2$
(3) If $H(\check{h}_1) = H(\check{h}_2)$ then $\check{h}_1 = \check{h}_2$

Definition 3.5. The normalized hamming distance between two IVHBFNs is given by,

$$d(\check{h}_1,\check{h}_2) = \frac{1}{4n}\left(|\tilde{p}_1^L-\tilde{p}_2^L|+|\tilde{p}_1^U-\tilde{p}_2^U|+|\tilde{q}_1^L-\tilde{q}_2^L|+|\tilde{q}_1^U-\tilde{q}_2^U|\right) \tag{3.6}$$

Definition 3.6. The normalized Euclidean distance between two IVHBFNs is given by,

$$d(\check{h}_1,\check{h}_2) = \sqrt{\frac{1}{n}\left((\tilde{p}_1(u)-\tilde{p}_2(u))^2+(\tilde{q}_1(u)-\tilde{q}_2(u))^2\right)} \tag{3.7}$$

IVHBF- Aggregation operators

In this section, we introduce two types of aggregation operators regarding interval-valued hesitant bipolar fuzzy sets.

Definition 3.7. Let $\tilde{h}_j =< u, (h^+_{B^*_j}, h^-_{B^*_j})/u \in U >= (\tilde{p}_j, \tilde{q}_j) = ([\tilde{p}^L, \tilde{p}^U], [\tilde{q}^L, \tilde{q}^U])(j = 1, 2, 3, ..., n)$ be a collection of all IVHBFEs and $w = (w_1, w_2, ..., w_n)^T$ be the weight vector of $\tilde{h}_j$ with $w_j \geq 0 (j = 1, 2, ...n)$ where $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$, then

(i) An Interval-Valued Hesitant Bipolar Fuzzy Weighted Averaging (IVHBFWA) operator is a mapping $IVHBFWA : IVHBFWA^n \to IVHBFWA$ is defined as,

$$\begin{aligned} IVHBFWA(\tilde{h_1}, \tilde{h_2}, ...\tilde{h_n}) &= \bigoplus_{j=1}^{n} w_j \tilde{h}_j \\ &= \left\langle \bigcup_{\tilde{p}^L \in \tilde{p}, \tilde{p}^U \in \tilde{p}} \left[1 - \prod_{j=1}^{n} (1 - \tilde{p}^L)^{w_j}, 1 - \prod_{j=1}^{n} (1 - \tilde{p}^U)^{w_j} \right], \right. \\ &\quad \left. \bigcup_{\tilde{q}^L \in \tilde{q}, \tilde{q}^U \in \tilde{q}} \left[- \prod_{j=1}^{n} (-\tilde{q}^L)^{w_j}, - \prod_{j=1}^{n} (-\tilde{q}^U)^{w_j} \right] \right\rangle \end{aligned}$$

(ii) An Interval-Valued Hesitant Bipolar Fuzzy Weighted Geometric (IVHBFWG) operator is a mapping $IVHBFWG : IVHBFWG^n \to IVHBFWG$ is defined as,

$$\begin{aligned} IVHBFWG(\tilde{h_1}, \tilde{h_2}, ..., \tilde{h_n}) &= \bigotimes_{j=1}^{n} \tilde{h_j}^{wj} \\ &= \left\langle \bigcup_{\tilde{p}^L \in \tilde{p}, \tilde{p}^U \in \tilde{p}} \left[\prod_{j=1}^{n} (\tilde{p}^L)^{w_j}, \prod_{j=1}^{n} (\tilde{p}^U)^{w_j} \right], \right. \\ &\quad \left. \bigcup_{\tilde{q}^L \in \tilde{q}, \tilde{q}^U \in \tilde{q}} \left[-(1 - \prod_{j=1}^{n} (1 - (-\tilde{q}^L))^{w_j}), \; -(1 - \prod_{j=1}^{n} (1 - (-\tilde{q}^U))^{w_j}) \right] \right\rangle \end{aligned}$$

Theorem 3.8. *Let $\tilde{h}_j (j = 1, 2, ..., n)$ represent a collection of Interval-Valued Hesitant Bipolar Fuzzy Elements (IVHBFEs). Their aggregated value, calculated using the IVHBF Weighted Average (IVHBFWA) operator, is also an IVHBFE.*

$$\begin{aligned} IVHBFWA(\tilde{h}_1, \tilde{h}_2, ..., \tilde{h}_n) = & \left\{ \bigcup_{\tilde{p}^L \in \tilde{p}, \tilde{p}^U \in \tilde{p}} \left[1 - \prod_{j=1}^n (1 - \tilde{p}^L)^{w_j}, 1 - \prod_{j=1}^n (1 - \tilde{p}^U)^{w_j} \right], \right. \\ & \left. \bigcup_{\tilde{q}^L \in \tilde{q}, \tilde{q}^U \in \tilde{q}} \left[- \prod_{j=1}^n (-\tilde{q}^L)^{w_j}, - \prod_{j=1}^n (-\tilde{q}^U)^{w_j} \right] \right\} \end{aligned} \tag{3.8}$$

Proof. Here, we use mathematical induction on n to demonstrate the result. First, we demonstrate equation (3.8) for n = 2
Because

$$\begin{aligned} w_1 \tilde{h}_1 = & \left\{ \left[1 - (1 - \tilde{p}_1^L)^{w_1}, 1 - (1 - \tilde{p}_1^U)^{w_1} \right], \left[(\tilde{q}_1^L)^{w_1}, (\tilde{q}_1^U)^{w_1} \right] \right\} \\ w_2 \tilde{h}_2 = & \left\{ \left[1 - (1 - \tilde{p}_2^L)^{w_2}, 1 - (1 - \tilde{p}_2^U)^{w_2} \right], \left[(\tilde{q}_2^L)^{w_2}, (\tilde{q}_2^U)^{w_2} \right] \right\} \end{aligned} \tag{3.9}$$

we have

$$\begin{aligned} w_1 \tilde{h}_1 \oplus w_2 \tilde{h}_2 = & \left\{ \left[1 - (1 - \tilde{p}_1^L)^{w_1}, 1 - (1 - \tilde{p}_1^U)^{w_1} \right], \left[(\tilde{q}_1^L)^{w_1}, (\tilde{q}_1^U)^{w_1} \right] \right\} \\ & \oplus \left\{ \left[1 - (1 - \tilde{p}_2^L)^{w_2}, 1 - (1 - \tilde{p}_2^U)^{w_2} \right], \left[(\tilde{q}_2^L)^{w_2}, (\tilde{q}_2^U)^{w_2} \right] \right\} \end{aligned}$$

$$= \Big\{\big[1-(1-\tilde{p}_1^L)^{w1}+1-(1-\tilde{p}_2^L)^{w2}-(1-(1-\tilde{p}_1^L)^{w1})(1-(1-\tilde{p}_2^L)^{w2}),1-(1-\tilde{p}_1^U)^{w1}+ 1-(1-\tilde{p}_2^U)^{w2}-(1-(1-\tilde{p}_1^U)^{w1})(1-(1-\tilde{p}_2^U)^{w2})\big],\big[(\tilde{q}_1^L)^{w_1}(\tilde{q}_2^L)^{w_2},(\tilde{q}_1^U)^{w_1}(\tilde{q}_2^U)^{w_2}\big]\Big\}$$

$$= \Big\{\big[1-(1-\tilde{p}_1^L)^{w1}(1-\tilde{p}_2^L)^{w2},1-(1-\tilde{p}_1^U)^{w1}(1-\tilde{p}_2^U)^{w2}\big],\big[(\tilde{q}_1^L)^{w_1}(\tilde{q}_2^L)^{w_2},(\tilde{q}_1^U)^{w_1}(\tilde{q}_2^U)^{w_2}\big]\Big\}$$

In other words, if equation (3.8) holds for n=k,
we have

$$IVHBFA(\tilde{h}_1,\tilde{h}_2,...,\tilde{h}_k) = \Big\{\big[1-\prod_{j=1}^k(1-\tilde{p}^L)^{w_j},1-\prod_{j=1}^k(1-\tilde{p}^U)^{w_j}\big], \big[\prod_{j=1}^k(\tilde{q}^L)^{w_j},\prod_{j=1}^k(\tilde{q}^U)^{w_j}\big]\Big\} \tag{3.10}$$

then, IVHBFE operations result when n=k+1.

$$\begin{aligned}
IVIHFA(\tilde{h}_1,\tilde{h}_2,...,\tilde{h}_k,\tilde{h}_{k+1}) =& \oplus_{j=1}^{k+1}(w_j\tilde{h}_j) = (\oplus_{j=1}^{k}(w_j\tilde{h}_j))\oplus(w_{k+1}\tilde{h}_{k+1})\\
=& \Big\{\big[1-\prod_{j=1}^k(1-\tilde{p}_j^L)^{w_j},1-\prod_{j=1}^k(1-\tilde{p}_j^U)^{w_j}\big],\\
& \big[\prod_{j=1}^k(\tilde{q}_j^L)^{w_j},\prod_{j=1}^k(\tilde{q}_j^U)^{w_j}\big]\Big\}\\
& \oplus\Big\{\big[1-(1-\tilde{p}_{k+1}^L)^{w_{k+1}},1-(1-\tilde{p}_{k+1}^U)^{w_{k+1}}\big]\\
& \big[(\tilde{q}_{k+1}^L)^{w_{k+1}},(\tilde{q}_{k+1}^U)^{w_{k+1}}\big]\Big\}\\
=& \Big\{\big[1-\prod_{j=1}^k(1-\tilde{p}_j^L)^{w_j}+1-(1-\tilde{p}_{k+1}^L)^{w_{k+1}}\\
& -(1-\prod_{j=1}^k(1-\tilde{p}_j^L)^{w_i})(1-(1-\tilde{p}_{k+1}^L)^{w_{k+1}}),\\
& 1-\prod_{j=1}^k(1-\tilde{p}_j^U)^{w_j}+1-(1-\tilde{p}_{k+1}^U)^{w_{k+1}}\\
& -(1-\prod_{j=1}^k(1-\tilde{p}_j^U)^{w_j})(1-(1-\tilde{p}_{k+1}^U)^{w_{k+1}})\big],\\
& \big[\prod_{j=1}^k(\tilde{q}_j^L)^{w_j},(\tilde{q}_{k+1}^L)^{w_{k+1}},(\prod_{j=1}^k\tilde{q}_j^U)^{w_j}(\tilde{q}_{k+1}^U)^{w_{k+1}}\big]\Big\}\\
=& \Big\{\big[1-\prod_{j=1}^{k+1}(1-\tilde{p}_j^L)^{w_j},1-\prod_{j=1}^{k+1}(1-\tilde{p}_j^U)^{wj}\big]\\
& \big[\prod_{j=1}^{k+1}(\tilde{q}_j^L)^{w_j},\prod_{j=1}^{k+1}(\tilde{q}_j^U)^{w_j}\big]\Big\}
\end{aligned}$$

Hence, (3.8) is valid for n=k+1. Hence, equation (3.8) is true for all n.
Hence the theorem. □

Theorem 3.9. *Let $\tilde{h}_j(j=1,2,...,n)$ be a collection of IVHBFEs. Then, their aggregated value calculated using the IVHBFWG operator is an IVHBFE and*

$$\begin{aligned}
IVHBFWG(\tilde{h}_1,\tilde{h}_2,...,\tilde{h}_n) =& \Big\{\bigcup_{\tilde{p}^L\in\tilde{p},\tilde{p}^U\in\tilde{p}}\Big[\prod_{j=1}^n(\tilde{p}^L)^{w_j},\prod_{j=1}^n(\tilde{p}^U)^{w_j}\Big],\\
& \bigcup_{\tilde{q}^L\in\tilde{q},\tilde{q}^U\in\tilde{q}}\Big[-\big(1-\prod_{j=1}^n(1-(-\tilde{q}^L))^{w_j}\big),\\
& -\big(1-\prod_{j=1}^n(1-(-\tilde{q}^U))^{w_j}\big)\Big]\Big\}
\end{aligned} \tag{3.11}$$

4 Proposed Model

4.1 Algorithm for Proposed Interval-Valued Hesitant Bipolar Fuzzy VIKOR Method

The IVHBF sets with MCDM problem have m alternatives $\mathcal{A}_i (i = 1, 2, ..., m)$ and n selection criteria $\mathcal{C}_j (j = 1, 2, ..., n)$ respectively. Let us consider the efficiency of alternatives $\mathcal{A}_i (i = 1, 2, ..., m)$ with respect to the criteria values $\mathcal{C}_j (j = 1, 2, ..., n)$.
Let $w = (w_1, w_2, ..., w_n)^T$ be the weight of $\mathcal{C}_j$ and $\sum_{j=1}^{n} w_j = 1$. Let $\hat{H} = (\hat{h}_{ij})_{m\times n} = \left(\langle \tilde{p}, \tilde{q}\rangle\right)_{m\times n} = \left(\langle [\tilde{p}^L, \tilde{p}^U][\tilde{q}^L, \tilde{q}^U]\rangle\right)_{m\times n}$ be a form of interval-valued hesitant bipolar fuzzy decision matrix, where $(\hat{h}_{ij})_{m\times n} = (\langle \tilde{p}, \tilde{q}\rangle)_{m\times n}$ is positive and negative membership hesitant value given by the experts the alternatives $\mathcal{A}_i$ and the criteria $\mathcal{C}_j$, $\tilde{p} \in [0, 1]$, $\tilde{q} \in [-1, 0]$. The proposed mathematical logic and the model presented as follows.

Step 1:

The interval-valued hesitant bipolar decision matrix is presented in Table 1.
$\hat{H} =< \tilde{p}, \tilde{q} >= \left([\tilde{p}^L, \tilde{p}^U], [\tilde{q}^L, \tilde{q}^U]\right)_{m\times n}$
Here, $\hat{h}_{ij} = \{\hat{\gamma}_{ij} \in \hat{h}_{ij}\}$, the decision matrix becomes,

Table 1. Interval-valued hesitant bipolar fuzzy decision matrix

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$
$\mathcal{A}_1$	$\tilde{p}^L(\gamma_{11}), \tilde{p}^U(\gamma_{11})$	$\tilde{p}^L(\gamma_{12}), \tilde{p}^U(\gamma_{12})$	$\tilde{p}^L(\gamma_{1n}), \tilde{p}^U(\gamma_{1n})$
	$\tilde{q}^L(\gamma_{11}), \tilde{q}^U(\gamma_{11})$	$\tilde{q}^L(\gamma_{12}), \tilde{q}^U(\gamma_{12})$	$\tilde{q}^L(\gamma_{1n}), \tilde{q}^U(\gamma_{1n})$
$\mathcal{A}_2$	$\tilde{p}^L(\gamma_{21}), \tilde{p}^U(\gamma_{21})$	$\tilde{p}^L(\gamma_{22}), \tilde{p}^U(\gamma_{22})$	$\tilde{p}^L(\gamma_{2n}), \tilde{p}^U(\gamma_{2n})$
	$\tilde{q}^L(\gamma_{21}), \tilde{q}^U(\gamma_{21})$	$\tilde{q}^L(\gamma_{22}), \tilde{q}^U(\gamma_{22})$	$\tilde{q}^L(\gamma_{2n}), \tilde{q}^U(\gamma_{2n})$
$\vdots$			
$\mathcal{A}_m$	$\tilde{p}^L(\gamma_{m1}), \tilde{p}^U(\gamma_{m1})$	$\tilde{p}^L(\gamma_{m2}), \tilde{p}^U(\gamma_{m2})$	$\tilde{p}^L(\gamma_{mn}), \tilde{p}^U(\gamma_{mn})$
	$\tilde{q}^L(\gamma_{m1}), \tilde{q}^U(\gamma_{m1})$	$\tilde{q}^L(\gamma_{m2}), \tilde{q}^U(\gamma_{m2})$	$\tilde{q}^L(\gamma_{mn}), \tilde{q}^U(\gamma_{mn})$

Step 2:

Utilize the IVHBFWA operator, as described in equation (4.1), to integrate the assessment.

$$\begin{aligned} \hat{H}_{ij} &= \{\langle \tilde{p}_{ij}, \tilde{q}_{ij}\rangle\} \\ &= \left\{\left\langle \left[\tilde{p}^L_{ij}, \tilde{p}^U_{ij}\right], \left[\tilde{q}^L_{ij}, \tilde{q}^U_{ij}\right] \right\rangle | i = 1, 2, ...m, j = 1, 2, ...n\right\}. \end{aligned}$$

$$\bigoplus_{j=1}^{n} w_j \tilde{h_j} = \left\langle \bigcup_{\tilde{p}^L\in\tilde{p}, \tilde{p}^U\in\tilde{p}} \left[1 - \prod_{j=1}^{n}(1-\tilde{p}^L)^{w_j}, 1 - \prod_{j=1}^{n}(1-\tilde{p}^U)^{w_j}\right], \bigcup_{\tilde{q}^L\in\tilde{q}, \tilde{q}^U\in\tilde{q}} \left[-\prod_{j=1}^{n}(-\tilde{q}^L)^{w_j}, -\prod_{j=1}^{n}(-\tilde{q}^U)^{w_j}\right] \right\rangle \quad (4.1)$$

Step 3:

Calculate the distance between the alternatives $\mathcal{A}_i$ by using the proposed normalized hamming distance measure by equation (3.6) and the entropy weight method. Also, obtain the basic at-

tribute weights $\omega_j(j = 1, 2, ...n, \omega_j \geq 0)$ and $\sum_{j=1}^{n} \omega_j = 1$ and the detailed procedure is shown in equations (4.2-4.5).
Determining the interval-valued hesitant bipolar fuzzy entropy values

$$d = \frac{1}{4n} \sum_{j=1}^{n} \Big[|\tilde{p}_{\alpha}^{L} - \tilde{p}_{\beta}^{L}| + |\tilde{p}_{\alpha}^{U} - \tilde{p}_{\beta}^{U}| + |\tilde{q}_{\alpha}^{L} - \tilde{q}_{\beta}^{L}| + |\tilde{q}_{\alpha}^{U} - \tilde{q}_{\beta}^{U}| \Big] \quad (4.2)$$

$$E_{w_j} = -\frac{1}{lnm} \sum_{i=1}^{m} \frac{d_j(\alpha_i, \alpha^*)}{\sum_{i=1}^{m} d_j(\alpha_i, \alpha^*)} ln \frac{d_j(\alpha_i, \alpha^*)}{\sum_{i=1}^{m} d_j(\alpha_i, \alpha^*)}, j = 1, 2, ...n \quad (4.3)$$

$$\tilde{w}_j = \frac{1 - E_{w_j}}{\sum_{j=1}^{n} 1 - E_{w_j}}, j = 1, 2, ...n \quad (4.4)$$

$$\begin{aligned} R^* &= \{\hat{H}_j | j = 1, 2, ...n|\} = \{ < \tilde{p}_j, \tilde{q}_j | j = 1, 2, ..n > \} \\ &= \{\langle [\tilde{p}_j^L, \tilde{p}_j^U], [\tilde{q}_j^L, \tilde{q}_j^U] \rangle | j = 1, 2, ...n\} \\ &= \{\langle [min_i \tilde{p}_{ij}^L, min_i \tilde{p}_{ij}^U], [min_i \tilde{q}_{ij}^L, min_i \tilde{q}_{ij}^U] \rangle | j = 1, 2, ...n\} \end{aligned} \quad (4.5)$$

Step 4:

Determine the score function based on the definition (3.3). The score value of positive and negative membership function is calculated by using the following equation, that is,

$$S(\hat{h}) = \frac{1}{\#l_h} \frac{[(1 + \tilde{p}^L + \tilde{p}^U)]}{4} \quad (4.6)$$

$$S(\hat{h}) = \frac{1}{\#l_h} \frac{[(1 + \tilde{q}^L + \tilde{q}^U)]}{4} \quad (4.7)$$

Step 5:

The IVHBF positive ideal solution denoted by $(\hat{h}_+)$ and IVHBF negative ideal solution denoted by $(\hat{h}_-)$ are given by the following conditions.

$$\hat{h}_+ = \Big\{ ([max\tilde{p}_{ij}^L, max\tilde{p}_{ij}^U], [min\tilde{p}_{ij}^L, min\tilde{p}_{ij}^U] / (\gamma_{ij}) \in \hat{h}_{ij} \in \mathcal{B}), \quad (4.8)$$
$$([min\tilde{q}_{ij}^L, min\tilde{q}_{ij}^U], [max\tilde{q}_{ij}^L, max\tilde{q}_{ij}^U] / (\gamma_{ij}) \in \hat{h}_{ij} \in \mathcal{H}) \Big\}$$

$$\hat{h}_- = \Big\{ ([min\tilde{p}_{ij}^L, min\tilde{p}_{ij}^U], [max\tilde{p}_{ij}^L, max\tilde{p}_{ij}^U] / (\gamma_{ij}) \in \hat{h}_{ij} \in \mathcal{B}), \quad (4.9)$$
$$([max\tilde{q}_{ij}^L, max\tilde{q}_{ij}^U], [min\tilde{q}_{ij}^L, min\tilde{q}_{ij}^U] / (\gamma_{ij}) \in \hat{h}_{ij} \in \mathcal{H}) \Big\}$$

where $\mathcal{B}$ and $\mathcal{H}$ are the set of benefit and cost criteria respectively.

Step 6:

Calculate the group utility S_i values for the highest level and the individual regret values R_i for the opponent.

$$S_i = \sum_{i=1}^{m} (w_j * \frac{d(h_i^+ - h_{ij})}{d(h_i^+ - h_i^-)}) \quad (4.10)$$

$$R_i = max \Big\{ (w_j * \frac{d(h_i^+ - h_{ij})}{d(h_i^+ - h_i^-)}) \Big\} \quad (4.11)$$

Step 7:

Calculate the index values G_i as follows

$$G_i = \vartheta * (\frac{S_i - S_i^N}{S_i^P - S_i^N}) + (1 - \vartheta)(\frac{R_i - R_i^N}{R_i^P - R_i^N}) \tag{4.12}$$

where $S_i^P = minS_i$, $S_i^N = maxS_i$, $R_i^P = minR_i$, $R_i^N = maxR_i$ and ϑ is the weight of the decision-making strategy.

Step 8:

Rank the alternatives by organizing each S_i, R_i, and G_i value in descending order. Investigate the alternative corresponding to the smallest G_i (smallest among G_i values) value to identify a compromise solution.
In Fig.:1, the IVHBF-VIKOR method's workflow is depicted as a hierarchical structure.

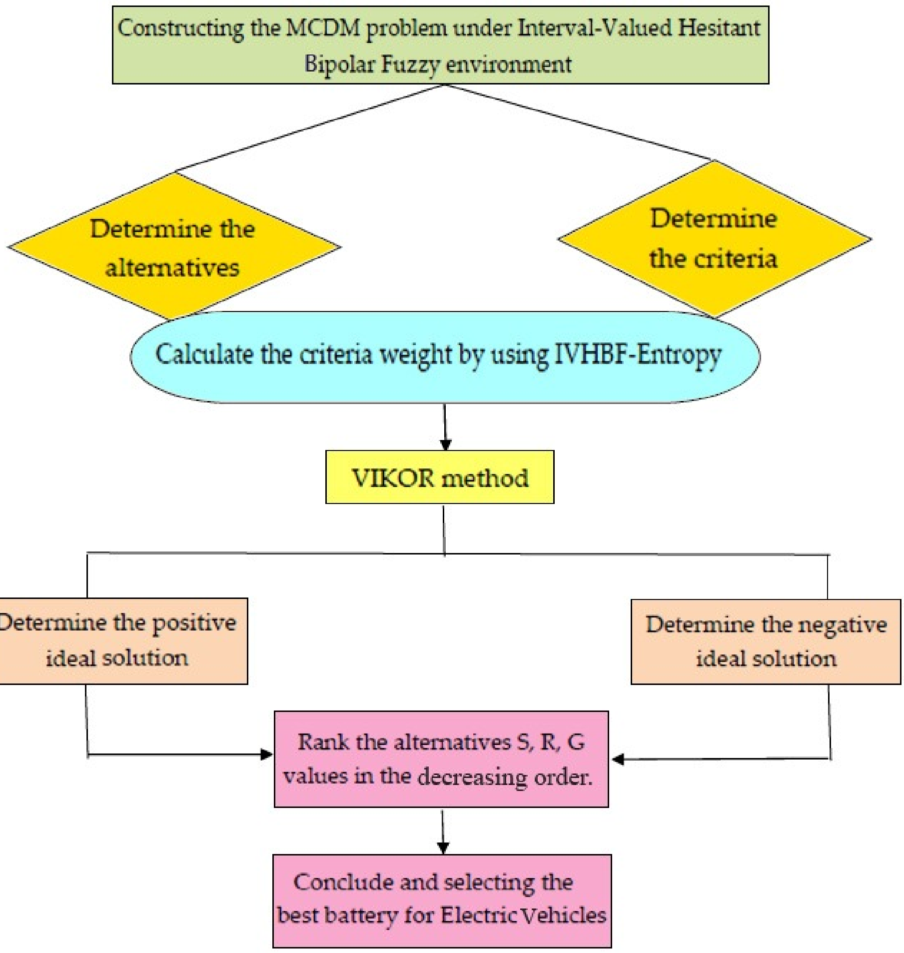

Figure 1. Flow chart of interval-valued hesitant bipolar fuzzy-VIKOR method

5 Illustrative Example

The extended IVHBFs describes the use of lithium battery material selection for electric vehicles. In this example, EVs play a very important role in smart cities, public transportation etc., requiring more efforts to facilitate the charging process and upgrade the batteries. The problem in the battery selection of EVs is the most important and sensitive in MCDM problem. Introduced MCDM methods to solve lithium battery selection of EVs problem. The alternatives utilized in this study are recommended based on individual feedback, with weighting vector w_i = 0.5 taking into account the following five aspects.

- $(\mathcal{C}_1) \rightarrow$ Cost
- $(\mathcal{C}_2) \rightarrow$ Reliability
- $(\mathcal{C}_3) \rightarrow$ Safety
- $(\mathcal{C}_4) \rightarrow$ Power
- $(\mathcal{C}_5) \rightarrow$ Energy density

Then, the five different batteries are chosen related to lithium battery are,

- $(\mathcal{A}_1) \rightarrow$ Li-ion Cobalt Oxide Battery ($LiCoO_2$)-LCOB
- $(\mathcal{A}_2) \rightarrow$ Li-ion Manganese Oxide Battery ($LiMn_2O_4$)-LMOB
- $(\mathcal{A}_3) \rightarrow$ Li-ion Nickel Manganese Cobalt Oxide Battery-($LiNiMnCoO_2$)-LNMCOB
- $(\mathcal{A}_4) \rightarrow$ Li-ion Iron Phosphate Battery ($LiFePO_4$)-LIPB
- $(\mathcal{A}_5) \rightarrow$ Li-ion Titanate Battery ($Li_4Ti_5O_{12}$)-LTOB

Table 2. Linguistic scale for rating the alternatives

Linguistic term	IVHBF-positive membership numbers	IVHBF-negative membership numbers
Extremely Good	1.0	0.1
Very Very Good	0.9	0.2
Very Good	0.8	0.3
Good	0.7	0.4
Medium Good	0.6	0.5
Fair	0.5	0.6
Medium Bad	0.4	0.7
Bad	0.3	0.8
Very Bad	0.2	0.9
Very Very Bad	0.1	1.0

Main Result

Now, we make a numerical evaluation under the proposed IVHBF environment. The decision matrix of IVHBF values are given in the Table 3. This decision matrix includes five criteria and five alternatives. Here are the steps and outcomes of the calculation for employing the suggested VIKOR method, which relies on Entropy and distance measures, to address the issue.

Step 1:

Standardize the initial matrix given by the decision maker independently according to the equation (3.1).

Table 3. Interval-valued hesitant bipolar fuzzy decision matrix

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$
$\mathcal{A}_1$	$\{[0.31, 0.46], [0.39, 0.57], [0.6, 0.71]\}, \{[-0.52, -0.4], [-0.5, -0.41], [-0.29, -0.2]\},$	$\{[0.66, 0.8], [0.7, 0.78], [0.73, 0.79]\}, \{[-0.22, -0.13], [-0.14, -0.1], [-0.12, -0.09]\},$	$\{[0.46, 0.55], [0.32, 0.41], [0.4, 0.54]\}, \{[-0.47, -0.36], [-0.54, -0.4], [-0.43, -0.34]\},$	$\{[0.32, 0.5], [0.47, 0.6], [0.51, 0.6]\} \{[-0.61, -0.49], [-0.44, -0.35], [-0.41, -0.32]\}$	$\{[0.28, 0.45], [0.51, 0.62], [0.71, 0.8]\}, \{[-0.5, -0.42], [-0.31, -0.26] [-0.22, -0.18]\},$
$\mathcal{A}_2$	$\{[0.56, 0.72], [0.64, 0.81], [0.63, 0.73]\}, \{[-0.32, -0.23], [-0.2, -0.14], [-0.24, -0.15]\},$	$\{[0.62, 0.75], [0.41, 0.6], [0.56, 0.67]\}, \{[-0.29, -0.19], [-0.42, -0.29], [-0.44, -0.31]\},$	$\{[0.24, 0.47], [0.2, 0.27], [0.27, 0.36]\}, \{[-0.65, -0.52], [-0.85, -0.8], [-0.63, -0.52]\},$	$\{[0.61, 0.73], [0.64, 0.76], [0.71, 0.86]\} \{[-0.26, -0.14], [-0.23, -0.11], [-0.16, -0.1]\}$	$\{[0.61, 0.74], [0.31, 0.51], [0.44, 0.62]\}, \{[-0.28, -0.18], [-0.47, -0.37] [-0.42, -0.31]\},$
$\mathcal{A}_3$	$\{[0.61, 0.82], [0.53, 0.65], [0.61, 0.75]\}, \{[-0.18, -0.11], [-0.4, -0.31], [-0.27, -0.17]\},$	$\{[0.55, 0.75], [0.5, 0.64], [0.57, 0.71]\}, \{[-0.35, -0.24], [-0.4, -0.33], [-0.4, -0.29]\},$	$\{[0.13, 0.27], [0.24, 0.36], [0.3, 0.4]\}, \{[-0.86, -0.81], [-0.66, -0.57], [-0.6, -0.5]\},$	$\{[0.55, 0.69], [0.72, 0.81], [0.67, 0.74]\} \{[-0.4, -0.33], [-0.15, -0.11], [-0.24, -0.13]\}$	$\{[0.58, 0.62], [0.6, 0.7], [0.71, 0.78]\}, \{[-0.14, -0.05], [-0.12, -0.09] [-0.11, -0.09]\},$
$\mathcal{A}_4$	$\{[0.47, 0.65], [0.35, 0.49], [0.46, 0.58]\}, \{[-0.37, -0.26], [-0.63, -0.47], [-0.5, -0.44]\},$	$\{[0.59, 0.7], [0.62, 0.69], [0.64, 0.73]\}, \{[-0.25, -0.17], [-0.26, -0.13], [-0.22, -0.14]\},$	$\{[0.33, 0.5], [0.25, 0.38], [0.38, 0.5]\}, \{[-0.63, -0.55], [-0.6, -0.52], [-0.39, -0.31]\},$	$\{[0.67, 0.8], [0.7, 0.84], [0.6, 0.71]\} \{[-0.22, -0.1], [-0.11, -0.06], [-0.31, -0.19]\}$	$\{[0.47, 0.61], [0.41, 0.62], [0.3, 0.4]\}, \{[-0.32, -0.25], [-0.4, -0.3] [-0.5, -0.42]\},$
$\mathcal{A}_5$	$\{[0.63, 0.84], [0.5, 0.67], [0.7, 0.7]\}, \{[-0.1, -0.08], [-0.42, -0.32], [-0.09, -0.04]\},$	$\{[0.63, 0.72], [0.54, 0.66], [0.61, 0.73]\}, \{[-0.24, -0.16], [-0.39, -0.3], [-0.35, -0.26]\},$	$\{[0.32, 0.5], [0.1, 0.21], [0.21, 0.29]\}, \{[-0.54, -0.45], [-0.9, -0.84], [-0.71, -0.63]\},$	$\{[0.7, 0.86], [0.8, 0.89], [0.63, 0.71]\} \{[-0.13, -0.07], [-0.09, -0.02], [-0.29, -0.22]\}$	$\{[0.45, 0.63], [0.61, 0.69], [0.45, 0.55]\}, \{[-0.35, -0.25], [-0.22, -0.12] [-0.3, -0.2]\},$

Step 2:

Combine the rating information in the matrix and use equation (4.1) to create a composite matrix $\hat{H}_{ij} = (\hat{h}_{ij})_{5\times5}$, shown in Table .4.

Table 4. Interval-valued hesitant bipolar fuzzy group matrix

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$
$\mathcal{A}_1$	$[0.58, 0.74]$ $[-0.27, -0.18]$	$[0.83, 0.91],$ $[-0.06, -0.03]$	$[0.53, 0.87],$ $[-0.33, -0.22]$	$[0.58, 0.72]$ $[-0.33, -0.23]$	$[0.68, 0.79]$ $[-0.18, -0.14]$
$\mathcal{A}_2$	$[0.76, 0.88],$ $[-0.12, -0.07]$	$[0.68, 0.82]$ $[-0.23, -0.13]$	$[0.33, 0.51],$ $[-0.58, -0.46]$	$[0.71, 0.91]$ $[-0.09, -0.04]$	$[0.61, 0.78]$ $[-0.24, -0.14]$
$\mathcal{A}_3$	$[0.73, 0.87],$ $[-0.14, -0.07]$	$[0.68, 0.84]$ $[-0.24, -0.15]$	$[0.32, 0.47],$ $[-0.34, -0.48]$	$[0.79, 0.87]$ $[-0.12, -0.06]$	$[0.78, 0.84]$ $[-0.04, -0.02]$
$\mathcal{A}_4$	$[0.56, 0.76],$ $[-0.34, -0.23]$	$[0.76, 0.84]$ $[-0.12, -0.05]$	$[0.44, 0.61],$ $[-0.38, -0.29]$	$[0.8, 0.9]$ $[-0.08, -0.03]$	$[0.53, 0.7]$ $[-0.25, -0.17]$
$\mathcal{A}_5$	$[0.76, 0.87],$ $[-0.06, -0.03]$	$[0.74, 0.84]$ $[-0.18, -0.11]$	$[0.31, 0.47],$ $[-0.34, -0.24]$	$[0.85, 0.93]$ $[-0.05, -0.02]$	$[0.88, 0.77]$ $[-0.15, -0.07]$

Step 3:

The basic attribute derives weights $w_j (j = 1, 2, 3, 4, 5), w_j \geq 0$ and $\sum_{j=1}^{5} w_j = 1$ are obtained based on the distance equation (4.2) and the entropy weight method discussed in equations (4.2-4.5). Then,
$w = (w_1, w_2, w_3, w_4, w_5)^T = (0.150, 0.272, 0.101, 0.355, 0.122)^T$.

Step 4:

We determine the score function using the equations (4.6 & 4.7) from the Table 3.

Table 5. Interval-valued hesitant bipolar score matrix

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$
$\mathcal{A}_1$	$(0.67, -0.22)$	$(0.91, 0.03)$	$(0.61, -0.26)$	$(0.66, -0.27)$	$(0.73, -0.15)$
$\mathcal{A}_2$	$(0.85, -0.04)$	$(0.77, -0.16)$	$(0.47, -0.49)$	$(0.88, 0)$	$(0.71, -0.17)$
$\mathcal{A}_3$	$(0.83, -0.07)$	$(0.79, -0.17)$	$(0.45, -0.5)$	$(0.86, -0.06)$	$(0.83, 0.06)$
$\mathcal{A}_4$	$(0.66, -0.28)$	$(0.83, -0.03)$	$(0.55, -0.33)$	$(0.88, 0.001)$	$(0.64, -0.19)$
$\mathcal{A}_5$	$(0.84, -0.008)$	$(0.82, -0.12)$	$(0.44, -0.51)$	$(0.93, -0.03)$	$(0.73, -0.07)$

Step 5:

Based on equation (4.8 & 4.9) the best and worst values of all attributes can be calculated then we have, $h_1^+ = (0.85, -0.04)$, $h_2^+ = (0.91, -0.03)$, $h_3^+ = (0.61, -0.26)$, $h_4^+ = (0.93, -0.03)$, $h_5^+ = (0.83, -0.06)$ $h_1^- = (0.66, -0.28)$, $h_2^- = (0.77, -0.16)$, $h_3^- = (0.44, -0.51)$, $h_4^- = (0.86, -0.06)$, $h_5^- = (0.64, -0.19)$

Step 6:

Calculate the group utility and individual regret values S, R and G, as provided in the table. Then, we sorted each S, R and G values from the minimum value and sorted the alternatives.

Table 6. Index values of each alternative

	S_i	R_i	G_i
$\mathcal{A}_1$	0.57	0.35	1
$\mathcal{A}_2$	0.52	0.26	0.642
$\mathcal{A}_3$	0.46	0.27	0.491
$\mathcal{A}_4$	0.47	0.15	0.242
$\mathcal{A}_5$	0.38	0.20	0.129

Step 7:

Ranking the alternatives and sorting by the value of G_i in the decreasing order, we get, $\mathcal{A}_5 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_1$.

Table 7. The evaluation value of each alternative

	G_i	Ranking order
$\mathcal{A}_1$	1	5
$\mathcal{A}_2$	0.642	4
$\mathcal{A}_3$	0.491	3
$\mathcal{A}_4$	0.242	2
$\mathcal{A}_5$	0.129	1

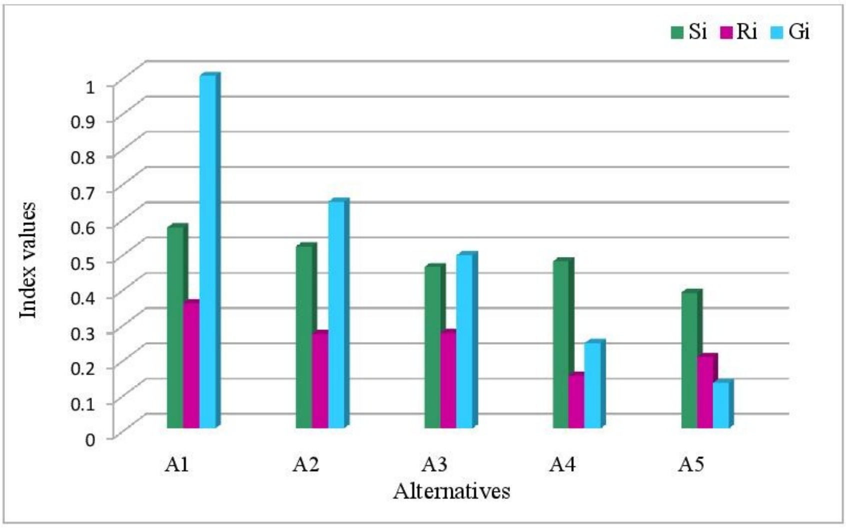

Figure 2. Graphical representation of IVHBF-VIKOR Method

6 Discussion

Hence, $\mathcal{A}_5$ emerges as the best alternative for the selection of electric vehicle batteries. The results supporting this conclusion are illustrated in Figure 2, while the detailed ranking values are provided in Table 7. $\mathcal{A}_5$ is preferred because it has the lowest ranking value among the alternatives, indicating its superior performance according to the criteria evaluated.

In the context of the IVHBF-VIKOR method, the alternatives that achieve the highest rankings are those that are closest to the ideal solution. This proximity to the ideal solution signifies that $\mathcal{A}_5$ aligns most closely with the optimal performance benchmarks set by the evaluation criteria. The IVHBF-VIKOR method simplifies the decision-making process by avoiding complex computations typically associated with other multi-criteria decision making methods. Instead, it derives the weight vector using the IVHBF entropy approach, which effectively captures the inherent vagueness and uncertainty of the available information.

The IVHBF entropy method is particularly adept at handling situations where information is imprecise or incomplete, which is often the case in real-world decision-making scenarios. By emphasizing the uncertainty and hesitancy in the data, this method ensures a more realistic and nuanced assessment of the alternatives.

Our proposed method demonstrates significant advantages, especially when dealing with high-dimensional fuzzy elements and situations where the attribute weights are entirely unknown. Traditional approaches often struggle in such conditions, either due to computational complexity or the inability to accurately capture the uncertainty in the data. In contrast, our method leverages the strengths of interval-valued hesitant bipolar fuzzy sets to provide a more effective and reliable decision-making framework. The effectiveness of our approach is evident in the results, where $\mathcal{A}_5$ stands out as the optimal choice. This outcome not only highlights the robustness of the IVHBF-VIKOR method but also underscores its practicality in real-world applications where decision-makers face high levels of uncertainty and conflicting criteria.

In summary, the proposed method offers a novel and efficient solution for selecting the best alternative in complex decision-making scenarios. By integrating IVHBF entropy and IVHBF-VIKOR, we provide a comprehensive framework that outperforms existing methods, particularly in handling vague and uncertain information. This makes our approach highly suitable for the dynamic and rapidly evolving field of electric vehicle battery selection, as well as other areas requiring sophisticated multi criteria decision making tools

7 Conclusion

In situations where decision-makers are initially uncertain or unwilling to disclose their preferences regarding criteria, the proposed method demonstrates significant effectiveness. This chapter applies our approach to the challenging problem of selecting lithium battery materials, crucial for the advancement of electric vehicles (EVs). Batteries are pivotal in enhancing EV quality while reducing production costs. However, the criteria for selecting batteries can vary widely depending on specific projects or applications.

The VIKOR method is particularly suited for this task as it excels in selecting and prioritizing alternatives amidst conflicting criteria. Our proposed method involves several key steps. First, we determine aggregate weights using interval-valued hesitant bipolar fuzzy entropy (IVHBF-entropy) measures. These measures allow us to capture the hesitant and often conflicting judgments of decision-makers more accurately.

Next, we apply the interval-valued hesitant bipolar fuzzy VIKOR (IVHBF-VIKOR) method to rank the alternatives based on five primary criteria. These criteria may include factors such as energy density, charge/discharge efficiency, lifespan, safety, and cost. By processing these criteria through the IVHBF-VIKOR method, we ensure a comprehensive and nuanced evaluation of each alternative.

The results from our approach are noteworthy because they closely approximate the ideal solution, reflecting a high degree of alignment with optimal decision-making. The ranking process is grounded in index values, which serve as the basis for evaluating and comparing the alternatives.

In this specific application, our findings indicate that the Li-ion Titanate Battery (A_5) emerges as the best substitute. This conclusion is derived from a systematic evaluation that places (A_5) at the top of the preference order. The overall order of preference for the alternatives is as follows: $\mathcal{A}_5 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_1$.

This ranking indicates that (A_5) is the most suitable choice given the criteria and the decision-making framework employed. This comprehensive assessment and ranking method not only facilitates the selection of optimal Li-ion battery materials but also provides a robust tool for addressing complex decision-making scenarios in various other applications. By incorporating IVHBF-entropy and IVHBF-VIKOR methods, we offer a novel and effective approach to managing hesitant and bipolar fuzzy information in multi criteria decision making contexts. In our future findings are extensions of fuzzy sets and comparative and sensitivity analysis of exsiting methods.

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