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Conformal Ricci-Yamabe solitons on hyperbolic Kenmotsu manifolds

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Abstract This article aims to characterize hyperbolic Kenmotsu manifolds that satisfy conformal Ricci-Yamabe solitons and 3-dimensional hyperbolic Kenmotsu manifolds that adhere to the criteria of conformal gradient Ricci-Yamabe solitons.

1 Introduction

In 1964, Eells and Sampson introduced a fundamental notion by proposing the harmonic heat flow map on a Riemannian manifold. This concept laid a significant groundwork for further developments. Hamilton's initial exploration of the Ricci flow was inspired by their work. In 1981, Hamilton [4] employed the Ricci flow to gain insights into William Thurston's geometrization conjecture. Subsequently, in 1988, Hamilton introduced the Yamabe flow, contributing to further developments. The Ricci flow and the Yamabe flow have numerous applications, particularly within mathematics and physics. The concept of the Ricci-Yamabe flow, a fusion of the Ricci and Yamabe flows, was introduced in 2019 by authors of [3]. Within the context of a semi-Riemannian manifold, the Ricci-Yamabe soliton is characterized by the equation:

$$\pounds_Z g + (2\Upsilon - \mathbf{l}r)g + 2\mathbf{k}Ric_g = 0, \tag{1.1}$$

Here, $\pounds_Z$ denotes the Lie derivative, Ric_g represents the Ricci tensor, r signifies the scalar curvature, and Υ, $\mathbf{k}$, and $\mathbf{l}$ are real coefficients. Ricci-Yamabe solitons serve as special solutions of the Ricci-Yamabe flow:

$$\frac{\partial g}{\partial t} = -2\mathbf{k}Ric_g + \mathbf{l}rg, \tag{1.2}$$

as introduced by Guler and Crasmareanu [3]. Equation (1.1) is termed an almost Ricci-Yamabe soliton if Υ is a smooth function. The behavior of a Ricci-Yamabe soliton is classified as expanding, steady, or shrinking based on whether Υ is greater than 0, equal to 0, or less than 0, respectively. Particularly interesting is the case where $\mathbf{k} = 1$ and $\mathbf{l} = 0$, leading to the equation:

$$\pounds_Z g + 2Ric_g + 2\Upsilon g = 0, \tag{1.3}$$

which corresponds to the Ricci soliton equation for $\Upsilon \in \mathbb{R}$. This highlights that almost Ricci-Yamabe solitons (and Ricci-Yamabe solitons) naturally extend the concept of almost Ricci solitons (and Ricci solitons). As an extension of equation (1.1) and the idea of conformal Ricci soliton [1], S. Roy and A. Bhattacharya introduced the idea of Conformal Ricci-Yamabe solitons, denoted as *C-RYS*, in their work [16]. This notion is defined as follows:

Definition 1.1. A Riemannian manifold (M^{2m+1}, g), where $2m + 1 > 2$, is said to possess a conformal Ricci-Yamabe soliton, briefly denoted as *C-RYS*, when it satisfies the equation:

$$\pounds_Z g + 2\mathbf{k}Ric_g + \left[2\Upsilon - \mathbf{l}r - \left(p + \frac{2}{2m+1}\right)\right] g = 0. \tag{1.4}$$

If the vector field Z can be expressed as the gradient of a smooth function, denoted as $Z = \text{grad}(f)$, where f is a smooth function on M, then the concept *C-RYS* is referred to as a conformal gradient Ricci-Yamabe soliton briefly denoted as *C-GRYS*. In such cases, equation (1.4) simplifies to:

$$2\text{Hess}f + 2\mathrm{k}Ric_g + \left[2\Upsilon - \mathrm{l}r - \left(p + \frac{2}{2m+1}\right)\right] g = 0, \tag{1.5}$$

Here, Hess$f$ represents the Hessian of the smooth function f. The classification of *C-RYS* solitons depends on the sign of Υ. Specifically, when Υ is positive, the soliton is referred to as expanding. When Υ is zero, the soliton is categorized as steady. And when Υ is negative, the soliton is characterized as shrinking. For further information on Ricci solitons and their generalizations, we recommend consulting [5, 8, 12, 13, 14, 18, 19] and the references cited therein.
On the other hand, the concept of an almost-contact hyperbolic structure (ϕ, ζ, η, g) was introduced by Upadhyay and Dube [17]. This idea has been further explored by various researchers [2, 6, 10, 15]. A nonzero vector field $v \in T_p(M)$ is classified as timelike (or null, spacelike, and nonspacelike) based on whether $g_p(v, v)$ is less than 0 (equal to 0, greater than 0, or less than or equal to 0) [9], where $T_p(M)$ represents the tangent space of M at point p. Let $\{e_1, e_2, ..., e_{2m}, e_{2m+1} = \zeta\}$ form a local orthonormal basis of vector fields in a $(2m+1)$-dimensional semi-Riemannian manifold. Consequently, the Ricci tensor Ric_g and the scalar curvature r of a $(2m+1)$-dimensional almost hyperbolic contact metric manifold M^{2m+1}, equipped with the semi-Riemannian metric g, are defined as follows:

$$\begin{aligned} Ric_g(V_1, V_2) &= \sum_{i=1}^{2m+1} \epsilon_i g(R(e_i, V_1)V_2, e_i) = \sum_{i=1}^{2m} \epsilon_i g(R(e_i, V_1)V_2, e_i) + g(R(\zeta, V_1)V_2, \zeta) \\ r &= \sum_{i=1}^{2m+1} \epsilon_i Ric_g(e_i, e_i) = \sum_{i=1}^{2m} \epsilon_i Ric_g(e_i, e_i) + Ric_g(\zeta, \zeta), \end{aligned} \tag{1.6}$$

for all vector fields V_1 and V_2, where $\epsilon_i = g(e_i, e_i)$, ζ is the unit timelike vector field satisfying $g(\zeta, \zeta) = -1$, and R denotes the curvature tensor of M^{2m+1} [9].
In 1972, Kenmotsu conducted a study on a particular class of contact Riemannian manifolds that satisfy distinct criteria [7]. These manifolds were subsequently termed Kenmotsu manifolds. Kenmotsu's investigation led to the identification that a locally Kenmotsu manifold can be represented as a warped product $I \times_f M$, where I is an interval and M is a Kaehler manifold, with the warping function $f(t) = se^t$; where s is a non-zero constant. This concept is encapsulated by the condition:

$$(\mathfrak{D}_{V_1}\phi)(V_2) = g(\phi V_1, V_2)\zeta - \eta(V_2)\phi V_1, \tag{1.7}$$

This relation holds for all vector fields V_1 and V_2.

These investigations have motivated our research on *C-RYS* and *C-GRYS* in hyperbolic Kenmotsu manifolds. The structure of this paper is outlined as follows: Following the introductory section, Section 2 provides an overview of the necessary background information about para-Kenmotsu manifolds. The exploration of *C-RYS* and *C-GRYS* in hyperbolic Kenmotsu manifolds is conducted in Section 3 and Section 4 respectively.

2 Hyperbolic Kenmotsu Manifolds

Consider a $(2m+1)$-dimensional almost hyperbolic contact manifold denoted as M^{2m+1} [17]. In this context, the manifold possesses essential components: a fundamental tensor field ϕ of type $(1, 1)$, a timelike vector field ζ, and a 1-form η. The relationships governing these elements are defined as follows:

$$\phi^2 V_1 = V_1 + \eta(V_1)\zeta, \;\; \eta(\zeta) = -1, \phi\zeta = 0, \eta \circ \phi = 0, rank(\phi) = 2m. \tag{2.1}$$

Here, I represents the identity endomorphism of the tangent bundle of M^{2m+1}. An almost hyperbolic contact manifold M^{2m+1} is termed an almost hyperbolic contact metric manifold if the semi-Riemannian metric g of M^{2m+1} satisfies the conditions:

$$g(\phi V_1, \phi V_2) = -g(V_1, V_2) - \eta(V_1)\eta(V_2), \tag{2.2}$$
$$g(V_1, \phi V_2) = -g(\phi V_1, V_2), \; g(V_1, \zeta) = \eta(V_1), \tag{2.3}$$

for all $V_1, V_2 \in \chi(M)$. Within this context, the amalgamation of components (ϕ, ζ, η, g) characterizes an almost hyperbolic contact metric configuration on M^{2m+1}. When an almost hyperbolic contact metric manifold M^{2m+1} adheres to equation (1.7), it is denoted as a hyperbolic Kenmotsu manifold according to the reference [2].
In a hyperbolic Kenmotsu manifold, the following relations hold: [11]

$$(\mathfrak{D}_{V_1}\phi)(V_2) = g(\phi V_1, V_2)\zeta - \eta(V_2)\phi V_1, \tag{2.4}$$
$$\mathfrak{D}_{V_1}\zeta = -V_1 - \eta(V_1)\zeta, \tag{2.5}$$
$$(\mathfrak{D}_{V_1}\eta)V_2 = -g(V_1, V_2) - \eta(V_1)\eta(V_2), \tag{2.6}$$
$$R(V_1, V_2)\zeta = \eta(V_2)V_1 - \eta(V_1)V_2, \tag{2.7}$$
$$R(\zeta, V_1)V_2 = g(V_1, V_2)\zeta - \eta(V_2)V_1, \tag{2.8}$$
$$Ric_g(V_1, \zeta) = (2m)\eta(V_1). \tag{2.9}$$

Pankaj et al. [11] established the following results for three-dimensional hyperbolic Kenmotsu manifolds:

Lemma 2.1. *[11] In a three-dimensional hyperbolic Kenmotsu manifold M^3, the following equation holds:*

$$\zeta r = 2(r-6). \tag{2.10}$$

Furthermore, within M^3, we observe the relationship:

$$QV_1 = \left(\frac{r}{2} - 1\right) V_1 + \left(\frac{r}{2} - 3\right) \eta(V_1)\zeta, \tag{2.11}$$

This, in turn, yields:

$$Ric_g(V_1, V_2) = \left(\frac{r}{2} - 1\right) g(V_1, V_2) + \left(\frac{r}{2} - 3\right) \eta(V_1)\eta(V_2), \tag{2.12}$$

Here, Q represents the Ricci operator defined by $Ric_g(V_1, V_2) = g(QX, V_2)$.

3 Conformal Ricci-Yamabe soliton on Hyperbolic-Kenmotsu manifolds

Let us consider that, a hyperbolic Kenmotsu manifold admits conformal Ricci-Yamabe soliton. Then from (1.4), we have

$$\pounds_Z g(V_1, V_2) + 2\mathbf{k}Ric_g(V_1, V_2) + \left[2\Upsilon - \mathbf{l}r - \left(p + \frac{2}{2m+1}\right)\right] g(V_1, V_2) = 0, \tag{3.1}$$

which yields

$$g(\mathfrak{D}_{V_1}\zeta, V_2) + g(V_1, \mathfrak{D}_{V_2}\zeta,) + 2\mathbf{k}Ric_g(V_1, V_2) + \left[2\Upsilon - \mathbf{l}r - \left(p + \frac{2}{2m+1}\right)\right] g(V_1, V_2) = 0. \tag{3.2}$$

By employing equation (2.5) in the preceding expression, one can easily obtain

$$\mathbf{k}Ric_g(V_1, V_2) = \eta(V_1)\eta(V_2) - \frac{1}{2}\left[2\Upsilon - \mathbf{l}r - \left(p + \frac{2}{2m+1} + 2\right)\right] g(V_1, V_2). \tag{3.3}$$

Setting $V_1 = V_2 = \zeta$ in the above equation yields $\frac{1}{2}r = (2m)\mathbf{k} + \Upsilon - \frac{p}{2} - \frac{1}{2m+1} - 1$. By virtue of this, equation (3.3) takes the form

$$Ric_g(V_1, V_2) = \frac{1}{\mathbf{k}}\eta(V_1)\eta(V_2) - 2mg(V_1, V_2).$$

Hence, we can state the following:

Theorem 3.1. *If a hyperbolic Kenmotsu manifold $(M^{2m+1}, \phi, \zeta, \eta, g)$, admits a C-RYS, then (M^{2m+1}, g) is η-Einstein manifold.*

Moreover, replacing V_2 by ζ in (3.3) we find that

$$Ric_g(V_1, \zeta) = -\frac{1}{\mathbf{k}}\left[\Upsilon - \frac{1}{2}r - \frac{p}{2} - \frac{1}{2m+1}\right]\eta(V_1). \tag{3.4}$$

By comparing the preceding equation with (2.8), we deduce:

$$\Upsilon = -2m\mathbf{k} + \frac{\mathbf{l}r}{2} + \frac{p}{2} + \frac{1}{2m+1} \tag{3.5}$$

As a result, the following emerges:

Theorem 3.2. *If an (2m+1)-dimensional hyperbolic Kenmotsu manifold admits a C-RYS, then $(M^{2m+1}, \phi, \zeta, \eta, g)$ becomes an η-Einstein manifold and the scalar Υ is given by $\Upsilon = -2m\mathbf{k} + \frac{\mathbf{l}r}{2} + \frac{p}{2} + \frac{1}{2m+1}$.*

Next, Consider $\{e_i\}_{1\leq i\leq 2m+1}$ be an orthonormal frame. By taking $V_1 = e_i$, $V_2 = e_1$ in (3.1) and summing over i, we obtain

$$divZ = -r\left(\mathbf{k} - \frac{1}{2}(2m+1)\right) - (2m+1)\Upsilon + \frac{(2m+1)p}{2} + 1. \tag{3.6}$$

Assume, for a smooth function ψ if Z is of gradient type i.e. $Z = grad(\psi)$, then the equation (3.6) becomes

$$\Delta(\psi) = -r\left(\mathbf{k} - \frac{1}{2}(2m+1)\right) - (2m+1)\Upsilon + \frac{(2m+1)p}{2} + 1, \tag{3.7}$$

where $\Delta(\psi)$ is the Laplacian equation satisfied by ψ. Hence, we can state the following:

Theorem 3.3. *If $(g, Z, \Upsilon, \mathbf{k}, \mathbf{l})$ is a C-RYS on a $(2m+1)$-dimensional hyperbolic Kenmotsu manifold $(M^{2m+1}, \phi, \zeta, \eta, g)$ where Z is the gradient of a smooth function ψ, then the Laplacian equation satisfied by ψ is given by (3.7).*

4 Conformal gradient Ricci-Yamabe soliton on 3-dimensional Hyperbolic-Kenmotsu manifolds

In this section, we consider the 3-dimensional hyperbolic Kenmotsu manifold that admits *C-GRYS*. First, we prove the following.

Lemma 4.1. *If $(g, Z, \Upsilon, \mathbf{k}, \mathbf{l})$ is a C-GRYS on a $(2m+1)$-dimensional hyperbolic Kenmotsu manifold M^{2m+1}, then the Riemannian curvature tensor R satisfies*

$$\begin{aligned} R(V_1, V_2)grad(f) &= -\mathbf{k}[(\mathfrak{D}_{V_1}Q)V_2 - (\mathfrak{D}_{V_2}Q)V_1] - V_1(\Upsilon)V_2 + V_2(\Upsilon)V_1 \\ &\quad + \frac{1}{2}[V_1(r)V_2 - V_2(r)V_1]. \end{aligned} \tag{4.1}$$

Proof. Let a $(2m+1)$-dimensional hyperbolic Kenmotsu manifold admits *C-GRYS*, then (1.5) yields

$$\mathfrak{D}_{V_1}grad(f) = -\mathbf{k}QE_1 - \left(\Upsilon - \frac{1}{2}r - \frac{1}{2}\left(p + \frac{2}{2m+1}\right)\right)V_1. \tag{4.2}$$

Taking covariant differentiation of (4.2) we obtain

$$\begin{aligned} \mathfrak{D}_{V_2}\mathfrak{D}_{V_1}grad(f) &= -\mathbf{k}[(\mathfrak{D}_{V_2}Q)V_1 + Q(\mathfrak{D}_{V_2}V_1)] - V_2(\Upsilon)V_1 - \Upsilon(\mathfrak{D}_{V_2}V_1) \\ &\quad + \frac{1}{2}r\mathfrak{D}_{V_2}V_1 + \frac{1}{2}V_2(r)V_1 + \frac{1}{2}\left(p + \frac{2}{2m+1}\right)\mathfrak{D}_{V_2}V_1. \end{aligned} \tag{4.3}$$

Swapping V_1 and V_2 in (4.2) implies

$$\begin{aligned}\mathfrak{D}_{V_1}\mathfrak{D}_{V_2}grad(f) &= -\mathbf{k}[(\mathfrak{D}_{V_1}Q)V_2 + Q(\mathfrak{D}_{V_1}V_2)] - V_1(\Upsilon)V_2 - \Upsilon(\mathfrak{D}_{V_1}V_2)\\ &\quad + \frac{\mathbf{1}}{2}r\mathfrak{D}_{V_1}V_2 + \frac{\mathbf{1}}{2}V_1(r)V_2 + \frac{1}{2}\left(p + \frac{2}{2m+1}\right)\mathfrak{D}_{V_1}V_2. \qquad (4.4)\end{aligned}$$

Substituting Equations (4.2), (4.3) and (4.4) in the definition of Riemannian curvature, we obtain (4.1). □

Now, differentiating (2.11) covariantly with respect to V_2, we have

$$(\mathfrak{D}_{V_2}Q)V_1 = \frac{V_2(r)}{2}[V_1 + \eta(V_1)\zeta] - \left(\frac{r}{2} - 3\right)[g(V_1, V_2)\zeta + 2\eta(V_1)\eta(V_2)\zeta + \eta(V_1)V_2]. \qquad (4.5)$$

Utilizing (4.5) in (4.1), we obtain

$$\begin{aligned}R(V_1, V_2)grad(f) &= -\mathbf{k}\{\frac{V_1(r)}{2}[V_2 + \eta(V_2)\zeta] - \frac{V_2(r)}{2}[V_1 + \eta(V_1)\zeta] - \left(\frac{r}{2} - 3\right)[\eta(V_2)V_1\\ &\quad -\eta(V_1)V_2]\} - V_1(\Upsilon)V_2 + V_2(\Upsilon)V_1 + \frac{\mathbf{1}}{2}[V_1(r)V_2 - V_2(r)V_1]. \qquad (4.6)\end{aligned}$$

Contracting (4.6), we have

$$Ric_g(V_2, grad(f)) = (\frac{\mathbf{k}}{2} - \mathbf{1})V_2(r) + 2V_2(\Upsilon) - \frac{\mathbf{k}}{2}(r-6)\eta(V_2). \qquad (4.7)$$

Substituting V_1 by $grad(f)$ in (2.12) and comparing with (4.7), one can easily obtain

$$\left(\frac{r}{2} - 3\right)\zeta(f)\eta(V_2) + \left(\frac{r}{2} - 1\right)V_2(f) = -(\frac{\mathbf{k}}{2} - \mathbf{1})V_2(r) - 2V_2(\Upsilon) + \frac{\mathbf{k}}{2}(r-6)\eta(V_2). \qquad (4.8)$$

Taking ζ in place of V_2 in the foregoing equation, we have

$$\zeta(f) = \left(\mathbf{1} - \frac{3\mathbf{k}}{4}\right)(r-6) - \zeta(\Upsilon). \qquad (4.9)$$

Applying inner product to (4.6) with ζ yields

$$\eta(V_2)V_1(f) - \eta(V_1)V_2(f) = -V_1(\Upsilon)\eta(V_2) + V_2(\Upsilon)\eta(V_1) + \frac{\mathbf{1}}{2}[V_1(r)\eta(V_2) - V_2(r)\eta(V_1)]. \quad (4.10)$$

Plugging $V_2 = \zeta$ in (4.10) and utilizing (4.9), we obtain

$$V_1(f) = -V_1(\Upsilon) + \frac{\mathbf{1}}{2}V_1(r) + \left(\frac{3\mathbf{k}}{4}\right)(r-6)\eta(V_1). \qquad (4.11)$$

Suppose that, the scalar curvature r remains constant. Substituting into (2.10), we obtain $r = -6$. Consequently, the preceding equation yields

$$V_1(f) = -V_1(\Upsilon), \qquad (4.12)$$

which means

$$grad(f) = -grad(\Upsilon). \qquad (4.13)$$

Utilizing (4.13) in (4.1) gives

$$-\mathfrak{D}_{V_1}grad(\Upsilon) = -\mathbf{k}QE_1 - \left(\Upsilon - \frac{\mathbf{1}}{2}r - \frac{1}{2}\left(p + \frac{2}{2m+1}\right)\right)V_1. \qquad (4.14)$$

This shows that M^3 is a *C-GRYS*, whose soliton function is $-\Upsilon$. Thus, we have

Theorem 4.2. *A 3-dimensional hyperbolic Kenmotsu manifold with constant scalar curvature admits C-GRYS whose soliton function is* $-\Upsilon$.

5 Conclusion

The results of this study on conformal Ricci-Yamabe solitons (*C-RYS*) and conformal gradient Ricci-Yamabe solitons (*C-GRYS*) in hyperbolic Kenmotsu manifolds align well with previous research. Pankaj et al. [11] showed that such manifolds display Einstein-like structures under certain soliton conditions. Our findings confirm that when a hyperbolic Kenmotsu manifold admits a *C-RYS*, it behaves as an η-Einstein manifold, adding clarity to the role of scalar curvature in this context.

Additionally, for 3-dimensional hyperbolic Kenmotsu manifolds admitting *C-GRYS*, we establish that they have constant scalar curvature, specifically $r = -6$. This builds on earlier studies, offering a deeper understanding of how the scalar curvature behaves within solitonic structures in hyperbolic Kenmotsu geometries.

References

[1] N. Basu and A. Bhattacharyya, *Conformal Ricci soliton in Kenmotsu manifold,* Glob. J. Adv. Res. Class. Mod. Geom., **4**, 15-21, (2015).

[2] L. Bhatt and K. K. Dube, *On CR-submanifolds of trans hyperbolic Sasakian manifold,* Acta Cienc. Indica, **29**, 91-96, (2003).

[3] S.L. Guler and M. Crasmareanu, *Ricci-Yamabe maps for Riemannian flow and their volume variation and volume entropy,* Turk. J. Math., **43 (2)**, 631–2641, (2019).

[4] R.S. Hamilton, *The Ricci flow on surfaces,* Math. Gen. Relativ. (Santa Cruz, C.A., 1986) Contemp. Math., **71**, 237-262, (1998).

[5] A. Haseeb, M.A. Khan and A. Scarfone, *Conformal η-Ricci-Yamabe solitons within the framework of ϵ-LP-Sasakian 3-manifolds,* Advances in Mat. Phy., 1-8, (2022).

[6] N.K. Joshi and K.K. Dube, *Semi invariant submanifolds of an almost r-contact hyperbolic metric manifold,* Demon. Math., **34**, 135-143, (2001).

[7] K. Kenmotsu, *A class of almost contact Riemannian manifolds,* Tohoku Math. J., **24**, 93-10, (1972).

[8] Li, Y., Kumara, H. A., Siddesha, M. S. and Naik, D. M. (2023), *Characterization of Ricci almost soliton on Lorentzian manifolds.* Symmetry, **15(6)**, 1175.

[9] B. O'Neill, *Semi-Riemannian Geometry with Applications to Relativity,* Academic Press, New York, (1983).

[10] R.B. Pal, *Submanifolds of hyperbolic contact metric manifold,* Acta Cien. Indica, **21** , 399-404, (1995).

[11] Pankaj, S. K. Chaubey and Gulhan Ayar, *Yamabe and gradient Yamabe solitons on 3-dimensional hyperbolic Kenmotsu manifolds,* Differential Geometry - Dynamical Systems, **23**, 175-188, (2021).

[12] M.D. Siddiqi, S.A. Siddiqui and Mobin Ahmad, *r-Almost Newton-Yamabe solitons on submanifolds of Kenmotsu space forms*, Palestine Journal of Mathematics, **11(1)**, 427-438, (2022).

[13] M.S. Siddesha, M.M. Praveena and A.B. Madhumohana Raju, *Kählerian Norden Space-time Manifolds and Ricci-Yamabe Solitons*, Palestine Journal of Mathematics, 14(1), xx-yy, (2025).

[14] A. Singh and S. Kishor, *Ricci solitons on para-Sasakian manifolds satisfying pseudo-symmetry curvature conditions*, Palestine Journal of Mathematics, **11(1)**, 583-593, (2022).

[15] B. B. Sinha and R. N. Singh, *Conformal infinitesimal transformations of almost contact metric structure,* Indian J. Pure Appl. Math., **15**, 609-612, (1984).

[16] S. Roy and A. Bhattacharyya, *A Kenmotsu metric as a $*$-conformal Yamabe soliton with torse forming potential vector field,* Acta Math. Sci., **37**, 1896-1908, (2021).

[17] M.D. Upadhyay and K.K. Dube, *Almost contact hyperbolic (ϕ, ξ, η, g)-structure,* Acta Math. Acad. Scient. Hung., **28**, 13-15, (1976).

[18] P. Zhang, Y. Li, S. Roy, S. Dey and A. Bhattacharyya, *Geometrical structure in a perfect fluid spacetime with conformal Ricci-Yamabe soliton,* Symmetry **14(3)**, 594, (2022).

[19] P. Zhang, Y. Li, S. Roy, and S. Dey, *Geometry of a-cosymplectic metric as $*$-conformal η-Ricci-Yamabe solitons admitting quarter-symmetric metric connection,* Symmetry, **13(11)**, 2189, (2021).

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