Gravitational Collapse and Spacetime Singularity with Λ Parametrization

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Abstract In this paper, we consider a phenomenological approach to study the spherically symmetric gravitational collapse of a massive star with Λ- parametrization. A collapsing model with a cosmological constant of the form $\Lambda = \frac{1}{3}\beta\Theta^2$ has been considered; here Θ is the expansion scalar and β is the model parameter to be determined. We have discussed singularity formation via the development of an apparent horizon. We obtained the exact solution to Einstein's field equations by using the Λ parametrization and junction conditions. We have calculated all the physical and geometrical quantities, which heavily depend on the model parameter β. We have estimated the values of the model parameter β for different ranges of masses and radii of collapsing stars. We examine the final fate of a collapsing star, which is a blackhole covered behind an apparent horizon.

1 Introduction

The expansion of the universe was first observed by the high-redshift supernova Ia[1] and confirmed later by cosmic microwave radiation[4]. In the context of this expansion, Einstein's general theory of relativity predicts a new component to the matter distribution of the universe known as dark energy. On the other hand, the gravitational collapse of astronomical objects and what will be the final fate of gravitational collapse is one of the most important issues in the general theory of relativity. Gravitational collapse occurs when an astronomical object is unable to counteract the pull of its own gravity. White dwarfs and neutron stars often develop as a result of collapse processes. A star is in equilibrium configuration during its thermonuclear burning when the pull of its own gravity is balanced by outward internal pressure. As a result, depending on the star's mass, the star may once again have an equilibrium configuration as a white dwarf or neutron star when the nuclear burning is finished. If M (mass of the star) is less than $1.4M_\odot$ (Chandrasekher limit), then electron degeneracy pressure will balance gravitational attraction and the star becomes a white dwarf, while if $1.4M_\odot < $ M$ < 2.5M_\odot$ then neutron degeneracy pressure will balance the gravitational attraction and the star becomes a neutron star. However, the result of gravitational collapse for massive stars gives the spacetime singularity (blackhole or naked singularity). According to Penrose's cosmic censorship conjecture (CCC), the space-time singularity formed by gravitational collapse should be hidden behind the horizon, which means the end state of the collapsing star must be a black hole (BH). However, there is no mathematical proof for CCC, and

different models have been put out in the literature that represent the collapsing star's ultimate demise as a naked singularity (NS). The strong cosmic censorship conjecture states that no past extendable non-spacelike geodesic may have a positive tangent at the singularity and cannot relate it to any point on the spacetime manifold. Thus, the final fate of the collapsing star is still an unresolved problem and has attracted the attention of many researchers for many years, starting with the seminal paper by Oppenheimer and Synder[5]. The gravitational collapse of a massive star is one of the few observable phenomena where the general theory of relativity is expected to play an important role, and the physics of it is interesting amongst astro-physicists. The study mainly focused on singularity formation within the framework of the general theory of relativity, but there are many other gravitational theories that are very important for theoretical astrophysicists [6]-[18]. The astrophysical observations suggest that the universe we live in today is made up of approximately 70 % of dark energy and 25% dark matter. The nature of dark energy as well as dark matter is unknown, and many different models, like quintessence[19][20], DGP branes[21][22], Gauss-Bonnet[23][24], dark energy in brane worlds[25]-[28], and cosmological constant[29] in Einstien's field equation, are proposed to explain the nature of dark energy. The cosmological constant (Λ) model is one of the most important among these.

The cosmological constant problem has a very fascinating and noteworthy history in cosmology. The old cosmological constant problem explains why vacuum energy density is negligibly small. According to recent type Ia supernova observation, the new cosmological constant problem understands why vacuum energy density is not only small but also of the same order as the present matter density of the universe. Instead of constant Λ, the dynamical character of Λ is preferred to explain the expansion of an accelerating universe. In a collapsing configuration, the mass of the collapsing star expands negatively, i.e., towards the core of the star. There are a number of questions that motivate our work: How does the cosmological constant act as a repulsive force? How does it affect the motion of a collapsing star? In our work, cosmological constants affect the motion and fate of a collapsing star, and we discuss the circumstances of whether cosmological constant prevent the collapse or favour a collapsing model. There are many different models that have been proposed in the literature to explain $\Lambda(t)$, in which natural dependence is $\Lambda \propto H^2$, i.e., $\Lambda \propto \Theta^2$[29]-[32]. In the present work, we have considered the model $\Lambda = \frac{\beta}{3}\Theta^2$, where β is a dimensionless constant parameter. Also, we have to use the junction condition to get the solution of the field equations and discuss the singularity formation. The paper is organised as follows: in Sec. 2, we have discussed the basic formalism for Einstein's field equations and junction conditions. In Sec. 3, we have calculated the solution of field equations with the use of Λ parametrization. In Sec. 4, we discussed the apparent horizon and singularity analysis. In Sec. 5, we have estimated the numerical value of the model parameter β. In Section 6, we have discussed the dynamics of the collapsing model for stiff matter and radiation fluid. Section 7 contains the concluding remarks of our work.

2 Basic formalism: Einstein's field equations and junction conditions

2.1 Metric and field equation

Let us consider the gravitational collapse of a masssive core of a star with a finite thickness. The spacetime divides into three different regions, Σ and $V^{\pm}$, where Σ denotes the surface of the star and $V^-(V^+)$ the interior (exterior) of the massive core. For the sake of simplicity, the interior spacetime of a massive core is assumed to be homogeneous, and isotropic is considered as

$$ds^2_- = dt^2 - a(t)^2(dr^2 + r^2d\Omega^2) \tag{2.1}$$

where $d\Omega^2 \equiv d\theta^2 + sin^2\theta d\phi$ and the function a depends on time $'t'$ only, a particular case of pioneering collapse model of Oppenheimer and Synder.

Consider a spherically symmetric system bounded by spherical surface Σ and interior fluid distribution is considered to be perfect fluid form

$$T_{ij} = (p + \rho)u_i u_j - p g_{ij} \tag{2.2}$$

where p and ρ are pressure and energy density of the fluid, respectively, and the vector u_i is the four velocity comoving vector satisfying $u^t = \delta_j^t$.

Since for collapsing configuration the $\frac{\dot{a}}{a} < 0$ and collapsing rate of the star is described by expansion scalar (Θ)

$$\Theta = v_{;i}^i = 3\frac{\dot{a}}{a} \tag{2.3}$$

where, dot (.) means the time derivative.

Let us consider the Einstein's field equations for the space-time described by metric(2.1) as:

$$G_-^{ij} = k\left[T_-^{ij} + \frac{\Lambda}{k}g_-^{ij}\right] \tag{2.4}$$

where ($k = \frac{8\pi G}{c^4}$), G_-^{ij} is the Einstein tensor, T_-^{ij} is the energy momentum tensor, and Λ is a cosmological constant describing the distribution of dark energy in the interior region. For systems (2.1) and (2.2), the non-vanishing component of Einstein's field equations are

$$\frac{\dot{\Theta}^2}{3} = \Lambda + k\rho \tag{2.5}$$

$$\frac{\Theta^2}{3} + \frac{2}{3}\dot{\Theta} = \Lambda - kp \tag{2.6}$$

The energy conservation equation (Bianchi's Identity) $T_{j;i}^i = 0$ have one non-vanishing equation

$$3\frac{p}{\rho}\dot{a} + \frac{\dot{\rho}}{\rho} + \Theta = 0 \tag{2.7}$$

Another important quantity, the mass function $m(t, r)$, which describes the total mass of collapsing fluid at any instant (t, r), is given by [33]

$$m(t, r) = \frac{1}{2}R(1 + R_{,\alpha}R_{,\beta}g^{\alpha\beta}) = \frac{1}{2}r^3 a\dot{a}^2 \tag{2.8}$$

2.2 The exterior metric and the junction condition

Since the present work concerns the gravitational collapse in a dark-energy background, then the exterior region of the spherical system is considered to be the Schwarzschild-de Sitter/anti-de Sitter metric [34, 35]

$$ds_+^2 = \alpha(R)dT^2 - \alpha^{-1}(R)dR^2 + R^2 d\Omega^2 \tag{2.9}$$

where $\alpha(R)$ is given by

$$\alpha(R) = 1 - \frac{2M}{R} \pm \frac{\Lambda R^2}{3} \tag{2.10}$$

where the $\pm$ sign corresponds to the Schwarzschild de-Sitter and Schwarzschild anti de-Sitter metric, and M represents the Newtonian mass of a star (also known as Schwarzschild mass), and Λ is a cosmological constant describing the distribution of dark energy in the exterior region, where the coordinate is taken as $x_+^i = (T, R, \theta, \phi)$. In particular, the metric (2.9) reduces to the Schwarzschild spacetime for $\Lambda = 0$. The boundary hypersurface Σ separates the interior (ds_-^2) and the exterior (ds_+^2) spacetime metric. The matching of interior metric(2.1) to the exterior Schwarzschild metric (2.9) on the hyper-surface Σ yields the junction conditions[36, 39]

$$m(t,r) \quad \underset{\Sigma}{\cong} \quad M \pm \frac{|\Lambda|}{6} R^3 \qquad (2.11)$$

and

$$p_{DE} \quad \underset{\Sigma}{\cong} \quad -\Lambda \qquad (2.12)$$

Equ.(2.11) shows that the mass of the collapsing system is equal to the generalised Schwarzschild mass over Σ. Thus, the equs.(2.11) and (2.12) are the required junction conditions[42].

Value of λ has an important role for collapsing mass. A positive value of Λ has an additive contribution, and a negative value of Λ has a deductive contribution to the collapsing mass. According to Einstein, the universe is contracted by gravity. In field equations, Einstein included a cosmological constant that would act as a repulsive force to counteract the attraction of gravity. Einstein rejected the cosmological constant after Hubble discovered that the universe expands instead of contracts. Einstein admitted that introducing the cosmological constant in field equations was his biggest mistake. However, research on type Ia supernovae [1, 2] suggests that the universe is expanding faster than before, i.e., accelerating. To overcome the acceleration, the cosmological constant must be added to the field equations. The application of the cosmological constant in field equations can help solve issues pertaining to the expansion, composition, and age of the universe[3].

A spacetime singularity is a breakdown in spacetime, either in its geometry or in some other basic physical structure. These are the points where physical quantities such as energy density or the geometrical quantities diverge. Kretschmann scalarcurvature (KS) is one such quantity. The Kretschmann curvature is a quadratic scalar invariant derived by the full contraction of Riemann curvature tensor[38]

$$\mathcal{K} = R_{ijkl} R^{ijkl} \qquad (2.13)$$

where R_{ijkl} denotes Riemann curvature tensor. For metric(2.1), we have

$$\mathcal{K} = 12 \left[\left(\frac{\dot{a}}{a} \right)^4 + \left(\frac{\ddot{a}}{a} \right)^2 \right] \qquad (2.14)$$

3 Solution of field equations: Λ Parametrization

In modern cosmology, dark fluids are commonly used as fundamental objects and, in some sense, replacing dynamical dark energy models. One of the primary goals of some important ongoing research in cosmology is to find a realistic way to correctly parameterise the equation of state (EoS) of this dark fluid. There are many models that propose the $\Lambda(t)$ decay law. It is difficult to imagine an expanding universe with a strictly constant vacuum energy density, so it makes the most sense that the cosmological "constant" in Einstein's equations is actually a time-dependent quantity in cosmology, i.e., $\Lambda = \Lambda(t)$. In the literature, numerous phenomenological functional forms have been put forth to describe a time-varying $\Lambda(t)$. Carvalho et al.[44] have demonstrated that natural dependence is $\Lambda \propto H^2$ (where H is Hubble parameter) based on dimensional arguments. Shapiro and Sola[43] later derived this functional dependence using a renormalisation group approach. In accordance with these authors, we shall assume that the $\Lambda \propto H^2$, i.e., the $\Lambda \propto \theta^2$ term is provided by Refs.[43, 44]. In the present work, the non-vanishing components of Einstien's field equations (2.5)-(2.6) posesses only two independent equations with four unknown physical parameters: $a(t), \rho(t), p(t)$, and $\Lambda(t)$. Here we have considered the variable cosmological constant $\Lambda(t)$. Therefore, to get the exact solution of field equations, we require two more constraints. Usually, we use the parametrisation of physical or geometrical parameters to get the solution of EFE's in the general theory of relativity. There are a number of parametrisations that have been used by many authors in literature[29][30][31]. In the collapsing process of any star, the internal thermal pressure, which acts outward to the core of the star (arises

due to a nuclear reaction at the core), decreases, and then the external pressure, which acts inward to the core of the star (arises due to the gravitational mass of the star), dominates over it. In collapsing configuration, $\frac{\dot{a}}{a} < 0$, i.e., $\Theta < 0$ ($\theta = 3H$). Since we are trying to study how Λ affects the final fate of collapsing stars, we have considered the Λ parameterisation in terms of Θ to study the collapsing configuration.

In order to solve field equations, we consider the $\Lambda(t)$ term, given by[30, 43, 44]

$$\Lambda = \frac{1}{3}\beta\Theta^2 \tag{3.1}$$

where β is a model parameter to be determined for the masssive stars.
We assume that matter component of star satisfies the equation of state (EoS)

$$p = \omega\rho \tag{3.2}$$

where $\omega(\neq 0)$ is a constant parameter.

By using equs.(3.1) and (3.2) into field equations (2.5)-(2.6) we get

$$(3\beta(\omega + 1) - 3\omega - 1)\dot{a}^2(t) = 2a(t)\ddot{a}(t) \tag{3.3}$$

on integration it gives,

$$a(t) = c_2 \left(2c_1 + 3(\beta - 1)t(\omega + 1)\right)^{\frac{2}{3(1-\beta)(\omega+1)}} \tag{3.4}$$

where c_1 and c_2 are arbitrary constants of integration.

By using junction conditions[42] given in equs.(2.11) and (2.12) we evaluate the value of arbitrary constants c_1 and c_2 as

$$c_1 = \frac{1}{2k\Lambda_0}\left(3k\Lambda_0 t_0(1 - \beta)(1 + \omega) - 2\sqrt{3k\omega\Lambda_0(\beta - 1)}\right) \tag{3.5}$$

and

$$c_2 = (6M)^{\frac{1}{3}}\left[\left(\frac{(1-\beta)\omega}{\sqrt{k\omega\Lambda_0(\beta-1)}}\right)^{\frac{2}{(1-\beta)(1+\omega)}}\left(\frac{kr^3}{\omega(\beta-1)}12^{\frac{1}{(1-\beta)(1+\omega)}} - 6r_0^3 6^{\frac{1}{(1-\beta)(1+\omega)}}\right)\Lambda_0\right]^{-\frac{1}{3}} \tag{3.6}$$

where Λ_0 is assumed to be the value of Λ at the junction of the exterior and interior of the star.
Using the value of c_1 and c_2 in equ.(3.4) we have the value of scale factor

$$a(t) = \left(\frac{6M\omega(\beta - 1)}{\Lambda_0(kr^3 + 6r_0^3(1 - \beta)\omega)}\right)^{\frac{1}{3}}\left(1 - \frac{\sqrt{3}}{2\omega}(1 + \omega)\sqrt{(\beta - 1)k\omega\Lambda_0}(t - t_0)\right)^{\frac{2}{3(1-\beta)(1+\omega)}} \tag{3.7}$$

By using equ. (3.7) in equ. (2.3), we have the value of expansion scalar Θ as

$$\Theta = -\frac{6}{(\beta - 1)\left(-\frac{2\sqrt{3}\omega}{\sqrt{k(\beta-1)\Lambda_0\omega}} + 3(t - t_0)(1 + \omega)\right)} \tag{3.8}$$

Also the value of energy density and pressure of collapsing star

$$\rho = \frac{12\Lambda_0\omega}{(\omega 2\sqrt{3} - 3(t - t_0)(1 + \omega)\sqrt{k\Lambda_0\omega(\beta - 1)})^2} \tag{3.9}$$

and

$$p = \frac{12\Lambda_0\omega^2}{(\omega 2\sqrt{3} - 3(t - t_0)(1 + \omega)\sqrt{k\Lambda_0\omega(\beta - 1)})^2} \tag{3.10}$$

Kretschmann curvature is given as

$$\mathcal{K} = \frac{48k^2\Lambda_0^2\omega^2\{5 + 6\omega + 9\omega^2(1 + \omega)^2 - 6\beta(1 + 4\omega + 3\omega^2)\}}{(\beta - 1)^2\{2\sqrt{3}\omega + 3(t_0 - t)(1 + \omega)\sqrt{k(\beta - 1)\Lambda_0\omega}\}} \tag{3.11}$$

Since, from equations (3.7), it can be seen that the value of scale factor is obtained in terms of the mass of the star (M), while all the physical and geometrical quantities from equations (3.4)-(3.11) are obtained by using the value of scale factor $a(t)$. Therefore, we obtain the solution of EFE for particular masses and radii of stars. By estimating the model parameter β, one can discuss the dynamics of a collapsing system.

4   Apparent horizon and singularity analysis

A spacetime singularity is a breakdown in spacetime, either in its geometry or in some other basic physical structure. In the gravitational collapse process, the spacetime singularity emerges when the physical parameter such as energy density (ρ) or geometrical parameter such as Kretschmann scalar curvature ($\mathcal{K}$) or both become divergent. The development of trapped surfaces in spacetime as a result of the gravitational collapse process will determine whether the singularity is a black hole (BH) or a naked singularity (NS). A naked singularity is a singularity without boundary, while the formation of BH is identified by the development of an apparent horizon. Black holes are regions of spacetime from which nothing, not even light, can escape. The gravitational attraction of a typical black hole is so intense that one would have to travel faster than light to escape its pull. Initially, no portion of spacetime is trapped when a star begins to collapse owing to the effect of its own gravity, but as collapse progresses, density (ρ) becomes divergent and trapped surfaces start to emerge. In the BH scenario, an apparent horizon is formed before the formation of a spacetime singularity. On the development of apparent horizons, we have

$$R_{,\alpha}R_{,\beta}g^{\alpha\beta} = (r\dot{a})^2 - 1 = 0 \tag{4.1}$$

We also assume that the collapsing star is not initially trapped at (t_0, r_0), then we should have

$$R_{,\alpha}R_{,\beta}g^{\alpha\beta}|_{(t_0,r_0)} = (r_0\dot{a}(t_0))^2 - 1 < 0 \tag{4.2}$$

By using the value of $a(t)$ from equ.(3.7) in the above equ.(4.1) we have the equation for the development of the apparent horizon

$$\left(-\frac{\omega}{2\sqrt{3}}\right)^{-\frac{4}{3(\beta-1)(1+\omega)}}\{3(t_{AH} - t_0)(1 + \omega)\sqrt{k(\beta - 1)\Lambda_0\omega}\}^{\left(-2-\frac{4}{3(\beta-1)(1+\omega)}\right)}$$

$$\left(\frac{6M\omega}{\Lambda_0\sqrt{\beta - 1}\{6r_{AH}^3 - 6r_0^3(\beta - 1)\omega\}}\right)^{\frac{2}{3}} = 1 \tag{4.3}$$

The time at which a spacetime singularity forms is the time when energy density and Kretschmann curvature diverge ($\rho \rightarrow \infty, \mathcal{K} \rightarrow \infty$) at a finite time given as

$$t_c = t_0 + \frac{2\omega}{(1 + \omega)\sqrt{3k\Lambda_0\omega(\beta - 1)}} \tag{4.4}$$

The geometrical radius of apparent horizon surface is

$$R_{AH} = r_{AH}a(t_{AH}) = r_{AH}\left(\frac{6M\omega(\beta-1)}{\Lambda_0(kr_{AH}^3 + 6r_0^3(1-\beta)\omega)}\right)^{\frac{1}{3}}$$

$$\left(1 - \frac{\sqrt{3}}{2\omega}(1+\omega)\sqrt{(\beta-1)k\omega\Lambda_0}(t_{AH} - t_0)\right)^{\frac{2}{3(1-\beta)(1+\omega)}} \quad (4.5)$$

The mass of the collapsing star on the apparent horizon region is

$$M_{AH} = m(t_{AH}, r_{AH}) = 12^{1+\frac{1}{(\beta-1)(1+\omega)}}\frac{Mr_{AH}^3\omega}{\Lambda_0(\beta-1)\{kr_{AH}^3 - 6r_0^3(\beta-1)\omega\}}$$

$$\left(-\frac{\omega}{\sqrt{k(\beta-1)\Lambda_0\omega}}\right)^{\frac{2}{(\beta-1)(1+\omega)}}\left[-\frac{2\sqrt{3}\omega}{\sqrt{k(\beta-1)\Lambda_0\omega}} + 3(t-t_0)(1+\omega)\right]^{-2-\frac{2}{(\beta-1)(1+\omega)}}$$

$$(4.6)$$

from equ.(4.3) one can obtain the time (t_{AH}) at which apparent horizon is developed given as

$$t_{AH} = \frac{1}{(1+\omega)\sqrt{3k(\beta-1)\Lambda_0\omega}}\left[2^{-\frac{4(\beta-1)(1+\omega)}{3\beta(1+\omega)-1-3\beta}}3^{\frac{2(\beta-\omega+\beta\omega)}{3\beta(1+\omega)-1-3\beta}}(1+\omega)(\beta-1)\right.$$

$$\sqrt{3}\{k\Lambda_0\omega(\beta-1)\}t_0 + 2\omega + t_0\omega\sqrt{3}\sqrt{k(\beta-1)\omega\Lambda_0}\Big]$$

$$+\frac{(r_{AH}M^{\frac{1}{3}})^{\frac{3(\beta-1)}{(3\beta-1)}}}{(\beta-1)(1+\omega)}2^{\frac{4(\beta-1)(1+\omega)}{3\beta(1+\omega)-1-3\beta}}3^{\frac{-2(\beta-\omega+\beta\omega)}{3\beta(1+\omega)-1-3\beta}}\left(\frac{\omega(1-\beta)}{\sqrt{k(\beta-1)\Lambda_0\omega}}\right)^{-\frac{2}{3\beta(1+\omega)-1-3\beta}}$$

$$\left[\frac{12^{\frac{1}{(1-\beta)(1+\omega)}}\Lambda_0}{\omega(\beta-1)}\{kr_{AH}^3 - 6r_0^3(\beta-1)\omega\}\right]^{\frac{(1-\beta)(1+\omega)}{3\beta(1+\omega)-1-3\beta}} \quad (4.7)$$

by assuming initial coordinate $(t_0, r_0) = (1,1)$, $k = 1$, $\Lambda_0 = -1$ and with the use of equ.(4.4) and equ.(4.7) we obtain

$$\frac{t_{AH}}{t_c} = \left(1 + \frac{2\omega}{(1+\omega)\sqrt{3\omega(1-\beta)}}\right)^{-1}\left[\frac{\omega}{(1+\omega)} - \sqrt{\omega}2^{\frac{4(\beta-1)(1+\omega)}{-1-3\omega+3\beta(1+\omega)}}3^{\frac{2(\beta-\omega+\beta\omega)}{-1-3\omega+3\beta(1+\omega)}}(1-\beta)^{\frac{3}{2}} + \right.$$

$$\frac{\{12\omega(1-\beta)\}^{\frac{1}{-1-3\omega+3\beta(1+\omega)}}}{3(1+\omega)(\beta-1)}\left(\frac{1}{8} + \frac{1}{48\omega(1-\beta)}\right)^{\frac{(1-\beta)(1+\omega)}{-1-3\omega+3\beta(1+\omega)}} + \frac{2}{1+\omega}\sqrt{\frac{\omega}{3(1-\beta)}}\Big] \quad (4.8)$$

When $\frac{t_{AH}}{t_c} < 1$, gravitational collapse leads to the formation of a **black hole**. In this case, the apparent horizon is developed before the collapse time t_c. When $\frac{t_{AH}}{t_c} > 1$, gravitational collapse does not favour the formation of black holes. In this case, an apparent horizon may be formed after the collapse time t_c, and we can not detect the horizon formation, so we cannot predict the formation of a black hole. This will be the case of the naked singularity.

5 Estimation of model parameter β

The surface radius of star is given as

$$R(t,r) \equiv ra(t)$$

$$= r\left(\frac{6M\omega(\beta-1)}{\Lambda_0(kr^3 + 6r_0^3(1-\beta)\omega)}\right)^{\frac{1}{3}} \left(1 - \frac{\sqrt{3}}{2\omega}(1+\omega)\sqrt{(\beta-1)k\omega\Lambda_0}(t-t_0)\right)^{\frac{2}{3(1-\beta)(1+\omega)}}$$

$$(5.1)$$

by assuming initial coordinte $(t_0, r_0) = (1,1)$, $\omega = 1$, $k = 1$, $\Lambda_0 = -1$ and using different ranges of masses (M) and radii (R) for massive stars (discussed in table1 and table2) in equ.(5.1) we obtain

$$6^{\frac{1}{3}}\left(\frac{8.2(1-\beta)}{7-6\beta}\right)^{\frac{1}{3}} = 1.9 \qquad \text{for} \quad M = 8.2M_\odot, R = 1.9R_\odot \qquad (5.2)$$

By solving equ.(5.2), we get $\beta = 0.1475$ approximately. Similarly, other values of model parameter β can be evaluated for different masses and radii $(R_\odot \leq R \leq 2R_\odot)$ given in table(1) and table(2) for different ω.

Table 1: Numerical value of model parameter β for estimated masses M and radii R_0 of the massive stars (for ω=1)

Massive star $M_0(M_\odot)$	Radius of star $R_0(R_\odot)$	β
$5.5 \leq$ M <6.5	$R_\odot \leq R \leq 2R_\odot$	0.7352
$6.5 \leq$ M < 8.2	$R_\odot \leq R \leq 2R_\odot$	0.4840
$8.2 \leq$ M < 9.2	$R_\odot \leq R \leq 2R_\odot$	0.1475
$9.2 \leq$ M < 10	$R_\odot \leq R \leq 2R_\odot$	0.5116
$10 \leq$ M < 15	$R_\odot \leq R \leq 2R_\odot$	0.6360
M ≥ 15	$R_\odot \leq R \leq 2R_\odot$	0.9

Table 2: Numerical value of model parameter β for estimated masses M and radii R_0 of the massive stars (for ω=$\frac{1}{3}$)

Massive star $M_0(M_\odot)$	Radius of star $R_0(R_\odot)$	β
$3.5 \leq$ M < 5	$R_\odot \leq R \leq 2R_\odot$	0.15
$5 \leq$ M < 5.9	$R_\odot \leq R \leq 2R_\odot$	0.87
$5.9 \leq$ M < 7	$R_\odot \leq R \leq 2R_\odot$	0.33
$7 \leq$ M < 7.5	$R_\odot \leq R \leq 2R_\odot$	0.53
$7.5 \leq$ M < 10	$R_\odot \leq R \leq 2R_\odot$	0.050
M ≥ 10	$R_\odot \leq R \leq 2R_\odot$	0.944

6 Dynamics of collapsing model

6.1 $\omega = 1$ (Stiff matter)

In the general theory of relativity,'stiff matter' is described by the relation $p = \rho$, where p is the fluid's pressure and ρ is its total energy density. It can also be described by a massless scalar field. For an understanding of this perfect fluid, one has to examine

the variation of energy density with scale factor $a(t)$. In this case, the energy density is proportional to $\frac{1}{a^6}$, whereas in the case of a radiative perfect fluid, the energy density is proportional to $\frac{1}{a^4}$. These findings suggest that our universe may have gone through a period where stiff matter dominated instead of radiation fluid. Due to its importance, many researchers have considered the presence of stiff matter in FRW cosmological models, and its importance was first recognised by Zeldovich[40]. Now in our model, we consider the case of stiff matter ($\omega=1$)[30][41] and get the value of scale factor, energy density, and other physical and geometrical parameters as:

$$a(t) = \left(\frac{6M(\beta - 1)}{\Lambda_0(kr^3 + 6r_0^3(1 - \beta))} \right)^{\frac{1}{3}} \left(1 - \sqrt{3(\beta - 1)k\Lambda_0}(t - t_0) \right)^{\frac{1}{3(1-\beta)}} \tag{6.1}$$
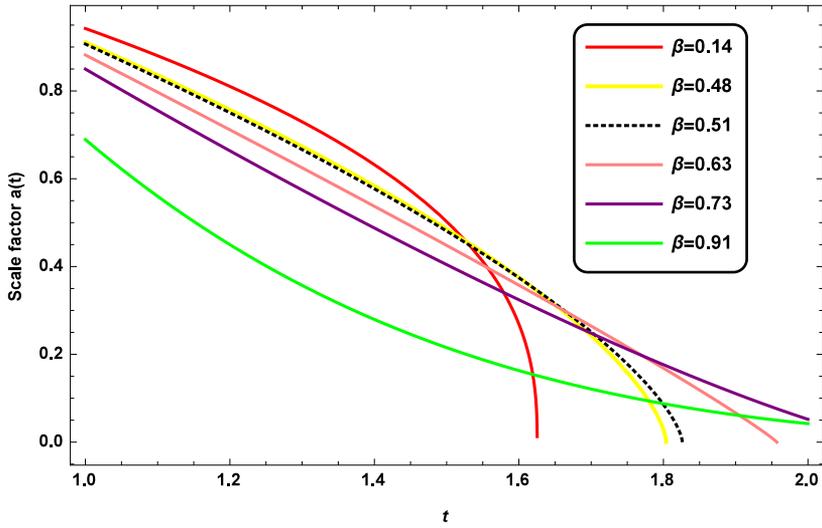
Figure 1: Collapsing configuration: The scale factor $a(t)$ given in equ.(6.1) is plotted with respect to time coordinate t for six values of model parameter β given in table 1

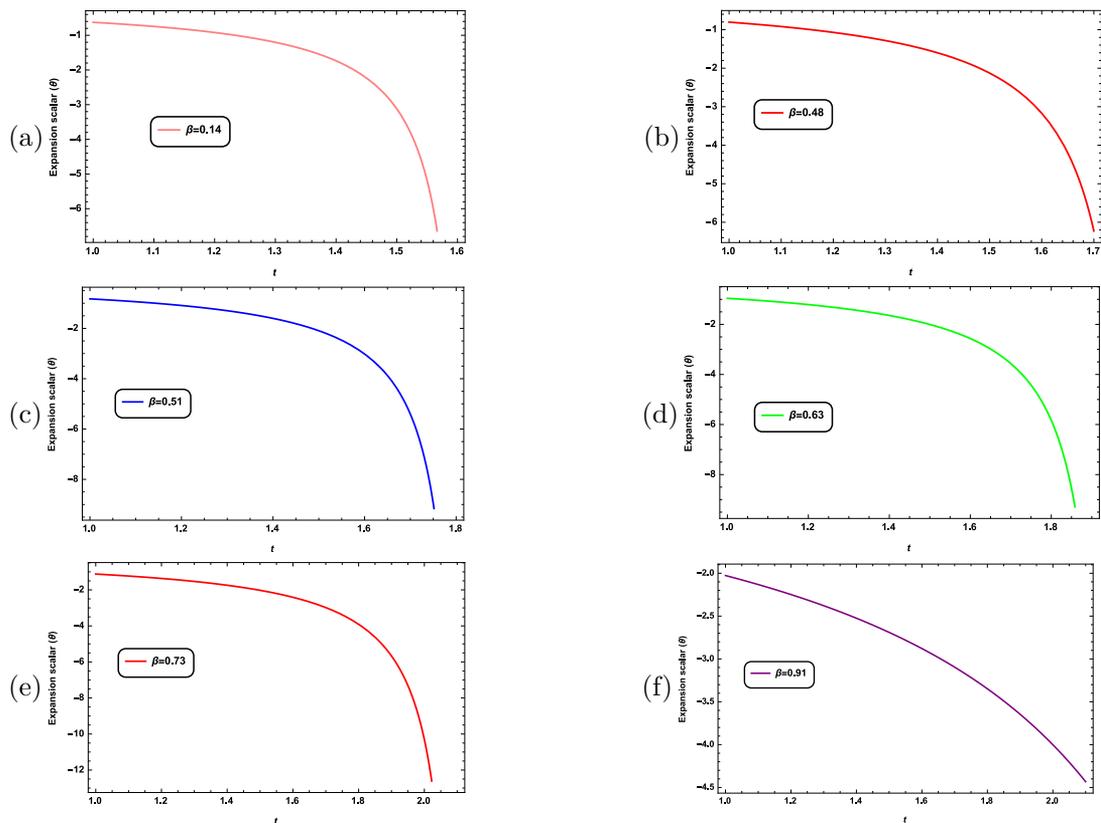$$\Theta = -\frac{6}{(\beta - 1)\left(-\frac{2\sqrt{3}}{\sqrt{k(\beta-1)\Lambda_0}} + 3(t - t_0) \right)} \tag{6.2}$$

Figure 2: Collapsing configuration: The expansion scalar (Θ) given in equ.(6.2) is plotted with respect to time coordinate t for six values of model parameter β given in table 1

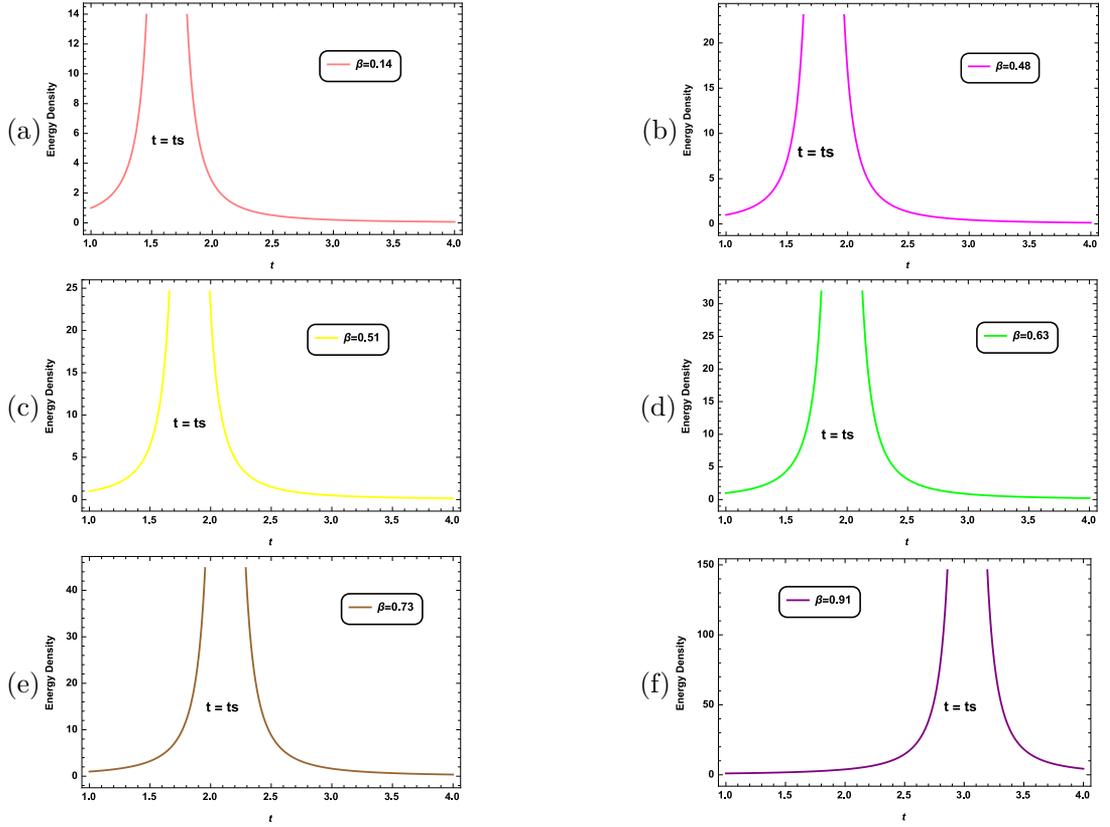$$p = \rho = \frac{3\Lambda_0}{(\sqrt{3} - 3(t - t_0)\sqrt{k\Lambda_0(\beta - 1)})^2} \qquad (6.3)$$

Figure 3: The energy density (ρ) given in equ.(6.3) is plotted with time coordinate t for six values of model parameter β given in table(1), with initial coordinate $(t_0, r_0) = (1, 1)$

$$\mathcal{K} = \frac{12k^2\Lambda_0^2(5 - 12\beta + 9\beta^2)}{(\beta - 1)^2(\sqrt{3} + 3(t_0 - t)\sqrt{k\Lambda_0(\beta - 1)})^4} \tag{6.4}$$
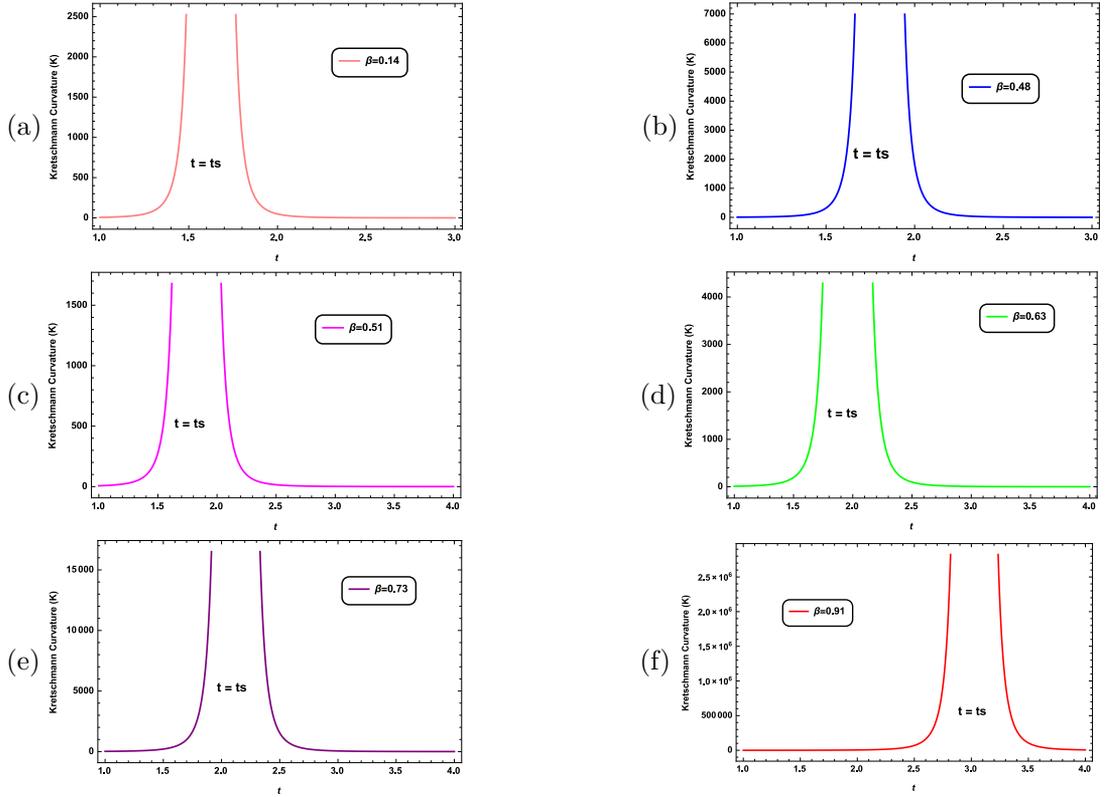
Figure 4: The Kretschmann curvature ($\mathcal{K}$) given in equ.(6.4) is plotted with time coordinate t for six values of model parameter β given in table(1), with initial coordinate $(t_0, r_0) = (1, 1)$

In fig(1), we show the behaviour of scale factor $a(t)$ with time coordinate t for some estimated value of model parameter β. In the same way, we display the behaviour of expansion scalar (Θ), energy density (ρ), and Kretschmann curvature ($\mathcal{K}$) with time coordinate t for some estimated value of model parameter β in fig(2), fig(3), and fig(4), respectively. We see that the nature of all these graphs heavily depends on the model parameters. The singularity formation allows for $0 < \beta < 1$. When $\beta \geq 1$, we will not find a feasible solution for the current model.

In this case the development of apparent horizon is given as

$$\frac{1}{2\sqrt{3k(\beta-1)\Lambda_0}}\left[2^{\frac{4(1-\beta)}{(3\beta-1)}}3^{\frac{1-2\beta}{2-3\beta}}t_0(\beta-1)2\sqrt{3}\{k(\beta-1)\Lambda_0\}+t_0\sqrt{3k(\beta-1)\Lambda_0}+2\right]+$$

$$\frac{\left(M^{\frac{1}{3}}r_{AH}\right)^{\frac{3(\beta-1)}{3\beta-2}}}{\beta-1}2^{\frac{\beta-2}{3\beta-2}}3^{\frac{1-2\beta}{3\beta-2}}\left(\frac{\Lambda_0}{(\beta-1)}12^{\frac{1}{2(1-\beta)}}\{kr_{AH}^3-6r_0^3(\beta-1)\}\right)^{\frac{1-\beta}{3\beta-2}}\left(\frac{\beta-1}{\sqrt{k(\beta-1)\Lambda_0}}(0.5)\right)^{\frac{1}{3\beta-2}}=0$$

by assuming initial coordintes $(t_0, r_0) = (1, 1)$, $\omega = 1$, $k = 1$, $\Lambda_0 = -1$ and with the use of equ.(4.4) and equ.(4.7) we obtain

$$\frac{t_{AH}}{t_c}=\left(1+\frac{1}{\sqrt{3(1-\beta)}}\right)^{-1}\left[\frac{1}{2}-2^{\frac{4(1-\beta)}{3\beta-2}}3^{\frac{1-2\beta}{2-3\beta}}(1-\beta)^{\frac{3}{2}}+\frac{1}{\sqrt{3(1-\beta)}}+\{12(1-\beta)\}^{\frac{1}{2(3\beta-2)}}\right.$$

$$\left.\frac{1}{(1-\beta)}\left\{\frac{7-6\beta}{288(\beta-1)}\right\}^{\frac{1-\beta}{3\beta-2}}\right] \quad (6.6)$$

We observe from equ.(6.6) that the value $\frac{t_{AH}}{t_c} < 1$ for the value of β given in table[1], it means for assumed masses and radii of stars $t_{AH} < t_c$, so we get the horizon formation

before the singularity formation, and this leads to the formation of black holes. The mass of the collapsing star in this case on the apparent horizon region is

$$M_{AH} = m(t_{AH}, r_{AH}) = 12^{1+\frac{1}{2(\beta-1)}} \frac{Mr_{AH}^3}{\Lambda_0(\beta-1)\{kr_{AH}^3 - 6r_0^3(\beta-1)\}}$$

$$\left(-\frac{1}{\sqrt{k(\beta-1)\Lambda_0}}\right)^{\frac{1}{(\beta-1)}} \left[-\frac{2\sqrt{3}}{\sqrt{k(\beta-1)\Lambda_0}} + 6(t-t_0)\right]^{-2-\frac{1}{(\beta-1)}} \quad (6.7)$$
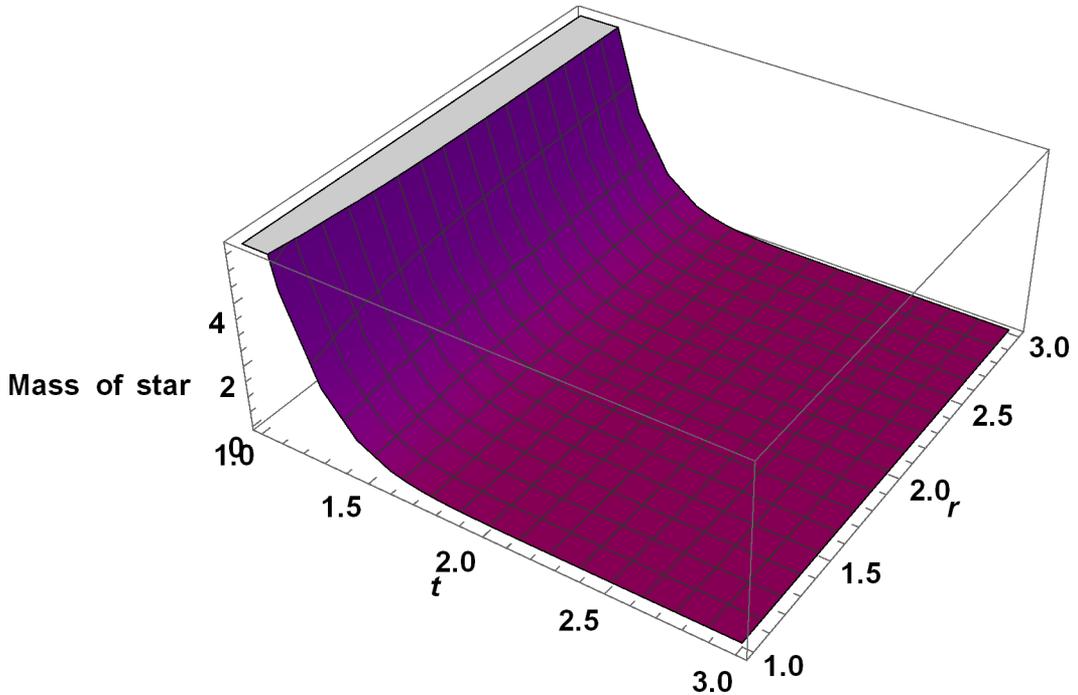

Figure 5: The mass function (M_{AH}) given in equ.(6.7) is plotted with time coordinate t radial coordinate r and for $\beta = 0.91$ (corresponding to $M = 15M_\odot$) with initial coordinate $(t_0, r_0) = (1,1)$, for all six values of model parameter β given in table(1) we will find the same configuration

6.2 $\omega = \frac{1}{3}$(Radiation fluid)

The Eos for ultra-relativistic 'radiation' is $\omega = \frac{1}{3}$. In this case, spacetime is dominated by radiation-induced pressure. In the case of a radiative perfect fluid, the energy density is proportional to $\frac{1}{a^4}$.In cosmology, for expanding the universe, the energy density of radiation decreases more quickly than the volume expansion because its wavelength is red-shifted, whereas in collapsing scenerio expansion, the scalar decreases with time and the energy density becomes divergent. Now in our model, we consider the case of radiation fluid ($\omega = \frac{1}{3}$)[29] and get the value of scale factor, energy density, and other physical and geometrical parameters as:

$$a(t) = \left(\frac{2M(\beta-1)}{\Lambda_0(kr^3 + 2r_0^3(1-\beta)\omega)}\right)^{\frac{1}{3}} \left(1 - 2\sqrt{(\beta-1)k\Lambda_0}(t-t_0)\right)^{\frac{1}{2(1-\beta)}} \quad (6.8)$$
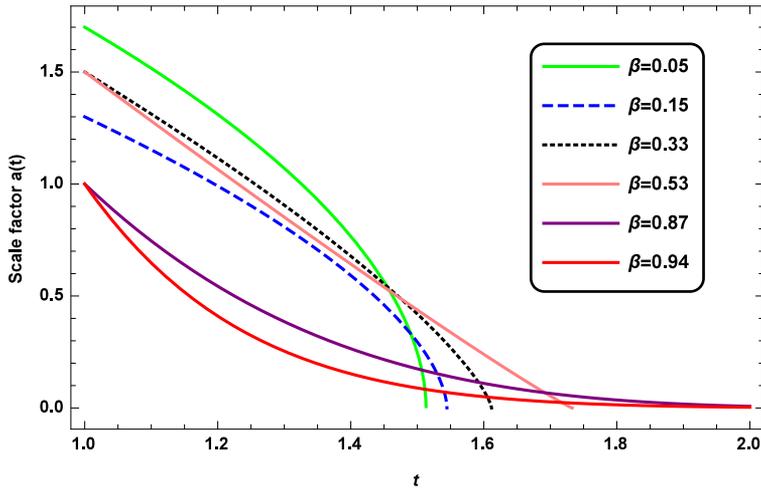
Figure 6: Collapsing configuration: The scale factor $a(t)$ given in equ.(6.8) is plotted with respect to time coordinate t for six value of model parameter β given in table 2

$$\Theta = -\frac{6}{(\beta - 1)\left(-\frac{2}{\sqrt{k(\beta - 1)\Lambda_0}} + 4(t - t_0)\right)} \tag{6.9}$$
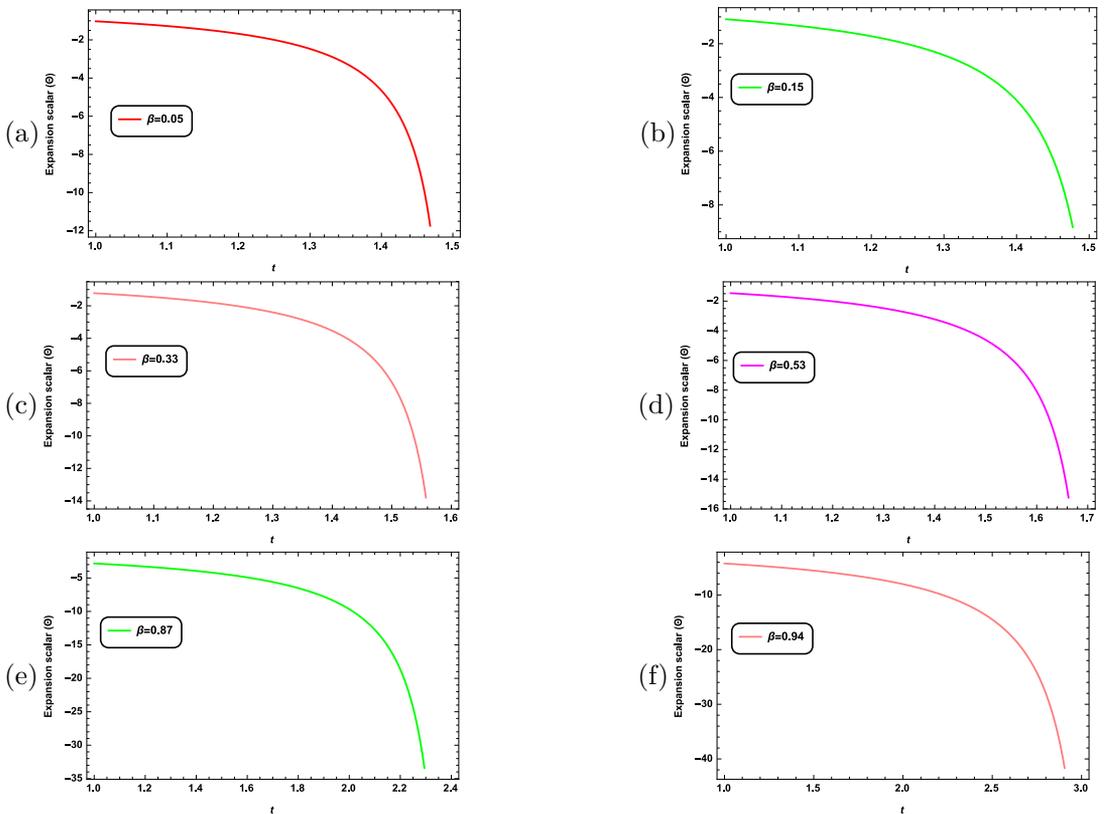

Figure 7: Collapsing configuration: The expansion scalar (Θ) given in equ.(6.9) is plotted with respect to time coordinate t for six values of model parameter β given in table 2

$$\rho = \frac{3\Lambda_0}{(1 - 2(t - t_0)\sqrt{k\Lambda_0(\beta - 1)})^2} \tag{6.10}$$

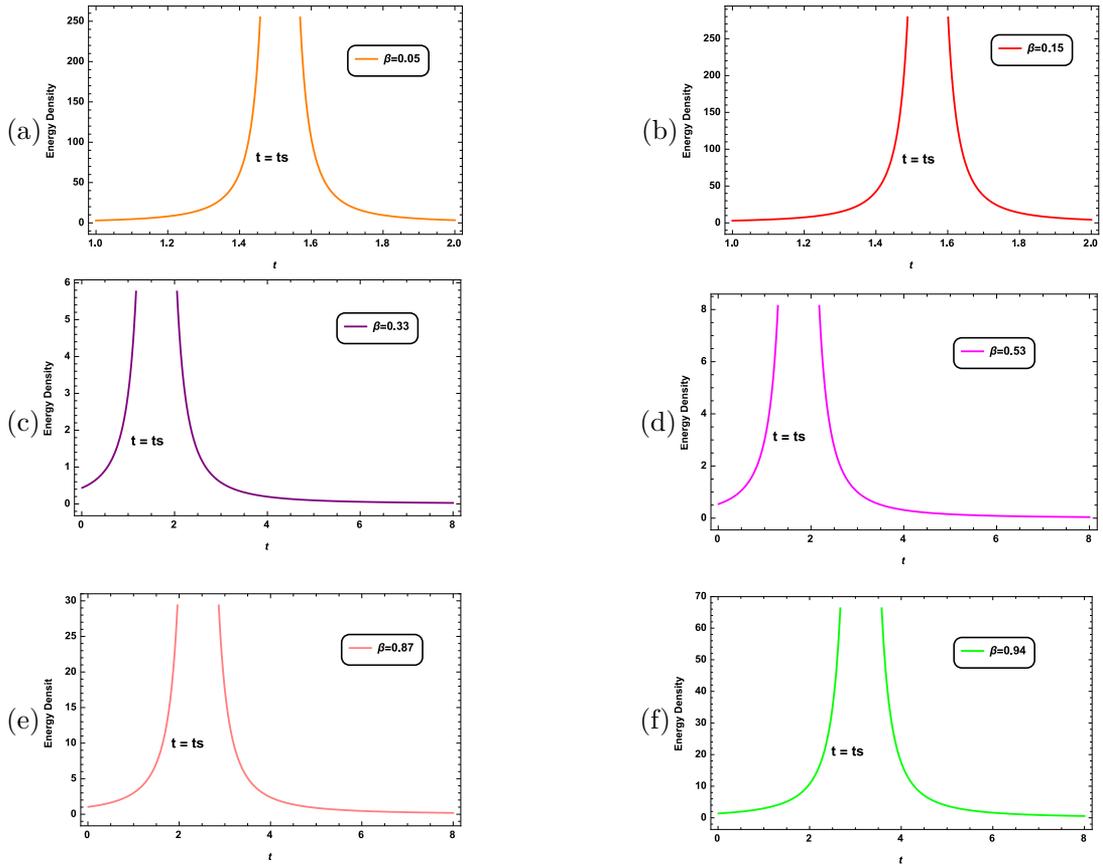$$p = \frac{\Lambda_0}{(1 - 2(t - t_0)\sqrt{k\Lambda_0(\beta - 1)})^2} \quad (6.11)$$

Figure 8: The energy density (ρ) given in equ.(6.10) is plotted with time coordinate t for six values of model parameter β given in table(2), with initial coordinate $(t_0, r_0) = (1, 1)$

and

$$\mathcal{K} = \frac{24k^2\Lambda_0^2(1 - 2\beta + 2\beta^2)}{(\beta - 1)^2(1 + 2(t_0 - t)\sqrt{k\Lambda_0(\beta - 1)})^4} \quad (6.12)$$
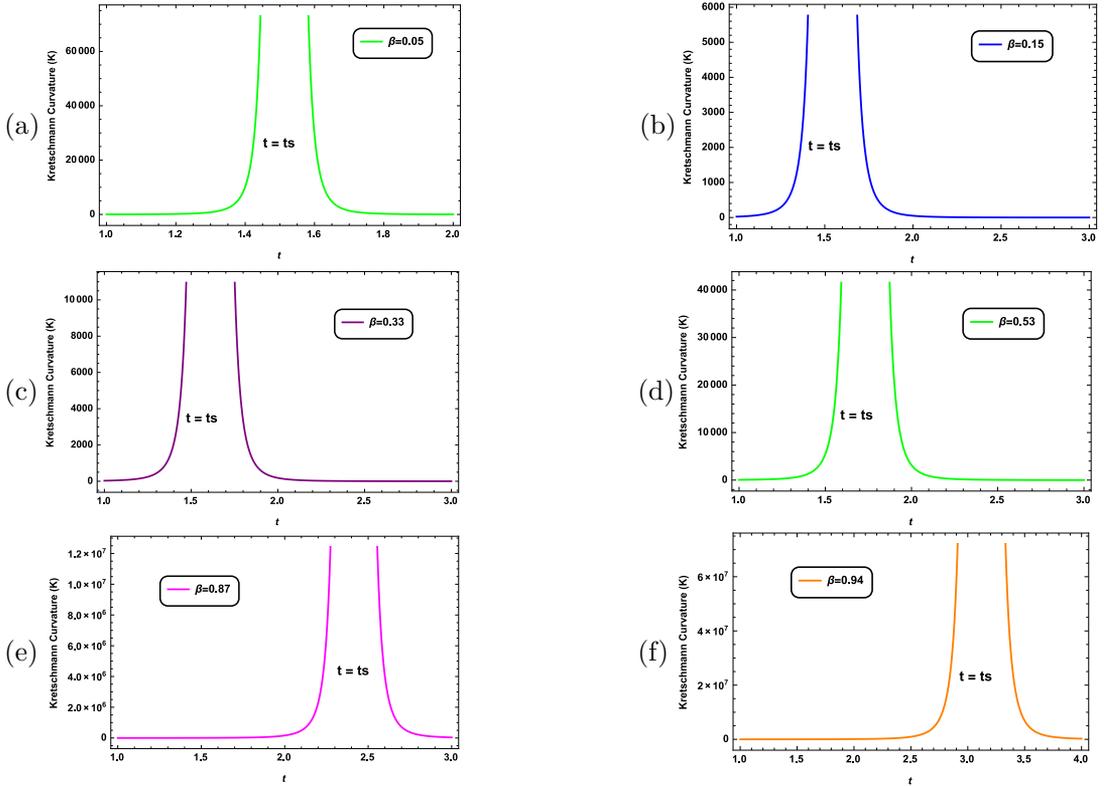
Figure 9: The Kretschmann curvature $(\mathcal{K})$ given in equ.(6.12) is plotted with time coordinate t for six values of model parameter β given in table(2), with initial coordinate $(t_0, r_0) = (1, 1)$

In fig(6), we display the behaviour of scale factor $a(t)$ with time coordinate t for some estimated values of model parameter β. In the same way, we display the behaviour of the expansion scalar (Θ), energy density (ρ), and Kretschmann curvature $(\mathcal{K})$ with time coordinate t for some estimated values of the model parameter β in fig(7), fig(8), and fig(9) respectively. We see that the nature of all these graphs heavily depends on the model parameters. The singularity formation allows for $0 < \beta < 1$. When $\beta \geq 1$, we will not find a feasible solution for the considered model.

In this case the development of apparent horizon is given as

$$\frac{1}{4\sqrt{3k(\beta-1)\Lambda_0}} \left[2^{\frac{8(1-\beta)}{3(2\beta-1)}} 3^{\frac{1+2\beta}{6(2\beta-1)}} 4t_0(\beta-1)\{k(\beta-1)\Lambda_0\} + t_0\sqrt{k(\beta-1)\Lambda_0} + 2 \right] +$$

$$\left(\frac{1-\beta}{\sqrt{k(\beta-1)\Lambda_0}} \right)^{\frac{1}{2\beta-1}} \frac{(M^{\frac{1}{3}} r_{AH})^{\frac{2(\beta-1)}{2\beta-1}}}{\beta-1} 2^{\frac{2(1+2\beta)}{3(1-2\beta)}} \left(\frac{\Lambda_0}{(\beta-1)} 8^{\frac{1}{2(1-\beta)}} \{kr_{AH}^3 - 2r_0^3(\beta-1)\} \right)^{-\frac{2(\beta-1)}{3(2\beta-1)}} = 0$$

$$(6.13)$$

by assuming initial coordintes $(t_0, r_0) = (1, 1)$, $\omega = \frac{1}{3}$, $k = 1$, $\Lambda_0 = -1$ and with the use of equ.(4.4) and equ.(4.7) we obtain

$$\frac{t_{AH}}{t_c} = \left(1 + \frac{1}{2\sqrt{1-\beta}} \right)^{-1} \left[\frac{1}{4} - 2^{\frac{8(1-\beta)}{6\beta-3}} 3^{\frac{1+2\beta}{12\beta-6}} (1-\beta)^{\frac{3}{2}} + \frac{1}{2\sqrt{1-\beta}} + \right.$$

$$\left. \left(\frac{3-2\beta}{4(\beta-1)} 2^{\frac{5-8\beta}{2(\beta-1)}} (1-\beta)^{-1+\frac{3}{4-4\beta}} \right)^{\frac{2(1-\beta)}{6\beta-3}} \right] \quad (6.14)$$

as above discussion in the case of $\omega = 1$ we get the same result in this case. We observe from equ.(6.14) that the value $\frac{t_{AH}}{t_c} < 1$ for value of β given in table[2], it means for

assumed masses and radii of stars $t_{AH} < t_c$, so we get the horizon formation before the singularity formation and this leads the formation of black hole. The mass of the collapsing star in this case on the apparent horizon region is

$$M_{AH} = m(t_{AH}, r_{AH}) = 2^{2 + \frac{3}{2(\beta - 1)}} \frac{M r_{AH}^3}{\Lambda_0(\beta - 1)\{k r_{AH}^3 - 2r_0^3(\beta - 1)\}}$$

$$\left(-\frac{1}{\sqrt{3k(\beta - 1)\Lambda_0}} \right)^{\frac{3}{2(\beta - 1)}} \left[-\frac{2}{\sqrt{k(\beta - 1)\Lambda_0}} + 4(t - t_0) \right]^{-2 - \frac{3}{2(\beta - 1)}} \quad (6.15)$$
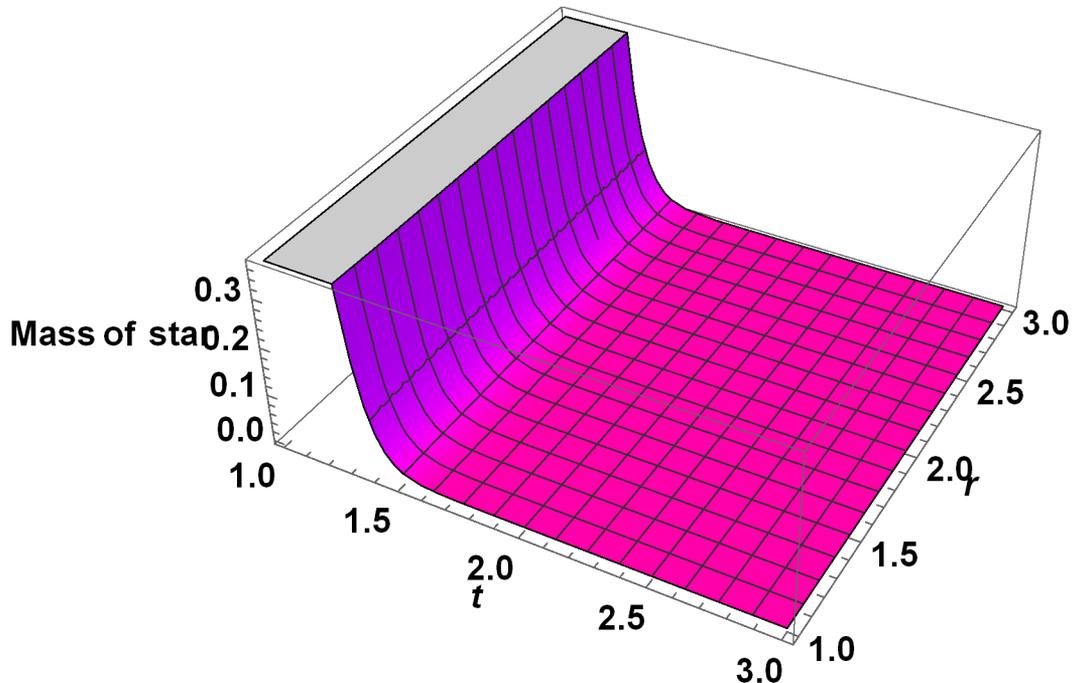

Figure 10: The mass function (M_{AH}) given in equ.(6.7) is plotted with time coordinate t radial coordinate r and for $\beta = 0.94$ (corresponding to $M = 10M_\odot$) with initial coordinate $(t_0, r_0) = (1,1)$, for all six values of model parameter β given in table(2) we will find the same configuration

7 Conclusion remarks

In this work, we have discussed the spherically symmetric homogeneous gravitational collapse of massive stars and their final fate. We have considered the interior of the star as FLRW spacetime, and the exterior region of the star is assumed to be Schwartzchild-de Sitter /anti-de Sitter. Since the dynamical character of the cosmological constant Λ is preferred in the recent cosmological model instead of the constant behaviour of the cosmological constant, we have used the Λ parametrisation to get the exact solution of field equations. We have considered the model $\Lambda = \frac{1}{3}\beta\Theta^2$ to obtain the solution of field equations and examine the evolution of scale factor a(t) in the presence of a variable cosmological constant. In addition, we have used junction conditions to obtain the exact solution of EFE's in terms of the mass of the star (M). We have discussed the two cases of gravitational collapse: the first is the case of stiff matter (ω=1), and the other is the case of radiation fluid ($\omega = \frac{1}{3}$) with their collapsing configuration. In order to assess the graphical representation of our result, we have considered different ranges of masses and radii of stars to estimate the numerical value of the model parameter β and observed that the nature of a collapsing star completely depends on this parameter β. The values

of $0 < \beta < 1$ favour the formation of black holes, while for $\beta \geq 1$, we will not favour the formation of black holes as we could not find a feasible solution for our model, i.e., not favour the formation of black holes. We have studied the collapsing configuration through a graphical representation of the model as follows:

• The expansion scalar Θ is negatively increasing with time coordinate t, showing that collapsing phenomena occur and motion of collapsing fluids towards the core of the star fig. [2] fig.(7) .

• The scale factor $a(t)$ takes the finite value for given masses of considered stars and is monotonically decreasing in nature [fig.(1) fig.(6)].

• The graphical representation of energy density (ρ) [fig.(3), Fig.(8)], and Kretschmann curvature ($\mathcal{K}$) [fig.(4), fig.(9)] shows that both take positive and finite values for considered masses of stars. Both are increasing in nature, and at a finite time t_c, both become divergent, which shows singularity formation.

• The graph of the mass function shows that it is regular, finite, and decreasing with time t and radial coordinate r [(fig.(5), fig.(10)].

The development of an apparent horizon for a collapsing star has been studied in equations (6.5, 6.13). Also, the mass (M_{AH}) of a collapsing star in the apparent horizon region for both stiff matter and radiation fluid has been calculated in equations (6.7, 6.15). To predict whether a black hole is formed or not, we have calculated the quantity $\frac{t_{AH}}{t_c}$ in both cases and observed that for the estimated value of model parameter β, the value $\frac{t_{AH}}{t_c} < 1$ in both cases considered suggested that the apparent horizon developed before the time t_c, and as blackholes must be hidden behind the apparent horizon, both cases (stiff matter and radiation fluid) lead to the formation of **Blackholes**.

References

[1] A. G. Riess et al. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998) [arXiv:astro-ph/9805201]; S. Perlmutter et al. [Su- pernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999) [arXiv:astro-ph/9812133]; A. G. Riess et al. [Supernova Search Team Collaboration], Astro- phys. J. 607, 665 (2004) [arXiv:astro-ph/0402512].

[2] Spergel, D. N., Verde, L., Peiris, H. V., Komatsu, E., Nolta, M. R., Bennett, C. L., ... & Wright, E. L. (2003). First-year Wilkinson Microwave Anisotropy Probe (WMAP)* observations: determination of cosmological parameters. The Astrophysical Journal Supplement Series, 148(1), 175.

[3] Perlmutter, S. et al.: Astrophys. J. 517(1999)565.

[4] C. L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003) [arXiv:astro-ph/0302207]; D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003) [arXiv:astro-ph/0302209].

[5] Oppenheimer, J. R., & Snyder, H. (1939). On continued gravitational contraction. Physical Review, 56(5), 455.

[6] Maiti, S. R. (1982). Fluid with heat flux in a conformally flat space-time. Physical Review D, 25(10), 2518.

[7] Modak, B. (1984). Cosmological solution with an energy flux. Journal of Astrophysics and Astronomy, 5, 317-322.

[8] Banerjee, A., Choudhury, S. D., & Bhui, B. K. (1989). Conformally flat solution with heat flux. Physical Review D, 40(2), 670.

[9] Schäfer, D., & Goenner, H. F. (2000). The gravitational field of a radiating and contracting spherically-symmetric body with heat flow. General Relativity and Gravitation, 32, 2119-2130.

[10] Ivanov, B. V. (2012). Collapsing shear-free perfect fluid spheres with heat flow. General Relativity and Gravitation, 44, 1835-1855.

[11] Herrera, L., Le Denmat, G., Santos, N. O., & Wang, A. (2004). Shear-free radiating collapse and conformal flatness. International Journal of Modern Physics D, 13(04), 583-592.

[12] Chakrabarti, S., & Banerjee, N. (2017). Scalar field collapse in a conformally flat spacetime. The European Physical Journal C, 77(3), 166.

[13] Cai, R. G., Ji, L. W., & Yang, R. Q. (2016). Collapse of self-interacting scalar field in anti-de Sitter space. Communications in Theoretical Physics, 65(3), 329.

[14] Roupas, Z. (2020). Relativistic gravitational collapse by thermal mass. Communications in Theoretical Physics, 73(1), 015401.

[15] Pradhan, A., Singh, A. K., & Chouhan, D. S. (2013). Anisotropic Bianchi Type-V Cosmology with Perfect Fluid and Heat Flow in Sáez-Ballester Theory of Gravitation. Palestine Journal of Mathematics, 2(2).

[16] Sharma, N. K. (2022). Anisotropic cosmological model involving null radiation flow and magnetic field. Palestine Journal of Mathematics, 11(1).

[17] Ali, M., & Salman, M. (2022). Conharmonic symmetry inheritance and its physical significance in general relativity. Palestine Journal of Mathematics, 11(4).

[18] Siddiqui, S. A. (2023). Estimation of η-Einstein solitons on Lorentzian concircular structure manifolds-[(LCS) n]. Palestine Journal of Mathematics, 12(2), 383-394.

[19] Liddle, A. R., & Scherrer, R. J. (1998). Classification of scalar field potentials with cosmological scaling solutions. Physical Review D, 59(2), 023509.

[20] Steinhardt, P. J., Wang, L., & Zlatev, I. (1999). Cosmological tracking solutions. Physical Review D, 59(12), 123504.

[21] Dvali, G., Gabadadze, G., & Porrati, M. (2000). 4D gravity on a brane in 5D Minkowski space. Physics Letters B, 485(1-3), 208-214.

[22] Deffayet, C. (2001). Cosmology on a brane in Minkowski bulk. Physics Letters B, 502(1-4), 199-208.

[23] Malafarina, D., Toshmatov, B., & Dadhich, N. (2020). Dust collapse in 4d einstein–gauss–bonnet gravity. Physics of the Dark Universe, 30, 100598.

[24] Banerjee, N., & Paul, T. (2018). Scalar field collapse in Gauss–Bonnet gravity. The European Physical Journal C, 78, 1-6.

[25] Cline, J. M., & Vinet, J. (2003). Problems with time-varying extra dimensions or "Cardassian expansion" as alternatives to dark energy. Physical Review D, 68(2), 025015.

[26] Gong, Y., Wang, A., & Wu, Q. (2008). Cosmological constant and late transient acceleration of the universe in the Horava–Witten heterotic M-theory on S1/Z2. Physics Letters B, 663(3), 147-151.

[27] Pereira, P. R., Da Silva, M. F. A., & Chan, R. (2006). Anisotropic self-similar cosmological model with dark energy. International Journal of Modern Physics D, 15(07), 991-999.

[28] Brandt, C. F. C., Chan, R., Silva, M. D., & Rocha, J. F. V. D. (2007). Inhomogeneous dark energy and cosmological acceleration. General Relativity and Gravitation, 39(10), 1675-1687.

[29] Campos, M., & Lima, J. A. S. D. (2012). Black hole formation with an interacting vacuum energy density. Physical Review D—Particles, Fields, Gravitation, and Cosmology, 86(4), 043012.

[30] Campos, M. (2014, March). Gravitational collapse and the vacuum energy. In Journal of Physics: Conference Series (Vol. 496, No. 1, p. 012021). IOP Publishing.

[31] Wang, P., & Meng, X. H. (2004). Can vacuum decay in our universe?. Classical and Quantum Gravity, 22(2), 283.

[32] Shah, H. H., Rahaman, F., Ali, A., & Molla, S. (2019). Gravitational collapse of an interacting vacuum energy density with an anisotropic fluid. Physics of the Dark Universe, 24, 100291.

[33] Cahill, M. E., & McVittie, G. C. (1970). Spherical Symmetry and Mass-Energy in General Relativity. II. Particular Cases. Journal of Mathematical Physics, 11(4), 1392-1401.

[34] Madhav, T. A., Goswami, R., & Joshi, P. S. (2005). Gravitational collapse in asymptotically anti–de Sitter or de Sitter backgrounds. Physical Review D—Particles, Fields, Gravitation, and Cosmology, 72(8), 084029.

[35] Malafarina, D., & Joshi, P. S. (2011). Gravitational collapse with tangential pressure. International Journal of Modern Physics D, 20(04), 463-495.

[36] Israel, W. (1966). Singular hypersurfaces and thin shells in general relativity. Il Nuovo Cimento B (1965-1970), 44(1), 1-14.

[37] Bonnor, W. B., & Vickers, P. A. (1981). Junction conditions in general relativity. General Relativity and Gravitation, 13, 29-36.

[38] Cherubini, C., Bini, D., Capozziello, S., & Ruffini, R. (2002). Second order scalar invariants of the Riemann tensor: applications to black hole spacetimes. International Journal of Modern Physics D, 11(06), 827-841.

[39] Misner, C. W., & Sharp, D. H. (1964). Relativistic equations for adiabatic, spherically symmetric gravitational collapse. Physical Review, 136(2B), B571.

[40] Doroshkevich, A. G., Zel'Dovich, Y. B., & Novikov, I. D. (1968). Weakly interacting particles in the anisotropic cosmological model. Soviet Physics JETP, 26(2).

[41] Oliveira-Neto, G., Monerat, G. A., Corrêa Silva, E. V., Neves, C., & Ferreira Filho, L. G. (2011). An early universe model with stiff matter and a cosmological constant. In International Journal of Modern Physics: Conference Series (Vol. 3, pp. 254-265). World Scientific Publishing Company.

[42] Pandey, A., Kumar, R., & Srivastava, S. K. (2022). Shear-free gravitational collapse of dust cloud and dark energy. International Journal of Geometric Methods in Modern Physics, 19(06), 2250092.

[43] Sola, J. (2014, July). Vacuum energy and cosmological evolution. In AIP Conference Proceedings (Vol. 1606, No. 1, pp. 19-37). American Institute of Physics.

[44] Carvalho, J. C., Lima, J. A. S., & Waga, I. (1992). Cosmological consequences of a time-dependent Λ term. Physical Review D, 46(6), 2404.

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