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Fuzzy Rough C-Means Based MRI Segmentation for Accurate Brain Tumor Detection

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Abstract *An improved method for brain tumor segmentation in MRI images using Fuzzy Rough C-Means (FRCM) clustering and compares its performance with two traditional clustering techniques: Fuzzy C-Means (FCM) and Fuzzy K-Means (FKM). FRCM integrates fuzzy logic with rough set theory, enabling better management of the uncertainties and ambiguities in MRI data compared to FCM and FKM, which rely solely on fuzzy logic or hard clustering principles. The performance of these methods is validated against manually annotated ground truth masks using evaluation metrics such as Dice Coefficient, Jaccard Index, Precision, Recall, and F-Score. The experimental results show that FRCM significantly outperforms both FCM and FKM in terms of accuracy and robustness in segmenting tumor regions. FRCM demonstrates improved precision and reliability, particularly in handling complex and noisy MRI images, where FCM and FKM tend to struggle with boundary delineation and uncertainty handling. This makes FRCM a more effective tool for brain tumor diagnosis and treatment planning, offering clinicians a more reliable method for identifying tumor regions. The comparison highlights FRCM's potential as an advanced segmentation technique in medical image analysis, with promising applications in neuro-oncology for improving patient outcomes.*

1 Introduction

Benign or malignant brain tumor are abnormal cell growths that can occur within the brain. For efficient diagnosis, treatment planning, and disease progression monitoring[24], rapid and accurate tumor identification is essential. When it comes to observing the structures of brain and detecting abnormalities, such as tumor, magnetic resonance imaging (MRI) [16] is one of the most useful non-invasive imaging methods. Nonetheless, the technique of manually separating brain tumor from MRI images is difficult, subjective, and significantly dependent on radiologists experience. The necessity for automated, reliable, and accurate techniques to support clinical decision-making is highlighted by the range of options in manual segmentation.

Since it directly affects the diagnosis, prognosis, and treatment planning of brain tumor patients, accurate segmentation of brain tumors in MRI images is crucial to the medical [29] community. The accurate delineation of tumor boundaries is crucial for determining the tumor [43] extent, organizing surgical procedures, and implementing targeted medicines. This can only be achieved by high-quality segmentation [44]. It also helps with the regular tracking of tumor growth or shrinking over time, which is essential for determining how well treatment plans work. The complex and varied characteristics of brain tumors, along with the variability in MRI picture quality, make tumor segmentation difficult to do consistently, even with advances in imaging technologies.

Fuzzy Rough C-Means (FRCM) clustering is a sophisticated image segmentation [36] method that addresses the inherent uncertainties and ambiguities in MRI images by combining the ideas of fuzzy logic with rough set theory. By giving each data point a degree of membership rather

than a binary categorization fuzzy logic makes it possible to describe inaccurate information. However, rough set theory approximates the data into lower and upper bounds in order to deal with the ambiguity and vagueness [5]. Combining these two methods in FRCM [2] offers a potent tool for controlling the erratic and inaccurate quality of MRI images, resulting in tumor segmentation that is more exact and dependable.
This study main goal is to create a better technique for segmenting brain tumors in MRI images by employing Fuzzy Rough C-Means (FRCM) clustering [18]. The suggested method seeks to outperform conventional clustering techniques in segmentation [28] accuracy by utilizing the advantages of fuzzy logic and rough set theory [7]. The performance of the suggested method is also intended to be validated against manually annotated ground truth masks through the use of multiple evaluation measures, including as the Dice Coefficient, Jaccard Index, Precision, Recall, and F-Score. This work makes two contributions: first, it presents a novel use of FRCM clustering for medical image processing; second, it offers a thorough validation of the method's efficacy in locating brain tumor locations in magnetic resonance imaging.
FCM relies only on fuzzy membership functions [41], and FKM uses hard boundaries, FRCM enhances segmentation by incorporating rough sets to better define unclear boundaries and manage noisy data. This is especially important in MRI brain images, where tumors often have irregular shapes and unclear edges. FRCM provides more precise segmentation of tumor regions, which is crucial for accurate diagnosis and treatment planning. Its ability to better delineate uncertain areas makes it superior to FCM and FKM [26], offering a more robust tool for medical image analysis [25] and improving the reliability of brain tumor detection.

2 Related Works

Brain tumor segmentation in MRI images has evolved significantly, with numerous approaches being developed to improve accuracy and efficiency. Traditional methods [23], including thresholding and region-growing techniques, have paved the way for more sophisticated methods. Recently, machine learning and deep learning approaches [9], such as Support Vector Machines (SVM) and convolutional neural networks (CNNs) [14] , have demonstrated substantial improvements.
The paper by Reddy Poli V.S. [34], titled "Fuzzy C-Means and Fuzzy K-Means Algorithms using Fuzzy Functional Dependencies," presents a novel approach by incorporating fuzzy functional dependencies (FFDs) into Fuzzy C-Means (FCM) and Fuzzy K-Means (FKM) algorithms to enhance clustering performance. This integration helps manage data uncertainty and redundancy, leading to improved clustering accuracy, particularly in noisy or incomplete datasets, with applications in areas like medical imaging and bioinformatics.
Seyed Emadedin Hashemi et al [38], in their paper, A Fuzzy C-Means Algorithm for Optimizing Data Clustering, address the challenge of FCM sensitivity to initial centroids. They propose a heuristic-based initialization technique that reduces time complexity and enhances clustering robustness. The algorithm's improved speed and accuracy, particularly in high-dimensional datasets, make it suitable for applications such as customer segmentation and image processing.
In their paper titled ,The Comparison of Clustering Algorithms K-Means and Fuzzy C-Means for Segmentation Retinal Blood Vessels, Wiharto and Suryani [42] present a comparative study between two popular clustering algorithms—K-Means and Fuzzy C-Means (FCM)—for the task of segmenting retinal blood vessels. The study is motivated by the importance of retinal vessel segmentation in diagnosing various eye-related diseases, particularly diabetic retinopathy. The authors explore the strengths and limitations of both algorithms in terms of segmentation accuracy, computational efficiency, and robustness to noise in medical images. The paper highlights that FCM, due to its ability to assign membership levels to each data point, yields better accuracy in segmenting complex structures like retinal vessels compared to the hard clustering approach of K-Means. However, FCM is noted to be more computationally intensive. The authors conduct experiments using a retinal image dataset and evaluate the performance based on parameters such as segmentation accuracy and processing time, concluding that FCM offers superior accuracy, but at the cost of increased computational complexity.
Ganesh, Naresh, and Arvind [10], in their work, MRI Brain Image Segmentation Using Enhanced Adaptive Fuzzy K-Means Algorithm, focus on improving the accuracy of brain tumor segmentation in MRI images by enhancing the traditional Fuzzy K-Means (FKM) clustering algorithm.

The paper addresses the challenge of accurately delineating tumor boundaries in noisy MRI images, which is crucial for effective treatment planning. The authors propose an Enhanced Adaptive Fuzzy K-Means (EAFKM) algorithm that dynamically adjusts clustering parameters based on local image characteristics, thereby improving segmentation precision, especially in regions with subtle intensity variations. The algorithm incorporates adaptive membership updates to reduce the impact of noise and improve convergence speed. Through experimental validation on MRI datasets, the authors demonstrate that the EAFKM algorithm outperforms both traditional FKM and other segmentation techniques in terms of accuracy and computational efficiency. The paper concludes that the proposed method significantly enhances tumor detection accuracy while maintaining low computational overhead, making it suitable for clinical applications.

The paper by Imran et al. [17] presents a decision-making method for selecting robots using interval-valued intuitionistic fuzzy information and a specific averaging technique called Aczel-Alsina Bonferroni means. This approach considers multiple criteria, making it suitable for complex decisions where various factors must be evaluated. The use of interval-valued intuitionistic fuzzy information allows for better handling of uncertainty and imprecision in the data. The authors demonstrate how their method can improve the robot selection process by providing clearer insights and more reliable results. Overall, the study contributes valuable techniques to enhance decision-making in robotics and related fields.

The literature highlights several shortcomings in traditional FCM and FKM approaches, such as sensitivity to initial centroids, increased computational complexity, and difficulty in handling noisy or incomplete datasets. FCM, while more accurate in segmenting complex structures, suffers from high computational cost, and K-Means struggles with assigning precise membership levels, leading to less accurate clustering. Enhanced methods like heuristic-based initialization [1] and adaptive updates improve performance but still face limitations in managing uncertainty and noise. Fuzzy Rough C-Means (FRCM) overcomes these issues by integrating rough set theory with fuzzy logic [39], enabling better uncertainty management, handling of noise, and reducing the sensitivity to initial conditions, resulting in more precise and robust segmentation with lower computational overhead.

Gazi et al.[11] apply the Pentagonal Fuzzy DEMATEL methodology to identify critical criteria influencing women's empowerment in the sports sector. By assessing various empowerment factors, the study provides insights into enhancing decision-making strategies for promoting gender equality in sports. Asra Riaz et al.[3] explores the development of codes based on Lattice-Valued Intuitionistic Fuzzy Sets of Type-3, applying these concepts to complex DNA analysis. This approach enhances data representation and uncertainty handling in bioinformatics, providing a novel framework for genetic code interpretation. The studies summarized above connect to image segmentation through their focus on advanced techniques that enhance decision-making and data analysis in medical imaging. For instance, Anjum et al. [37] emphasize precise data representation in mutation detection, which parallels the need for accurate pixel representation in segmenting medical images. Palanikumar et al. [32] contribute to medical diagnosis with complex Pythagorean normal interval-valued fuzzy aggregation operators, improving the handling of uncertainty, crucial for segmenting noisy images. Similarly, Ejegwa et al. [33] highlight the importance of accurate data interpretation for diagnosing medical emergencies, akin to effective image segmentation facilitating quicker diagnoses.

Kausar et al. [31] focus on a selection method for medical robotics that emphasizes precise decision-making, which mirrors how accurate segmentation informs robotic-assisted treatments. Khalil et al. [40] present a bipolar interval-valued neutrosophic optimization model for healthcare systems, underscoring the need for optimizing processes that rely on image data. Riaz et al. [4] improve data representation and processing through algebraic codes, leading to more accurate segmentation results, while Munir et al. [30] demonstrate how generalized fuzzy sets enhance decision-making in medical imaging. Collectively, these studies underscore the significance of advanced computational techniques and fuzzy logic in improving image segmentation outcomes, ultimately aiding in better clinical decision-making.

3 Preliminaries

3.1 Fuzzy Logic

It provides a way to handle the partial membership of elements to cluster rather than assigning them to a single cluster. This is particularly important in medical image segmentation [15], where tissue boundaries are often not sharply defined.

3.2 Rough Set

Rough set theory deals with uncertainty and vagueness in data by providing a mathematical Fuzzy framework to approximate sets [19] that are not crisply defined. In rough sets, any given set is characterized by a lower approximation (elements that definitely belong to the set) and an upper approximation (elements that possibly belong to the set). This theory is valuable when dealing with imprecise or incomplete information, as it allows for a systematic way of handling ambiguities.

Definition 3.1 (Lower Approximation). Lower approximation [20] of set X (denoted as $\underline{X}$) consists of all elements that are certainly members of X. in other words, every element of the lower approximation is entirely contained within X.

$$\underline{X} = \{x \in U | [x] \subseteq X\} \tag{3.1}$$

Where U is the universal set and $[x]$ represents the equivalence class of element x, which is the set of all elements indistinguishable from x based on available attributes.

Definition 3.2 (Upper Approximation). Upper Approximation [13] of set X (denoted as $\overline{X}$) includes all elements that could possibly belong to X. It includes elements that have some overlap with X.

$$\overline{X} = \{x \in U | [x] \cap X \neq \phi\} \tag{3.2}$$

where $[x] \cap X$ refers to the intersection of the equivalence class $[x]$ with the set X.

Definition 3.3 (Boundary Region). The boundary region [8] of X is the difference between the upper and lower approximation. It represents the uncertainty or the roughness of the set, where elements cannot be definitely classified as inside or outside X.

$$Boundary(X) = \overline{X} - \underline{X} \tag{3.3}$$

3.3 Integration of Fuzzy set and Rough Set in Fuzzy Rough C Means

The Fuzzy Rough C-Means (FRCM) algorithm integrates fuzzy logic [21] and rough set theory to enhance clustering performance in complex datasets, such as medical images [22]. In FRCM, fuzzy logic is used to assign degrees of membership to different clusters, allowing for soft clustering where data points can belong to multiple clusters simultaneously. Rough sets , on the other hand, help in defining the boundaries of these clusters more accurately, especially in cases where the data is noisy or overlaps between clusters. This integration enables FRCM to effectively differentiate between tumor and non-tumor regions in MRI images by leveraging the strengths of both fuzzy logic [27] and rough set theory.

3.4 Usefulness in Tumor Segmentation

- *Handling Uncertainity: FRCM effectively manage the uncertainity and vagueness present in MRI images, particularly when distinguish between tumor and non-tumor tissues, which may have similar intensity level or overlapping boundaries.*
- *Improved Accuracy: By defining both lower and upper approximation, FRCM can better capture the complex structure of tumor region, leading to more accurate segmentation. The boundary regions, where the most uncertainity exists are explicitly handled, reducing the changes of misclassification between tumor and non-tumor.*

- *Adaptive Segmentation: FRCM can adapt to the varying levels of fuzziness in different regions of the brain, making it a robust tool for segmenting [35] MRI images with different levels of noise and intensity variations. This adaptability is crucial for correctly identifying and segmenting tumor, which can have irregular shapes and diffuse boundaries.*

4 Proposed Methodology

Fuzzy Rough C-Means (FRCM) is a sophisticated clustering algorithm that integrates fuzzy logic principles [14] with rough set theory to enhance data segmentation, particularly in scenarios involving uncertainty and imprecision, such as MRI image analysis. It combines the principle of Fuzzy C means with rough set theory. It utilizes the lower and upper approximations of rough sets to handle uncertainty and ambiguity in data more effectively. FRCM is more robust to outliner and noise compared of FCM , which can be crucial in medical image processing and other critical applications. FRCM can provide better segmentation quality in complex images with overlapping regions and high noise levels.

4.1 Mathematical Formula

FRCM extends the FCM objective function by incorporating rough set approximation:

$$J_{FRCM} = \sum_{i=1}^{N}\sum_{j=1}^{C}(u_{ij}^{m}\|x_i - v_j\|^2 + \lambda\left(\mu_{ij}^{lower} + \mu_{ij}^{upper}\right)\|x_i - v_j\|^2) \tag{4.1}$$

where J_{FRCM} represent objective function, μ_{ij}^{lower} and μ_{ij}^{upper} are the lower and upper approximation of the membership values in segment, u_{ij}^{m} represent the fuzzy membership value for i_{th} row and j_{th} column, x_i represent the i_{th} pixel, v_j represent centroid intensity value for the j_{th} segment,λ is the regularization parameter that balances the contributions of the fuzzy and rough components, N represent the number of pixel in image , C represent number of segments and $\|x_i - v_j\|^2$ represent squared Euclidean distance between pixel intensity and the centroid of a segment.
Steps in FRCM clustering is given by:

1. *Initialization: Randomly initialize the cluster center v_j and membership values μ_{ij}*
2. *Calculate Membership Values: For each data point x_i and each cluster j update the membership value μ_{ij}, using:*
$$u_{ij} = \frac{1}{\sum_{K=1}^{C}\left(\frac{\|x_i - v_j\|}{\|x_i - v_k\|}\right)^{\frac{2}{m-1}}} \tag{4.2}$$
3. *Update Cluster Centers: Update the Cluster center v_j using the new membership values,*
$$v_j = \frac{\sum_{i=1}^{N} u_{ij}^{m} x_i}{\sum_{i=1}^{N} u_{ij}^{m}} \tag{4.3}$$
4. *Calculate Lower and Upper Approximation*
$$\mu_{ij}^{lower} = \min_{K\neq j}\left(\frac{1}{1 + \left(\frac{\|x_i - v_j\|}{\|x_i - v_k\|}\right)^2}\right) \tag{4.4}$$
$$\mu_{ij}^{upper} = \max_{K\neq j}\left(\frac{1}{1 + \left(\frac{\|x_i - v_j\|}{\|x_i - v_k\|}\right)^2}\right) \tag{4.5}$$
5. *Iterate until convergence: Repeat steps 2 to 4 until the changes in the cluster centers v_j and membership values u_{ij} are below a certain threshold or until maximum number of iterations is reached.*

6. *Segment the tumor region: Once the clustering is complete, use the membership values u_{ij} to assign each pixel in the MR image to a cluster. The cluster with highest membership value corresponding to tumor characteristics is identified as the tumor region.*

FRCM clustering is effective for MRI brain tumor segmentation due to its robustness to noise and ability to handle uncertainty in the data, resulting in more accurate tumor detection.

5 Performance Metrics and Evaluation

To assess the effectiveness of segmentation method, several key performance metrics, like the Dice Coefficient, Jaccard Index, Precision, Recall and F-Score are calculated. These metrics offer a comprehensive evaluation of the segmentation quality. The Dice Coefficient and Jaccard Index measure the overlap between the segmented regions and the ground truth, providing insights into the accuracy and completeness of the segmentation. Precision and Recall metrics further quantify the algorithms ability to correctly identify tumor regions while minimizing false positives and negatives. The F-Score, which combines Precision and Recall into a single metric, offers a balanced measure of each method's overall performance. The mathematical formula for the metrics are given below:

$$\textit{Dice Coefficient} = \frac{2 \times |\textit{Predicted} \cap \textit{Ground Truth}|}{|\textit{Predicted}| + |\textit{Ground Truth}|} \tag{5.1}$$

$$\textit{Jaccard index} = \frac{|\textit{Predicted} \cap \textit{Ground Truth}|}{|\textit{Predicted} \cup \textit{Ground Truth}|} \tag{5.2}$$

$$\textit{Precision} = \frac{\textit{True Positive}}{\textit{True Positive} + \textit{False Positive}} \tag{5.3}$$

$$\textit{Recall} = \frac{\textit{True Positive}}{\textit{True Positive} + \textit{False Negative}} \tag{5.4}$$

$$\textit{F-Score} = 2 \times \frac{\textit{Precision} \times \textit{Recall}}{\textit{Precision} + \textit{Recall}} \tag{5.5}$$

6 Experimental Setup

The experimental setup was carefully designed to ensure accurate and reliable segmentation of MRI brain images. MATLAB 2017b was utilized as a primary software environment for implementing the segmentation algorithm. The ground truth annotation process involved selecting and resizing MRI brain images. The ground truth mask for both normal and abnormal images were detected using MATLAB. These masks served as the benchmark for evaluating the segmentation performance of Fuzzy Rough C Means (FRCM), Fuzzy C Means (FCM) [12], Fuzzy K Means (FKM) [6]. The Pseudo code for segmentation using FRCM is shown in Table 1.

1. Initialize cluster centers randomly.
2. Initialize fuzzy membership matrix U.
3. Repeat until Convergence:
a. For each pixel $P(i,j)$ in the image:
i. Compute the distance between $P(i,j)$ and all cluster centers.
ii. Update the fuzzy membership values U for $P(i,j)$.
iii. Calculate the lower and upper approximations for each cluster.
b. Update cluster centers by minimizing the objective function.
c. Check for convergence.
4. Output the final segmented image based on the highest membership values.

Table 1: Pseudo Code of the FRCM algorithm

7 Results

Fuzzy Rough C-Means (FRCM) is ideal for MRI tumor segmentation due to its ability to effectively manage uncertainty, imprecision, and noise, which are inherent in MRI images. Its robustness against noise ensures more accurate segmentation, even in images with poor contrast or irregular tumor shapes. Additionally, FRCM capability to manage vague and uncertain data improves the precision of tumor detection, making it highly reliable for critical applications such as medical diagnosis and treatment planning. Its balance of accuracy, flexibility, and robustness makes FRCM particularly well-suited for MRI tumor segmentation. Figure 1 represents the ground truth mask of the MRI images which was taken from using MATLAB 2017.

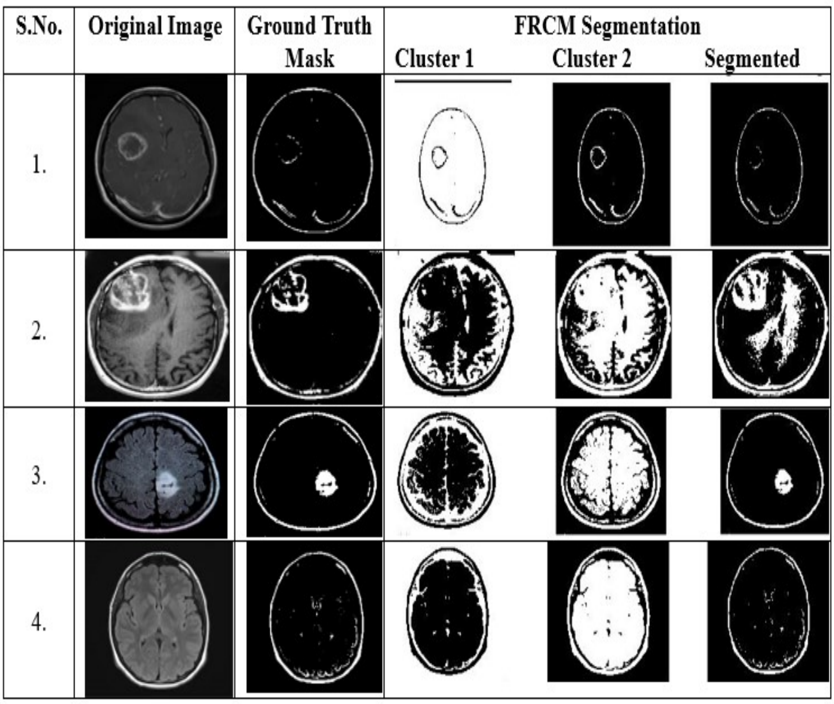


Figure 1: Displaying the ground truth mask

The segmentation results were evaluated by comparing the performance of Fuzzy Rough C-Means (FRCM) with Fuzzy C-Means (FCM) and Fuzzy K-Means (FKM) algorithms. The FRCM algorithm demonstrated superior accuracy in delineating tumor regions, achieving higher Dice Coefficient, Jaccard Index, and F-Score values compared to FCM and FKM. The FRCM method was particularly effective in handling overlapping and noisy data, leading to more precise segmentation of the tumor boundaries. While FCM and FKM also produced reasonable results, they were less effective in cases where the tumor boundaries were ambiguous. The results underscore the robustness of FRCM in medical image segmentation tasks, offering a reliable approach for identifying tumor and non-tumor regions.

From Figure 2, the cluster 1 represent the non-tumor and cluster 2 represent the tumor pixels. It is concluded that, Fuzzy Rough C-Means (FRCM) generally outperforms Fuzzy C-Means (FCM) and Fuzzy K-Means (FKM) in terms of overlap with the ground truth, particularly in cluster 2. This is evidenced by higher values for the Dice Coefficient, Jaccard Index, and F-Score in several instances. The high Recall values (1.0) across all methods indicate that the segmentation successfully identifies all relevant regions, but the differences in Precision, Dice Coefficient, and Jaccard Index suggest varying levels of accuracy in identifying true positives versus false positives. Cluster centers differ significantly across the methods, reflecting how each algorithm defines and separates the tumor regions from normal tissue.

FRCM cluster 2 consistently yields better metrics (Dice Coefficient, Jaccard Index, Precision, F-Score), suggesting that it accurately captures the tumor region compared to FCM and FKM. The

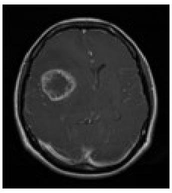
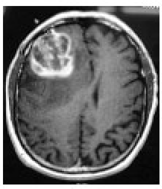
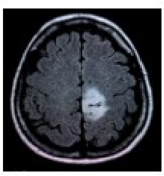
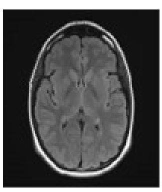
S.No.	Sample MRI Images	Metrics	Fuzzy Rough C Means (FRCM)		Fuzzy C Means (FCM)		Fuzzy K Means (FKM)	
			Cluster 1	Cluster 2	Cluster 1	Cluster 2	Cluster 1	Cluster 2
1.		Center	0.0656	0.7248	0.0121	0.2723	0.0092	0.2994
		Dice Coefficient	0	0.9627	0	0.8232	0	0.8354
		Jaccard Index	0	0.9143	0	0.8762	0	0.8971
		Precision	0	0.9143	0	0.8962	0	0.8571
		Recall	0	0.9899	0	0.8956	0	0.9076
		F-Score	0	0.9552	0	0.1755	0	0.8770
2.		Center	0.1432	0.5625	0.1340	0.5610	0.1500	0.5610
		Dice Coefficient	0	0.9824	0	0.8868	0	0.8019
		Jaccard Index	0	0.9131	0	0.8066	0	0.8241
		Precision	0	0.9131	0	0.8266	0	0.8441
		Recall	0	0.9998	0	0.8900	0	0.9000
		F-Score	0	0.9782	0	0.8725	0	0.6878
3.		Center	0.0198	0.3664	0.0223	0.3594	0.0221	0.3625
		Dice Coefficient	0	0.9721	0	0.8709	0	0.2717
		Jaccard Index	0	0.9458	0	0.8451	0	0.8456
		Precision	0	0.9458	0	0.8845	0	0.8456
		Recall	0	0.9998	0	0.8912	0	0.8867
		F-Score	0	0.9545	0	0.9253	0	0.8542
4.		Center	0.0790	0.4200	0.0787	0.4220	0.0796	0.4315
		Dice Coefficient	0	0.9653	0	0.8754	0	0.8761
		Jaccard Index	0	0.9703	0	0.9213	0	0.9032
		Precision	0	0.9703	0	0.9000	0	0.9003
		Recall	0	0.9798	0	0.9600	0	0.9500
		F-Score	0	0.9657	0	0.8652	0	0.8557

Figure 2: Results of Different Segmentation Methods

high Precision and F-Score in FRCM cluster 2 indicate a lower rate of false positives, which is crucial for clinical applications where accuracy is paramount. The comparative analysis highlights FRCM superior capability in handling the complexities of MRI brain image segmentation, making it a potentially more reliable choice for clinical applications where precise tumor detection is crucial.

8 Future Work

There is considerable scope for improving the segmentation performance of the explored methods. One promising direction is the integration of advanced optimization techniques to enhance cluster initialization and improve convergence in algorithms like Fuzzy Rough C-Means (FRCM). Incorporating deep learning with fuzzy clustering could also achieve greater accuracy in segmenting complex MRI images. Additionally, experimenting with different membership functions and adjusting the fuzzification parameter may provide deeper insights into algorithmic behavior. Applying these methods to larger, more diverse datasets would help validate their robustness and generalizability in clinical scenarios. These improvements can lead to more precise and reliable tumor segmentation, ultimately aiding in better diagnosis and treatment planning.

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