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Enhanced Cubic Spline Interpolation Method Incorporating Trapezoidal Fuzzy Number

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Abstract

This paper introduces an advanced method for cubic spline interpolation aimed at accurately approximating fuzzy functions defined over discrete data points. The proposed technique integrates trapezoidal fuzzy numbers to effectively model the uncertainty and imprecision inherent in fuzzy data. By leveraging the flexibility and smoothness of cubic spline polynomials, the method delivers an efficient and accurate means of representing nonlinear fuzzy functions with varying degrees of uncertainty. The use of trapezoidal fuzzy numbers ensures a balance between computational simplicity and the expressive power needed to handle vague or imprecise information. Through detailed analysis and illustrative examples, the effectiveness of the approach is demonstrated in capturing the behavior of fuzzy functions. The results underscore the method's potential in advancing fuzzy spline approximation and highlight its applicability in areas such as decision-making, optimization, and computational intelligence. This work contributes to the broader understanding of fuzzy interpolation techniques and supports improved handling of uncertainty in mathematical modeling and real-world problem solving.

1 Introduction

Cubic splines approximate data by connecting consecutive points with third-degree polynomial segments, ensuring smoothness through the continuity of both the first and second derivatives across all subintervals. Unlike high-degree interpolating polynomials, which can introduce undesirable oscillations and pose computational and stability issues when applied to large datasets, cubic splines offer a more stable and accurate alternative. The concept of splines is rooted in traditional drafting, where a flexible strip was used to draw smooth curves—an idea that inspired their mathematical development for interpolation tasks. Several researchers have contributed significantly to the study of fuzzy interpolation using splines. Abbasbandy [1] conducted an in-depth study on the interpolation of fuzzy data using complete splines, focusing on their application in approximating fuzzy functions. Building on this work, Abbasbandy and Babolian [2] explored the use of natural splines for fuzzy data interpolation, highlighting their computational efficiency and accuracy in handling uncertainty in fuzzy systems. In a similar direction, Behforooz et al.[3] investigated introducing an innovative approach that enhanced interpolation techniques within fuzzy environments. Kaleva [4] made notable contributions to the mathematical analysis of fuzzy data interpolation, discussing various theoretical frameworks and methodologies in his paper published in Fuzzy Sets and Systems. Klir et al.[5] extended the development of fuzzy set theory, emphasizing its fundamental principles and broad efforts in fields such as fuzzy logic and decision-making. Karpagam and Vijayalakshmi [6] conducted comparative studies on numerical methods, including Trapezoidal, Simpson's 3/8 Rule, and Weddle's Rule, to evaluate their effectiveness in fuzzy interpolation across different contexts. Expanding on this research, Karpagam and Vijayalakshmi[7] explored fuzzy cubic spline interpolation using triangular fuzzy numbers, demonstrating its practical applications in fuzzy data analysis and presenting new perspectives for improving interpolation accuracy. Further advancements in fuzzy interpolation were made by Lowen [8], who introduced the concept of fuzzy Lagrange interpolation and formulated the fuzzy Lagrange interpolation theorem, broadening its applicability to fuzzy datasets. Zimmermann [9] provided a comprehensive explanation of numerous efforts, serving as a foundational reference for researchers working in fuzzy logic and system modeling. Finally, L.A. Zadeh [10] made pioneering contributions to fuzzy set theory with his seminal paper "Fuzzy Sets", published in Information and Control, laying the theoretical groundwork for the development of fuzzy logic and its widespread application across multiple disciplines. Shrivastava [11] explored various trajectory planning schemes for robotic manipulators, optimizing motion strategies for efficient trajectory selection. The physical mechanism behind this involves dynamic modeling of robotic arms, incorporating kinematic constraints, actuator dynamics, and energy minimization principles. The study considers various trajectory optimization techniques such as minimum jerk, minimum torque, and energy-efficient pathways to ensure smooth, collision-free, and precise movement. The integration of advanced control systems like PID, adaptive control, and reinforcement learning further refines the robotic manipulator's response to environmental disturbances and task constraints. Suma et al. [12] proposed the mechanism relies on the representation of uncertainty in search spaces using fuzzy sets, allowing for an adaptive approach to optimization. The mathematical formulation ensures the minimization of cost functions under imprecise conditions, improving efficiency in path selection. By using decagonal fuzzy numbers, the model effectively captures imprecision in search parameters, allowing dynamic adjustments in real-time decision-making scenarios, such as logistics and network routing. Revathi et al. [13] developed an effective method to analyze student performance using fuzzy numbers. The approach integrates uncertainty modeling in academic assessments, capturing variations in grading, learning effectiveness, and student engagement. The fuzzy logic framework enables the ranking of students based on linguistic variables such as "high," "medium," and "low" performance, supported by interval-based performance evaluation. This method aids educators in identifying learning gaps, designing personalized learning plans, and making data-driven decisions to improve academic outcomes. Dhuraiv et al. [14] introduced a novel ranking function for octagonal fuzzy numbers applied to solving transportation problems. This method leverages the geometric properties of fuzzy numbers to rank alternatives effectively, ensuring an optimal allocation of resources within transportation networks under uncertainty. The ranking function considers parameters such as transport costs, demand variability, and supply chain disruptions, offering a robust decision-making tool for logistics management and operational research. Suguna et al. [15] explored generalized the physical mechanism utilizes multi-criteria aggregation operators to analyze disease symptoms, integrating expert knowledge and probabilistic uncertainty in diagnostic decision-making. By combining neutrosophic logic, which allows for truth, indeterminacy, and falsehood values, the model enhances medical diagnostics by accommodating incomplete, contradictory, or vague clinical data, leading to more accurate disease classification and patient management. Kousar et al. [16] applied an integrated fuzzy-rough approach to multi-criteria decision-making in sustainable agritourism. This method models sustainability factors such as environmental impact, economic viability, and social acceptance by incorporating imprecise data into decision frameworks. The fuzzy-rough hybrid model refines sustainability assessments by leveraging historical data and expert opinions, providing a structured approach for policymakers and agribusinesses to develop eco-friendly tourism initiatives. Kausar et al. [17] investigated the study models mechanical deformations, vibrations, and material uncertainties using fuzzy differential equations. The scheme enhances predictive accuracy in engineering designs by incorporating fuzzy constraints into elasticity equations, thereby improving reliability in applications such as structural health monitoring, automotive engineering, and aerospace systems. Indira et al. [18] analyzed the prediction of stochastic transportation problems in a multi-objective rough interval environment. The physical mechanism incorporates stochastic programming techniques, addressing uncertainty in transportation costs, demand fluctuations, and network reliability. The rough interval approach allows for adaptive decision-making by considering real-world constraints such as fuel price variability, weather disruptions, and traffic congestion, improving supply chain resilience. Kousar et al. [19] examined leveraging air pollution indices, meteorological parameters, and epidemiological data, the model helps policymakers implement effective smog control strategies, balancing economic and public health considerations. Ejegwa et al. [20] introduced the model utilizes Pythagorean fuzzy sets to quantify supplier attributes under uncertainty, ensuring robust decision-making in procurement processes. This approach enhances supplier evaluation by incorporating multiple factors such as cost efficiency, reliability, and sustainability, thus optimizing the supply chain for industries like manufacturing and retail. Jayakumar et al. [21] analyzed the study employs lattice structures to represent spatial relationships and sustainability criteria in urban planning. By integrating spatial-temporal analysis with multi-fuzzy decision models, the approach optimizes land use planning, transportation networks, and urban green space management, promoting sustainable city development. Kousar et al. [22] proposed the models resource recycling, waste reduction, and energy efficiency using fuzzy mathematical programming. The framework aids industries in transitioning towards circular economy models by optimizing raw material utilization, reducing carbon footprints, and enhancing production lifecycle sustainability. Shams et al. [23] developed a fractional calculus principles to model memory effects and hereditary properties in complex systems. This approach is particularly useful in engineering and physics applications where processes exhibit non-local behavior, such as diffusion phenomena in porous media and viscoelastic material behavior. Sangodapo et al. [24] enhances the representation of uncertainty by incorporating neutrality alongside positive and negative membership functions. The model improves decision-making accuracy in applications such as financial forecasting, project risk assessment, and expert consensus. Rasool et al. [25] explored the approach accounts for subjective customer preferences, service quality, and environmental sustainability. The model refines hotel ranking systems by incorporating user reviews, operational efficiency, and sustainability criteria, aiding the tourism and hospitality industry. Moussaoui et al. [26] investigated fixed point results for TR-weak fuzzy contractions using binary relations. The study models stability conditions in iterative mapping processes within mathematical analysis frameworks. This theory has implications in applied mathematics, optimization, and engineering problems involving iterative algorithms. Kumari and Sharma [27] examined machine repair problems using Gaussian fuzzy numbers. The model incorporates failure rates, maintenance strategies, and resource allocation under uncertain conditions, improving predictive maintenance strategies and industrial reliability. Zyoud et al. [29] developed a two-stage fuzzy AHP model for prioritizing sustainable sanitation services under uncertainty. The framework integrates subjective judgments, sanitation infrastructure reliability, and environmental sustainability factors. Nouiri and Abdelkebi [31] explored the physical mechanism incorporates anomalous diffusion and memory effects in signal transmission modeling, enhancing the accuracy of wave propagation predictions in communication systems and biological signal processing. This paper begins with an overview of the fundamental concepts and preliminary discussions in Section 2, providing the necessary background for the proposed methodology. Section 3 introduces the new approach in detail, accompanied by numerical examples to illustrate its effectiveness and applicability. Section 4 presents the key conclusions depicted from the study, summarizing the main findings and their implications. At last, Section 5 defines prospective guidelines for future research and discusses the limits of this current work, highlighting areas that require further exploration and improvement.

1.1 Motivation of this Research

This research is motivated by the need for effective mathematical tools to handle imprecise and uncertain data in real-world applications such as decision-making, optimization, and computational intelligence. Traditional cubic spline interpolation, while effective for smooth function approximation, struggles to represent the inherent uncertainty of fuzzy functions. To address this limitation, the study proposes a novel approach that incorporates trapezoidal fuzzy numbers into cubic spline approximation, ensuring both computational efficiency and accuracy in representing complex fuzzy data. By combining the flexibility of fuzzy numbers with the precision of cubic splines, this method enhances the approximation of non-linear fuzzy functions and broadens the applicability of spline techniques in uncertain environments, contributing to advancements in fuzzy mathematics and practical problem-solving.

1.2 Applications in Real life

The proposed cubic spline approximation using trapezoidal fuzzy numbers has numerous real-life applications where uncertainty and imprecise data play a crucial role. In engineering and manufacturing, it can be used for modeling uncertain measurements in quality control, ensuring better decision-making in production processes. In finance and risk assessment, it helps in forecasting stock trends and evaluating uncertain investment risks. In medical diagnostics, it aids in interpreting imprecise physiological data, improving diagnosis and treatment planning. In climate modeling, it enhances the approximation of uncertain environmental parameters, leading to better predictions of climate change effects. Additionally, in artificial intelligence and machine learning, it improves fuzzy logic-based decision systems by offering more accurate approximations of uncertain input data. This method thus provides a powerful tool for handling uncertainty across various fields, improving accuracy and efficiency in real-world problem-solving.

2 Preliminaries

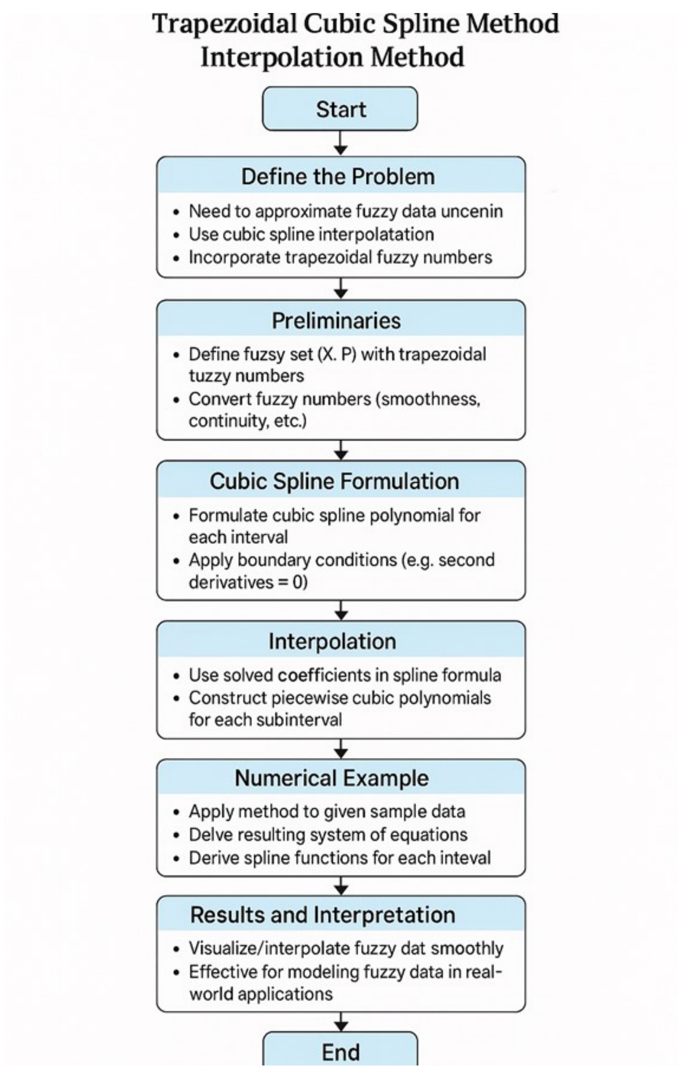
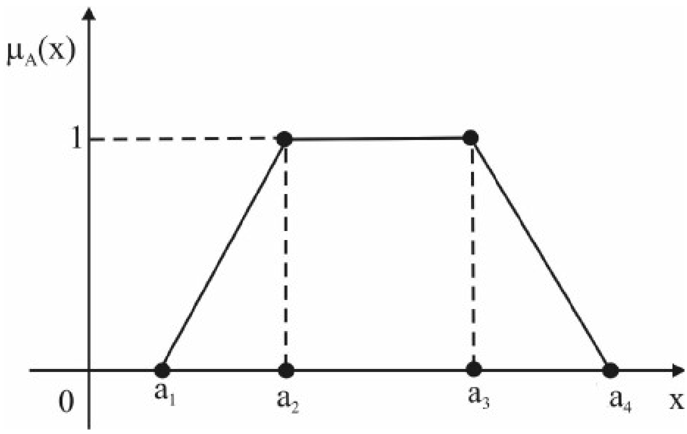


Figure 1. Flowchart of the Trapezoidal Cubic Spline Interpolation Method for modeling fuzzy data

Definition 2.1. Let X be a Universe of Discourse. A Fuzzy Set a is defined as follows: $A = < x, \mu_A(x) > /x \in X$, Where $\mu_A : X \to [0, 1], \mu_A(X)$ denotes the Membership Degree of the element X to the Set A.

The Goal of Cubic Splines is to derive a Third-Order Polynomial for each interval Between Knots, represented as $f_v(l) = a_v l^3 + b_v l^2 + c_v l + D_v$. Given N+1 Data Points $(v = 0, 1, 2, \dots)$, There are N intervals, resulting in 4N unknown constants to determine. similar to quadratic splines, 4N conditions are necessary to evaluate these unknowns.



Definition 2.2. Given N Data Points $(l_1, m_1)\dots(l_N, m_N)$ Where x_i are distinct and arranged in increasing order, a cubic spline z(l) the passes through these points is defined as a collection of cubic polynomials.
$Z_1(l) : m_1 + b_1(l - l_1) + c_1(l - l_1)^2 + d_1(l - l)^3 on[l_1, l_1]$

Z$_2(l) : m_2 + b_2(l - l_2) + c_2(l - l_2)^2 + d_2(l - l)^3 on[l_2, l_2]$

Z$_{(N-1)}(x) : m_{(N-1)} + b_{(N-1)}(l - l_{(N-1)}) + c_{(N-1)}(l - l_{(N-1)})^2 + d_{(N-1)}(l - l)^3 on[l_{(N-1)}, l_N]$

With the following conditions:
$Z_v(l_v) = m$ and $Z_v(l_{(v+1)}) = m_{(v+1)}$ for $v = 1, 2, \dots, N - 1$.
This characteristic ensures that the spline z (l) effectively interpolates the given data points.

$Z'_{(v-1)}(l_v) = Z'_v(l_v)$ for $v = 2\dots N - 1$.

The continuity of $Z'(l)$ on the interval $[l_1, l_N]$ ensures that the slopes of adjacent segments are consistent at their junctions.

$Z''_{(v-1)}(l_i) = Z''_v(l_i)$ for $v = 2\dots N - 1$.

$Z''(l)$ is continuous on the interval $[l_1, l_N]$ which also forces the neighboring spline to have the same curvature to guarantee the smoothness.

Let z(l) is a cubic polynomial, $Z''(l)$ is linear in each interval. In the interval $(l_{(v-1)}, l_v)$.

Assume $Z''(l) = 1/h[(l_{v-l})Z''(l_{(v-1)}) + (l - l)Z''(l_v)]$

This equation valid for $l = l_{(v-1)}$ and $l = l_v$

$Z(l) = 1/h[(l_{v-l})^3/3!Z''(l_{(v-1)}) + (l - l_{(v-1)})^3/3!Z(l_v)] + a_v(l_v - l) + b_v(l - l_{(v-1)})$ where a_v, b_v are constants to be found out by using the conditions

$Z(l_v) = m_v, v = 0, 1, 2, \dots, N$

Put $l = l_{(v-1)}$,we get

$m_{(v-1)} = 1/h[h^{3/3!}Z^{''}(l_{(v-1)})] + ha_v$

$a_v = 1/(h)[m_{(v-1)} - h^{2/3!}Z^{''}(l_{(v-1)})]$

Put $l = l_{(i)}$, we get

$b_v = 1/(h)[m_{v-h^{2/3!}}Z^{''}(l_v)]$

Hence the equation reduces to
$Z(l) = 1/h[((l_v-l)^3/3!)Z^{''}(l_{(v-1)})+(l-l_{((v-1)})^3/3!)Z^{''}(l_v)]+1/h(l_v-l)[m_{(v-1)}-h^2/3!Z^{''}(l_{(v-1)}))]+1/h(l-l_{(v-1)})[m_{v-h^2/3!}Z^{''}(l_v)]$

Put $Z^{''}(l) = P_v$, the above equation becomes,

$Z(l) = 1/6h[(l_{v-l})^3P_{(v-1)} + (l - l_{(v-1)})^3P_v] + 1/h(l_{v-l})[m_{(v-1)} - h^{2/6}P_{(v-1)}] + 1/h(l - l_{(v-1)})[m_{v-h^{2/6}}P_v]$

The quantities P_v represent the second derivatives of the spline, which are not yet known. Now, we resolve the continuity of $Z^{'}(l)$ based on the first conditions, which state that:
$Z^{'}(l) = 1/6h[3(l^2_{v-l)}(-P_{(v-1)})+3(l-l^2_{(v-1)}P_v]+1/h[-m_{(v-1)}+h^{2/6}P_{(v-1)}]+1/h[m_{v-h^{2/6}P_v]}$

$Z^{'}(l_{v-}) = h/3P_v + h/6P_{(v-1)} + 1/h(m_{v-}m_{(v-1)})$

Similarly, $Z^{'}(l_{v+}) = h/3P_v - h/6P_{(v-1)} + 1/h(m_{(v+1)} - m_v)$

Equating above equations $P_{(v-1)} + 4P_v + P_{(v+1)} = 6/h^2[m_{(v-1)} - 2m_v + m_{(v+1)}]$ For $v = 1, 2, 3 \ldots (N - 1)$

Furthermore, based on the first conditions, we have Z(l) is linear for $l \prec l_0$ and $l \succ l_n$, since $Z^{''}(l_v) = 0$ at $l = l_0$ and $l = l_n$

Hence $P_0 = 0, P_N = 0$

We can derive (n+1) equations in (n+1) unknowns $P_0, P_1, P_2, \ldots, P_N$.
Therefore, we can solve for $P_0, P_1, P_2 \ldots P_N$. By substituting, we obtain the cubic spline for each interval.

3 Examples

To obtain a cubic spline estimate for the function m = z (l) from the following statistics

R	(3,4,5,6)	(4,5,6,7)	(5,6,7,8)	(6,7,8,9)	(7,8,9,10)
P	0	1	1	1	0

Solution. Initially, the trapezoidal fuzzy number convert into crisp number by ranking function.

R	4.5	5.5	6.5	7.5	8.5
P	0	1	1	1	0

Given that h is the length of the interval and n is the number of intervals, and considering that the values of l are uniformly spaced with $h = 1$ and $n = 3$, we have:

$P_{(v-1)} + 4P_v + P_{(v+1)} = 6[m_{(v-1)} - 2m_v + m_{(v+1)}]v = 1, 2 \ldots (N - 1)$

$P_0 = m^{''}_0 = 0; P_4 = m^{''}_4 = 0.$

$For v = 1$ $P_0 + 4P_1 + P_2 = 6[m_0 - 2m_1 + m_2]$
$4P_1 + P_2 = -6$

$For v = 2$ $P_1 + 4P_2 + P_3 = 6[m_1 - 2m_2 + m_3]$
$P_1 + 4P_2 + P_3 = 0$

$For v = 3$ $P_2 + 4P_3 + P_4 = 6[m_2 - 2m_3 + m_4]$
$P_2 + 4P_3 = 0$

Solving the above equations, we get $P_1 = -12/7, P_2 = 6/7, P_3 = -12/7$

The Cubic Spline in $l_{(v-1)} \leq l \leq l_v$, is given by
$M = Z(l) = 1/6[(l_v - l)^3 P_{(v-1)} + (l - l_{(v-1)})^3 P_v] + (l_v - l)[m_{(v-1)} - 1/6P_{(v-1)}] + (l - l_{(v-1)})[m_v - 1/6P_v]$

Put v =1 in the above equation, The Cubic Spline, For $4.5 \leq l \leq 5.5$ is given by
$M = -1/21[-6l^3 + 81l^2 - 9.45l + 425.25]$

Put v =2, The Cubic Spline, For $5.5 \leq l \leq 6.5$ is given by
$M = 1/7[3l^3 - 55.5l^2 + 341.25l - 690.125]$

Put v = 3,The Cubic Spline, For $6.5 \leq l \leq 7.5$ is given by
$M = 1/7[-3l^3 + 61.5l^2 - 419.25l + 957.625]$

Put v = 4, The Cubic Spline, For $7.5 \leq l \leq 8.5$ is given by
$M = 1/7[2l^3 - 51l^2 + 424.5l - 1151.75]$

Therefore, The Cubic Spline Approximation for the given function is:

M= $-1/21[-6l^3 + 81l^2 - 9.45l + 425.25]$ for $4.5 \leq l \leq 5.5$
$1/7[3l^3 - 55.5l^2 + 341.25l - 690.125]$ for $5.5 \leq l \leq 6.5$
$1/7[-3l^3 + 61.5l^2 - 419.25l + 957.625]$ for $6.5 \leq l \leq 7.5$
$1/7[2l^3 - 51l^2 + 424.5l - 1151.75]$ for $7.5 \leq l \leq 8.5$

4 Conclusion

This paper introduces a Novel Cubic Spline Interpolation Method designed specifically for fuzzy data, providing an optimal approximation at discrete points with enhanced accuracy and computational efficiency. The method ensures smooth and reliable interpolation while effectively handling uncertainty and imprecision inherent in fuzzy datasets. A numerical illustration is presented to justify the efficiency of the intended method, demonstrating its superior accuracy compared to traditional interpolation methods, with significantly reduced approximation error. This improvement makes the method particularly useful in applications where precise modeling of uncertain data is critical. The proposed interpolation technique has broad applications across various domains, including decision-making, engineering, finance, and artificial intelligence. In decision-making, it can help refine predictive models that involve uncertain inputs. In engineering, it aids in optimizing control systems and structural analysis where fuzzy parameters are involved. In finance, it can be applied to risk assessment and portfolio optimization, while in artificial intelligence, it enhances machine learning models that rely on imprecise or uncertain data.

5 Future Scope

Future research will focus on enhancing the cubic spline interpolation method to address various challenges and expand its applicability to complex datasets. One major direction is the extension of the method to higher-dimensional fuzzy data, making it more versatile for multidimensional problems in engineering, finance, and artificial intelligence. This expansion would allow for more accurate interpolation and approximation in cases where data exists in multiple correlated dimensions. Another promising avenue is the integration of adaptive and hybrid methods to improve accuracy and performance. By incorporating machine learning techniques, optimization algorithms, or other advanced computational strategies, the method can become more robust in handling complex, uncertain datasets. Research will also investigate the robustness of the method under different levels of uncertainty and noise, ensuring reliability in real-world applications. This involves testing its performance across diverse datasets and refining it to maintain precision under varying conditions. Overall, while the proposed cubic spline interpolation method for fuzzy data shows great promise, future research will focus on expanding its capa2bilities, improving efficiency, and addressing computational and theoretical challenges to make it a more powerful and practical tool for handling uncertainty in various real-world applications.

References

[1] S.Abbasbandy, “Interpolation of Fuzzy Data By Complete Splines: J.Appl.Math.Comput., Vol.8, Pp. 587-594, 2001.

[2] S.Abbasbandy and E. Babolian, “Interpolation of Fuzzy Data by Natural Splines,”J.Appl.Math.Comput., Vol.5, Pp.457-463,1998.

[3] H.Behforooz, R.Ezzati And S.Abbasbandy. “Interpolation Of Fuzzy Data by Using E(3) Cubic Splines,” International Journal of Pure and Applied Mathematics, Vol.60, No.4, Pp.383-392, 2010.

[4] O.Kaleva, “Interpolation of Fuzzy Data”, Fuzzy Sets and Systems, Vol.61, Pp. 63-70, 1994.

[5] G.J.Klir, U.S.Clair and B.Yuan, “ Fuzzy Set Theory: Foundations and Applications,” Prentice –Hall Inc., 1997.

[6] A. Karpagam, Dr. V. Vijayalakshmi., “Comparison Results of Trapezoidal, Simpson’s 1/3 Rule, Simpson’s 3/8 Rule and Weddle’s Rule “A Journal of Composition Theory “ Volume 12, Issue Ix PP 1884-1887.

[7] Yadav, P. K., Roshan, M. (2024). Heat transfer analysis of a peristaltically induced creeping magnetohydrodynamic flow through an inclined annulus using homotopy perturbation method. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik, 104(11), e202400198.

[8] R.Lowen, “A Fuzzy Lagrange Interpolation Therorem,” Fuzzy Sets and Systems, Vol.34, Pp.33-38, 1990.

[9] H.J. Zimmermann, “ Fuzzy Sets Theory and Its Application,” Kluwer Academic Press, Dordrecht, 1991.

[10] L.A. Zadeh, “Fuzzy Sets” Information and Control, 8(30 Pp. 338-353, 1965.

[11] Shrivastava, A. (2025). Exploring Optimal Motion Strategies: A Comprehensive Study of Various Trajectory Planning Schemes for Trajectory Selection of Robotic Manipulator. Journal of the Institution of Engineers (India): Series C, 1-20.

[12] Suma, S., Karpagam, A., Vijayalakshmi, V., Isaiyarasi, T., Ramya, S. (2024). FUZZY OPTIMAL SOLUTION FOR THE SHORTEST MAPPING IN THE UNIVARIATE SEARCH APPROACH USING DECAGONAL FUZZY NUMBERS. JOURNAL OF BASIC SCIENCE AND ENGINEERING, 21(1), 1471-1479.

[13] Revathi, A. N., Karpagam, A., Suguna, M., Salahuddin. (2024). An Effective Method of Analysing Student Performance Using Fuzzy Numbers. Advances In Fuzzy Systems, 2024(1), 1513765.

[14] Dhurai, K., Karpagam, A. (2018). New Ranking Function on Octagonal Fuzzy Number for Solving Fuzzy Transportation Problem. International Journal of Pure and Applied Mathematics, 119(9), 125-131.

[15] Palanikumar, M., Suguna, M., Jana, C. (2023). Generalized Neutrosophic Sets and Their Applications for Aggregated Operators Based on Diagnostic Disease Problem. In Fuzzy Optimization, Decision-Making and Operations Research: Theory and Applications (Pp. 219-240). Cham: Springer International Publishing.

[16] Kousar, S., Kausar, N. (2025). Multi-Criteria Decision-Making for Sustainable Agritourism: An Integrated Fuzzy-Rough Approach. Spectrum Of Operational Research, 2(1), 134-150.

[17] Kausar, N., Garg, H. (2024). Contra-Hormonic Generalized Fuzzy Numerical Scheme for Solving Mechanical Engineering Problems. Journal Of Applied Mathematics and Computing, 70(5), 4629-4653.

[18] Indira, P., Jayalakshmi, M. (2024). Prediction Of Stochastic Transportation Problem with Fixed Charge in Multi-Objective Rough Interval Environment. International Journal of Analysis and Applications, 22, 117-117.

[19] Kousar, S., Ansar, A., Kausar, N., Freen, G. (2025). Multi-Criteria Decision-Making for Smog Mitigation: A Comprehensive Analysis of Health, Economic, And Ecological Impacts. Spectrum Of Decision Making and Applications, 2(1), 53-67.

[20] Ejegwa, P. A., Kausar, N., Aydin, N., Deveci, M. (2025). A Novel Pythagorean Fuzzy Correlation Coefficient Based on Spearman's Technique of Correlation Coefficient with Applications in Supplier Selection Process. Journal Of Industrial Information Integration, 44, 100762.

[21] Jayakumar, V., Pethaperumal, M., Kausar, N., Pamucar, D., Simic, V., Salman, M. A. (2025). Lattice-Based Decision Models for Green Urban Development: Insights From Q-Rung Orthopair Multi-Fuzzy Soft Set. International Journal of Computational Intelligence Systems, 18(1), 1-27.

[22] Kousar, S., Alvi, A., Kausar, N., Garg, H., Kadry, S., Kim, J. (2025). Fuzzy Multi-Objective Optimization Model to Design A Sustainable Closed-Loop Manufacturing System. Peerj Computer Science, 11, E2591.

[23] Shams, M., Kausar, N., Agarwal, P., Jain, S. (2024). Fuzzy Fractional Caputo-Type Numerical Scheme for Solving Fuzzy Nonlinear Equations. In Fractional Differential Equations (Pp. 167-175). Academic Press.

[24] Sangodapo, T. O., Kausar, N., Chreif, M. Y. (2024). A New Decision-Making Analysis Model Based on The Transformation of Picture Fuzzy Sets into Fuzzy Sets. In Analytical Decision Making and Data Envelopment Analysis: Advances and Challenges (Pp. 455-464). Singapore: Springer Nature Singapore.

[25] Rasool, E., Kausar, N., Ahmad, S. S. S., Aydin, N., Olanrewaju, O. A. (2024). Multi-Attribute Decision-Making Based on Pythagorean Fuzzy Numbers and Its Application in Hotel Evaluations. Decision Making: Applications In Management and Engineering, 7(2), 559-571.

[26] Moussaoui, A., Melliani, S. (2024). FIXED POINT RESULTS FOR TR-WEAK FUZZY CONTRACTION VIA BINARY RELATION. Palestine Journal of Mathematics, 13(2).

[27] Kumari, U., Sharma, D. C. (2024). Fuzzy Analysis of a Retrial Machine Repair Problem Using Gaussian Fuzzy Number. Palestine Journal of Mathematics, 13(3).

[28] Zyoud, S., Omair, S. M., Jarrad, S. A. (2025). A Two-Stage Model of The Fuzzy Analytic Hierarchy Process and The Fuzzy Synthetic Evaluation Technique to Prioritize Sustainable Sanitation Services Under Uncertainty. Scientific Reports, 15(1), 3736.

[29] Jansirani, M. M., Sujatha, S. (2024). LUKASIEWICZ FUZZY IDEALS APPLIED TO BCK/BCI-ALGEBRAS. Palestine Journal of Mathematics, 13.

[30] Palanikumar, M., Sreelatha Devi, V., Jana, C., Weber, G. W. (2023). Multi-Criteria Group Decision-Making Q-Rung Neutrosophic Interval-Valued Soft Set TOPSIS Aggregating Operator for The Selection of Diagnostic Health Imaging. In Fuzzy Optimization, Decision-Making and Operations Research: Theory and Applications (Pp. 499-517). Cham: Springer International Publishing.

[31] Nouiri, B., Abdelkebir, S. (2024). A Numerical Approach of The Space-Time-Fractional Telegraph Equations with Variable Coefficients. Palestine Journal of Mathematics, 13(3).

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