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An integrated weighted aggregating operators via generalized trigonometric spherical fuzzy set

M. Prakash and S.V. Manemaran

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Corresponding Author: S.V. Manemaran

Abstract We introduce new methods for the sin trigonometric spherical fuzzy interaction aggregating operator in this study. A combination of the sin trigonometric operator and the spherical fuzzy set. The universal aggregation function is used to study the novel averaging and geometric operations. The sin trigonometric spherical fuzzy interaction operators are commutative, associative, idempotent, and boundedness compatible. The sin trigonometric spherical fuzzy interaction weighted averaging, weighted geometric, generalized weighted averaging, and generalized weighted geometric are the four new aggregating operators that are introduced. The Euclidean distance, Hamming distance, and score values are often assumed to represent the aggregation functions.

1 Introduction

The fuzzy set (FS) [1], intuitionistic FS (IFS) [2], Pythagorean FS (PFS) [3, 4], neutrosophic set (NSS) [5] and Fermatean FS (FFS) [6] have all been developed as a consequence of the uncertainties. Zadeh [1] proposes a membership degree (MD) for decision-makers. An IFS concept was established by Atanassov [2]. Since each object possesses MD $\varkappa$ and non-membership degree (NMD) ς and satisfies $0 \leq \varkappa + \varsigma \leq 1$, where $\varkappa, \varsigma \in [0,1]$. According to the criterion that $\varkappa^2 + \varsigma^2 \leq 1$, PFSs are defined by their MDs and NMDs, as developed by Yager [3]. Cuong et al. [7] creating the idea of picture FSs (PiFSs). Since PiFSs is an extended form of IFSs, it has been observed that it may support certain extra ambiguities. It is noted in PiFSs that the MD $\varkappa$, neutral ς, and NMD $\breve{a}$ have $0 \leq \varkappa + \varsigma + \breve{a} \leq 1$; for $\varkappa, \varsigma, \breve{a} \in [0,1]$. It will guarantee that expert opinion messages like "yes," "abstain," "no," and "refusal" are sent over the PiFS. Additionally, it will prevent evaluation information from being omitted and guarantee that the evaluation data and the real decision environment are consistent. Although there are many applications and research on PiFSs, the notion has not been thoroughly investigated. For certain AOs with MADM, Shahzaib et al. [8] defined the spherical FS (SFS). The SFS requires that $0 \leq \varkappa^2 + \varsigma^2 + \breve{a}^2 \leq 1$ instead of $0 \leq \varkappa + \varsigma + \breve{a} \leq 1$. Jin et al. [9] presented the linguistic SFS AOs, which were talked about in MADM difficulties. In DM, Rafiq et al. presented SFSs and their uses [10]. Decision-making (DM) and the fact that $\varkappa^2 + \varsigma^2 \geq 1$ are problematic. Senapati et al. [6] introduced the idea of an FFS in 2019. Both the MD and NMD with the property that $0 \leq \varkappa^3 + \varsigma^3 \leq 1$.

2 Basic concepts

Definition 2.1. [3] Let $\digamma$ be the universe set. The PFS $\beth = \left\{x, \langle \varkappa^{\urcorner}[x], \upsilon^{\dashv}[x] \rangle | x \in \digamma \right\}$, where $\varkappa^{\urcorner}, \upsilon^{\dashv} : \digamma \to [0,1]$ refers the MD and NMD of $x \in \digamma$ to $\beth$, respectively and $0 \leq [\varkappa^{\urcorner}[x]]^2 + [\upsilon^{\dashv}[x]]^2 \leq 1$. For, $\beth = \langle \varkappa^{\urcorner}, \varkappa^{\dashv} \rangle$ is called the Pythagorean fuzzy number (PFN).

Definition 2.2. [6] A Fermatean fuzzy set $\beth = \left\{x, \langle \varkappa^{\urcorner}[x], \upsilon^{\dashv}[x] \rangle | x \in \digamma \right\}$, where $\varkappa^{\urcorner}[x]$ and $\upsilon^{\dashv}[x]$ denote MD and NMD of u respectively, where $\varkappa^{\urcorner}, \upsilon^{\dashv} : \digamma \to [0,1]$ and $0 \leq [\varkappa^{\urcorner}[x]]^3 +$

$[\upsilon^{\dashv}[x]]^3 \leq 1$. Here, $\sqsupset = \langle \varkappa^{\urcorner}, \varkappa^{\dashv} \rangle$ is represent a Fermatean fuzzy number [FFN].

Definition 2.3. For any PFNs, $\sqsupset = \langle \varkappa^{\urcorner}, \upsilon^{\dashv} \rangle$, $\sqsupset_1 = \langle \varkappa_1^{\urcorner}, \upsilon_1^{\dashv} \rangle$ and $\sqsupset_2 = \langle \varkappa_2^{\urcorner}, \upsilon_2^{\dashv} \rangle$, $\varkappa^{\urcorner}, \upsilon^{\dashv}$ denote MD and NMD of u respectively. Then

(i) $\sqsupset_1 \curlyvee \sqsupset_2 = \left[\sqrt{[\varkappa_1^{\urcorner}]^2 + [\varkappa_2^{\urcorner}]^2 - [\varkappa_1^{\urcorner}]^2 \cdot [\varkappa_2^{\urcorner}]^2}, [\upsilon_1^{\dashv} \cdot \upsilon_2^{\dashv}]\right]$

(ii) $\sqsupset_1 \curlywedge \sqsupset_2 = \left[[\varkappa_1^{\urcorner} \cdot \varkappa_2^{\urcorner}], \sqrt{[\upsilon_1^{\dashv}]^2 + [\upsilon_2^{\dashv}]^2 - [\upsilon_1^{\dashv}]^2 \cdot [\upsilon_2^{\dashv}]^2}\right]$

(iii) $\mho \cdot \sqsupset = \left[\sqrt{\imath - [\imath - [\varkappa^{\urcorner}]^2]^{\mho}}, [\upsilon^{\dashv}]^{\mho}\right]$

(iv) $\sqsupset^{\mho} = \left[[\varkappa^{\urcorner}]^{\mho}, \sqrt{\imath - [\imath - [\upsilon^{\dashv}]^2]^{\mho}}\right]$

Definition 2.4. If $\sqsupset_1 = \langle \varkappa_1^{\urcorner}, \upsilon_1^{\dashv} \rangle$ and $\sqsupset_2 = \langle \varkappa_2^{\urcorner}, \upsilon_2^{\dashv} \rangle$ are any two PFNs. Then the interaction AO is defined as

(i) $$\sqsupset_1 \curlyvee \sqsupset_2 = \begin{bmatrix} \sqrt{[\varkappa_1^{\urcorner}]^2 + [\varkappa_2^{\urcorner}]^2 - [\varkappa_1^{\urcorner}]^2 \cdot [\varkappa_2^{\urcorner}]^2}, \\ \sqrt{[\upsilon_1^{\dashv}]^2 + [\upsilon_2^{\dashv}]^2 - [\upsilon_1^{\dashv}]^2 \cdot [\upsilon_2^{\dashv}]^2 - [\upsilon_1^{\dashv}]^2 \cdot [\varkappa_2^{\urcorner}]^2 - [\varkappa_1^{\urcorner}]^2 \cdot [\upsilon_2^{\dashv}]^2} \end{bmatrix}$$

(ii) $$\sqsupset_1 \curlywedge \sqsupset_2 = \begin{bmatrix} \sqrt{[\varkappa_1^{\urcorner}]^2 + [\varkappa_2^{\urcorner}]^2 - [\varkappa_1^{\urcorner}]^2 \cdot [\varkappa_2^{\urcorner}]^2 - [\varkappa_1^{\urcorner}]^2 \cdot [\upsilon_2^{\dashv}]^2 - [\upsilon_1^{\dashv}]^2 \cdot [\varkappa_2^{\urcorner}]^2}, \\ \sqrt{[\upsilon_1^{\dashv}]^2 + [\upsilon_2^{\dashv}]^2 - [\upsilon_1^{\dashv}]^2 \cdot [\upsilon_2^{\dashv}]^2} \end{bmatrix}$$

(iii) $\mho \cdot \sqsupset_1 = \left[\sqrt{\imath - [\imath - [\varkappa_1^{\urcorner}]^2]^{\mho}}, \sqrt{[\imath - [\varkappa_1^{\urcorner}]^2]^{\mho} - [\imath - [\varkappa_1^{\urcorner} + \upsilon_1^{\dashv}]^2]^{\mho}}\right]$

(iv) $\sqsupset_1^{\mho} = \left[\sqrt{[\imath - [\upsilon_1^{\dashv}]^2]^{\mho} - [\imath - [\varkappa_1^{\urcorner} + \upsilon_1^{\dashv}]^2]^{\mho}}, \sqrt{\imath - [\imath - [\upsilon_1^{\dashv}]^2]^{\mho}}\right]$

where $\mho$ be a positive integers.

3 Different AOs for TSFN

Throughout this papar $\sin \pi/2 = \measuredangle$.

Definition 3.1. Suppose that $\sqsupset_1 = \langle \varkappa_1^{\urcorner}, \backepsilon_1^{\dashv}, \upsilon_1^{\dashv} \rangle$ and $\sqsupset_2 = \langle \varkappa_2^{\urcorner}, \backepsilon_2^{\dashv}, \upsilon_2^{\dashv} \rangle$ be the any two TSFNs. Then

(i) $$\sqsupset_1 \curlyvee \sqsupset_2 = \begin{bmatrix} \sqrt{[\measuredangle\varkappa_1^{\urcorner}]^2 + [\measuredangle\varkappa_2^{\urcorner}]^2 - [\measuredangle\varkappa_1^{\urcorner}]^2 \cdot [\measuredangle\varkappa_2^{\urcorner}]^2}, \\ \sqrt{[\measuredangle\, \backepsilon_1^{\dashv}]^2 + [\measuredangle\, \backepsilon_2^{\dashv}]^2 - [\measuredangle\, \backepsilon_1^{\dashv}]^2 \cdot [\measuredangle\, \backepsilon_2^{\dashv}]^2}, \\ \sqrt{\begin{array}{c} [\measuredangle\upsilon_1^{\dashv}]^2 + [\measuredangle\upsilon_2^{\dashv}]^2 - [\measuredangle\upsilon_1^{\dashv}]^2 \cdot [\measuredangle\upsilon_2^{\dashv}]^2 \\ -[\measuredangle\upsilon_1^{\dashv}]^2 \cdot [\measuredangle\varkappa_2^{\urcorner}]^2 - [\measuredangle\varkappa_1^{\urcorner}]^2 \cdot [\measuredangle\upsilon_2^{\dashv}]^2 \end{array}} \end{bmatrix}$$

(ii) $$\sqsupset_1 \curlywedge \sqsupset_2 = \begin{bmatrix} \sqrt{\begin{array}{c} [\measuredangle\varkappa_1^{\urcorner}]^2 + [\measuredangle\varkappa_2^{\urcorner}]^2 - [\measuredangle\varkappa_1^{\urcorner}]^2 \cdot [\measuredangle\varkappa_2^{\urcorner}]^2 \\ -[\measuredangle\varkappa_1^{\urcorner}]^2 \cdot [\measuredangle\upsilon_2^{\dashv}]^2 - [\measuredangle\upsilon_1^{\dashv}]^2 \cdot [\measuredangle\varkappa_2^{\urcorner}]^2 \end{array}}, \\ \sqrt{[\measuredangle\, \backepsilon_1^{\dashv}]^2 + [\measuredangle\, \backepsilon_2^{\dashv}]^2 - [\measuredangle\, \backepsilon_1^{\dashv}]^2 \cdot [\measuredangle\, \backepsilon_2^{\dashv}]^2} \\ \sqrt{[\measuredangle\upsilon_1^{\dashv}]^2 + [\measuredangle\upsilon_2^{\dashv}]^2 - [\measuredangle\upsilon_1^{\dashv}]^2 \cdot [\measuredangle\upsilon_2^{\dashv}]^2} \end{bmatrix}$$

(iii) $$\mho \cdot \sqsupset_1 = \begin{bmatrix} \sqrt{\imath - [\imath - [\measuredangle\varkappa_1^{\urcorner}]^2]^{\mho}}, \sqrt{\imath - [\imath - [\measuredangle\, \backepsilon_1^{\dashv}]^2]^{\mho}}, \\ \sqrt{[\imath - [\measuredangle\varkappa_1^{\urcorner}]^2]^{\mho} - [\imath - [\measuredangle\varkappa_1^{\urcorner} + \measuredangle\upsilon_1^{\dashv}]^2]^{\mho}} \end{bmatrix}$$

(iv) $$\sqsupset_1^{\mho} = \begin{bmatrix} \sqrt{[\imath - [\measuredangle\upsilon_1^{\dashv}]^2]^{\mho} - [\imath - [\measuredangle\varkappa_1^{\urcorner} + \measuredangle\upsilon_1^{\dashv}]^2]^{\mho}} \\ \sqrt{\imath - [\imath - [\measuredangle\, \backepsilon_1^{\dashv}]^2]^{\mho}}, \sqrt{\imath - [\imath - [\measuredangle\upsilon_1^{\dashv}]^2]^{\mho}} \end{bmatrix}$$

3.1 TSFIWA operator

Definition 3.2. Let $\sqsupset_{\propto} = \left\langle \varkappa_{\propto}^{\daleth}, \backepsilon_{\propto}^{\beth}, \upsilon_{\propto}^{\dashv} \right\rangle$ be the TSFNs, $\varsigma_{\propto}$ be the weight of $\sqsupset_{\propto}$ and $\varsigma_{\propto} \geq 0$, $\biguplus_{\propto=1}^{\ell} \varsigma_{\propto} = 1$. Then the TSFIWA operator $[\sqsupset_1, \sqsupset_2, ..., \sqsupset_{\ell}] = \biguplus_{\propto=1}^{\ell} \varsigma_{\propto} \sqsupset_{\propto}$.

Theorem 3.3. *Let* $\sqsupset_{\propto} = \left\langle \varkappa_{\propto}^{\daleth}, \backepsilon_{\propto}^{\beth}, \upsilon_{\propto}^{\dashv} \right\rangle$ *be the TSFNs, .*

Then, TSFIWA$[\sqsupset_1, \sqsupset_2, ..., \sqsupset_{\ell}] = \left[\begin{array}{c} \sqrt{\shortmid - \Diamond_{\propto=1}^{\ell}\left[\shortmid - [\measuredangle \varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}}, \sqrt{\shortmid - \Diamond_{\propto=1}^{\ell}\left[\shortmid - [\measuredangle \backepsilon_{\propto}^{\beth}]^2\right]^{\varsigma_{\propto}}}, \\ \sqrt{\Diamond_{\propto=1}^{\ell}\left[\shortmid - [\measuredangle \varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}} - \Diamond_{\propto=1}^{\ell}\left[\shortmid - [\measuredangle \varkappa_{\propto}^{\daleth} + \measuredangle \upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}} \end{array}\right].$

Proof. If $\propto = 2$, TSFIWA$[\sqsupset_1, \sqsupset_2] = \varsigma_1 \sqsupset_1 \curlyvee \varsigma_2 \sqsupset_2$,
where,

$$\varsigma_1 \sqsupset_1 = \left[\begin{array}{c} \sqrt{\shortmid - \left[\shortmid - [\measuredangle \varkappa_1^{\daleth}]^2\right]^{\varsigma_1}}, \sqrt{\shortmid - \left[\shortmid - [\measuredangle \backepsilon_1^{\beth}]^2\right]^{\varsigma_1}}, \\ \sqrt{\left[\shortmid - [\measuredangle \varkappa_1^{\daleth}]^2\right]^{\varsigma_1} - \left[\shortmid - [\measuredangle \varkappa_1^{\daleth} + \measuredangle \upsilon_1^{\dashv}]^2\right]^{\varsigma_1}} \end{array}\right]$$

and

$$\varsigma_2 \sqsupset_2 = \left[\begin{array}{c} \sqrt{\shortmid - \left[\shortmid - [\measuredangle \varkappa_2^{\daleth}]^2\right]^{\varsigma_2}}, \sqrt{\shortmid - \left[\shortmid - [\measuredangle \backepsilon_2^{\beth}]^2\right]^{\varsigma_2}}, \\ \sqrt{\left[\shortmid - [\measuredangle \varkappa_2^{\daleth}]^2\right]^{\varsigma_2} - \left[\shortmid - [\measuredangle \varkappa_2^{\daleth} + \measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2}} \end{array}\right]$$

We get

$$\begin{aligned} \varsigma_1 \sqsupset_1 \curlyvee \varsigma_2 \sqsupset_2 &= \left[\begin{array}{c} \sqrt{\begin{array}{c} \left[\shortmid - [\shortmid - [\measuredangle \varkappa_1^{\daleth}]^2]^{\varsigma_1}\right] + \left[\shortmid - [\shortmid - [\measuredangle \varkappa_2^{\daleth}]^2]^{\varsigma_2}\right] \\ - \left[\shortmid - [\shortmid - [\measuredangle \varkappa_1^{\daleth}]^2]^{\varsigma_1}\right] \cdot \left[\shortmid - [\shortmid - [\measuredangle \varkappa_2^{\daleth}]^2]^{\varsigma_2}\right] \end{array}}, \\ \sqrt{\begin{array}{c} \left[\shortmid - [\shortmid - [\measuredangle \backepsilon_1^{\beth}]^2]^{\varsigma_1}\right] + \left[\shortmid - [\shortmid - [\measuredangle \backepsilon_2^{\beth}]^2]^{\varsigma_2}\right] \\ - \left[\shortmid - [\shortmid - [\measuredangle \backepsilon_1^{\beth}]^2]^{\varsigma_1}\right] \cdot \left[\shortmid - [\shortmid - [\measuredangle \backepsilon_2^{\beth}]^2]^{\varsigma_2}\right] \end{array}}, \\ \sqrt{\begin{array}{c} \left[\shortmid - [\shortmid - \measuredangle \upsilon_1^{\dashv}]^2]^{\varsigma_1}\right] + \left[\shortmid - [\shortmid - \measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2}\right] \\ - \left[\shortmid - [\shortmid - \measuredangle \upsilon_1^{\dashv}]^2]^{\varsigma_1}\right] \cdot \left[\shortmid - [\shortmid - \measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2}\right] \\ - [\shortmid - [\measuredangle \varkappa_1^{\daleth} + \measuredangle \upsilon_1^{\dashv}]^2]^{\varsigma_1} \cdot [\shortmid - [\measuredangle \varkappa_2^{\daleth} + \measuredangle \upsilon_2^{\dashv}]^2]^{\varsigma_2} \end{array}} \end{array}\right] \\ &= \left[\begin{array}{c} \sqrt{\shortmid - \Diamond_{\propto=1}^{2}\left[\shortmid - [\measuredangle \varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}}, \sqrt{\shortmid - \Diamond_{\propto=1}^{2}\left[\shortmid - [\measuredangle \backepsilon_{\propto}^{\beth}]^2\right]^{\varsigma_{\propto}}}, \\ \sqrt{\Diamond_{\propto=1}^{2}\left[\shortmid - [\measuredangle \varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}} - \Diamond_{\propto=1}^{2}\left[\shortmid - [\measuredangle \varkappa_{\propto}^{\daleth} + \measuredangle \upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}} \end{array}\right]. \end{aligned}$$

Using induction $\propto \geq 3$, TSFIWA$[\sqsupset_1, \sqsupset_2, ..., \sqsupset_{\ell}]$

$$= \left[\begin{array}{c} \sqrt{\shortmid - \Diamond_{\propto=1}^{\ell}\left[\shortmid - [\measuredangle \varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}}, \sqrt{\shortmid - \Diamond_{\propto=1}^{\ell}\left[\shortmid - [\measuredangle \backepsilon_{\propto}^{\beth}]^2\right]^{\varsigma_{\propto}}}, \\ \sqrt{\Diamond_{\propto=1}^{\ell}\left[\shortmid - [\measuredangle \varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}} - \Diamond_{\propto=1}^{\ell}\left[\shortmid - [\measuredangle \varkappa_{\propto}^{\daleth} + \measuredangle \upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}}, \end{array}\right].$$

If $\propto = \ell+1$, then $TSFIWA[\sqsupset_1, \sqsupset_2, ..., \sqsupset_\ell, \sqsupset_{\ell+1}]$

$$= \left[\begin{array}{c} \sqrt{\begin{array}{c}\biguplus_{\propto=1}^{\ell}\left[\imath-\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^{\varsigma_\propto}\right]+\left[\imath-\left[\imath-[\measuredangle\varkappa_{\ell+1}^{\urcorner}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[\imath-\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^{\varsigma_\propto}\right]\cdot\left[\imath-\left[\imath-[\measuredangle\varkappa_{\ell+1}^{\urcorner}]^2\right]^{\varsigma_{\ell+1}}\right]\end{array}}, \\ \sqrt{\begin{array}{c}\biguplus_{\propto=1}^{\ell}\left[\imath-\left[\imath-[\measuredangle \backepsilon_{\propto}^{\lrcorner}]^2\right]^{\varsigma_\propto}\right]+\left[\imath-\left[\imath-[\measuredangle \backepsilon_{\ell+1}^{\lrcorner}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[\imath-\left[\imath-[\measuredangle \backepsilon_{\propto}^{\lrcorner}]^2\right]^{\varsigma_\propto}\right]\cdot\left[\imath-\left[\imath-[\measuredangle \backepsilon_{\ell+1}^{\lrcorner}]^2\right]^{\varsigma_{\ell+1}}\right]\end{array}}, \\ \sqrt{\begin{array}{c}\biguplus_{\propto=1}^{\ell}\left[\imath-\left[\imath-\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\right]+\left[\imath-\left[\imath-\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[\imath-\left[\imath-\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\right]\cdot\left[\imath-\left[\imath-\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\cdot\left[\imath-[\measuredangle\varkappa_{\ell+1}^{\urcorner}+\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\end{array}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^{\varsigma_\propto}\cdot\left[\imath-[\measuredangle\varkappa_{\ell+1}^{\urcorner}]^2\right]^{\varsigma_{\ell+1}}}, \\ \sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle \backepsilon_{\propto}^{\lrcorner}]^2\right]^{\varsigma_\propto}\cdot\left[\imath-[\measuredangle \backepsilon_{\ell+1}^{\lrcorner}]^2\right]^{\varsigma_{\ell+1}}}, \\ \sqrt{\begin{array}{c}\left[\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\right]\cdot \\ \left[\left[[\measuredangle\varkappa_{\ell+1}^{\urcorner}]^2\right]^{\varsigma_{\ell+1}}-[\measuredangle\varkappa_{\ell+1}^{\urcorner}+\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\right]\end{array}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^{\varsigma_\propto}}, \sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-[\measuredangle \backepsilon_{\propto}^{\lrcorner}]^2\right]^{\varsigma_\propto}}, \\ \sqrt{\Diamond_{\propto=1}^{\ell+1}\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell+1}\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}} \end{array}\right]$$

Theorem 3.4. *If* $\sqsupset_\propto = \langle \varkappa_{\propto}^{\urcorner}, \backepsilon_{\propto}^{\lrcorner}, \upsilon_{\propto}^{\dashv}\rangle$ *be the TSFNs and* $\sqsupset_\propto = \sqsupset$ *and* $\varkappa^{\urcorner}\cdot\upsilon^{\dashv}=0$*, then the TSFIWA*$[\sqsupset_1, \sqsupset_2, ..., \sqsupset_\ell] = \sqsupset$, .

Proof. Note that, $[\measuredangle\varkappa_{\propto}^{\urcorner}, \backepsilon_{\propto}^{\lrcorner}, \measuredangle\upsilon_{\propto}^{\dashv}] = [\measuredangle\varkappa^{\urcorner}, \backepsilon^{\lrcorner}, \measuredangle\upsilon^{\dashv}]$, and $\biguplus_{\propto=1}^{\ell}\varsigma_\propto = 1$.
We get, $TSFIWA[\sqsupset_1, \sqsupset_2, ..., \sqsupset_\ell]$

$$= \left[\begin{array}{c} \sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\varkappa^{\urcorner}]^2\right]^{\varsigma_\propto}}, \sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle \backepsilon^{\lrcorner}]^2\right]^{\varsigma_\propto}}, \\ \sqrt{\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\varkappa^{\urcorner}]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\varkappa^{\urcorner}+\measuredangle\upsilon^{\dashv}]^2\right]^{\varsigma_\propto}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{\imath-\left[\imath-[\measuredangle\varkappa^{\urcorner}]^2\right]^{\biguplus_{\propto=1}^{\ell}\varsigma_\propto}}, \sqrt{\imath-\left[\imath-[\measuredangle \backepsilon^{\lrcorner}]^2\right]^{\biguplus_{\propto=1}^{\ell}\varsigma_\propto}}, \\ \sqrt{\left[\imath-[\measuredangle\varkappa^{\urcorner}]^2\right]^{\biguplus_{\propto=1}^{\ell}\varsigma_\propto}-\left[\imath-[\measuredangle\varkappa^{\urcorner}+\measuredangle\upsilon^{\dashv}]^2\right]^{\biguplus_{\propto=1}^{\ell}\varsigma_\propto}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{\imath-\left[\imath-[\measuredangle\varkappa^{\urcorner}]^2\right]}, \sqrt{\imath-\left[\imath-[\measuredangle \backepsilon^{\lrcorner}]^2\right]}, \\ \sqrt{\left[\imath-[\measuredangle\varkappa^{\urcorner}]^2\right]-\left[\imath-[\measuredangle\varkappa^{\urcorner}+\measuredangle\upsilon^{\dashv}]^2\right]} \end{array}\right]$$

$$= \left[\measuredangle\varkappa^{\urcorner}, \backepsilon^{\lrcorner}, \measuredangle\upsilon^{\dashv}\right] = \sqsupset$$

3.2 Interaction weighted geometric[TSFIWG] operator

Definition 3.5. Let $\beth_\propto = \left\langle \varkappa_\propto^{\urcorner}, \backepsilon_\propto^{\lrcorner}, \upsilon_\propto^{\dashv} \right\rangle$ be the TSFNs, $\varsigma_\propto$ be the weight of $\beth_\propto$. Then the TS-FIWG operator $[\beth_1, \beth_2, ..., \beth_\ell] = \diamond_{\propto=1}^{\ell} \beth_\propto^{\varsigma_\propto}$.

Theorem 3.6. *If* $\beth_\propto = \left\langle \varkappa_\propto^{\urcorner}, \backepsilon_\propto^{\lrcorner}, \upsilon_\propto^{\dashv} \right\rangle$ *be the TSFNs. Then,*

$$TSFIWG[\beth_1, \beth_2, ..., \beth_\ell] = \left[\begin{array}{c} \sqrt{\diamond_{\propto=1}^{\ell}\left[\imath - [\measuredangle \upsilon_\propto^{\dashv}]^2\right]^{\varsigma_\propto} - \diamond_{\propto=1}^{\ell}\left[\imath - [\measuredangle \varkappa_\propto^{\urcorner} + \measuredangle \upsilon_\propto^{\dashv}]^2\right]^{\varsigma_\propto}}, \\ \sqrt{\imath - \diamond_{\propto=1}^{\ell}\left[\imath - [\measuredangle \backepsilon_\propto^{\lrcorner}]^2\right]^{\varsigma_\propto}}, \sqrt{\imath - \diamond_{\propto=1}^{\ell}\left[\imath - [\measuredangle \upsilon_\propto^{\dashv}]^2\right]^{\varsigma_\propto}} \end{array} \right]$$

Proof. If $\propto = 2$, then TSFIWG$[\beth_1, \beth_2] = \beth_1^{\varsigma_1} \curlywedge \beth_2^{\varsigma_2}$,
where,

$$\beth_1^{\varsigma_1} = \left[\begin{array}{c} \sqrt{\left[\imath - [\measuredangle \upsilon_1^{\dashv}]^2\right]^{\varsigma_1} - \left[\imath - [\measuredangle \varkappa_1^{\urcorner} + \measuredangle \upsilon_1^{\dashv}]^2\right]^{\varsigma_1}}, \\ \sqrt{\imath - \left[\imath - [\measuredangle \backepsilon_1^{\lrcorner}]^2\right]^{\varsigma_1}}, \sqrt{\imath - \left[\imath - [\measuredangle \upsilon_1^{\dashv}]^2\right]^{\varsigma_1}} \end{array} \right]$$

$$\beth_2^{\varsigma_2} = \left[\begin{array}{c} \sqrt{\left[\imath - [\measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2} - \left[\imath - [\measuredangle \varkappa_2^{\urcorner} + \measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2}}, \\ \sqrt{\imath - \left[\imath - [\measuredangle \backepsilon_2^{\lrcorner}]^2\right]^{\varsigma_2}} \quad \sqrt{\imath - \left[\imath - [\measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2}} \end{array} \right]$$

We get,

$$\beth_1^{\varsigma_1} \curlywedge \beth_2^{\varsigma_2} = \left[\begin{array}{c} \sqrt{\begin{array}{r} \left[\imath - \left[\imath - [\measuredangle \upsilon_1^{\dashv}]^2\right]^{\varsigma_1}\right] + \left[\imath - \left[\imath - [\measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2}\right] \\ - \left[\imath - \left[\imath - [\measuredangle \upsilon_1^{\dashv}]^2\right]^{\varsigma_1}\right] \cdot \left[\imath - \left[\imath - [\measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2}\right] \\ - \left[\left[\imath - [\measuredangle \varkappa_1^{\urcorner} + \measuredangle \upsilon_1^{\dashv}]^2\right]^{\varsigma_1} \cdot \left[\imath - [\measuredangle \varkappa_2^{\urcorner} + \measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2}\right] \end{array}} \\ \sqrt{\begin{array}{r} \left[\imath - \left[\imath - [\measuredangle \backepsilon_1^{\lrcorner}]^2\right]^{\varsigma_1}\right] + \left[\imath - \left[\imath - [\measuredangle \backepsilon_2^{\lrcorner}]^2\right]^{\varsigma_2}\right] \\ - \left[\imath - \left[\imath - [\measuredangle \backepsilon_1^{\lrcorner}]^2\right]^{\varsigma_1}\right] \cdot \left[\imath - \left[\imath - [\measuredangle \backepsilon_2^{\lrcorner}]^2\right]^{\varsigma_2}\right] \end{array}} \\ \sqrt{\begin{array}{r} \left[\imath - \left[\imath - [\measuredangle \upsilon_1^{\dashv}]^2\right]^{\varsigma_1}\right] + \left[\imath - \left[\imath - [\measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2}\right] \\ - \left[\imath - \left[\imath - [\measuredangle \upsilon_1^{\dashv}]^2\right]^{\varsigma_1}\right] \cdot \left[\imath - \left[\imath - [\measuredangle \upsilon_2^{\dashv}]^2\right]^{\varsigma_2}\right] \end{array}} \end{array} \right]$$

$$\text{Hence, TSFIWG}[\beth_1, \beth_2] = \left[\begin{array}{c} \sqrt{\diamond_{\propto=1}^{2}\left[\imath - [\measuredangle \upsilon_\propto^{\dashv}]^2\right]^{\varsigma_\propto} - \diamond_{\propto=1}^{2}\left[\imath - [\measuredangle \varkappa_\propto^{\urcorner} + \measuredangle \upsilon_\propto^{\dashv}]^2\right]^{\varsigma_\propto}}, \\ \sqrt{\imath - \diamond_{\propto=1}^{2}\left[\imath - [\measuredangle \backepsilon_\propto^{\lrcorner}]^2\right]^{\varsigma_\propto}}, \sqrt{\imath - \diamond_{\propto=1}^{2}\left[\imath - [\measuredangle \upsilon_\propto^{\dashv}]^2\right]^{\varsigma_\propto}} \end{array} \right]$$

$$\text{TSFIWG}[\beth_1, \beth_2, ..., \beth_\ell] = \left[\begin{array}{c} \sqrt{\diamond_{\propto=1}^{\ell}\left[\imath - [\measuredangle \upsilon_\propto^{\dashv}]^2\right]^{\varsigma_\propto} - \diamond_{\propto=1}^{\ell}\left[\imath - [\measuredangle \varkappa_\propto^{\urcorner} + \measuredangle \upsilon_\propto^{\dashv}]^2\right]^{\varsigma_\propto}}, \\ \sqrt{\imath - \diamond_{\propto=1}^{\ell}\left[\imath - [\measuredangle \backepsilon_\propto^{\lrcorner}]^2\right]^{\varsigma_\propto}}, \sqrt{\imath - \diamond_{\propto=1}^{\ell}\left[\imath - [\measuredangle \upsilon_\propto^{\dashv}]^2\right]^{\varsigma_\propto}} \end{array} \right]$$

If $\propto = \ell+1$, then TSFIWG$[\sqsupset_1, ..., \sqsupset_\ell, \sqsupset_{\ell+1}]$

$$= \left[\begin{array}{c} \sqrt{\begin{array}{c}\biguplus_{\propto=1}^{\ell}\left[\imath-\left[\imath-[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\right]+\left[\imath-\left[\imath-[\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[\imath-\left[\imath-[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\right]\cdot\left[\imath-\left[\imath-[\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\cdot\left[\imath-[\measuredangle\varkappa_{\ell+1}^{\urcorner}+\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\end{array}} \\ \sqrt{\begin{array}{c}\biguplus_{\propto=1}^{\ell}\left[\imath-\left[\imath-[\measuredangle\backepsilon_{\propto}^{\daleth}]^2\right]^{\varsigma_\propto}\right]+\left[\imath-\left[\imath-[\measuredangle\backepsilon_{\ell+1}^{\daleth}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[\imath-\left[\imath-[\measuredangle\backepsilon_{\propto}^{\daleth}]^2\right]^{\varsigma_\propto}\right]\cdot\left[\imath-\left[\imath-[\measuredangle\backepsilon_{\ell+1}^{\daleth}]^2\right]^{\varsigma_{\ell+1}}\right]\end{array}} \\ \sqrt{\begin{array}{c}\biguplus_{\propto=1}^{\ell}\left[\imath-\left[\imath-[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\right]+\left[\imath-\left[\imath-[\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[\imath-\left[\imath-[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\right]\cdot\left[\imath-\left[\imath-[\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\right]\end{array}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{\begin{array}{c}\left[\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\right]\cdot \\ \left[\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}-[\measuredangle\varkappa_{\ell+1}^{\urcorner}+\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\end{array}}, \\ \sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\backepsilon_{\propto}^{\daleth}]^2\right]^{\varsigma_\propto}\cdot\left[\imath-[\measuredangle\backepsilon_{\ell+1}^{\daleth}]^2\right]^{\varsigma_{\ell+1}}} \\ \sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}\cdot\left[\imath-[\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{\Diamond_{\propto=1}^{\ell+1}\left[\imath-[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell+1}\left[\imath-[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}}, \\ \sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-[\measuredangle\backepsilon_{\propto}^{\daleth}]^2\right]^{\varsigma_\propto}}\sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_\propto}} \end{array}\right]$$

Corollary 3.7. *Let* $\sqsupset_\propto = \langle \varkappa_\propto^{\urcorner}, \backepsilon_\propto^{\daleth}, \upsilon_\propto^{\dashv}\rangle$ *be the TSFNs and all are equal and and* $\varkappa^{\urcorner}\cdot\upsilon^{\dashv} = 0$. *Then TSFIWG*$[\sqsupset_1, \sqsupset_2, ..., \sqsupset_\ell] = \sqsupset$.

3.3 generalized TSFIWA [GTSFIWA] operator

Definition 3.8. Let $\sqsupset_\propto = \langle \varkappa_\propto^{\urcorner}, \varkappa_\propto^{\daleth}, \upsilon_\propto^{\dashv}\rangle$ be the TSFNs, $\varsigma_\propto$ be a weight of $\sqsupset_\propto$. Then, the GTSFIWA operator $[\sqsupset_1, \sqsupset_2, ..., \sqsupset_\ell] = \left[\biguplus_{\propto=1}^{\ell}\varsigma_\propto\sqsupset_\propto^2\right]^{1/2}$.

Theorem 3.9. *Let* $\sqsupset_\propto = \langle \varkappa_\propto^{\urcorner}, \varkappa_\propto^{\daleth}, \upsilon_\propto^{\dashv}\rangle$ *be the TSFNs. Then GTSFIWA* $[\sqsupset_1, \sqsupset_2, ..., \sqsupset_\ell] =$

$$\left[\begin{array}{c}\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_\propto^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}}\right],\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\backepsilon_\propto^{\daleth}]^2\right]^2\right]^{\varsigma_\propto}}\right], \\ \left[\sqrt{\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_\propto^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_\propto^{\urcorner}+\measuredangle\upsilon_\propto^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}}\right]\end{array}\right].$$

Proof. First, we have find that

$$\biguplus_{\propto=1}^{\ell}\varsigma_\propto\sqsupset_\propto^2 = \left[\begin{array}{c}\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_\propto^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}},\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\backepsilon_\propto^{\daleth}]^2\right]^2\right]^{\varsigma_\propto}}, \\ \sqrt{\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_\propto^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_\propto^{\urcorner}+\measuredangle\upsilon_\propto^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}}\end{array}\right].$$

If $\propto = 2$, then $\varsigma_1\sqsupset_1^2 = \left[\begin{array}{c}\sqrt{\imath-\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}, \sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_1^{\daleth}]^2\right]^2\right]^{\varsigma_1}}, \\ \sqrt{\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_1}-\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}+\measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\end{array}\right]$

and

$$\varsigma_2\sqsupset_2^2 = \left[\begin{array}{c}\sqrt{\imath-\left[\imath-\left[[\measuredangle\varkappa_2^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}, \sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_2^{\daleth}]^2\right]^2\right]^{\varsigma_1}}, \\ \sqrt{\left[\imath-\left[[\measuredangle\varkappa_2^{\urcorner}]^2\right]^2\right]^{\varsigma_1}-\left[\imath-\left[[\measuredangle\varkappa_2^{\urcorner}+\measuredangle\upsilon_2^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\end{array}\right].$$

We get, $\varsigma_1\sqsupset_1 \curlyvee \varsigma_2\sqsupset_2=$

$$\left[\begin{array}{c}
\sqrt{\begin{array}{c}\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2+\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\varkappa_2^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2, \\ -\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\varkappa_2^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2\end{array}} \\
\sqrt{\begin{array}{c}\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_1^{\daleth}]^2\right]^2\right]^{\varsigma_1}}\right]^2+\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_2^{\daleth}]^2\right]^2\right]^{\varsigma_1}}\right]^2, \\ -\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_1^{\daleth}]^2\right]^2\right]^{\varsigma_1}}\right]^2\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_2^{\daleth}]^2\right]^2\right]^{\varsigma_1}}\right]^2\end{array}} \\
\sqrt{\begin{array}{c}\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2+\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_2^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2, \\ -\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_2^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \\ -\left[\begin{array}{c}\left[\sqrt{\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_1}-\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}+\measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2\cdot \\ \left[\sqrt{\left[\imath-\left[[\measuredangle\varkappa_2^{\urcorner}]^2\right]^2\right]^{\varsigma_1}-\left[\imath-\left[[\measuredangle\varkappa_2^{\urcorner}+\measuredangle\upsilon_2^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2\end{array}\right]\end{array}}
\end{array}\right]$$

$$= \left[\begin{array}{c}\sqrt{\imath-\Diamond_{\propto=1}^{2}\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}}, \sqrt{\imath-\Diamond_{\propto=1}^{2}\left[\imath-\left[[\measuredangle\,\backepsilon_1^{\daleth}]^2\right]^2\right]^{\varsigma_\propto}}, \\ \sqrt{\Diamond_{\propto=1}^{2}\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{2}\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}+\measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}}\end{array}\right]$$

In general,

$$= \left[\begin{array}{c}\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}}, \sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\,\backepsilon_1^{\daleth}]^2\right]^2\right]^{\varsigma_\propto}} \\ \sqrt{\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_1^{\urcorner}+\measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}}\end{array}\right].$$

If $\propto = \ell+1$, then $\biguplus_{\propto=1}^{\ell}\varsigma_\propto\sqsupset_\propto^2+\varsigma_{\ell+1}\sqsupset_{\ell+1}^2=\biguplus_{\propto=1}^{\ell+1}\varsigma_\propto\sqsupset_\propto^2$.

Now, $\biguplus_{\propto=1}^{\ell}\varsigma_\propto\sqsupset_\propto^2+\varsigma_{\ell+1}\sqsupset_{\ell+1}^2=\varsigma_1\sqsupset_1^2\curlyvee\varsigma_2\sqsupset_2^2\curlyvee\ldots\curlyvee\varsigma_\ell\sqsupset_\ell^2\curlyvee\varsigma_{\ell+1}\sqsupset_{\ell+1}^2$

$$
= \left[\begin{array}{c}
\sqrt{\dfrac{\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}}\right]^2+\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\varkappa_{\ell+1}^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2, \\ -\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}}\right]^2\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\varkappa_{\ell+1}^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2}{}} \\
\sqrt{\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\backepsilon_{\propto}^{\lrcorner}]^2\right]^2\right]^{\varsigma_\propto}}\right]^2+\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\backepsilon_{\ell+1}^{\lrcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2, \\ -\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\backepsilon_{\propto}^{\lrcorner}]^2\right]^2\right]^{\varsigma_\propto}}\right]^2\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\backepsilon_{\ell+1}^{\lrcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2} \\
\sqrt{\begin{array}{c}\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\lrcorner}]^2\right]^2\right]^{\varsigma_\propto}}\right]^2+\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\varkappa_{\ell+1}^{\lrcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2, \\ -\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\lrcorner}]^2\right]^2\right]^{\varsigma_\propto}}\right]^2\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\varkappa_{\ell+1}^{\lrcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \\ -\left[\begin{array}{c}\left[\sqrt{\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}}\right]^2\cdot \\ \left[\sqrt{\left[\imath-\left[[\measuredangle\varkappa_{\ell+1}^{\urcorner}]^2\right]^2\right]^{\varsigma_1}-\left[\imath-\left[[\measuredangle\varkappa_{\ell+1}^{\urcorner}+\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2\end{array}\right]\end{array}}
\end{array} \right]
$$

$$
= \left[\begin{array}{c}
\sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\varkappa_{1}^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}}, \sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\backepsilon_{1}^{\lrcorner}]^2\right]^2\right]^{\varsigma_\propto}}, \\
\sqrt{\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\varkappa_{1}^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\varkappa_{1}^{\urcorner}+\measuredangle\upsilon_{1}^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}}
\end{array} \right]
$$

and $\biguplus_{\propto=1}^{\ell+1}\left[\varsigma_\propto \sqsupset_{\propto}^{2}\right] =$

$$
\left[\begin{array}{c}
\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}}\right], \\
\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\backepsilon_{\propto}^{\lrcorner}]^2\right]^2\right]^{\varsigma_\propto}}\right], \\
\left[\sqrt{\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}}\right]
\end{array} \right].
$$

Corollary 3.10. *Let* $\sqsupset_\propto = \left\langle \varkappa_{\propto}^{\urcorner}, \backepsilon_{\propto}^{\lrcorner}, \upsilon_{\propto}^{\dashv} \right\rangle$ *be the TSFNs and all are equal. Then GTSFIWA* $[\sqsupset_1, \sqsupset_2, ..., \sqsupset_\ell] = \sqsupset$*.*

3.4 Generalized TSFIWG [GTSFIWG] operator

Definition 3.11. Let $\sqsupset_\propto = \left\langle \varkappa_{\propto}^{\urcorner}, \varkappa_{\propto}^{\lrcorner}, \upsilon_{\propto}^{\dashv} \right\rangle$ be the TSFNs, $\varsigma_\propto]$ be the weight of $\sqsupset_\propto$. Then, the GTSFIWG$[\sqsupset_1, \sqsupset_2, ..., \sqsupset_\ell] = \frac{1}{\mho}\left[\Diamond_{\propto=1}^{\ell}[\mho\sqsupset_\propto]^{\varsigma_\propto}\right]$.

Theorem 3.12. *Let* $\sqsupset_\propto = \left\langle \varkappa_{\propto}^{\urcorner}, \varkappa_{\propto}^{\lrcorner}, \upsilon_{\propto}^{\dashv} \right\rangle$ *be the collection of TSFNs.*
Then the GTSFIWG operator $[\sqsupset_1, \sqsupset_2, ..., \sqsupset_\ell] =$

$$
\left[\begin{array}{c}
\left[\sqrt{\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}}\right], \\
\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\backepsilon_{\propto}^{\lrcorner}]^2\right]^2\right]^{\varsigma_\propto}}\right]\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}}\right]
\end{array} \right]
$$

Proof. Using the induction method,

$$\diamond_{\propto=1}^{\ell}[\mho\sqsupset_{\propto}]^{\varsigma_{\propto}} = \left[\frac{\sqrt{\diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_{\propto}} - \diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_{\propto}}}}{\sqrt{\imath-\diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\,\backepsilon_{\propto}^{\lrcorner}]^2\right]^2\right]^{\varsigma_{\propto}}}\sqrt{\imath-\diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_{\propto}}}},\right]$$

If $\propto = 2$, then

$$[\mho\sqsupset_{1}]^{\varsigma_1} = \left[\frac{\sqrt{\left[\imath-\left[[\measuredangle\upsilon_{1}^{\dashv}]^2\right]^2\right]^{\varsigma_1} - \left[\imath-\left[[\measuredangle\varkappa_{1}^{\urcorner}+\measuredangle\upsilon_{1}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}}{\sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_{1}^{\lrcorner}]^2\right]^2\right]^{\varsigma_1}}\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{1}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}}\right]$$

and

$$[\mho\sqsupset_{2}]^{\varsigma_2} = \left[\frac{\sqrt{\left[\imath-\left[[\measuredangle\upsilon_{2}^{\dashv}]^2\right]^2\right]^{\varsigma_1} - \left[\imath-\left[[\measuredangle\varkappa_{2}^{\urcorner}+\measuredangle\upsilon_{2}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}}{\sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_{2}^{\lrcorner}]^2\right]^2\right]^{\varsigma_1}}\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{2}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}}\right]$$

We get, $[\mho\sqsupset_{1}]^{\varsigma_1} \curlywedge [\mho\sqsupset_{2}]^{\varsigma_2}$

$$= \left[\frac{\sqrt{\begin{array}{l}\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{1}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 + \left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{2}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \\ -\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{1}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{2}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \\ -\left[\begin{array}{l}\left[\sqrt{\left[\imath-\left[[\measuredangle\upsilon_{1}^{\dashv}]^2\right]^2\right]^{\varsigma_1}-\left[\imath-\left[[\measuredangle\varkappa_{1}^{\urcorner}+\varkappa_{1}^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2\cdot \\ \left[\sqrt{\left[\imath-\left[[\measuredangle\upsilon_{2}^{\dashv}]^2\right]^2\right]^{\varsigma_1}-\left[\imath-\left[[\measuredangle\varkappa_{2}^{\urcorner}+\measuredangle\upsilon_{2}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2\end{array}\right]\end{array}},}{\sqrt{\begin{array}{l}\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_{1}^{\lrcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2 + \left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_{2}^{\lrcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \\ -\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_{1}^{\lrcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\,\backepsilon_{2}^{\lrcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2\end{array}}\sqrt{\begin{array}{l}\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{1}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 + \left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{2}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \\ -\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{1}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{2}^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2\end{array}}}\right]$$

$$= \left[\frac{\sqrt{\diamond_{\propto=1}^{2}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_{\propto}} - \diamond_{\propto=1}^{2}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_{\propto}}},}{\sqrt{\imath-\diamond_{\propto=1}^{2}\left[\imath-\left[[\measuredangle\,\backepsilon_{\propto}^{\lrcorner}]^2\right]^2\right]^{\varsigma_{\propto}}},\sqrt{\imath-\diamond_{\propto=1}^{2}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^2\right]^{\varsigma_{\propto}}}}\right]$$

If $\propto = \ell$, then

$$= \left[\begin{array}{c}\sqrt{\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\\ \sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\backepsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\end{array}\right]$$

If $\propto = \ell+1$, then $\Diamond_{\propto=1}^{\ell}[\mho\sqsupset_{\propto}]^{\varsigma_{\propto}}\cdot[\mho\sqsupset_{\ell+1}]^{\varsigma_{\ell+1}} = \Diamond_{\propto=1}^{\ell+1}[\mho\sqsupset_{\propto}]^{\varsigma_{\propto}}$.
Now, $\Diamond_{\propto=1}^{\ell}[\mho\sqsupset_{\propto}]^{\varsigma_{\propto}}\cdot[\mho\sqsupset_{\ell+1}]^{\varsigma_{\ell+1}} = [\mho\sqsupset_{1}]^{\varsigma_{1}} \curlywedge [\mho\sqsupset_{2}]^{\varsigma_{2}} \curlywedge \ldots \curlywedge [\mho\sqsupset_{\ell}]^{\varsigma_{\ell}} \curlywedge [\mho\sqsupset_{\ell+1}]^{\varsigma_{\ell+1}}$

$$= \left[\begin{array}{c}\sqrt{\begin{array}{c}\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}+\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\begin{array}{c}\left[\sqrt{\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\\ \left[\sqrt{\left[\imath-\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}-\left[\imath-\left[[\measuredangle\varkappa_{\ell+1}^{\urcorner}+\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\end{array}\right]\end{array}},\\ \sqrt{\begin{array}{c}\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\backepsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}+\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\backepsilon_{\ell+1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\backepsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\backepsilon_{\ell+1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\end{array}}\\ \sqrt{\begin{array}{c}\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}+\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\left[\sqrt{\imath-\left[\imath-\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\end{array}}\end{array}\right]$$

$$= \left[\begin{array}{c}\sqrt{\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\varkappa_{1}^{\urcorner}+\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\\ \sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\backepsilon_{1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\end{array}\right]$$

Hence

$$\frac{1}{\mho}\left[\Diamond_{\propto=1}^{\ell+1}[\mho\sqsupset_{\propto}]^{\varsigma_{\propto}}\right] =$$

$$\left[\begin{array}{c}\left[\sqrt{\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\varkappa_{\propto}^{\urcorner}+\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right],\\ \left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\backepsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]\left[\sqrt{\imath-\Diamond_{\propto=1}^{\ell+1}\left[\imath-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]\end{array}\right]$$

Corollary 3.13. *Let $\sqsupset_{\propto} = \left\langle \varkappa_{\propto}^{\urcorner}, \backepsilon_{\propto}^{\gimel}, \upsilon_{\propto}^{\dashv}\right\rangle$ be the collection of TSFNs and all are equal. Then the GTSFIWG $[\sqsupset_{1}, \sqsupset_{2}, ..., \sqsupset_{\ell}] = \sqsupset$.*

4 Conclusion:

An important benefit of ED and HD for TSFNs is their algebraic accessibility. Data analysis may be greatly enhanced by HD of TSFNs. The benefits of HD are demonstrated through the use of powerful statistics. We presented proposed models and examples of TSFIWA, TSFIWG, GTS-FIWA, and GTSFIWG. The following topics will be further discussed in the future: There will be a more thorough discussion of the following subjects: (1) There is a connection between the cubic NS and IVPFS via interaction AOs. (2) Complex TSFIWA, complex TSFIWG, complex GTSFIWA, and complex GTSFIWG can all be used to fix the problem.

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Author information

M. Prakash, Research Scholar, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai, Tamilnadu, India.
E-mail: mprakashmagi@gmail.com

S.V. Manemaran, Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai, Tamilnadu, India.
E-mail: svmanemaran@gmail.com

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