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NONLINEAR IMPULSIVE DYNAMIC SINGULAR STURM-LIOUVILLE PROBLEMS

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Abstract In this study, some existence and uniqueness theorems are obtained by considering the nonlinear impulsive dynamic singular Sturm–Liouville problem in Weyl's limit-circle case.

1 Introduction

The Sturm–Liouville theory has plagued mathematicians for nearly 200 years. Especially while solving partial differential equations with the Fourier method, the emergence of such equations has increased the importance of the subject. Equations of this type have been studied by researchers in many species and under various boundary conditions. One of these boundary conditions is the impulsive boundary condition. Such equations are studied intensively under impulsive boundary conditions. The emergence of equations containing such boundary conditions in some physical problems has made the subject interesting (see [1, 4, 5, 13, 14, 15, 18, 17, 7, 19, 20]).

With the help of the time scale concept, which was introduced to the literature by Hilger [10] in the 1990s, differential equations and difference equations wanted to be examined under a single structure. In this case, it has become necessary to investigate all the cases considered in the classical derivative case on the time scale (see [6]). Equations with impulsive boundary conditions have started to be investigated on the time scale (see [2, 8, 21, 23, 24, 11, 22, 12]). Recently, in [2], the authors studied some spectral properties of impulsive dynamic Sturm–Liouville problems. Some existence theorems for second-order impulsive time scale boundary value problems are given in [11, 12, 21, 22, 23, 24]. In [3], existence and uniqueness of solutions of non-linear impulsive dynamic Sturm–Liouville problems on the whole axis were investigated. In this study, the nonlinear singular Sturm–Liouville problem is discussed under impulsive boundary conditions on the interval $(a,d)\cup(d,\infty)$. Some existence and uniqueness theorems are obtained by considering such an equation in Weyl's limit-circle case over an infinite interval. The difference of this manuscript from the studies in the literature is that the equation is given in Weyl's limit-circle state.

2 Statement of the problem

Firstly, we assume that the reader is familiar with the basic facts of time scales (see [6]). Now, we consider the following equation

$$Ly := -\left[p(\zeta)\,y^{\Delta}(\zeta)\right]^{\nabla} + q(\zeta)\,y(\zeta) = \Upsilon(\zeta, y(\zeta))\,,\ \zeta\in(a,d)\cup(d,\infty), \tag{2.1}$$

where y is a sought solution, $(a,d)\cup(d,\infty)\subset\mathbb{T}$, and $\mathbb{T}$ is a time scale.

$H=L^2_\nabla(J)$ is a Hilbert space of real-valued functions endowed with the following inner product

$$\langle y,z\rangle:=\int_a^d y^{(1)}(\zeta)z^{(1)}(\zeta)\nabla\zeta+\delta\int_d^\infty y^{(2)}(\zeta)z^{(2)}(\zeta)\nabla\zeta,$$

where

$$y(\zeta)=\begin{cases} y^{(1)}(\zeta), & \zeta\in[a,d)\\ y^{(2)}(\zeta), & \zeta\in(d,\infty),\end{cases}\quad z(\zeta)=\begin{cases} z^{(1)}(\zeta), & \zeta\in[a,d)\\ z^{(2)}(\zeta), & \zeta\in(d,\infty).\end{cases}$$

Our basic assumption throughout the paper is the following:

(K1) q is a real-valued continuous function on $J:=[a,d)\cup(d,\infty)\subset\mathbb{T}$, p is nabla differentiable function on J, p^∇ is continuous on J and $p(t)\neq 0$ for all $t\in J$. The point $d\in\mathbb{T}$ is regular for L and one-sided limits $q(d\pm)$, $p^\nabla(d\pm)$ exist. Furthermore, Weyl's limit-circle case holds for L.

(K2) $\Upsilon(\zeta,y)$ is a real-valued and continuous function in $(\zeta,\gamma)\in J\times\mathbb{R}$, and

$$|\Upsilon(\zeta,\gamma)|\leq g(\zeta)+\vartheta|\gamma| \tag{2.2}$$

for all (ζ,γ) in $J\times\mathbb{R}$, where $g(\zeta)\geq 0$, $g\in H$, and ϑ is a positive constant.

Let

$$\phi(\zeta)=\begin{cases}\phi^{(1)}(\zeta), & \zeta\in[a,d)\\ \phi^{(2)}(\zeta), & \zeta\in(d,\infty),\end{cases}$$

$$\chi(\zeta)=\begin{cases}\chi^{(1)}(\zeta), & \zeta\in[a,d)\\ \chi^{(2)}(\zeta), & \zeta\in(d,\infty)\end{cases}$$

be the solutions of Eq. (2.1) satisfying

$$\phi^{(1)}(a)=0,\ p(a)\phi^{(1)^\Delta}(a)=1,\ \chi^{(1)}(a)=-1,\ p(a)\chi^{(1)^\Delta}(a)=0, \tag{2.3}$$

and impulsive conditions

$$\begin{aligned}\Psi(d+)&=\Omega\Psi(d-),\ \Psi(\zeta):=\begin{pmatrix}\phi(\zeta)\\ p(\zeta)\phi^\Delta(\zeta)\end{pmatrix},\\ \Xi(d+)&=\Omega\Xi(d-),\ \Xi(\zeta):=\begin{pmatrix}\chi(\zeta)\\ p(\zeta)\chi^\Delta(\zeta)\end{pmatrix},\end{aligned} \tag{2.4}$$

where Ω is the 2×2 matrix with entries from $\mathbb{R}$ and $\det\Omega=1/\delta>0$.

If we set $W_\Delta^{(i)}:=W_\Delta\left(\phi^{(i)},\chi^{(i)}\right)$ $(i=1,2)$, then we have $W_\Delta^{(1)}=\delta W_\Delta^{(2)}$. Let

$$W_\Delta:=W_\Delta^{(1)}=\delta W_\Delta^{(2)}.$$

Consider the following sets

$$\mathcal{D}_{\max}=\left\{y\in H:\begin{array}{c}y\text{ is }\Delta\text{-absolutely continuous and }py^\Delta\\ \text{is }\nabla\text{-absolutely continuous functions}\\ \text{on all subintervals of }J,\text{ one-sided}\\ \text{limits }y(d\pm)\text{ and }\left(py^\Delta\right)(d\pm)\text{ exist and}\\ \text{finite, }Y(d+)=\Omega Y(d-)\text{ and }Ly\in H\end{array}\right\},$$

where

$$Y(\zeta):=\begin{pmatrix}y(\zeta)\\ p(\zeta)y^\Delta(\zeta)\end{pmatrix}.$$

Then the *maximal operator* $\mathcal{L}_{\max}$ on $\mathcal{D}_{\max}$ is given by

$$\mathcal{L}_{\max}y=Ly.$$

Let $y, z \in \mathcal{D}_{\max}$. Then we have

$$\langle Ly_1, y_2\rangle - \langle y_1, Ly_2\rangle$$
$$= \int_a^d \left[(Ly_1)(\zeta)y_2(\zeta) - y_1(\zeta)(Ly_2)(\zeta)\right] \nabla\zeta$$
$$+\delta \int_d^\infty \left[(Ly_1)(\zeta)y_2(\zeta) - y_1(\zeta)(Ly_2)(\zeta)\right] \nabla\zeta$$
$$= \delta[y_1, y_2]\,(\infty) - \delta[y_1, y_2]\,(d+) + [y_1, y_2]\,(d-) - [y_1, y_2]\,(a)\,, \tag{2.5}$$

where

$$[y_1, y_2]\,(\zeta) := W_\Delta\,(y_1, y_2)\,(\zeta) = p\,(\zeta)\left\{y_1(\zeta)y_2^\Delta(\zeta) - y_1^\Delta(\zeta)y_2(\zeta)\right\}. \tag{2.6}$$

It follows from (2.3) and the condition (**K1**) that $\phi,\, \chi \in H$ and moreover, $\phi, \chi \in \mathcal{D}_{\max}$. Thus, for every $y \in \mathcal{D}_{\max},\ [y, \phi]_\infty$ and $[y, \chi]_\infty$ exist and are finite.

Now, we consider Eq. (2.1) with conditions

$$\begin{aligned} y\,(a) \cos\alpha + p\,(a)\, y^\Delta\,(a) \sin\alpha &= \xi_1, \\ [y, \phi]_\infty \cos\beta + [y, \chi]_\infty \sin\beta &= \xi_2, \end{aligned} \tag{2.7}$$

$$Y\,(d+) = \Omega Y\,(d-)\,, \tag{2.8}$$

where $Y = \begin{pmatrix} y \\ py^\Delta \end{pmatrix}$,

(**K3**)

$$\omega := \cos\alpha \sin\beta - \cos\beta \sin\alpha \neq 0,$$

$\alpha, \beta, \xi_1, \xi_2 \in \mathbb{R}$.

Thus, we get

$$[y, \phi]_\infty = y\,(a) + \int_a^\infty \phi\,(\zeta)\,\Upsilon\,(\zeta, y\,(\zeta))\,\nabla\zeta,$$

$$[y, \chi]_\infty = p\,(a)\, y^\Delta\,(a) + \int_a^\infty \chi\,(\zeta)\,\Upsilon\,(\zeta, y\,(\zeta))\,\nabla\zeta.$$

because the function y in (2.7) satisfies Eq. (2.1).

Remark 2.1. When $\mathbb{T} = \mathbb{R}$, similar problems have been studied in [1] and in [9] without impulsive boundary conditions.

3 Green's function for the problem

In this section, problem (2.1), (2.7), (2.8) will be transformed into a fixed point problem with the help of Green's function.

Let us consider the following problem

$$-\left[p\,(\zeta)\, y^\Delta\,(\zeta)\right]^\nabla + q\,(\zeta)\, y\,(\zeta) = h\,(\zeta)\,,\ \zeta \in (a, d) \cup (d, \infty),\ h \in H, \tag{3.1}$$

$$\left.\begin{aligned} y\,(a) \cos\alpha + p\,(a)\, y^\Delta\,(a) \sin\alpha = 0, \\ [y, \phi]_\infty \cos\beta + [y, \chi]_\infty \sin\beta = 0,\ \alpha, \beta \in \mathbb{R}, \\ Y\,(d+) = \Omega Y\,(d-)\,, \end{aligned}\right\} \tag{3.2}$$

where

$$Y = \begin{pmatrix} y \\ py^{\Delta} \end{pmatrix}.$$

Set

$$\Lambda(\zeta) = \cos\alpha\phi(\zeta) + \sin\alpha\chi(\zeta), \ \Theta(\zeta) = \cos\beta\phi(\zeta) + \sin\beta\chi(\zeta), \tag{3.3}$$

where $W_\Delta(\Lambda, \Theta) = \omega$. Then, Λ and Θ are solutions of Eq. (3.1) and $\Lambda, \Theta \in H$. Further, we see that

$$[\Lambda, \phi]_\zeta = \Lambda(a) = -\sin\alpha, \ [\Lambda, \chi]_\zeta = p(a)\Lambda^{\Delta}(a) = \cos\alpha, \ (\zeta \in [a, d)), \tag{3.4}$$

$$[\Theta, \phi]_\zeta = \Theta(a) = -\sin\beta, \ [\Theta, \chi]_\zeta = p(a)\Theta^{\Delta}(a) = \cos\beta, \ (\zeta \in [a, d)), \tag{3.5}$$

$$[\Theta, \phi]_\infty = -\frac{1}{\alpha}\sin\beta, \ [\Theta, \chi]_\infty = \frac{1}{\alpha}\cos\beta,$$

$$\Phi(d+) = \Omega\Phi(d-), \ \Phi(\zeta) := \begin{pmatrix} \Lambda(\zeta) \\ p(\zeta)\Lambda^{\Delta}(\zeta) \end{pmatrix}, \tag{3.6}$$

$$\Psi(d+) = \Omega\Psi(d-), \ \Psi(\zeta) := \begin{pmatrix} \Theta(\zeta) \\ p(\zeta)\Theta^{\Delta}(\zeta) \end{pmatrix}. \tag{3.7}$$

Let

$$G(\zeta, \gamma) = \frac{1}{\omega}\begin{cases} \Lambda(\zeta)\Theta(\gamma), & \text{if } a \leq \zeta \leq \gamma < \infty, \ \zeta \neq d, \ \gamma \neq d, \\ \Lambda(\gamma)\Theta(\zeta), & \text{if } a \leq \gamma \leq \zeta < \infty, \ \zeta \neq d, \ \gamma \neq d. \end{cases} \tag{3.8}$$

Then, we obtain

$$\int_a^\infty \int_a^\infty |G(\zeta, \gamma)|^2 \nabla\zeta\nabla\gamma < \infty, \tag{3.9}$$

due to $\Lambda, \Theta \in H$.

Theorem 3.1. *The function*

$$y(\zeta) = \int_a^d G(\zeta, \gamma)h(\gamma)\nabla\gamma + \delta\int_d^\infty G(\zeta, \gamma)h(\gamma)\nabla\gamma, \ \zeta \in [a, d) \cup (d, \infty), \tag{3.10}$$

is the unique solution of (3.1)-(3.2).

Proof. One can obtain the solution of Eq. (3.1) by the method of variation of constants as follows

$$y(\zeta) = \begin{cases} y^{(1)}(\zeta), & \zeta \in [a, d) \\ y^{(2)}(\zeta), & \zeta \in (d, \infty), \end{cases}$$

$$= \begin{cases} c_1\Lambda^{(1)}(\zeta) + c_2\Theta^{(1)}(\zeta) \\ +\frac{1}{\omega}\int_a^\zeta \Theta^{(1)}(\zeta)\Lambda^{(1)}(\gamma)h(\gamma)\nabla\gamma \\ +\frac{1}{\omega}\int_\zeta^d \Lambda^{(1)}(\zeta)\Theta^{(1)}(\gamma)h(\gamma)\nabla\gamma, \ \ \zeta \in [a, d), \\ c_3\Lambda^{(2)}(\zeta) + c_4\Theta^{(2)}(\zeta) \\ +\frac{\delta}{\omega}\int_d^\zeta \Theta^{(2)}(\zeta)\Lambda^{(2)}(\gamma)h(\gamma)\nabla\gamma \\ +\frac{\delta}{\omega}\int_\zeta^\infty \Lambda^{(2)}(\zeta)\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma, \ \ \zeta \in (d, \infty), \end{cases} \tag{3.11}$$

where c_i $(i = 1, 2, 3, 4)$ are arbitrary constants.

By (3.11), we have

$$p(\zeta)\, y^{\Delta}(\zeta) = \begin{cases} c_1 p(\zeta)\, \Lambda^{(1)^{\Delta}}(\zeta) + c_2 p(\zeta)\, \Theta^{(1)^{\Delta}}(\zeta) \\ +\frac{1}{\omega}\int_a^{\zeta} p(\zeta)\, \Theta^{(1)^{\Delta}}(\zeta)\, \Lambda^{(1)}(\gamma)\, h(\gamma)\, \nabla\gamma \\ +\frac{1}{\omega}\int_{\zeta}^{d} p(\zeta)\, \Lambda^{(1)^{\Delta}}(\zeta)\, \Theta^{(1)}(\gamma)\, h(\gamma)\, \nabla\gamma,\ \zeta \in [a,d), \\ c_3 p(\zeta)\, \Lambda^{(2)^{\Delta}}(\zeta) + c_4 p(\zeta)\, \Theta^{(2)^{\Delta}}(\zeta) \\ +\frac{\delta}{\omega}\int_d^{\zeta} p(\zeta)\, \Theta^{(2)^{\Delta}}(\zeta)\, \Lambda^{(2)}(\gamma)\, h(\gamma)\, \nabla\gamma \\ +\frac{\delta}{\omega}\int_{\zeta}^{\infty} p(\zeta)\, \Lambda^{(2)^{\Delta}}(\zeta)\, \Theta^{(2)}(\gamma)\, h(\gamma)\, \nabla\gamma,\ \zeta \in (d,\infty). \end{cases}$$

Hence

$$\begin{aligned} y(a) &= c_1\Lambda^{(1)}(a) + c_2\Theta^{(1)}(a) + \frac{1}{\omega}\int_a^d \Lambda^{(1)}(a)\, \Theta^{(1)}(\gamma)\, h(\gamma)\, \nabla\gamma \\ &= -c_1 \sin\alpha - c_2 \sin\beta - \frac{1}{\omega}\sin\alpha \int_a^d \Lambda^{(1)}(\gamma)\, h(\gamma)\, \nabla\gamma, \text{ and} \\ p(a)\, y^{\Delta}(a) &= c_1 p(a)\, \Lambda^{(1)^{\Delta}}(a) + c_2 p(a)\, \Theta^{(1)^{\Delta}}(a) \\ &\quad +\frac{1}{\omega}\int_a^d p(a)\, \Lambda^{(1)^{\Delta}}(a)\, \Theta^{(1)}(\gamma)\, h(\gamma)\, \nabla\gamma \\ &= c_1\cos\alpha + c_2\cos\beta + \frac{1}{\omega}\cos\alpha\int_a^d \Lambda^{(1)}(\gamma)\, h(\gamma)\, \nabla\gamma. \end{aligned} \tag{3.12}$$

From (3.12) and (3.2), we get $c_2 = 0$. Likewise, we obtain

$$[y,\phi]_{\zeta} = p(\zeta)\left\{y(\zeta)\phi^{\Delta}(\zeta) - y^{\Delta}(\zeta)\phi(\zeta)\right\}$$

$$= \begin{cases} c_1[\Lambda^{(1)},\phi]_{\zeta} + \frac{1}{\omega}[[\Theta^{(1)}(\zeta),\phi]_{\zeta}\int_a^{\zeta}\Lambda^{(1)}(\gamma)\, h(\gamma)\, \nabla\gamma \\ +\frac{1}{\omega}[\Lambda^{(1)}(\zeta),\phi]_{\zeta}\int_{\zeta}^{d}\Theta^{(1)}(\gamma)\, h(\gamma)\, \nabla\gamma,\ \zeta\in[a,d), \\ c_3[\Lambda^{(2)},\phi]_{\zeta} + c_4[\Theta^{(2)},\phi]_{\zeta} \\ +\frac{\delta}{\omega}[\Theta^{(2)},\phi]_{\zeta}\int_d^{\zeta}\Lambda^{(2)}(\gamma)\, h(\gamma)\, \nabla\gamma \\ +\frac{\delta}{\omega}[\Lambda^{(2)},\phi]_{\zeta}\int_{\zeta}^{\infty}\Theta^{(2)}(\gamma)\, h(\gamma)\, \nabla\gamma,\ \zeta\in(d,\infty). \end{cases}$$

Therefore, we see that

$$[y,\phi]_{\infty} = -c_3\frac{1}{\delta}\sin\alpha - c_4\frac{1}{\delta}\sin\beta - \frac{\delta}{\omega}\frac{1}{\delta}\sin\beta\int_d^{\infty}\Lambda^{(2)}(\gamma)\, h(\gamma)\, \nabla\gamma.$$

Similarly, we obtain

$$[y,\chi]_{\zeta} = p(\zeta)\left\{y(\zeta)\chi^{\Delta}(\zeta) - y^{\Delta}(\zeta)\chi(\zeta)\right\}$$

$$\begin{cases} c_1[\Lambda^{(1)},\chi]_\zeta+\frac{1}{\omega}[[\Theta^{(1)}(\zeta),\chi]_\zeta\int_a^\zeta\Lambda^{(1)}(\gamma)h(\gamma)\nabla\gamma \\ +\frac{1}{\omega}[\Lambda^{(1)}(\zeta),\chi]_\zeta\int_\zeta^d\Theta^{(1)}(\gamma)h(\gamma)\nabla\gamma,\ \zeta\in[a,d), \\ c_3[\Lambda^{(2)},\chi]_\zeta+c_4[\Theta^{(2)},\chi]_\zeta \\ +\frac{\delta}{\omega}[\Theta^{(2)},\chi]_\zeta\int_d^\zeta\Lambda^{(2)}(\gamma)h(\gamma)\nabla\gamma \\ +\frac{\delta}{\omega}[\Lambda^{(2)},\chi]_\zeta\int_\zeta^\infty\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma,\ \zeta\in(d,\infty), \end{cases}$$

and

$$[y,\chi]_\infty=c_3\frac{1}{\delta}\cos\alpha+c_4\frac{1}{\delta}\cos\beta+\frac{\delta}{\omega}\frac{1}{\delta}\cos\beta\int_d^\infty\Lambda^{(2)}(\gamma)h(\gamma)\nabla\gamma.$$

It follows from (3.2) that $c_3=0$.

Likewise, we obtain

$$Y(d+)=\begin{pmatrix} y(d+) \\ p(d+)y^\Delta(d+)\end{pmatrix}=\begin{pmatrix} c_4\Theta^{(2)}(d+) \\ c_4p(d+)\Theta^{(2)^\Delta}(d+)\end{pmatrix}$$

$$+\begin{pmatrix} \frac{\delta}{\omega}\int_d^\infty\Lambda^{(2)}(d+)\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma \\ \frac{\delta}{\omega}\int_d^\infty p(d+)\Lambda^{(2)^\Delta}(d+)\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma \end{pmatrix}$$

$$=c_4\begin{pmatrix}\Theta^{(2)}(d+) \\ p(d+)\Theta^{(2)^\Delta}(d+)\end{pmatrix}$$

$$+\frac{\delta}{\omega}\int_d^\infty\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma\begin{pmatrix}\Lambda^{(2)}(d+) \\ p(d+)\Lambda^{(2)^\Delta}(d+)\end{pmatrix}$$

$$=c_4\Psi(d+)+\left\{\frac{\delta}{\omega}\int_d^\infty\Theta^{(2)}(\gamma)h(\gamma)d_q\gamma\right\}\Phi(d+)$$

and

$$Y(d-)=\begin{pmatrix} y(d-) \\ p(d-)y^\Delta(d-)\end{pmatrix}$$

$$=\begin{pmatrix} c_1\Lambda^{(1)}(d-) \\ c_1p(d-)\Lambda^{(1)^\Delta}(d-)\end{pmatrix}$$

$$+\begin{pmatrix} \frac{1}{\omega}\int_a^d\Theta^{(1)}(d-)\Lambda^{(1)}(\gamma)h(\gamma)\nabla\gamma \\ \frac{1}{\omega}\int_a^d p(d-)\Theta^{(1)^\Delta}(d-)\Lambda^{(1)}(\gamma)h(\gamma)\nabla\gamma \end{pmatrix}$$

$$=c_1\begin{pmatrix}\Lambda^{(1)}(d-) \\ p(d-)\Lambda^{(1)^\Delta}(d-)\end{pmatrix}$$

$$+\frac{1}{\omega}\int_a^d\Lambda^{(1)}(\gamma)h(\gamma)d_q\gamma\begin{pmatrix}\Theta^{(1)}(d-) \\ p(d-)\Theta^{(1)^\Delta}(d-)\end{pmatrix}$$

$$=c_1\Phi(d-)+\left\{\frac{1}{\omega}\int_a^d\Lambda^{(1)}(\gamma)h(\gamma)d_q\gamma\right\}\Psi(d-).$$

By (3.2), we conclude that

$$c_4\Psi(d+)+\left\{\frac{\delta}{\omega}\int_d^\infty\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma\right\}\Phi(d+)$$

$$=\Omega\left\{c_1\Phi(d-)+\left\{\frac{1}{\omega}\int_a^d\Lambda^{(1)}(\gamma)h(\gamma)\nabla\gamma\right\}\Psi(d-)\right\}.$$

From (3.6) and (3.7), we get

$$\Phi(d-)\left\{\frac{\delta}{\omega}\int_d^\infty\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma-c_1\right\}$$

$$=\Psi(d-)\left\{\frac{1}{\omega}\int_a^d\Lambda^{(1)}(\gamma)h(\gamma)\nabla\gamma-c_4\right\},\text{ and}$$

$$\begin{pmatrix}\Lambda^{(1)}(d-)\\ p(d-)\Lambda^{(1)^\Delta}(d-)\end{pmatrix}\left\{\frac{\delta}{\omega}\int_d^\infty\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma-c_1\right\}$$

$$=\begin{pmatrix}\Theta^{(1)}(d-)\\ p(d-)\Theta^{(1)^\Delta}(d-)\end{pmatrix}\left\{\frac{1}{\omega}\int_a^d\Lambda^{(1)}(\gamma)h(\gamma)\nabla\gamma-c_4\right\}.$$

Therefore we obtain

$$c_4\Theta^{(1)}(d-)-c_1\Lambda^{(1)}(d-)$$

$$=\Theta^{(1)}(d-)\left\{\frac{1}{\omega}\int_a^d\Lambda^{(1)}(\gamma)h(\gamma)\nabla\gamma\right\}$$

$$-\Lambda^{(1)}(d-)\left\{\frac{\delta}{\omega}\int_d^\infty\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma\right\},\text{ or}$$

$$c_4p(d-)\Theta^{(1)^\Delta}(d-)-c_1p(d-)\Lambda^{(1)^\Delta}(d-)$$

$$=p(d-)\Theta^{(1)^\Delta}(d-)\left\{\frac{1}{\omega}\int_a^d\Lambda^{(1)}(\gamma)h(\gamma)\nabla\gamma\right\}$$

$$-p(d-)\Lambda^{(1)^\Delta}(d-)\left\{\frac{\delta}{\omega}\int_d^\infty\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma\right\}.$$

Thus, we have

$$c_1=\frac{\delta}{\omega}\int_d^\infty\Theta^{(2)}(\gamma)h(\gamma)\nabla\gamma,$$

$$c_4=\frac{1}{\omega}\int_a^d\Lambda^{(1)}(\gamma)h(\gamma)\nabla\gamma.$$

So, we obtain

$$y(\zeta)=\begin{cases}\delta\int_d^\infty \frac{\Lambda^{(1)}(\zeta)\Theta^{(2)}(\gamma)}{\omega}h(\gamma)\nabla\gamma \\ +\int_a^\zeta \frac{\Theta^{(1)}(\zeta)\Lambda^{(1)}(\gamma)}{\omega}h(\gamma)\nabla\gamma \\ +\int_\zeta^d \frac{\Lambda^{(1)}(\zeta)\Theta^{(1)}(\gamma)}{\omega}h(\gamma)\nabla\gamma,\ \zeta\in[a,d), \\ \int_a^d \frac{\Theta^{(2)}(\zeta)\Lambda^{(1)}(\gamma)}{\omega}h(\gamma)\nabla\gamma \\ +\delta\int_d^\zeta \frac{\Theta^{(2)}(\zeta)\Lambda^{(2)}(\gamma)}{\omega}h(\gamma)\nabla\gamma \\ +\delta\int_\zeta^\infty \frac{\Lambda^{(2)}(\zeta)\Theta^{(2)}(\gamma)}{\omega}h(\gamma)\nabla\gamma,\ \zeta\in(d,\infty).\end{cases}$$

□

Theorem 3.2. *The function*

$$y(\zeta)=w(\zeta)+\langle G(\zeta,.),h(.)\rangle,$$

where $w(\zeta)=\frac{1}{\omega}(\xi_1\Lambda(\zeta)-\xi_2\Theta(\zeta))$, *is the unique solution of the problem (3.1), (2.7), (2.8).*

Proof. By virtue of (3.4)-(3.7), $w(\zeta)$ is a unique solution of the equation $Ly=0$, $\zeta\in(0,d)\cup(d,\infty)$ satisfying the boundary conditions (2.7), (2.8). From Theorem 3.1, we see that the function $\langle G(\zeta,.),h(.)\rangle$ is a unique solution of Eq. (3.1) satisfying (3.2), and the proof is completed. □

4 An existence theorem for the problem

Theorem 4.1. *Suppose that the hypotheses (**K1**)-(**K3**) hold. In addition, let there exist a number* $K>0$ *such that*

$$\begin{aligned}&\int_a^d\left|\Upsilon\left(\zeta,y^{(1)}(\zeta)\right)-\Upsilon\left(\zeta,z^{(1)}(\zeta)\right)\right|^2\nabla\zeta\\&+\delta\int_d^\infty\left|\Upsilon\left(\zeta,y^{(2)}(\zeta)\right)-\Upsilon\left(\zeta,z^{(2)}(\zeta)\right)\right|^2\nabla\zeta\\&\leq K^2\left(\int_a^d\left|y^{(1)}(\zeta)-z^{(1)}(\zeta)\right|^2\nabla\zeta+\delta\int_d^\infty\left|y^{(2)}(\zeta)-z^{(2)}(\zeta)\right|^2\nabla\zeta\right)\\&=K^2\|y-z\|^2 \end{aligned}\tag{4.1}$$

for all $y,z\in H$. *If*

$$K\left(\int_a^d\int_a^d|G(\zeta,\gamma)|^2\nabla\zeta\nabla\gamma+\delta\int_d^\infty\int_d^\infty|G(\zeta,\gamma)|^2\nabla\zeta\nabla\gamma\right)<1,\tag{4.2}$$

then the problem (2.1), (2.7), (2.8) has a unique solution in H.

Proof. From Theorem 3.2, we infer that the equation defined as

$$y(\zeta)=w(\zeta)+\langle G(\zeta,.),\Upsilon(.,y(.))\rangle,\ \zeta\in[a,d)\cup(d,\infty),\tag{4.3}$$

is equivalent to problem (2.1), (2.7), (2.8). □

Let us define the operator $T:H\to H$ as

$$(Ty)(\zeta)=w(\zeta)+\langle G(\zeta,.),\Upsilon(.,y(.))\rangle,\tag{4.4}$$

where $\zeta \in [a,d) \cup (d,\infty)$ and $y, w \in H$. Then, we have $Ty = y$.

Now, we prove that T is a contraction operator. For $y, z \in H$ and $\zeta \in [a,d) \cup (d,\infty)$, we obtain

$$\begin{aligned} |(Ty)(\zeta) - (Tz)(\zeta)|^2 &= |\langle G(\zeta,.), [\Upsilon(., y(.)) - \Upsilon(., z(.))]\rangle|^2 \\ &\leq \|G(\zeta,.)\|^2 \|\Upsilon(., y(.)) - \Upsilon(., z(.))\|^2 \\ &\leq K^2 \|G(\zeta,.)\|^2 \|y-z\|^2. \end{aligned}$$

Consequently, we see that

$$\|Ty - Tz\| \leq \varrho \|y-z\|,$$

where

$$\varrho = K\left(\int_a^d \int_a^d |G(\zeta,\gamma)|^2 \nabla\zeta\nabla\gamma + \delta \int_d^\infty \int_d^\infty |G(\zeta,\gamma)|^2 \nabla\zeta\nabla\gamma\right)^{\frac{1}{2}} < 1.$$

Theorem 4.2. *Let assumptions (**K1**)-(**K3**) hold. In addition, let there exist numbers M, $K > 0$ such that*

$$\begin{aligned} &\int_a^d \left|\Upsilon\left(\zeta, y^{(1)}(\zeta)\right) - \Upsilon\left(\zeta, z^{(1)}(\zeta)\right)\right|^2 \nabla\zeta \\ &+ \delta \int_d^\infty \left|\Upsilon\left(\zeta, y^{(2)}(\zeta)\right) - \Upsilon\left(\zeta, z^{(2)}(\zeta)\right)\right|^2 \nabla\zeta \\ &\leq K^2 \left(\int_a^d \left|y^{(1)}(\zeta) - z^{(1)}(\zeta)\right|^2 \nabla\zeta + \delta \int_d^\infty \left|y^{(2)}(\zeta) - z^{(2)}(\zeta)\right|^2 \nabla\zeta\right) \\ &= K^2 \|y-z\|^2,\ y, z \in S_M, \end{aligned} \tag{4.5}$$

where $S_M = \{\gamma \in H : \|\gamma\| \leq M\}$ is a closed set of H and K may depend on M. If

$$\begin{aligned} &\left\{\int_a^d \left|w^{(1)}(\zeta)\right|^2 \nabla\zeta + \delta \int_d^\infty \left|w^{(2)}(\zeta)\right|^2 \nabla\zeta\right\}^{1/2} \\ &+ \left(\int_a^d \int_a^d |G(\zeta,\gamma)|^2 \nabla\zeta\nabla\gamma + \delta \int_d^\infty \int_d^\infty |G(\zeta,\gamma)|^2 \nabla\zeta\nabla\gamma\right)^{\frac{1}{2}} \\ &\times \sup_{y \in S_M} \left\{\begin{array}{c} \int_a^d \left|\Upsilon\left(\gamma, y^{(1)}(\gamma)\right) - \Upsilon\left(\gamma, z^{(1)}(\gamma)\right)\right|^2 \nabla\gamma \\ +\delta \int_d^\infty \left|\Upsilon\left(\gamma, y^{(2)}(\gamma)\right) - \Upsilon\left(\gamma, z^{(2)}(\gamma)\right)\right|^2 \nabla\gamma \end{array}\right\}^{1/2} \\ &\leq M \end{aligned} \tag{4.6}$$

and

$$K\left(\int_a^d \int_a^d |G(\zeta,\gamma)|^2 \nabla\zeta\nabla\gamma + \delta \int_d^\infty \int_d^\infty |G(\zeta,\gamma)|^2 \nabla\zeta\nabla\gamma\right)^{\frac{1}{2}} < 1, \tag{4.7}$$

then problem (2.1), (2.7), (2.8) has unique solution y with

$$\int_a^d \left|y^{(1)}(\zeta)\right|^2 \nabla\zeta + \delta \int_d^\infty \left|y^{(2)}(\zeta)\right|^2 \nabla\zeta \le M^2.$$

Proof. For $y \in S_M$, we have

$$\|Ty\| = \|w(\zeta) + \langle G(\zeta,.), \Upsilon(., y(.))\rangle\|$$

$$\le \|w\| + \|\langle G(\zeta,.), \Upsilon(., y(.))\rangle\|$$

$$\le \|w\| + \left(\int_a^d\int_a^d |G(\zeta,\gamma)|^2 \nabla\zeta\nabla\gamma + \delta\int_d^\infty\int_d^\infty |G(\zeta,\gamma)|^2\nabla\zeta\nabla\gamma\right)^{\frac{1}{2}}$$

$$\times \sup_{y\in S_M}\left\{\begin{array}{c} \int_a^d \left|\Upsilon\left(\gamma, y^{(1)}(\gamma)\right) - \Upsilon\left(\gamma, z^{(1)}(\gamma)\right)\right|^2\nabla\gamma \\ +\delta\int_d^\infty \left|\Upsilon\left(\gamma, y^{(2)}(\gamma)\right) - \Upsilon\left(\gamma, z^{(2)}(\gamma)\right)\right|^2\nabla\gamma \end{array}\right\}^{1/2} \le M,$$

that is, $T : S_M \to S_M$.

As in the proof of Theorem 4.2, we get

$$\|Ty - Tz\| \le \varrho\,\|y - z\|,\ \varrho < 1,$$

where $y, z \in S_M$. By the Banach fixed point theorem, the theorem follows. □

5 An existence theorem without uniqueness

Theorem 5.1. *Under conditions (**K1**)-(**K3**), T is a completely continuous operator.*

Proof. Let $y_0 \in H$. Then, we obtain

$$|(Ty)(\zeta) - (Ty_0)(\zeta)|^2 = |\langle G(\zeta,.), [\Upsilon(., y(.)) - \Upsilon(., y_0(.))]\rangle|^2$$

$$\le \|G(\zeta,.)\|^2 \left\{\begin{array}{c} \int_a^d \left|\Upsilon\left(\gamma, y^{(1)}(\gamma)\right) - \Upsilon\left(\gamma, y_0^{(1)}(\gamma)\right)\right|^2\nabla\gamma \\ +\delta\int_d^\infty \left|\Upsilon\left(\gamma, y^{(2)}(\gamma)\right) - \Upsilon\left(\gamma, y_0^{(2)}(\gamma)\right)\right|^2\nabla\gamma \end{array}\right\}^2.$$

Hence, we get

$$\|Ty - Ty_0\|^2 \le K\left\{\begin{array}{c} \int_a^d \left|\Upsilon\left(\gamma, y^{(1)}(\gamma)\right) - \Upsilon\left(\gamma, y_0^{(1)}(\gamma)\right)\right|^2\nabla\gamma \\ +\delta\int_d^\infty \left|\Upsilon\left(\gamma, y^{(2)}(\gamma)\right) - \Upsilon\left(\gamma, y_0^{(2)}(\gamma)\right)\right|^2\nabla\gamma \end{array}\right\}^2, \tag{5.1}$$

where

$$K = \left(\int_a^d\int_a^d |G(\zeta,\gamma)|^2\nabla\zeta\nabla\gamma + \frac{1}{\eta}\int_d^\infty\int_d^\infty |G(\zeta,\gamma)|^2\nabla\zeta\nabla\gamma\right).$$

Let us define the operator F as

$$(Fy)(\zeta) = \Upsilon(\zeta, y(\zeta)).$$

Under the condition (K2), F is continuous in H (see [16]). Then, for the given $\epsilon > 0$, we can find a $\eta > 0$ such that $\|y - y_0\| < \eta$ implies

$$\left\{ \begin{array}{c} \int_a^d \left|\Upsilon\left(\gamma, y^{(1)}(\gamma)\right) - \Upsilon(\gamma, y_0^{(1)}(\gamma))\right|^2 \nabla\gamma \\ \\ +\delta \int_d^\infty \left|\Upsilon\left(\gamma, y^{(2)}(\gamma)\right) - \Upsilon(\gamma, y_0^{(2)}(\gamma))\right|^2 \nabla\gamma \end{array} \right\} < \frac{\epsilon^2}{K^2}.$$

From (5.1), we deduce that

$$\|Ty - Ty_0\| < \epsilon.$$

Let $Y = \{y \in H : \|y\| \le m\}$. By (4.6), for all $y \in Y$, we obtain

$$\|Ty\| \le \|w\| + \left\{ \begin{array}{c} K \int_a^d \left|\Upsilon\left(\gamma, y^{(1)}(\gamma)\right)\right|^2 \nabla\gamma \\ \\ +\delta K \int_d^\infty \left|\Upsilon\left(\gamma, y^{(2)}(\gamma)\right)\right|^2 \nabla\gamma \end{array} \right\}^{1/2}.$$

Moreover, by (2.2), we infer that

$$\int_a^d \left|\Upsilon\left(\gamma, y^{(1)}(\gamma)\right)\right|^2 \nabla\gamma + \delta \int_d^\infty \left|\Upsilon\left(\gamma, y^{(2)}(\gamma)\right)\right|^2 \nabla\gamma$$

$$\le \int_a^d \left[\left|g^{(1)}(\gamma)\right| + \vartheta \left|y^{(1)}(\gamma)\right|\right]^2 \nabla\gamma + \delta \int_d^\infty \left[\left|g^{(2)}(\gamma)\right| + \vartheta \left|y^{(2)}(\gamma)\right|\right]^2 \nabla\gamma$$

$$\le 2 \int_a^d [\left|g^{(1)}(\gamma)\right|^2 + \vartheta^2 \left|y^{(1)}(\gamma)\right|^2] \nabla\gamma$$

$$+ 2\delta \int_d^\infty [\left|g^{(2)}(\gamma)\right|^2 + \vartheta^2 \left|y^{(2)}(\gamma)\right|^2] \nabla\gamma$$

$$= 2(\|g\|^2 + \vartheta^2 \|y\|^2) \le 2(\|g\|^2 + \vartheta^2 m^2),$$

that is, $T(Y)$ is bounded.

Moreover, for all $y \in Y$, we have

$$\int_a^d \left|(Ty^{(1)})(\zeta + h) - (Ty^{(1)})(\zeta)\right|^2 \nabla\zeta$$

$$+\delta \int_d^\infty \left|(Ty^{(2)})(\zeta + h) - (Ty^{(2)})(\zeta)\right|^2 \nabla\zeta$$

$$= \|\langle [G(\zeta + h, .) - G(\zeta, .)], \Upsilon(., y(.))\rangle\|^2$$

$$\le \left(\begin{array}{c} \int_a^d \int_a^d |G(\zeta + h, \gamma) - G(\zeta, \gamma)|^2 \nabla\zeta\nabla\gamma \\ \\ +\delta \int_d^\infty \int_d^\infty |G(\zeta + h, \gamma) - G(\zeta, \gamma)|^2 \nabla\zeta\nabla\gamma \end{array} \right)$$

$$\times \left\{ \begin{array}{c} \int_a^d \left|\Upsilon\left(\gamma, y^{(1)}(\gamma)\right)\right|^2 \nabla\gamma \\ \\ +\delta \int_d^\infty \left|\Upsilon\left(\gamma, y^{(2)}(\gamma)\right)\right|^2 \nabla\gamma \end{array} \right\}^2$$

$$\leq 2\left(\|g\|^{2}+\vartheta^{2}m\right)\left(\begin{array}{c}\int_{a}^{d}\int_{a}^{d}\left|G\left(\zeta+h,\gamma\right)-G\left(\zeta,\gamma\right)\right|^{2}d_{q}\zeta d_{q}\gamma\\ +\delta\int_{d}^{\infty}\int_{d}^{\infty}\left|G\left(\zeta+h,\gamma\right)-G\left(\zeta,\gamma\right)\right|^{2}d_{q}\zeta d_{q}\gamma\end{array}\right).$$

From (3.9), for given $\epsilon>0$, all $y\in Y$ and all $h<\eta$, there exists a $\eta>0$ such that

$$\int_{a}^{d}\left|Ty^{(1)}\left(\zeta+h\right)-Ty^{(1)}\left(\zeta\right)\right|^{2}\nabla\zeta$$

$$+\delta\int_{d}^{\infty}\left|Ty^{(2)}\left(\zeta+h\right)-Ty^{(2)}\left(\zeta\right)\right|^{2}\nabla\zeta<\epsilon^{2}.$$

Furthermore, we have

$$\int_{N}^{\infty}\left|\left(Ty^{(2)}\right)\left(\zeta\right)\right|^{2}\nabla\zeta\leq\int_{N}^{\infty}\left|w^{(2)}\left(\zeta\right)\right|^{2}\nabla\zeta$$

$$+2\left(\|g\|^{2}+\vartheta^{2}m\right)\int_{N}^{\infty}\left\|G\left(\zeta,.\right)\right\|^{2}\nabla\zeta,$$

where $y\in Y$ and $N>d$. By (3.9), we deduce that for given $\epsilon>0$ there exists a positive number N, depending only on ϵ such that

$$\int_{N}^{\infty}\left|\left(Ty^{(2)}\right)\left(\zeta\right)\right|^{2}\nabla\zeta<\epsilon^{2},$$

for all $y\in Y$.

Therefore, $T\left(Y\right)$ is relatively compact in H. □

Theorem 5.2. *Assume that conditions **(K1)**-**(K3)** hold. Further, let there exists a number $M>0$ such that inequality*

$$\left\{\int_{a}^{d}\left|w^{(1)}\left(\zeta\right)\right|^{2}\nabla\zeta+\delta\int_{d}^{\infty}\left|w^{(2)}\left(\zeta\right)\right|^{2}\nabla\zeta\right\}^{1/2}$$

$$+\left(\int_{a}^{d}\int_{a}^{d}\left|G\left(\zeta,\gamma\right)\right|^{2}\nabla\zeta\nabla\gamma+\delta\int_{d}^{\infty}\int_{d}^{\infty}\left|G\left(\zeta,\gamma\right)\right|^{2}\nabla\zeta\nabla\gamma\right)$$

$$\times\sup_{y\in S_{M}}\left\{\begin{array}{c}\int_{a}^{d}\left|\Upsilon\left(\gamma,y^{(1)}\left(\gamma\right)\right)-\Upsilon\left(\gamma,z^{(1)}\left(\gamma\right)\right)\right|^{2}\nabla\gamma\\ +\delta\int_{d}^{\infty}\left|\Upsilon\left(\gamma,y^{(2)}\left(\gamma\right)\right)-\Upsilon\left(\gamma,z^{(2)}\left(\gamma\right)\right)\right|^{2}\nabla\gamma\end{array}\right\}^{1/2}$$

$$\leq M, \tag{5.2}$$

where $S_{M}=\left\{y\in H:\|y\|\leq M\right\}$. Then the problem (2.1), (2.7), (2.8) has at least one solution y with

$$\int_{a}^{d}\left|y^{(1)}\left(\zeta\right)\right|^{2}\nabla\zeta+\delta\int_{d}^{\infty}\left|y^{(2)}\left(\zeta\right)\right|^{2}\nabla\zeta\leq M^{2}.$$

Proof. Let us define the operator $T:H\rightarrow H$ as (4.4). From Theorems 4.1, 4.2 and (5.2), we see that $T:S_{M}\rightarrow S_{M}$. Clearly, S_{M} is convex, closed, and bounded. Using the Schauder fixed point theorem, the theorem follows. □

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