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INTUITIONISTIC FUZZY SEMI-MAGIC LABELING OF HUMAN CHAIN GRAPH

J. Komathi, D.Meiyappan and K.Anitha

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Corresponding Author: K. Anitha

Abstract The non-membership and membership functions of an intuitionistic fuzzy graph are said to be intuitionistic fuzzy semi-labeling graphs if they are bijective and have distinct values for each vertex. By connecting a Y-tree $Y_{(m+1)}$, where $m \leq 3$, and a cycle C_m to each u_{2i} for $1 \leq i \leq n$ in a path $u_1, u_2, \ldots, u_{2n+1}$, we obtain a human chain network $HC_{(n,m)}$. In this research, we establish the admissibility of Intuitionistic Fuzzy Bi-magic Labeling on the Human Chain Graph, Circular Human Chain Graph, and Strong Human Chain Graph, and calculate its bi-magic number.

1 Introduction

Graph Magic labelling [14],[25] assigns values to each vertex or edge of a graph based on specific requirements. Fuzzy magic labeling takes this concept into fuzzy space by including degrees of uncertainty, resulting in useful applications in optimization, telecommunications, and network modelingis a widely studied topic in discrete mathematics, where each graph element (vertex or edge) is assigned values subject to certain conditions. Fuzzy magic labelling extends this concept into fuzzy space by incorporating degrees of uncertainty, leading to practical applications in optimization, telecommunications, and network modelling [18]. With the advent of intuitionistic fuzzy sets introduced by Atanassov (1986) [4], presented intuitionistic fuzzy sets, which have been further developed to include membership and non-membership degrees. [5] [10] [29] [32] [63]. This work applies these expansions to human chain graph families, offering fresh insights into graph labeling under uncertainty

Several studies have advanced the notion of fuzzy magic graphs. Gani and Rajalaxmi(2012) [6] and Gani and Akram (2014) [7] explored the algebraic characteristics and practical applications of fuzzy labeling graphs. Fathalian et al.. (2019) [20] improved the idea by investigating magic labeling for several types of fuzzy graphs [37],[39],[42]. Intuitive fuzzy magic labeling is based on Mehra and Singh's (2017) [43], approach, which we created in 2019 [1].Anitha and ArunaDevi expanded the notion of rough space in [12].

Researchers such as Ameenal Bibi and Devi (2017) [2] and Chellamani [23] provided an account of the history of graph labeling algorithms. We established bi-magic labeling in intuitionistic space (2023) [24] and Gallian (2014) [27] provided an account of the history of graph labeling algorithms. We established bi-magic labeling in intuitionistic space [8].

Human chain graphs, first investigated by Anitha and Selvam (2018) [3] [46], represent networks with sequential or circular connectedness, such as social chains, supply networks, and communication protocols. These structures are ideal for labeling techniques such as D-lucky edge labelling and others [9] [11] [53] [56] Exploring fuzzy labeling inside these graphs offers a more sophisticated portrayal of real-world systems with ambiguous or imprecise edges. Hence extending the theory to cover intuitionistic human chain graphs

Magic labeling techniques have also found practical use in communication networks and health-care systems [22]. Riaz et al. (2022)[26] used fuzzy approaches to code DNA sequences, whereas Munir et al. (2021) [44] used them to hemodialysis models. Fuzzy labeling is important in multi-criteria decision-making systems [61], including smog mitigation techniques [21], [59], and robotics engineering selection [40]. Researchers examined the generalization of intuitionistic fuzzy graphs to different graph families [40], [41], [45], [49], [56], [58], [60].

This study draws from recent advancements in intuitionistic fuzzy systems, including fuzzy non-linear programming by Vanaja et al. (2024) [16] and integral equations over fuzzy bipolar spaces by Mani et al. (2023) [28]. Additionally, the computational aspects of fuzzy graphs, as explored by Pamucar et al. (2022) [19] and Liu et al. (2023) [24], highlight the relevance of fuzzy graph theory in addressing complex optimization problems [35]. These advancements [60, 62, 64, 66, 58]inspire further investigations into the human chain graph family under intuitionistic fuzzy labelling, filling a critical gap in the literature.

Several recent research have focused on human chain networks, including D-lucky edge labeling by Ram et al. (2021) [11] [57]], topological indices by Devi and Anitha (2020) [51], and dominance theory by Shanthi and Anitha (2022) [48]. Real-world network uncertainty can be more accurately modeled by using these labeling techniques to an intuitionistic fuzzy context. Anitha and Datta (2023) [33]found that fuzzy sets are beneficial in picture processing, indicating that similar techniques might improve network theory. This study expands on Murugesan's (2023) [41],work, which highlights the significance of intuitionistic fuzzy labeling in real-world applications..

This work aims to explore intuitionistic fuzzy magic labeling on human chain graph structures to enhance decision-making in uncertain systems and contribute to the field of fuzzy and intuitionistic graph labeling [26, 31, 36, 44, 52, 56] Using these approaches on human chain networks allows for better understanding and optimization of networks with less information [13, 15, 17, 21, 24]. This study, which bridges the gap between fuzzy labeling theory and intuitionistic fuzzy systems, [28, 34, 38, 47, 50, 54] provides the framework for future research in mathematical theory and practical network science.

This study bridges a gap in the field of fuzzy and intuitionistic fuzzy graph labeling by examining intuitionistic fuzzy magic labeling for structured graphs such as human chain networks. Traditional techniques in structured graph systems lack complete frameworks for dealing with uncertainty. This paper introduces a scalable technique for intuitionistic fuzzy labeling on human chain graphs, enhancing consistency, uniqueness, and optimization in uncertain contexts.

2 Intuitionistic fuzzy Bi-magic semi-labeling

Definition 2.1. An IFG is said to be Semi-labeling graph on Intuitionistic space if the non-membership and membership functions are bi-jective with all unique values for each vertex.

Definition 2.2. A semi-labeling graph is said to be a magic graph on intuitionistic space if

$$\sigma_m(v_i) + \mu_m(v_i, v_j) + \sigma_m(v_j) = \sigma_n(v_i) + \mu_n(v_i, v_j) + \sigma_n(v_j) \quad \text{for all } v_i, v_j \in V,$$

where $\sigma_m(v_i)$ and $\mu_m(v_i, v_j)$ represent the membership functions of vertices and edges of the intuitionistic fuzzy graph, while $\sigma_n(v_i)$ and $\mu_n(v_i, v_j)$ represent the non-membership functions of vertices and edges of the intuitionistic fuzzy graph.

Definition 2.3. A semi-labeling graph which admits a Bi-magic labeling on Intuitionistic space is called a Intuitionistic Fuzzy Bi-magic semi-labeling(IFBSL)

Definition 2.4. By connecting a Y-tree $Y_{(m+1)}$ with $m \leq 3$ and a cycle C_m to each u_{2i} for $1 \leq i \leq n$, we obtain a path $\{u_1, u_2, \ldots, u_{2n+1}\}$ that yields a human chain network $HC_{(n,m)}$. The vertices of the Y-tree $Y_{(m+1)}$ and the cycle C_m are labeled as $w_1, w_2, \ldots, w_{nm}$ and $v_1, v_2, \ldots, v_{n(m-1)}$, respectively.

Theorem 2.5. For $n \geq 2$ and $m \geq 3$, every Human Chain Graph admits Intuitionistic Fuzzy Bi-magic semi-labeling.

Proof. Let's define z in the interval $(0,1]$ such that: z = $\begin{cases} 0.1, & \text{if } n < 2 \\ 0.01, & \text{if } n \geq 2 \end{cases}$

Bi-magic Semi-labeling for Non-membership Functions of Vertices in $HC_{(n,m)}$:

$$\begin{aligned}
\sigma_m(u_{2i-1}) &= (3m+4-i)z, \quad 1 \leq i \leq n+1, \\
\sigma_m(v_{2i}) &= [2n(m-1)-(i-1)]z, \quad 1 \leq i \leq n, \\
\sigma_m(v_{2i-1}) &= \min\left\{\sigma_m(v_{2i}) \mid 1 \leq i \leq \frac{n(m-1)}{2}\right\} - iz, \quad 1 \leq i \leq n, \\
\sigma_m(w_{mi+j+1}) &= [4(m+i)+j+2]z, \quad 1 \leq j \leq m-1, \quad 0 \leq i \leq n, \\
\sigma_m(w_{mi-2}) &= \max\{\sigma_m(w_{mi}), \sigma_m(w_{mi-1})\} + iz, \quad 1 \leq i \leq n, \\
\sigma_m(u_{2i}) &= \min\{\sigma_m(w_{mi-2}), \sigma_m(u_{2i-1}), \sigma_m(u_{2i+1}), \sigma_m(v_{2i-1}), \sigma_m(v_{2i})\} - 4z, \quad 1 \leq i \leq n.
\end{aligned}$$

Bi-magic Semi-labeling for Non-membership Functions of Vertices in $HC_{(n,m)}$:

$$\begin{aligned}
\sigma_n(u_{2i-1}) &= (3m+2+2i)z, \quad 1 \leq i \leq n+1, \\
\sigma_n(v_{2i}) &= [2n(m-1)+2(i-1)]z, \quad 1 \leq i \leq n, \\
\sigma_n(v_{2i-1}) &= \min\left\{\sigma_n(v_{2i}) \mid 1 \leq i \leq \frac{n(m-1)}{2}\right\} - (i+1)z, \quad 1 \leq i \leq n, \\
\sigma_n(w_{mi+j+1}) &= [4(m+i)+j+5]z, \quad 1 \leq j \leq m-1, \quad 0 \leq i \leq n, \\
\sigma_n(w_{mi-2}) &= \max\{\sigma_n(w_{mi}), \sigma_n(w_{mi-1})\} + iz, \quad 1 \leq i \leq n, \\
\sigma_n(u_{2i}) &= \min\{\sigma_n(w_{mi-2}), \sigma_n(u_{2i-1}), \sigma_n(u_{2i+1}), \sigma_n(v_{2i-1}), \sigma_n(v_{2i})\} - 4z, \quad 1 \leq i \leq n.
\end{aligned}$$

Bi-magic Semi-labeling for Membership Functions of Edges in $HC_{(n,m)}$:

$$\begin{aligned}
\mu_m(w_{(mi-2)}, u_{2i}) &= \sigma_m(w_{(mi-2)}) + \sigma_m(u_{2i}) + [nm+2m-6i]z, \quad 1 \leq i \leq n, \\
\mu_m(u_{2i}, u_{(2i-1)}) &= \max\left\{\sigma_m(w_{(mi-2)}), \sigma_m(u_{(2i-1)}), \sigma_m(u_{(2i+1)}), \sigma_m(v_{(2i-1)}), \sigma_m(v_{2i})\right\} \\
&\quad - \min\left\{\sigma_m(w_{(mi-2)}), \sigma_m(u_{(2i-1)}), \sigma_m(u_{(2i+1)}), \sigma_m(v_{(2i-1)}), \sigma_m(v_{2i})\right\} \\
&\quad + [2n-3i+3]z, \quad 1 \leq i \leq n, \\
\mu_m(u_{2i}, u_{(2i+1)}) &= \max\left\{\sigma_m(w_{(mi-2)}), \sigma_m(u_{(2i-1)}), \sigma_m(u_{(2i+1)}), \sigma_m(v_{(2i-1)}), \sigma_m(v_{2i})\right\} \\
&\quad - \min\left\{\sigma_m(w_{(mi-2)}), \sigma_m(u_{(2i-1)}), \sigma_m(u_{(2i+1)}), \sigma_m(v_{(2i-1)}), \sigma_m(v_{2i})\right\} \\
&\quad + [2n-3i+4]z, \quad 1 \leq i \leq n, \\
\mu_m(u_{2i}, v_{2i}) &= \max\left\{\sigma_m(w_{(mi-2)}), \sigma_m(u_{(2i-1)}), \sigma_m(u_{(2i+1)}), \sigma_m(v_{(2i-1)}), \sigma_m(v_{2i})\right\} \\
&\quad - \min\left\{\sigma_m(w_{(mi-2)}), \sigma_m(u_{(2i-1)}), \sigma_m(u_{(2i+1)}), \sigma_m(v_{(2i-1)}), \sigma_m(v_{2i})\right\} \\
&\quad + [mn-3(i-1)+2]z, \quad 1 \leq i \leq n, \\
\mu_m(u_{2i}, v_{2i-1}) &= \max\left\{\sigma_m(w_{(mi-2)}), \sigma_m(u_{(2i-1)}), \sigma_m(u_{(2i+1)}), \sigma_m(v_{(2i-1)}), \sigma_m(v_{2i})\right\} \\
&\quad - \min\left\{\sigma_m(w_{(mi-2)}), \sigma_m(u_{(2i-1)}), \sigma_m(u_{(2i+1)}), \sigma_m(v_{(2i-1)}), \sigma_m(v_{2i})\right\} \\
&\quad + [mn-3(i-1)+4]z, \quad 1 \leq i \leq n, \\
\mu_m(v_{2i}, v_{(2i-1)}) &= \max\left\{\sigma_m(w_{(mi-2)}), \sigma_m(u_{(2i-1)}), \sigma_m(u_{(2i+1)}), \sigma_m(v_{(2i-1)}), \sigma_m(v_{2i})\right\} \\
&\quad - \min\left\{\sigma_m(w_{(mi-2)}), \sigma_m(u_{(2i-1)}), \sigma_m(u_{(2i+1)}), \sigma_m(v_{(2i-1)}), \sigma_m(v_{2i})\right\} \\
&\quad + [mn-3i+2]z, \quad 1 \leq i \leq n, \\
\mu_m(w_{mi}, w_{(mi-2)}) &= [3nm-8i]z, \quad 1 \leq i \leq n, \\
\mu_m(w_{(mi+j)}, w_{(mi+j+1)}) &= [3nm-8i+1]z, \quad 1 \leq i \leq n.
\end{aligned}$$

Bi-magic Semi-labeling for Non-membership Functions of edges of $HC_{(n,m)}$:

$$\begin{aligned}
\mu_n(w_{(mi-2)}, u_{2i}) &= \sigma_n(w_{(mi-2)}) + \sigma_n(u_{2i}) + [2n + 2m(i-1)]z, \quad 1 \le i \le n,\\
\mu_n(u_{2i}, u_{(2i-1)}) &= \max\{\sigma_n(w_{(mi-2)}), \sigma_n(u_{(2i-1)}), \sigma_n(u_{(2i+1)}), \sigma_n(v_{(2i-1)}), \sigma_n(v_{2i})\}\\
&\quad - \min\{\sigma_n(w_{(mi-2)}), \sigma_n(u_{(2i-1)}), \sigma_n(u_{(2i+1)}), \sigma_n(v_{(2i-1)}), \sigma_n(v_{2i})\}\\
&\quad + [2mn - 6i + 1]z, \quad 1 \le i \le n,\\
\mu_n(u_{2i}, u_{(2i+1)}) &= \max\{\sigma_n(w_{(mi-2)}), \sigma_n(u_{(2i-1)}), \sigma_n(u_{(2i+1)}), \sigma_n(v_{(2i-1)}), \sigma_n(v_{2i})\}\\
&\quad - \min\{\sigma_n(w_{(mi-2)}), \sigma_n(u_{(2i-1)}), \sigma_n(u_{(2i+1)}), \sigma_n(v_{(2i-1)}), \sigma_n(v_{2i})\}\\
&\quad + [2m(1-i) + 5]z, \quad 1 \le i \le n,\\
\mu_n(u_{2i}, v_{2i}) &= \max\{\sigma_n(w_{(mi-2)}), \sigma_n(u_{(2i-1)}), \sigma_n(u_{(2i+1)}), \sigma_n(v_{(2i-1)}), \sigma_n(v_{2i})\}\\
&\quad - \min\{\sigma_n(w_{(mi-2)}), \sigma_n(u_{(2i-1)}), \sigma_n(u_{(2i+1)}), \sigma_n(v_{(2i-1)}), \sigma_n(v_{2i})\}\\
&\quad + [2m(n+1-i)]z, \quad 1 \le i \le n,\\
\mu_n(u_{2i}, v_{(2i-1)}) &= \max\{\sigma_n(w_{(mi-2)}), \sigma_n(u_{(2i-1)}), \sigma_n(u_{(2i+1)}), \sigma_n(v_{(2i-1)}), \sigma_n(v_{2i})\}\\
&\quad - \min\{\sigma_n(w_{(mi-2)}), \sigma_n(u_{(2i-1)}), \sigma_n(u_{(2i+1)}), \sigma_n(v_{(2i-1)}), \sigma_n(v_{2i})\}\\
&\quad + [2mn - m(i-1) + 2]z, \quad 1 \le i \le n,\\
\mu_n(v_{2i}, v_{(2i-1)}) &= \max\{\sigma_n(w_{(mi-2)}), \sigma_n(u_{(2i-1)}), \sigma_n(u_{(2i+1)}), \sigma_n(v_{(2i-1)}), \sigma_n(v_{2i})\}\\
&\quad - \min\{\sigma_n(w_{(mi-2)}), \sigma_n(u_{(2i-1)}), \sigma_n(u_{(2i+1)}), \sigma_n(v_{(2i-1)}), \sigma_n(v_{2i})\}\\
&\quad + [mn - 6i + 8]z, \quad 1 \le i \le n,\\
\mu_n(w_{mi}, w_{(mi-2)}) &= [3nm - 8i - 1]z, \quad 1 \le i \le n,\\
\mu_n(w_{(mi+j)}, w_{(mi+j+1)}) &= [3nm - 8i]z, \quad 1 \le i \le n.
\end{aligned}$$

Now, we check whether these labeling satisfy Intuitionistic Fuzzy Bi-magic semi-labeling for all human chain graphs.
Let us split the vertices and edges of the human chain graph into four parts:

(i) Head with vertices $\{u_{2i}, v_i\}$ and edges $\{u_{2i}v_{2i}, u_{2i}v_i, v_iv_{i+1}\}$.

(ii) Hands with vertices $\{u_{2i+1}, u_{2i}\}$ and edges $\{u_iu_{i+1} \mid 1 \le i \le 2n\}$.

(iii) Legs with vertices $\{w_{m(i-1)+j} \mid 1 \le j \le 3, 1 \le i \le n\}$ and edges $\{w_{m(i-1)+1}w_{m(i-1)+2}, w_{m(i-1)+1}w_{m(i-1)+3} \mid 1 \le i \le n\}$.

(iv) Trunk with vertices $\{u_{2i}, w_{m(i-1)+1} \mid 1 \le i \le n\}$ and edges $\{u_{2i}w_{m(i-1)+1} \mid 1 \le i \le n\}$.

Magicness of Heads

To verify that heads of Human chain graph satisfies the Bi-magic Semi-labeling condition we prove that the magic value is same for odd and even cases of vertex index and edge index.

Magic value for Membership Function:

$$M_m(P) = \sigma_m(v_{2i}) + \mu_m(v_{2i}, v_{2i-1}) + \sigma_m(v_{2i-1})$$

When i is even:

$$\begin{aligned}
M_m(P) = &\left\{2[n(m-1)] - (2x-1)\right\} z \\
&+ \max\left\{z\sigma_m(w_{2mx-2}, \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\right\} \\
&- \min\left\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\right\} \\
&+ [mn + 2 - 3(2x)]z \\
&+ \min\left\{\sigma_m(v_{4x}) \;\middle|\; 1 \leq 2x \leq \frac{n(m-1)}{2}\right\} - 2xz \\
= &[3mn - 2n - 10x + 2]z \\
&+ \max\left\{\sigma_m(w_{2mx-2}, \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\right\} \\
&- \min\left\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\right\} \\
&+ \min\left\{\sigma_m(v_{4x}) \;\middle|\; 1 \leq 2x \leq \frac{n(m-1)}{2}\right\}
\end{aligned}$$

When i is odd:

$$\begin{aligned}
M_m(P) = &\left\{2[n(m-1)] - (2x-2)\right\} z \\
&+ \max\left\{z\sigma_m(w_{2mx-2}, \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\right\} \\
&- \min\left\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\right\} \\
&+ [mn + 2 - 3(2x-1)]z \\
&+ \min\left\{\sigma_m(v_{4x}) \;\middle|\; 1 \leq 2x \leq \frac{n(m-1)}{2}\right\} - 2xz \\
= &[3mn - 2n - 10x + 2]z \\
&+ \max\left\{\sigma_m(w_{2mx-2}, \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\right\} \\
&- \min\left\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\right\} \\
&+ \min\left\{\sigma_m(v_{4x}) \;\middle|\; 1 \leq 2x \leq \frac{n(m-1)}{2}\right\}
\end{aligned}$$

Magic value for Non-Membership Function:

$$M_n(P) = \sigma_n(v_{2i}) + \mu_n(v_{2i}, v_{2i-1}) + \sigma_n(v_{2i-1})$$

When i is even:

$$\begin{aligned}
M_n(P) = &\left\{2[n(m-1)] + 2x\right\} z \\
&+ \max\left\{z\sigma_n(w_{2mx-2}, \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\right\} \\
&- \min\left\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\right\} \\
&+ [mn - 2x]z \\
&+ \min\left\{\sigma_n(v_{4x}) \;\middle|\; 1 \leq 2x \leq \frac{n(m-1)}{2}\right\} - 2xz \\
= &\left\{3mn - 2n - 2x\right\} z \\
&+ \max\left\{\sigma_n(w_{2mx-2}, \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\right\} \\
&- \min\left\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\right\} \\
&+ \min\left\{\sigma_n(v_{4x}) \;\middle|\; 1 \leq 2x \leq \frac{n(m-1)}{2}\right\}
\end{aligned}$$

When i is odd:

$$\begin{aligned} M_n(P) &= \{2[n(m-1)] + 2x - 1\}\, z \\ &\quad + \max\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad - \min\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad + [mn - 2x + 1]z \\ &\quad + \min\left\{\sigma_n(v_{4x}) \;\middle|\; 1 \le 2x \le \frac{n(m-1)}{2}\right\} - 2xz \\ &= \{3mn - 2n - 2x\}\, z \\ &\quad + \max\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad - \min\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad + \min\left\{\sigma_n(v_{4x}) \;\middle|\; 1 \le 2x \le \frac{n(m-1)}{2}\right\} \end{aligned}$$

Hence, the magic value is the same in both cases.

Magicness of Hands

To verify that Hands of Human chain graph satisfies the Bi-magic Semi-labeling condition we prove that the magic value is same for odd and even cases of vertex index and edge index.

Magic value for Membership Function:

$$M_m(P) = \sigma_m(u_{2i}) + \mu_m(u_{2i}, u_{2i-1}) + \sigma_m(u_{2i-1})$$

When i is even:

$$\begin{aligned} M_m(P) &= \{3m + 4 - 2x\}\, z \\ &\quad + \max\{\sigma_m(w_{2mx-2}, \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} \\ &\quad - \min\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} \\ &\quad + [2n - 6x + 3]z \\ &\quad + \min\left\{\sigma_m(v_{4x}) \;\middle|\; 1 \le 2x \le \frac{n(m-1)}{2}\right\} - 3z \\ &= \{3m + 2n - 8x + 4\}\, z \\ &\quad + \max\{\sigma_m(w_{2mx-2}, \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} \\ &\quad - \min\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} \\ &\quad + \min\left\{\sigma_m(v_{4x}) \;\middle|\; 1 \le 2x \le \frac{n(m-1)}{2}\right\} \quad \text{for } 1 \le i \le n \end{aligned}$$

When i is odd:

$$\begin{aligned} M_m(P) &= \{3m + 4 - 2x - 1\}\, z \\ &\quad + \max\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} \\ &\quad - \min\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} \\ &\quad + [2n - 6x - 3]z \\ &\quad + \min\left\{\sigma_m(v_{4x}) \;\middle|\; 1 \le 2x \le \frac{n(m-1)}{2}\right\} - 2xz \end{aligned}$$

$$\begin{aligned} &= \{3m + 2n - 8x + 4\}\, z \\ &\quad + \max\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} \\ &\quad - \min\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} \\ &\quad + \min\left\{\sigma_m(v_{4x}) \;\middle|\; 1 \le 2x \le \frac{n(m-1)}{2}\right\} \quad \text{for } 1 \le i \le n. \end{aligned}$$

Magic value for Non-Membership Function:

$$M_m(P) = \sigma_m(u_{2i}) + \mu_m(u_{2i}, u_{2i-1}) + \sigma_m(u_{2i-1})$$

When i is odd:

$$\begin{aligned} M_n(P) &= \{(3m+2+4x+2)\}\, z \\ &\quad + \max\{\sigma_n(w_{2mx-2}, \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad - \min\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad + [2nm - 12x - 6 + 1]z \\ &\quad + \min\left\{\sigma_n(v_{4x}) \;\middle|\; 1 \le 2x \le \frac{n(m-1)}{2}\right\} - 2xz \\ &= \{[2nm + 3m - 8x - 1]z\}\, z \\ &\quad + \max\{\sigma_n(w_{2mx-2}, \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad - \min\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad + \min\left\{\sigma_n(v_{4x}) \;\middle|\; 1 \le 2x \le \frac{n(m-1)}{2}\right\} \end{aligned}$$

When i is even:

$$\begin{aligned} M_n(P) &= \{3m+2+4x\}\, z \\ &\quad + \max\{\sigma_n(w_{2mx-2}, \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad - \min\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad + [2nm - 12x + 1]z \\ &\quad + \min\left\{\sigma_n(v_{4x}) \;\middle|\; 1 \le 2x \le \frac{n(m-1)}{2}\right\} - 2xz \\ &= \{[2nm + 3m - 8x - 1]z\}\, z \\ &\quad + \max\{\sigma_n(w_{2mx-2}, \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad - \min\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} \\ &\quad + \min\left\{\sigma_n(v_{4x}) \;\middle|\; 1 \le 2x \le \frac{n(m-1)}{2}\right\} \end{aligned}$$

Hence, the magic value is the same in both cases.

Magicness of Legs

To verify that Legs of Human chain graph satisfies the Bi-magic Semi-labeling condition we prove that the magic value is same for odd and even cases of vertex index and edge index.

Magic value for Membership Function:

When i is even:

$$\begin{aligned} M_m(P) &= \sigma_m(w_{mi-2}) + \mu_m(w_{mi-2}, w_{mi-1}) + \sigma_m(w_{mi-1}) \\ &= \min\{\sigma_m(w_{2mx}), \sigma_m(w_{2mx-1})\} - 2xz; \quad 1 \le i \le n \\ &\quad + [3nm - 16x]z + [4(m+2x)]z \\ &= \min\{\sigma_m(w_{2mx}), \sigma_m(w_{2mx-1})\} + [3mn + 4m - 10x]z \end{aligned}$$

When i is odd:

$$\begin{aligned} M_m(P) &= \sigma_m(w_{mi-2}) + \mu_m(w_{mi-2}, w_{mi-1}) + \sigma_m(w_{mi-1}) \\ &= \min\{\sigma_m(w_{2mx}), \sigma_m(w_{2mx-1})\} - (2x-1)z; \quad 1 \le i \le n \\ &\quad + [3nm - 16x + 8]z + [4(m + 2x - 1)]z \\ &= \min\{\sigma_m(w_{2mx}), \sigma_m(w_{2mx-1})\} + [3mn + 4m - 10x]z \end{aligned}$$

Magic value for Non-Membership Function:
When i is even:

$$\begin{aligned} M_m(P) &= \sigma_n(w_{mi-2}) + \mu_n(w_{mi-2}, w_{mi-1}) + \sigma_n(w_{mi-1}) \\ &= \min\{\sigma_n(w_{2mx}), \sigma_n(w_{2mx-1})\} - 2xz; \quad 1 \le i \le n \\ &\quad + [4(m + 2x) + 3]z + [nm(2 - 2x) + 1]z \\ &= \min\{\sigma_n(w_{2mx}), \sigma_n(w_{2mx-1})\} + [2nm - 2mnx + 10x + 4m + 3]z \end{aligned}$$

When i is odd:

$$\begin{aligned} M_m(P) &= \sigma_n(w_{mi-2}) + \mu_n(w_{mi-2}, w_{mi-1}) + \sigma_n(w_{mi-1}) \\ &= \min\{\sigma_n(w_{2mx}), \sigma_n(w_{2mx-1})\} - (2x+1)z; \quad 1 \le i \le n \\ &\quad + [4(m + 2x + 1) + 3]z + [nm(2 - 2x - 1) + 1]z \\ &= \min\{\sigma_n(w_{2mx}), \sigma_n(w_{2mx-1})\} + [2nm - 2mnx + 10x + 4m + 3]z \end{aligned}$$

Hence, the magic value is the same in both cases.

Magicness of Trunk

To verify that Trunk of Human chain graph satisfies the Bi-magic Semi-labeling condition we prove that the magic value is same for odd and even cases of vertex index and edge index.

Magic value for Membership Function:
When i is even:

$$\begin{aligned} M_m(P) &= \sigma_m(w_{mi-2}) + \mu_m(w_{mi-2}, u_{2i}) + \sigma_m(u_{2i}) \\ &= \min\{\sigma_m(w_{2mx}), \sigma_m(w_{2mx-1})\} + (2x)z; \quad 1 \le i \le n \\ &\quad + \sigma_m(w_{mi-2}) + \sigma_m(u_{2i}) + [nm + 2m - 12x]z \\ &\quad + \min\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} - 3z; \quad 1 \le 2x \le n \\ &= \min\{\sigma_m(w_{2mx}), \sigma_m(w_{2mx-1})\} + \sigma_m(w_{mi-2}) + \sigma_m(u_{2i}) \\ &\quad + [nm + 2m - 10x - 3]z \\ &\quad + \min\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} - 3z; \quad 1 \le 2x \le n \end{aligned}$$

When i is odd:

$$\begin{aligned} M_m(P) &= \sigma_m(w_{mi-2}) + \mu_m(w_{mi-2}, u_{2i}) + \sigma_m(u_{2i}) \\ &= \min\{\sigma_m(w_{2mx}), \sigma_m(w_{2mx-1})\} + (2x)z; \quad 1 \le i \le n \\ &\quad + \sigma_m(w_{mi-2}) + \sigma_m(u_{2i}) + [nm + 2m - 12x - 6]z \\ &\quad + \min\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} - 3z; \quad 1 \le 2x \le n \\ &= \min\{\sigma_m(w_{2mx}), \sigma_m(w_{2mx-1})\} + \sigma_m(w_{mi-2}) + \sigma_m(u_{2i}) \\ &\quad + [nm + 2m - 10x - 3]z \\ &\quad + \min\{\sigma_m(w_{2mx-2}), \sigma_m(u_{4x-1}), \sigma_m(u_{4x+1}), \sigma_m(v_{4x-1}), \sigma_m(v_{4x})\} - 3z; \quad 1 \le 2x \le n \end{aligned}$$

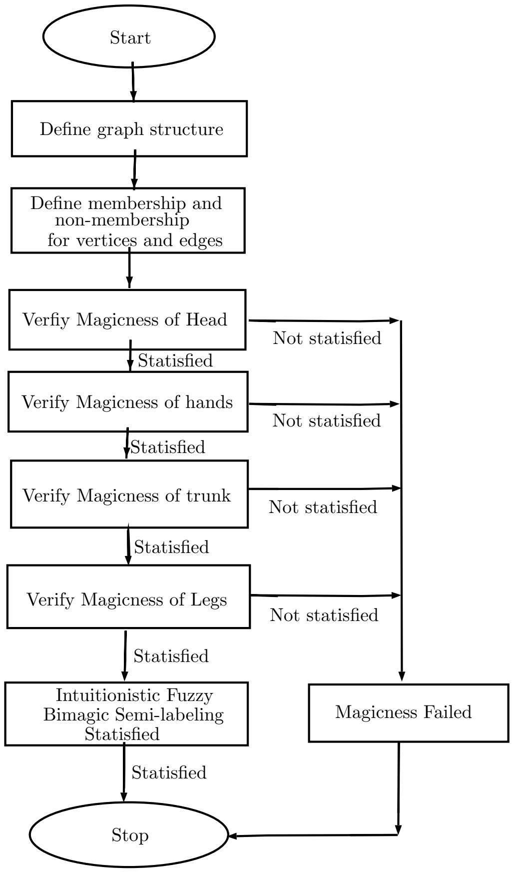

Figure 1. Verification of Intuitionistic Fuzzy Bimagic Semi-labeling of $HC_{m,n}$

Magic value for Non-Membership Function:
When i is even:

$$\begin{aligned} M_n(P) &= \sigma_n(w_{mi-2}) + \mu_n(w_{mi-2}, u_{2i}) + \sigma_n(u_{2i}) \\ &= \min\{\sigma_n(w_{2mx}), \sigma_n(w_{2mx-1})\} + (2x)z + \sigma_n(w_{mi-2}) + \sigma_n(u_{2i}) + [nm + 2m - 12x]z \\ &\quad + \min\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} - 3z; \quad 1 \leq 2x \leq n \\ &= \min\{\sigma_n(w_{2mx}), \sigma_n(w_{2mx-1})\} + \sigma_n(w_{mi-2}) + \sigma_n(u_{2i}) + [nm + 2m - 10x - 3]z \\ &\quad + \min\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} - 3z; \quad 1 \leq 2x \leq n \end{aligned}$$

When i is odd:

$$\begin{aligned} M_n(P) &= \sigma_n(w_{mi-2}) + \mu_n(w_{mi-2}, u_{2i}) + \sigma_n(u_{2i}) \\ &= \min\{\sigma_n(w_{2mx}), \sigma_n(w_{2mx-1})\} + (2x)z; \quad 1 \leq i \leq n \\ &\quad + \sigma_n(w_{mi-2}) + \sigma_n(u_{2i}) + [nm + 2m - 12x - 6]z \\ &\quad + \min\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} - 3z; \quad 1 \leq 2x \leq n \\ &= \min\{\sigma_n(w_{2mx}), \sigma_n(w_{2mx-1})\} + \sigma_n(w_{mi-2}) + \sigma_n(u_{2i}) + [nm + 2m - 10x - 3]z \\ &\quad + \min\{\sigma_n(w_{2mx-2}), \sigma_n(u_{4x-1}), \sigma_n(u_{4x+1}), \sigma_n(v_{4x-1}), \sigma_n(v_{4x})\} - 3z; \quad 1 \leq 2x \leq n \end{aligned}$$

Hence, the magic value is the same in both cases.

Therefore, the Human chain graph admits Intuitionistic fuzzy bi-magic semi-labeling.

□

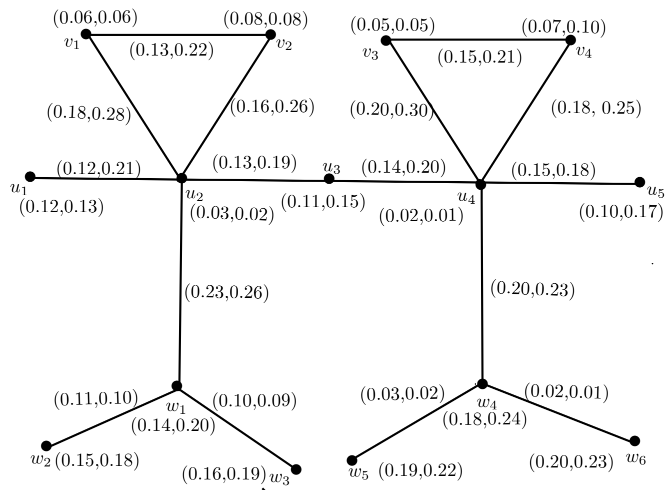

Figure 2. Intuitionistic Fuzzy Human Chain Graph
Here, the intuitionistic fuzzy magic number of the head and hands is (0.27, 0.36), and for the trunk and legs, it is (0.40, 0.48). Hence, $HC_{(2,3)}$ is an intuitionistic fuzzy bi-magic graph.

3 Intuitionistic fuzzy Bi-magic semi-labeling for Circular Human Chain Graph

Definition 3.1. A circular human chain graph $CHC_{(n,m)}(p, q)$ is obtained by joining a Y-tree $Y_{(m+1)}$ (where $m \geq 3$) and a cycle of length m (denoted C_m). For $1 \leq i \leq n$, each u_{2i} (where $1 \leq i \leq n$) belongs to a circle formed by the vertices $u_1, u_2, \ldots, u_{2n}, u_1$, with $n \geq 3$.

Theorem 3.2. For $n \geq 3$ and $m \geq 3$, every Circular Human Chain Graph admits Intuitionistic Fuzzy Bimagic Semi-Labeling.

Proof. The circular human chain graph is obtained by merging vertices u_1 and u_{2n+1} in the human chain graph. Hence, the Intuitionistic fuzzy semi-labeling for the remaining vertices and edges remains the same, and so the proof is obvious from the above theorem. □

4 Intuitionistic fuzzy Bi-magic semi-labeling for Strong Human chain graph

Definition 4.1. The strong human chain graph $SHC_{n,m}, n > 1$ and $m \geq 3$ is obtained from Human chain network by merging the vertex w_{mi} and $w_{m(i+1)-1}$;$1 \leq i \leq n-1$ with common vertices in Y-tree. .

Theorem 4.2. For $n \geq 2$ and $m \geq 3$, every strong human chain graph admits an Intuitionistic Fuzzy Bi-magic semi-labeling.

Proof. Strong human chain graph is obtained by merging the vertex w_{mi} and $w_{m(i+1)-1}$ in human chain graph. So, we redefine the labeling for legs alone, Intuitionistic fuzzy semi-labeling for remaining vertices and edges will be the same as mentioned in human chain graph.
Let's define z in the interval (0,1] such that

$$z = \begin{cases} 0.1 & \text{if } n < 2, \\ 0.01 & \text{if } n \geq 2 \end{cases}$$

Bi-magic Semi-labeling for membership function of vertices in $SHC_{(n,m)}$:

$$\sigma_m(w_{2i-1}) = [4m + 2i]z; \quad 1 \leq i \leq m,$$
$$\sigma_m(w_{2i}) = [4m + 2i + 1]; \quad 1 \leq i \leq m,$$

Bi-magic Semi-labeling for non-membership function of vertices in $SHC_{(n,m)}$:

$$\sigma_n(w_{2i-1}) = [5m + 2i + 1]z; \quad 1 \le i \le m,$$
$$\sigma_n(w_{2i}) = [5m + 2i + 2]; \quad 1 \le i \le n,$$

Bi-magic Semi-labeling for membership function of edges in $SHC_{(n,m)}$:

$$\mu_m(w_{2i-1}, w_{2i}) = [2m - 4i + 4]z,$$
$$\mu_m(w_{2i+1}, w_{2i}) = [nm - 3i + 1]z.$$

Bi-magic Semi-labeling for non-membership function of edges in $SHC_{(n,m)}$:

$$\mu_n(w_{2i-1}, w_{2i}) = [4m - 4i]z,$$
$$\mu_n(w_{2i-1}, w_{2i}) = [3m - 4i + 1]z.$$

Magicness of legs
To verify that Legs of Strong Human chain graph satisfies the Bi-magic Semi-labeling condition we prove that the magic value is same for odd and even cases of vertex index and edge index.
Magic value for the membership function:
When i is even:

$$\begin{aligned} M_m(P) &= \sigma_m(w_{4x-1}) + \mu_m(w_{4x-1}, w_{4x}) + \sigma_m(w_{4x}) \\ &= [4m + 4x]z + [4m + 4x + 1]z + [2m - 8x + 4] \\ &= [10m + 5]z. \end{aligned}$$

When i is odd:

$$\begin{aligned} M_m(P) &= \sigma_m(w_{4x}) + \mu_m(w_{4x}, w_{4x+1}) + \sigma_m(w_{4x+1}) \\ &= [4m + 4x + 2]z + [4m + 4x + 2 + 1]z + [2m - 8x - 4 + 4] \\ &= [10m + 5]z. \end{aligned}$$

Magic value for the non-membership function:
When i is even:

$$\begin{aligned} M_n(P) &= \sigma_n(w_{4x-1}) + \mu_n(w_{4x-1}, w_{4x}) + \sigma_n(w_{4x}) \\ &= [5m + 4x + 1]z + [4m - 8x]z + [5m + 4x + 2]z \\ &= [14m + 3]z. \end{aligned}$$

When i is odd:

$$\begin{aligned} M_n(P) &= \sigma_n(w_{4x+1}) + \mu_n(w_{4x+1}, w_{4x}) + \sigma_n(w_{4x}) \\ &= [5m + 4x + 2 + 1]z + [4m - 8x - 4]z + [5m + 4x + 2 + 2]z \\ &= [14m + 3]z. \end{aligned}$$

Hence, the magic value is the same in both cases.

□

The intuitionistic fuzzy bi-magic labeling approach is adaptable to many types of human chain graphs and does not require any adjustments. Merging vertices w_{mi} and $w_{m(i+1)-1}$ in the strong human chain network did not affect the labeling approach's attributes.Bi-magic labeling maintained its integrity when shifting from linear to circular chain structures, proving its versatility across structural changes. All human chain graphs studied (standard, circular, and strong) met the intuitionistic fuzzy bi-magic labeling characteristics. Unique and consistent membership and non-membership values were assigned to each vertex v_j and edge e_{ij} using a bi-jective

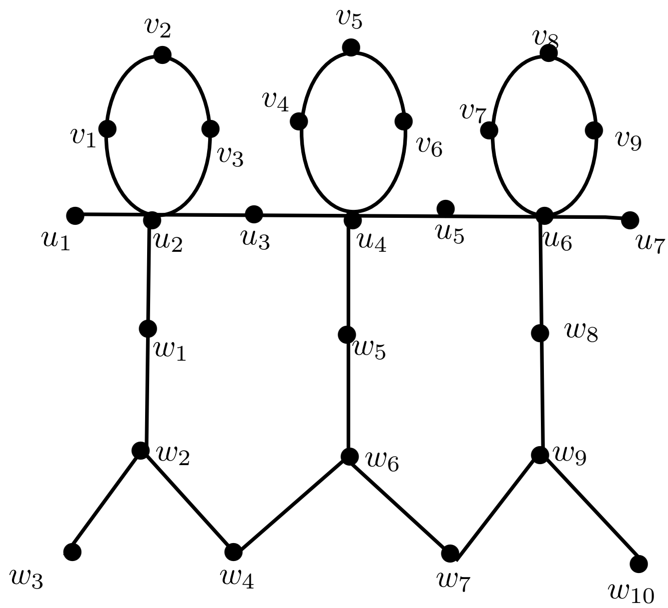

Figure 3. Strong Human Chain Graph $SHC_{3,4}$

approach.The bi-magic values were stable across different types of human chain graphs. The bi-magic labeling method's consistency across graph types demonstrates its durability and ability to preserve label integrity in many setups. The streamlined computation of magic values for membership and non-membership functions simplifies the labeling procedure. The function computations for head, hands, legs, and trunk segments are stable and need minimum recalculations as graph size grows, demonstrating efficient scaling.

5 Applications of Intuitionistic Fuzzy Bi-magic Semi-labeling in Healthcare Systems

To handle complex interactions between entities in modern healthcare, effective and dynamic decision-making and resource management systems are required.The Intuitionistic Fuzzy Bi-magic Semi-labeling Approach for the Human Chain Graph $HC_{(2,3)}$ offers a framework for correctly defining interactions.The framework efficiently balances membership (trust in interactions) with non-membership (ambiguity), making it suitable for circumstances involving uncertainty and various levels of engagement.This section applies Theorem 2.5 in healthcare to enhance patient care, staff allocation, and resource management.

5.1 Patient Care Optimization

In a hospital, patients go through several phases of therapy, which include several procedures and encounters with clinical human resources. In the Human Chain Graph, the path vertices u_1, u_2, u_3, u_4, u_5 represent patients at various phases, whereas the cycle vertices v_1, v_2, v_3, v_4 represent medical resources (e.g., equipment, pharmaceuticals) (referring to Figure 2).The Y-tree vertices $w_1, w_2, ..., w_6$ indicate healthcare workers (e.g., physicians, nurses).
For a patient represented by vertex u_2 undergoing a surgical procedure, the participation of a surgeon (w_1) and a helping nurse (w_3) is portrayed using membership and non-membership values. The model minimizes procedure delays by guaranteeing that $\sigma_m(w_1)+\mu_m(w_1, w_3)+\sigma_m(w_3) = 0.27$ and $\sigma_n(w_1) + \mu_n(w_1, w_3) + \sigma_n(w_3) = 0.36$.

- **Consistency in Treatment:** :Labeling clinical staff (head and hands) with equal magic numbers (0.27,0.36) gives consistency and balance in patient treatment.
- **Reduced Treatment Delays:** The model's balance of membership and non-membership

amounts identifies disparities and delays in the treatment process, allowing for timely reallocation of people and resources.

5.2 Staff Allocation and Workload Balancing:

Efficient healthcare personnel allocation is crucial for ensuring high-quality patient care, particularly in big institutions with several departments. The Human Chain Graph's Y-tree model portrays staff as vertices, with edges representing interactions with patients and resources. The theorem assures that all edges meet the bi-magic labeling criteria. Consider the following situation where two nurses (w_2 and w_5) provide assistance in different departments.
Consider a scenario where two nurses(w_2 and w_5) assist in different departments.
The equation $\sigma_m(w_1) + \mu_m(w_1, w_3) + \sigma_m(w_3) = 0.27$ provides equal participation in patient care activities, whereas $\sigma_n(w_1) + \mu_n(w_1, w_3) + \sigma_n(w_3) = 0.36$, incorporates for uncertainties like shift changes and emergencies. This allows them to efficiently balance their duties.

- **Workload Balance:** This strategy maintains consistent magic numbers for clinical staff encounters, reducing tiredness and improving overall performance.
- **Dynamic Reallocation:** In case of unexpected changes (e.g., a sudden increase in patient admissions), the intuitionistic fuzzy labeling allows for real-time adjustments in staff assignments based on updated non-membership and membership values.

5.3 Resource Allocation and Utilization:

Effective use of medical resources (such as surgical equipment, ICU beds, and medications) is critical in hospital management. resources are represented by the Human Chain Graph's cycle. Theorem's bi-magic labeling optimizes resource allocation for patients.
In a surgical operation including an ICU bed (vertex v_1) and a ventilator (vertex v_3), the total $\sigma_m(v_1) + \mu_m(v_1, v_3) + \sigma_m(v_3) = 0.40$ and $\sigma_n(v_1) + \mu_n(v_1, v_3) + \sigma_m(v_3) = 0.48$ accommodate for uncertainties such equipment failure or increasing demand. .

- **Optimal Utilization:**The model maintains magic numbers (0.40,0.48) for resources (trunk and legs) to prevent misuse or under-use of medical equipment.
- **Reduction of Wastage:** Balanced labeling reduces waste by recognizing resource allocation inconsistencies.

5.4 Real-time Monitoring and Dynamic Adjustment

Hospital procedures require regular evaluations and improvements in order to meet changing situations, such as patient crises or personnel shortages. The Intuitionistic Fuzzy Bi-magic Semi-labeling model enables real-time decision-making by adjusting membership and non-membership values based on current hospital state. If an ICU bed (vertex v_2) is unavailable since it requires maintenance, the non-membership value $\sigma_m v_2$ is raised to account for the uncertainty. This modification re-evaluates the edges connected to v_2, letting the hospital system to provide alternative resources without affecting patient care quality.

- **Proactive Decision-making:** This approach lets administrators in hospitals identify possible difficulties earlier in order to make intelligent choices.
- **Adaptive Resource Management:** Bi-magic labeling permits flexible modifications to patient care and resource allocation, without affecting hospital procedures.

Intuitionistic Fuzzy Bi-magic Semi-labeling (Theorem 2.5) offers a flexible framework for optimizing operations in complex systems, including healthcare. This strategy improves patient outcomes by modeling uncertainty and balancing interactions between patients, healthcare personnel, and resources. It is particularly useful in dynamic and unpredictable contexts, such as hospitals, where accuracy and agility are essential for success.
The Human Chain Graph (HCm,n) structure, which represents healthcare operations intuitively, utilizes intuitionistic fuzzy labeling. This combination optimizes resource allocation and patient flow, opening the path for sophisticated decision-support systems in healthcare management.

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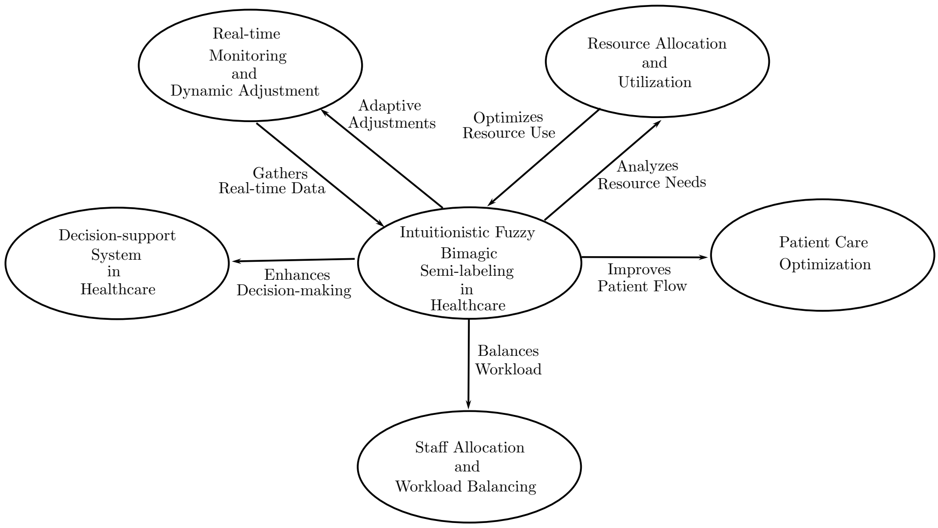

Figure 4. Applications of Intuitionistic Fuzzy Bimagic Semi-labeling in Healthcare

Theorem 2.5 provides a balanced approach to managing trust outside of healthcare settings. These examples demonstrate the adaptability and efficacy of intuitionistic fuzzy magic tagging in addressing complicated issues across several areas. This strategy optimizes resources, trust, and relationships in dynamic and complex situations by handling uncertainty and ensuring balanced distributions. The method improves efficiency, stability, and reliability in areas such as cloud resource optimization, healthcare system management, and blockchain network security.

Conclusion

This research uses intuitionistic fuzzy semi-labeling to investigate the unique characteristics of circular, human, and strong human chain graphs. We developed intuitive fuzzy magic labeling, which extends existing graph labeling approaches by including membership and non-membership degrees to better depict uncertainty. This labeling technique effectively addresses complex real-world difficulties by offering an architecture for decision-making that balances multiple variables and controls uncertainty in dynamic situations, as demonstrated by the examples mentioned. Our findings indicate that it has the potential to optimize processes in several domains, including communication networks, energy management, cloud computing, healthcare, environmental monitoring, cryptography, marketing, and social systems.

Future Work

Our aim is to reduce the computational cost of the intuitionistic fuzzy bi-magic labeling approach, particularly as network sizes increase. Our focus will be on optimizing algorithms and heuristics to improve model scalability for bigger networks. We plan to extend the model's application to additional graph architectures, including bipartite and tree-based networks, to widen its utility across disciplines. We want to enhance the model's flexibility to dynamic contexts, such as load balancing in smart grids or controlling variable risks in blockchain networks, by using adaptive or dynamic membership functions instead of fixed values. These additions improve the model's scalability, versatility, and resilience, making it suited for more real-world applications.

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Author information

J. Komathi, Department of Mathematics, Sri Sairam Engineering college, India.
E-mail: komathi.math@sairam.edu.in

D.Meiyappan, Department of Mathematics, Sri Venkateswara College of Engineering, India.
E-mail: mei@svce.ac.in

K.Anitha, Department of Mathematics, Amirtha school of engineering, Amirta Vishwa Vidyapeetham,Chennai., India.
E-mail: k_anitha@ch.amirta.edu

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