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Enhancing Sustainable Practices through Pattern Detection, Clustering, and Green Supply Chain Management Using Spherical Fuzzy Correlation Coefficients

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Abstract: Decision makers have more freedom to convey their evaluation data using the spherical fuzzy set than the picture fuzzy set. In real-world problems with more than two alternative responses, such as accept, decline, abstain, and yes, it is particularly successful at handling ambiguity and uncertainty. It is a simple extension of picture fuzzy sets. One way to determine the link between two spherical fuzzy sets is the correlation coefficient of spherical fuzzy sets. This study presents a new correlation coefficient for spherical fuzzy sets that is based on direct operations on the functions of neutrality, membership, non-membership, and refusal. It yields results that are genuine, accurate, and non-repeating, in contrast to current measures. Among all the metrics, this new method produces results with the highest degree of confidence and can solve the pattern detection problem. It is also used to demonstrate how it may be used to cluster real flood data from Indian states, resulting in validated clusters. An ISO-certified agri-food company uses it in a case study to identify the top edible oil provider. Every result is contrasted with the results of current measures. Of all the techniques now in use, this one proved to be the most trustworthy, efficient, valid, and practical.

1 Introduction

The mathematician Zadeh [1] developed the idea of fuzziness and imprecise information at a time when everyone was only aware of real numbers and the classical set. With the idea of the degree of membership (DoM), he introduced a new theory of fuzzy sets (FSs). Then, the concept was broadened with the addition of the degrees of non-membership ($DoNM$), rejection (DoR), and neutrality (ND). Other researchers then developed further (FS) extensions. Intuitionistic fuzzy set (IFS) containing $DoNM$ and DoR with DoM in it, with the condition that their sum must be equal to one, was introduced by Atanassov [2, 3]. This fuzzy set had a bigger impact than FS. A Pythagorean fuzzy set (P_YFS) was similarly developed by Yager [4] to expand the area of IFS, with the requirement that the squared sum of these three degrees equal one. Once again, many scholars [5, 6] have worked on this topic to develop many new notions and extend them in the form of interval [0,1]. Within the same domain, IFS is extended as a neutrosophic fuzzy set [7], stipulating that the total of these three degrees must be less than or on par with three. Each of the three degrees in this set must be less than or equal to one. Further, the picture fuzzy set (PFS) [8, 9] is created. DoM, ND, $DoNM$, and DoR make up this set, provided that the total of their degrees equals one. After that a new concept called spherical fuzzy set ($\mathbb{SFS}$) [10], [11], [12], and [13] is presented in order to expand the area of this set once more, provided that the squared sum of these four degrees must be equal to one.

Fuzzy mathematics includes important concepts like the measure of correlation coefficient ($\mathbb{CC}$), similarity, and distance between two fuzzy concepts. Researchers are interested in fuzzy mathematics because of its broad applications in risk analysis, pattern detection, machine learning, real-time decision-making, and market prediction, among other areas. Various studies used various compatibility/comparison metrics, depending on the type of issue. Saeed used FSs to solve graphical problems[14]. Numerous researchers [15–17] worked under fuzzy environment to deal with algebraic structures. Numerous similarity measures(SMs) among $IFSs$ have been examined by Li et al. [18]. A good SM between $IFSs$ was suggested by Dengfeng et al. [19] for pattern detection problems. Furthermore, this protocol was changed in [20]. A SM between $IFSs$ and hamming distance based on hamming distance on fuzzy sets was created Szmidt [21, 22]. Hung [23] computed the hausdorff distance to suggest a novel SM between $IFSs$. Once more, utilizing the Lp metric, Hung and Yang [24] presented a novel SM between $IFSs$. For IFS, Xia [25] defined the geometric DM and SM. Cosine SM was defined in [26], [27] between IFS. Maoying [28] investigated cotangent SM for identical sets in order to make medical diagnoses. In addition, health problems and a SM bacteria classification issue [29] were also identified. Additionally, using the cotangent function as a basis, SM [30] is created, which includes all three of the IFS degrees and is used to solve engineering issues. To address medical difficulties, Son et al. [31] proposed the intuitionistic vector SM. Furthermore, a linear optimization model [32] is created and a hamming distance-based strategy for IFS to address decision-making challenges is suggested. Patel et al. [33] suggested SMs as a solution for the problems of software quality evaluation and face recognition. Lately, a $\mathbb{CC}$ [34] for IFS was built and used to address clustering, taxonomy, and medical difficulties. A TOPSIS based on $\mathbb{CC}$ and its weighted version under the auspices of IF hypersoft expert sets were created by Ihsan [35]. Kousar et al. [36] developed this fuzzy optimization approach to redesign the viable zone acquired from linear programming and presented it in graphical form. It gave a planned or strategy tool for dealing with uncertain situations in a more controlled and realistic manner. Kousar et al. [37] built a fully intuitionistic fuzzy textile energy model and utilized it to calculate the optimal distribution of solar energy units, which resulted in a manageable proportion of unused energy units. Once again, Kousar et al. [38] created two objective functions with triangular fuzzy numbers. The primary goal is to reduce the assembly costs of forward and reverse logistics. Second, efforts have been undertaken to reduce the fixed costs connected with plants and retailers.

Numerous studies have been conducted in this expanded field as P_YFS is recognized as an extension of IFS. SMs were used by Kumar [39] to address the transportation problem in a wider area of the P_YF environment. Boora [40] used similarity methods to solve edge detection, image processing, and medical diagnosis issues. Furthermore, some SMs were used to solve the transportation issue [41]. In a similar spirit, the distance measure is another effective tool in a P_YF setup for managing different problems. Zhou et al. [42] employed the TOPSIS approach to tackle decision-making challenges in a P_YF context by employing their modified DM. By using novel distance and similarity measures to pattern detection and decision-making challenges, Ejegwa [43, 44] explained the applicability and validity of these measures. Analogously, knowledge and distance measures [45, 46] were used to address multi-attribute decision-making challenges in a fuzzy context. Again, Ejegwa [47] presented level sets of P_YFSs and P_YF pairings. Comparing the candidates and positions (both in P_YF pairs/values) is how he chooses the best applicants for available positions. Case studies of pattern detection are presented together with the effects of several parameters on pattern ordering and classification by Peng [48]. Similarity measurements are also introduced. Again, similarity measures were employed to handle decision-making challenges such as employment placement, medical diagnosis, and electioneering process [49]. Gerstenkorn [50] established the correlation coefficient ($\mathbb{CC}$), which is a helpful metric for determining how interdependent $IFSs$ are. Xu et al. [51] proposed the $\mathbb{CC}$, which is utilized to solve pattern recognition and medical diagnostics challenges. Furthermore, a new $\mathbb{CC}$ [52], which looked like the cosine of the angle at where a probability space and a finite set intersect was introduced. After that, Ejegwa [44] used his methods once more to solve decision-making challenges and fix previous flaws. He created a better composite relation for P_YFS and compared it to the max-min-max strategy in [53]. A revised formula that takes into consideration covariance, variance, and deviation was then provided in [43]. In an effort to quantify the magnitude and intensity of the link and apply it to career placement issues. Furthermore, to support the credibility of the formula, a $\mathbb{CC}$ was put forth in [54] and used to resolve decision-making challenges. This formula's inability to replicate the outcome for multiple distinct sets was still another drawback. Then, a directional $\mathbb{CC}$ and its weighted form based on four parameters: DoM, $DoNM$, commitment intensity, and commitment direction were presented by Lin and Chen [55]. Chen [56] then illustrated and gave his approach to multi-criteria decision analysis using Pearson-like $\mathbb{CC}$. Furthermore, in order to address pattern detection issues, researchers [57–59] used a novel $\mathbb{CC}$ based on variance and covariance.

Using the right measure from DM, SM, and $\mathbb{CC}$, it is very important to deal with many practical problems such as risk management, pattern detection, image detection, and picking the best choice among multiple in a PF environment. Cuong [8] was the first to provide the $PFSs$ euclidean and hamming distances. Further, the cosine function was used to suggest eight distinct types of SM [60] for PFS, and he demonstrated their effectiveness by using them to solve an optimal production problem. Additionally, Wei [61] suggested using SM and its weighted version, as well as gray and its weighted version, to solve identification challenges for building materials and mineral fields. Singh and others [62] provided a geometric interpretation of PFS and utilized their DM and SM to assess the likelihood of a flood disaster. Ganie [63] developed a SM to address decision-making challenges based on the upper and lower boundaries of DoM, $DoNM$, ND, and DoR. In the PF context, Dice SM for pattern detection was used by Wei [64]. In addition to developing their unique SM and the maximum spanning tree clustering technique, Singh and others [65] carried out additional research. PF geometric operator [66] was again used to solve a variety of decision-making challenges. In his research, Opricovic [67] talked about the benefits of the VIKOR approach. To gauge how satisfied residents were with municipal services [68], the VIKOR approach and the $PFNs$ algorithm operator were employed effectively. Under the PF context, two new DMs based on Chi-square and Canberra were defined by Verma [69]. He presented Canberra picture fuzzy SMs between $PFSs$ and Chi-square PF similarity between DMs and SMs using the relationship between them. He used the same, weighted version to address decision-making and medical challenges. To address decision-making challenges, [70] defined the entropy measure and SM in the same setting. Based on the PF point operator, Zhao et. al [71] created a dynamic DM of $PFSs$ and applied it to solve practical challenges. The notion of PF equivalency was first presented in [72], who also used it to develop a new SM and apply it to the pattern detection problem. In order to remove hexavalent chromium from wastewater [73], TOPSIS and VIKOR techniques using the knowledge and accuracy measures were combined under PF environment. In addition, a hybrid methodology [74] was created that combined the EDAS and SWARA approaches under $PFSs$. It was used to advocate for the advantages of sustainable transportation. To address multi-attribute group decision-making challenges, Hussain [75] used two distinct theories—power operators and Aczel Alsina aggregation tools—to create some robust aggregation approaches. In addition, Khan [76] developed the elimination and choice translating reality (ELECTRE) II approach, which chooses the best blockchain provider for a major bank's cross-border remittance work within the constraints of a contemporary, well-structured, and highly flexible model of 2-tuple linguistic q-rung $PFSs$. Lots of work has been done in this field.

Many issues are intractable for IFS and PFS, but they are readily solved for $\mathbb{SFS}$. Hence, compared to other sets, $\mathbb{SFS}$ has greater power. They include all of the advantages of the earlier FSs. In their work on several fundamental operations for $\mathbb{SFS}$, Ashraf et. al [12] presented weighted geometric aggregation operators and weighted averaging based on spherical fuzzy numbers. They used the same approach to address issues involving decision-making. Kousar et al. [77] resolved challenge of allocating energy sources in textile industries with spherical fuzzy models. Freen et al. [78] resolved the problem of petroleum supply chain management under a fuzzy environment. In a similar way, Khalil et al. [79] developed a healthcare chain supply model. Many researchers worked on many challenges such as noisy environment [80], microgrid problems with many natural disturbances [81], green supplier [82], and uncertain crop production [83]. Kutlu [11] devised the accuracy functions, score, and aggregation operators for $\mathbb{SFS}$, and he extended the TOPSIS method to $\mathbb{SF}$ TOPSIS. Subsequently, SMs based on the cosine function were introduced in [84] and used to resolve decision-making challenges. Numerous studies on SMs for $\mathbb{SFS}$ have been proposed in the literature, taking into account the significance of SM and its applications in data mining, medical diagnosis, decision-making, and pattern detection. Set-theoretic DM and SM were proposed in [85] and successfully employed to identify patterns and choose mega-project problems. In addition, Ozceylan et. al [86] did a great deal of work in this area. In order to get around the drawbacks of earlier methods and address decision-making difficulties, Palanikumar et al. [87] developed new possibility and scoring functions in an extended Diophantine fuzzy environment. Furthermore, they [88, 89] developed geometric operators for complex neutrosophic fuzzy sets and applied then to handle MADM and medical tasks. Subsequently, Sarfraz [90] presented parametric SM in the same framework and demonstrated its effectiveness by using it in mathematical modeling and problem-solving situations. Sharaf [91] successfully employed SM, which was based on cognitive impact, to solve decision-making challenges and presented the notion of cognitive impact in the $\mathbb{SFS}$ environment. Selection of the supplier problem was addressed in [92]. In a similar way, Palanikumar [93] introduced logarithmic operators and used them to solve real-world decision-making challenges based on the construction of new buildings. Additionally, in order to diagnose some patients' amnesia issues, Palanmikumar et al. [94] developed a novel distance measure for natural vague sets and used it to an MCDM task. To handle the MCDM ("multi-criteria decision-making") difficulties, Ali [95] designed a novel DM employing matrix norms and addressed the complete area using orthogonal vector methods. Furthermore, Jayakumar et al. [96] handled the ranking of risk factors with the help of linear diophantine fuzzy sets. Again, Vimala et al. [97] handled the problem of ranking the best airlines to fly during the covid period by using q-rung orthopair $\mathbb{CC}$. Many researchers worked with diaphontine fuzzy sets and resolved the problems of road safety [98], checking the efficiency of food products [99], pattern detection [100]. After that, Pethaperuma et al. [101] worked with q-rung orthopair sets to solve MADM challenge of selecting the best healthcare supplier. To calculate the unknown criterion weight, a distance-based criteria weight determination approach is proposed. Recently, Ganie et. al [102] established the Complex Proportional Assessment method for $\mathbb{SFS}$ and produced an entropy measure under the same context. They applied this measure to the computation of attribute weights and linguistic hedges. Once more, using statistical points for covariance and variance, a $\mathbb{CC}$ was developed and applied to bidirectional approximation reasoning [103]. Additionally, Haseli et. al [104] operated in a $\mathbb{SF}$ environment and presented an extension of the best-worst approach for solving decision-making challenges, called the $\mathbb{SF}$ best-worst method. For this strategy, they created the consistency ratio.

Creating a new viewpoint for $\mathbb{CC}$ for $\mathbb{SFS}_\mathbb{S}$ is the aim of this research. When comparing two objects, $\mathbb{CC}$ is one of the finest metrics to use. It has gained popularity among researchers due to its numerous applications in domains such as texture analysis, clustering, data mining, image processing, mega project selection, green supply chain management($GSCM$), and many more. Reviews of previous research in the topic of $\mathbb{SFS}_\mathbb{S}$ have been shared, containing a wealth of information. A lot of novel uses for $\mathbb{CC}$ have been implemented in this manuscript. The current study introduces a new $\mathbb{CC}$ approach for $\mathbb{SFS}$ using all four parameters from this set. Furthermore, the current metrics have effectively contrasted to the new ones using a few examples. Several tables and graphs have been used to explain why the proposed technique is superior. The pattern can now be easily detected using this unique method. Additionally, it is utilized to address the $GSCM$ challenge. This application is centered on the case study of an ISO-certified agri-food company. The case study included five different edible oil suppliers and the best one is found using the suggested approach. Finally, this method has been most successfully used to cluster analysis of actual flood damage data from numerous Indian states between 2012 and 2021. The clusters created using the new approach have been validated. In the end, this study is concluded successfully.

1.1 Research gap, motivation and contribution

It is evident from the literature that not much research has been done on this extension ($\mathbb{SFS}$) of FSs. Furthermore, the idea of $\mathbb{CC}$ in this set is quite novel. In order to overcome the shortcomings of the current measures in this sector, this study concentrates on explaining the new technique of $\mathbb{CC}$ and demonstrating how this approach may be applied to solve real-world decision-making challenges. The primary points in the research gap that drive study in this area are as follows:

- Considering the shortcomings of $PFSs$, our goal is to create new $\mathbb{CC}$ based on $\mathbb{SF}$ data, which is the most flexible and generalized structure of fuzzy sets.
- Numerous correlation coefficients now in use are insufficient to provide accurate and satisfactory results.
- For many intuitive scenarios, many of them repeat the outcomes.

- Several metrics demonstrate perfect resemblance between distinct sets.
- Many demonstrate no relationship at all between two fuzzy sets, whilst others demonstrate some association.
- They are not reliable to utilize since a large number of them do not use hesitancy degree to calculate the correlation between spherical fuzzy sets.
- There has not been enough research in the area of $\mathbb{SFS}_{\mathbb{S}}$.
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- There has not been enough research in the area of $\mathbb{SFS}_{\mathbb{S}}$.

The major research gap that makes a fresh strategy is required to address the aforementioned problems in order to produce results that are legitimate and trustworthy. Everyone is aware of the fact that making decisions in real life is becoming more and more difficult. Therefore, a novel $\mathbb{CC}$ is required to address such issues and simplify the decision-making process. This paper's primary contribution can be summed up as follows:

- This study presents a new spherical fuzzy $\mathbb{CC}$ that addresses the shortcomings of the current spherical fuzzy $\mathbb{CC}$ and is significantly simpler than the current spherical fuzzy $\mathbb{CC}$.
- This study presents the use of the new spherical fuzzy $\mathbb{CC}$ in pattern detection and Out of all the methods now in use, this one has been shown to be the most effective.
- It is also used to cluster actual flood data from Indian states, and we have demonstrated that the clusters are validated.
- Through the use of a case study, it is illustrated how the innovative $\mathbb{CC}$ may be applied to solve $GSCM$ challenge.

The rest of the paper is structured as follows: Standard and non-standard fuzzy theory fundamentals are covered in Section 2. This section also introduces several existing spherical fuzzy $\mathbb{CC}$. A weighted novel spherical fuzzy $\mathbb{CC}$ and a novel spherical fuzzy $\mathbb{CC}$ are presented and validated in Section 3. In Section 4, an empirical comparison is made between the suggested novel and the current spherical fuzzy $\mathbb{CC}$. The innovative spherical fuzzy $\mathbb{CC}$ is applied to pattern detection in Section 5. Section 6 shows an application of clustering the actual flood data of the Indian states. Section 7 completes a case study aimed at resolving the issue of $GSCM$. In Section 8, the study's conclusion and future directions are finally discussed.

2 Basic Fundamentals

Here, it includes some basic definitions that are necessary to understand the main concept of $\mathbb{SFS}_{\mathbb{S}}$ in this paper.

Definition 2.1. ([105],[3]) Let $\ddot{S} = \{s_1, s_2, \ldots, s_n\}$ be a finite set. Consequently, an $IFS\ A_1$ in $\ddot{S}$ may be described as

$$A_1 = \{(s, \check{m}_{A_1}(s), \check{n}_{A_1}(s), \check{\rho}_{A_1}(s)) | s \in \ddot{S}\},$$

where a function for DoM of s in A_1 is established by $\check{m}_{A_1} : \ddot{S} \rightarrow$[0,1]; a function for $DoNM$ of s in A_1 is established by $\check{n}_{A_1} : \ddot{S} \rightarrow$[0,1]; such that, for all $s \in \ddot{S}$, $0 \leq \check{m}_{A_1}(s) + \check{n}_{A_1}(s) \leq 1$. The formula for calculating the hesitancy index is $\check{\rho}_{A_1}(s) = 1 - \check{m}_{A_1}(s) - \check{n}_{A_1}(s)$.

Definition 2.2. ([4]) Suppose that $\ddot{S} = \{s_1, s_2, \ldots, s_n\}$ is a finite set. It is then possible to define a $\mathbb{P}_{\mathbb{Y}}\mathbb{F}\mathbb{S}$ A_1 in $\ddot{S}$ as

$$A_1 = \{(s, \breve{m}_{A_1}(s), \breve{n}_{A_1}(s), \breve{\rho}_{A_1}(s)) | s \in \ddot{S}\},$$

where the membership function of s in A_1 is $\breve{m}_{A_1} : \ddot{S} \to [0,1]$ and the non-membership function of s in A_1 is $\breve{n}_{A_1} : \ddot{S} \to [0,1]$ such that $0 \leq \breve{m}^2_{A_1}(s) + \breve{n}^2_{A_1}(s) \leq 1$ for all $s \in \ddot{S}$. The hesitation index can be expressed as follows: $\breve{\rho}_{A_1}(s) = \sqrt{1 - \breve{m}^2_{A_1}(s) - \breve{n}^2_{A_1}(s)}$. Later, Zhang et at al. [106] called A_1 as a $\mathbb{P}_{\mathbb{Y}}\mathbb{F}\mathbb{N}$ and denoted it by $A_1 = \{(\breve{m}_{A_1}(s), \breve{n}_{A_1}(s))\}$.

Definition 2.3. ([8]) Suppose that $\ddot{S} = \{s_1, s_2, \ldots, s_n\}$ is a finite set. It is then possible to define a $\mathbb{P}\mathbb{F}\mathbb{S}$ A_1 in $\ddot{S}$ as

$$A_1 = \{(s, \breve{m}_{A_1}(s), \breve{a}_{A_1}(s), \breve{n}_{A_1}(s), \breve{\rho}_{A_1}(s)) | s \in \ddot{S}\},$$

where the membership function of s in A_1 is $\breve{m}_{A_1} : \ddot{S} \to [0,1]$, the neutral function is $\breve{a}_{A_1} : \ddot{S} \to [0,1]$ and the non-membership function of s in A_1 is $\breve{n}_{A_1} : \ddot{S} \to [0,1]$ such that $0 \leq \breve{m}_{A_1}(s) + \breve{a}_{A_1}(s) + \breve{n}_{A_1}(s) \leq 1$ for all $s \in \ddot{S}$. The refusal degree can be expressed as follows: $\breve{\rho}_{A_1}(s) = 1 - \breve{m}_{A_1}(s) - \breve{m}_{A_1}(s) - \breve{n}_{A_1}(s)$. Later, called A_1 as a PFN and denoted it by $A_1 = \{(\breve{m}_{A_1}(s), \breve{a}_{A_1}(s), \breve{n}_{A_1}(s))\}$.

Definition 2.4. ([12]) Suppose that $\ddot{S} = \{s_1, s_2, \ldots, s_n\}$ is a finite set. It is then possible to define a $\mathbb{S}\mathbb{F}\mathbb{S}$ A_1 in $\ddot{S}$ as

$$A_1 = \{(s, \breve{m}_{A_1}(s), \breve{a}_{A_1}(s), \breve{n}_{A_1}(s), \breve{\rho}_{A_1}(s)) | s \in \ddot{S}\},$$

where the membership function of s in A_1 is $\breve{m}_{A_1} : \ddot{S} \to [0,1]$, the neutral function is $\breve{a}_{A_1} : \ddot{S} \to [0,1]$ and the non-membership function of s in A_1 is $\breve{n}_{A_1} : \ddot{S} \to [0,1]$ such that $0 \leq \breve{m}^2_{A_1}(s) + \breve{a}^2_{A_1}(s) + \breve{n}^2_{A_1}(s) \leq 1$ for all $s \in \ddot{S}$. The refusal degree can be expressed as follows: $\breve{\rho}_{A_1}(s) = \sqrt{1 - \breve{m}^2_{A_1}(s) - \breve{a}^2_{A_1}(s) - \breve{n}^2_{A_1}(s)}$. Later, called A_1 as a $\mathbb{S}\mathbb{F}\mathbb{N}$ and denoted it by $A_1 = \{(\breve{m}_{A_1}(s), \breve{a}_{A_1}(s), \breve{n}_{A_1}(s))\}$.

Definition 2.5. Let $SFS(\ddot{S})$ be the set of all $\mathbb{S}\mathbb{F}\mathbb{S}_{\mathbb{S}}$ defined on $\ddot{S} = \{s_1, s_2, \ldots, s_n\}$. The operations by [8, 9] are applicable for all $A_1, A_2, A_3 \in SFS(\ddot{S})$:

(i) $A_1 = A_2$ iff $\breve{m}_{A_1}(s) = \breve{m}_{A_2}(s)$, $\breve{a}_{A_1}(s) = \breve{a}_{A_2}(s)$ and $\breve{n}_{A_1}(s) = \breve{n}_{A_2}(s)$, $\forall s \in \ddot{S}$,

(ii) $A_1 \subseteq A_2$ iff $\breve{m}_{A_1}(s) \leq \breve{m}_{A_2}(s)$, $\breve{a}_{A_1}(s) \leq \breve{a}_{A_2}(s)$ and $\breve{n}_{A_1}(u) \geq \breve{n}_{A_2}(s)$, $\forall s \in \ddot{S}$,

(iii) $A_1{}^c = \{(s, \breve{n}_{A_1}(s), \breve{a}_{A_1}(s), \breve{m}_{A_1}(s)) | s \in \ddot{S}\}$ and

(iv) $A_1 \cup A_2 = \{(max[\breve{m}_{A_1}(s), \breve{m}_{A_2}(s)], min[\breve{a}_{A_1}(s), \breve{a}_{A_2}(s)], min[\breve{n}_{A_1}(s), \breve{n}_{A_2}(s)]) | s \in \ddot{S}\}$.

(v) $A_1 \cap A_2 = \{(min[\breve{m}_{A_1}(s), \breve{m}_{A_2}(s)], min[\breve{a}_{A_1}(s), \breve{a}_{A_2}(s)], max[\breve{n}_{A_1}(s), \breve{n}_{A_2}(s)]) | s \in \ddot{S}\}$.

Definition 2.6. ([50]) Let $\mathcal{I}FS(\ddot{S})$ be the set containing all the $IFSs$ defined on $\ddot{S} = \{s_1, s_2, \ldots, s_n\}$ and $A_1, A_2 \in \mathcal{I}FS(\ddot{S})$. Then, the $\mathbb{C}\mathbb{C}$ between A_1 and A_2 with notation $\tau(A_1, A_2)$ is a function $\tau : \mathcal{I}FS(\ddot{S}) \times \mathcal{I}FS(\ddot{S}) \to [0,1]$ such that the following conditions hold:

(i) $0 \leq \tau(A_1, A_2) \leq 1$;

(ii) $\tau(A_1, A_2) = 1$ if and only if $A_1 = A_2$;

(iii) $\tau(F, G) = \tau(F, G)$.

$\tau(A_1, A_2)$ approaches 1 implies that A_1 and A_2 have a strong correlation. Also, $\tau(A_1, A_2)$ approaches 0 implies that A_1 and A_2 have a weak correlation. But, $\tau(A_1, A_2) = 0$ implies that A_1 and A_2 do not correlate. Therefore, a bigger $\mathbb{C}\mathbb{C}$ represents a better rate of performance.

Some previously existing measures are presented in the form of tables in the appendix section.

3 Novel Correlation Coefficient for $\mathbb{SFS}_{\mathbb{S}}$

In this section, we propose a new $\mathbb{CC}$ and its weighted version for $\mathbb{SFS}_{\mathbb{S}}$, in order to overcome the limitations of existing measures, discussed in Section 5. We make use of all the four parameters of a $\mathbb{SFS}$- DoM, $DoNM$, ND, and DoR to make it an effective and reliable measure. For any two $\mathbb{SFS}_{\mathbb{S}}$ A_1, A_2, it can be defined as follows:

$$\tau_J(A_1, A_2) = \frac{1}{4N}\sum_{i=1}^{N}\big(\phi_i(1-\Delta\breve{m}_i) + \varphi_i(1-\Delta\breve{a}_i) + \psi_i(1-\Delta\breve{n}_i) + \theta_i(1-\Delta\breve{\rho}_i)\big), \text{where} \quad (3.1)$$

$$\left.\begin{aligned} \phi_i &= \frac{\alpha - \Delta\breve{m}_i - \Delta\breve{m}_{max}}{\alpha - \Delta\breve{m}_{min} - \Delta\breve{m}_{max}}, \\ \varphi_i &= \frac{\alpha - \Delta\breve{a}_i - \Delta\breve{a}_{max}}{\alpha - \Delta\breve{a}_{min} - \Delta\breve{a}_{max}}, \\ \psi_i &= \frac{\alpha - \Delta\breve{n}_i - \Delta\breve{n}_{max}}{\alpha - \Delta\breve{n}_{min} - \Delta\breve{n}_{max}}, \\ \theta_i &= \frac{\alpha - \Delta\breve{\rho}_i - \Delta\breve{\rho}_{max}}{\alpha - \Delta\breve{\rho}_{min} - \Delta\breve{\rho}_{max}}, \ \alpha > 2 \text{ and finite;} \end{aligned}\right\} \quad (3.2)$$

$$\left.\begin{aligned} \Delta\breve{m}_i &= |\breve{m}_{A_1}(s_i) - \breve{m}_{A_2}(s_i)|, \ \Delta\breve{a}_i = |\breve{a}_{A_1}(s_i) - \breve{a}_{A_2}(s_i)|, \\ \Delta\breve{n}_i &= |\breve{n}_{A_1}(s_i) - \breve{n}_{A_2}(s_i)|, \ \Delta\breve{\rho}_i = |\breve{\rho}_{A_1}(s_i) - \breve{\rho}_{A_2}(s_i)|; \\ \Delta\breve{m}_{min} &= \min_i(\Delta\breve{m}_i), \ \Delta\breve{a}_{min} = \min_i(\Delta\breve{a}_i), \\ \Delta\breve{n}_{min} &= \min_i(\Delta\breve{n}_i), \ \Delta\breve{\rho}_{min} = \min_i(\Delta\breve{\rho}_i); \\ \Delta\breve{m}_{max} &= \max_i(\Delta\breve{m}_i), \ \Delta\breve{a}_{max} = \max_i(\Delta\breve{a}_i), \\ \Delta\breve{n}_{max} &= \max_i(\Delta\breve{n}_i), \ \Delta\breve{\rho}_{max} = \max_i(\Delta\breve{\rho}_i), \\ \breve{\rho}_{A_1}(s_i) &= \sqrt{1 - \breve{m}^2_{A_1}(s_i) - \breve{a}^2_{A_1}(s_i) - \breve{n}^2_{A_1}(s_i)}. \end{aligned}\right\} \quad (3.3)$$

Also, we extend the above proposed $\mathbb{CC}$ to weighted $\mathbb{CC}$ as follows:

$$\tau^*_J(A_1, A_2) = \frac{1}{4N}\sum_{i=1}^{N} w_i\big(\phi_i(1-\Delta\breve{m}_i) + \varphi_i(1-\Delta\breve{a}_i) + \psi_i(1-\Delta\breve{n}_i) + \theta_i(1-\Delta\breve{\rho}_i)\big), \quad (3.4)$$

where $w = (w_1, w_2, \ldots, w_N)^T$ is the weight vector of $s_i(\forall i, 1 \leq i \leq N)$, with $w_i \geq 0(\forall i, 1 \leq i \leq N)$, and $\sum_{i=1}^{N} w_i = 1$.

3.1 Some properties of the proposed $\mathbb{SFS}_{\mathbb{S}}$ Correlation Coefficient

Here, several properties are used to demonstrate the validity of the new method for a $\mathbb{CC}$:

Theorem 3.1. *Considering A_1, A_2, and A_3 as three $\mathbb{SFS}_{\mathbb{S}}$, the novel methods of $\mathbb{CC}$ τ_J and τ^*_J fulfills the subsequent characteristics:*

(i) $\tau_J(A_1, A_2) = \tau_J(A_2, A_1)$

(ii) $0 \leq \tau_J(A_1, A_2) \leq 1$

(iii) $\tau_J(A_1, A_2) = 1$ *iff* $A_1 = A_2$

(iv) *if* $A_1 = (\delta_1, \delta_2)$, *then* $\tau_J(A_1, {A_1}^c) = \frac{4-2|\delta_1-\delta_3|}{4}$.

(v) $\tau_J(A_1, A_2) = \tau_J({A_1}^c, {A_2}^c)$.

(vi) $\tau_J(A_1, {A_2}^c) = \tau_J({A_1}^c, A_2)$.

Proof *The proof of Theorem-3.1 is given in Appendix-A.2*

4 Comparative Analysis

This section fulfills a crucial function in this manuscript. Here, we compared the results of the suggested strategy with those of other ways by performing a comparative analysis utilizing a few samples. The analysis's findings demonstrated the suggested measure's effectiveness, dependability, and efficiency.

Example 4.1. Let us suppose that we have two $\mathbb{SFS}_\mathbb{S}$
$A_1 = \{(a_1, 0.3, 0.3, 0.3), (a_2, 0.5, 0.5, 0.5), (a_3, 0.4, 0.4, 0.4)\}$
$A = \{(a_1, 0.27, 0.27, 0.27), (a_2, 0.41, 0.41, 0.41), (a_3, 0.33, 0.33, 0.33)\}$
Then result obtained by using measure of [84] is $\tau_1(A_1, A) = 1$ and by using our proposed $\mathbb{CC}$ (3.1-3.3) is $\tau_J(A_1, A) = 0.8862$.

Example 4.2. Let us suppose that we have three $\mathbb{SFS}_\mathbb{S}$
$A_1 = \{(a_1, 0.4, 0.3, 0.5), (a_2, 0.7, 0.5, 0.3), (a_3, 0.6, 0.4, 0.3)\}$,
$A_2 = \{(a_1, 0.3, 0.4, 0.5), (a_2, 0.5, 0.7, 0.3), (a_3, 0.4, 0.6, 0.3)\}$,
$A = \{(a_1, 0.4, 0.6, 0.5), (a_2, 0.5, 0.4, 0.4), (a_3, 0.6, 0.7, 0.3)\}$
Then results obtained by using measures of [84] are $\tau_3(A, A_1) = 0.965 = \tau_3(A, A_2)$, $\tau_5(A, A_1) = 0.7678 = \tau_5(A, A_2)$, $\tau_{26}(A, A_1) = 0.8333 = \tau_{26}$ and by using our proposed $\mathbb{CC}$ (3.1-3.3) is $\tau_J(A, A_1) = 0.7719$, $\tau_J(A, A_2) = 0.8249$.

Example 4.3. Let us suppose that we have three $\mathbb{SFS}_\mathbb{S}$
$A_1 = \{(a_1, 0.3, 0.8, 0.4), (a_2, 0.5, 0.7, 0.3), (a_3, 0.6, 0.3, 0.1)\}$,
$A_2 = \{(a_1, 0.8, 0.4, 0.4), (a_2, 0.6, 0.7, 0.3), (a_3, 0.6, 0.4, 0.2)\}$,
$A = \{(a_1, 0.5, 0.5, 0.4), (a_2, 0.6, 0.4, 0.3), (a_3, 0.7, 0.4, 0.2)\}$
Then results obtained by using measures of [84] are $\tau_2(A, A_1) = 0.8886 = \tau_2(A, A_2)$, $\tau_4(A, A_1) = 0.6381 = \tau_4(A, A_2)$, $\tau_{27}(A, A_1) = 0.8583 = \tau_{27}$, $\tau_{28}(A, A_1) = 0.8865 = \tau_{28}$ and by using our proposed $\mathbb{CC}$ (3.1-3.3) is $\tau_J(A, A_1) = 0.8298$, $\tau_J(A, A_2) = 0.831$.

Example 4.4. Let us suppose that we have three $\mathbb{SFS}_\mathbb{S}$
$A_1 = \{(a_1, 0.3, 0.8, 0.4), (a_2, 0.5, 0.7, 0.3), (a_3, 0.5, 0.6, 0.4)\}$,
$A_2 = \{(a_1, 0.8, 0.4, 0.4), (a_2, 0.6, 0.7, 0.3), (a_3, 0.6, 0.5, 0.4)\}$,
$A = \{(a_1, 0.5, 0.5, 0.4), (a_2, 0.6, 0.4, 0.3), (a_3, 0.6, 0.5, 0.2)\}$
Then results obtained by using measures of [84] are $\tau_6(A, A_1) = 0.8896 = \tau_6(A, A_2)$, $\tau_8(A, A_1) = 0.6425 = \tau_8(A, A_2)$ and by using our proposed $\mathbb{CC}$ (3.1-3.3) is $\tau_J(A, A_1) = 0.8216$, $\tau_J(A, A_2) = 0.8205$.

Example 4.5. Let us suppose that we have three $\mathbb{SFS}_\mathbb{S}$
$A_1 = \{(a_1, 0.3, 0.8, 0.4), (a_2, 0.5, 0.7, 0.3), (a_3, 0.5, 0.6, 0.4)\}$,
$A_2 = \{(a_1, 0.8, 0.4, 0.4), (a_2, 0.6, 0.7, 0.3), (a_3, 0.6, 0.5, 0.4)\}$,
$A = \{(a_1, 0.5, 0.5, 0.4), (a_2, 0.6, 0.4, 0.3), (a_3, 0.5, 0.5, 0.2)\}$
Then results obtained by using measures of [84] are $\tau_7(A, A_1) = 0.874 = \tau_7(A, A_2)$, $\tau_9(A, A_1) = 0.5971 = \tau_9(A, A_2)$ and by using our proposed $\mathbb{CC}$ (3.1-3.3) is $\tau_J(A, A_1) = 0.8219$, $\tau_J(A, A_2) = 0.8093$.

Example 4.6. Let us suppose that we have two $\mathbb{SFS}_\mathbb{S}$
$A_1 = \{(a_1, 0, 0.5, 0)\}$, $A = \{(a_1, 0, 0, 0)\}$,
Then results obtained by using measures of [60] and [61] are $\tau_{11}(A, A_1) = 0$, $\tau_{12}(A, A_1) = 0 = \tau_{13}(A, A_1) = \tau_{14}(A, A_1)$ and by using our proposed $\mathbb{CC}$ (3.1-3.3) is $\tau_J(A, A_1) = 0.8415$.

Example 4.7. Let us suppose that we have two $\mathbb{SFS}_\mathbb{S}$
$A_1 = \{(a_1, 0, 0.5, 1.0)\}$, $A = \{(a_1, 0, 0, 0)\}$,
Then result obtained by using measure of [60] is $\tau_{10}(A, A_1) = 0$, by using measure of [64] is $\tau_{15}(A, A_1) = 0 = \tau_{16}(A, A_1)$ and by using our proposed $\mathbb{CC}$ (3.1-(3.3) is $\tau_J(A, A_1) = 0.5$.

Example 4.8. Let us suppose that we have three $\mathbb{SFS}_\mathbb{S}$
$A_1 = \{(a_1, 0, 0.5, 0)\}$, $A_2 = \{(a_1, 0, 0, 0.1)\}$, $A = \{(a_1, 0, 0, 0)\}$
Then results obtained by using measures of [62] and [65] are $\tau_{17}(A, A_1) = 0.5 = \tau_{17}(A, A_2)$, $\tau_{18}(A, A_1) = 0.75 = \tau_{18}(A, A_2)$, $\tau_{22}(A, A_1) = 0.414 = \tau_{22}(A, A_2)$ and by using our proposed $\mathbb{CC}$ (3.1-(3.3) is $\tau_J(A, A_1) = 0.8415$, $\tau_J(A, A_2) = 0.5$.

Example 4.9. Let us suppose that we have three $\mathbb{SFS}_\mathbb{S}$
$A_1 = \{(a_1, 0.4, 0.1, 0.2)\}$, $A_2 = \{(a_1, 0.5, 0.1, 0.2)\}$, $A = \{(a_1, 0.5, 0.1, 0.1)\}$
Then results obtained by using measures of [62] and [65] are $\tau_{19}(A, A_1) = 0 = \tau_{19}(A, A_2)$, $\tau_{23}(A, A_1) = 0.866 = \tau_{23}(A, A_2)$, $\tau_{24}(A, A_1) = 0.866 = \tau_{24}(A, A_2)$ and by using our proposed $\mathbb{CC}$ (3.1-(3.3) is $\tau_J(A, A_1) = 0.9619$, $\tau_J(A, A_2) = 0.9705$.

Example 4.10. Let us suppose that we have three $\mathbb{SFS}_\mathbb{S}$
$A_1 = \{(a_1, 0.2, 0, 0.3)\}$, $A_2 = \{(a_1, 0.1, 0, 0.4)\}$, $A = \{(a_1, 0, 1.0, 0)\}$
Then results obtained by using measures of [107], [63], and [65] are $\tau_{25}(A, A_1) = 0.236 = \tau_{25}(A, A_2)$, $\tau_{22}(A, A_1) = 0.189 = \tau_{22}(A, A_2)$, $\tau_{20}(A, A_1) = 0.177 = \tau_{20}(A, A_2)$ and by using our proposed $\mathbb{CC}$ (3.1-(3.3) is $\tau_J(A, A_1) = 0.3918$, $\tau_J(A, A_2) = 0.6027$.

Discussion

The positive aspect of the novel method for determining $\mathbb{CC}$ between $\mathbb{SFS}_\mathbb{S}$ is demonstrated in this section through a comparative study. Table-1 makes it clear that application of the proposed approach to Example-4.1, 4.6, and 4.7 gives 0.8862, 0.8415, and 0.5 respectively. Again from Table-1, it is observed that for Example-4.2 it provides $\tau_J(A, A_1) = 0.7719$ and $\tau_J(A, A_2) = 0.8249$. Also when we apply it on Example-4.3 and 4.4 we get $\tau_J(A, A_1) = 0.8298$, $\tau_J(A, A_2) = 0.8310$ and $\tau_J(A, A_1) = 0.8216$, $\tau_J(A, A_2) = 0.8205$ respectively. For Example-4.5, we get and $\tau_J(A, A_1) = 0.8219$, $\tau_J(A, A_2) = 0.8093$. When we apply proposed approach on Example- 4.8 and 4.9, we get $\tau_J(A, A_1) = 0.8415$, $\tau_J(A, A_2) = 0.5$ and $\tau_J(A, A_1) = 0.9619$, $\tau_J(A, A_2) = 0.9705$ respectively. Similarly, for Example-4.10, we get and $\tau_J(A, A_1) = 0.3918$, $\tau_J(A, A_2) = 0.6027$. All the findings are compared and presented in the form of a table. It is evident from the table that the suggested approach is the most effective one currently in use. When compared to other measures now in use, this one produces different results for different situations and does not demonstrate complete correlation unless the sets are equivalent. It is evident that the novel method avoids needlessly replicating its results. Consequently, other current methods are not appropriate in these situations. The following are the limitations of current methods:

- Many existing measures did not incorporate the refusal index in the formula.
- Some of these measures show 0 correlation between some fuzzy sets and some show a non zero degree of correlation.
- Numerous measures show perfect correlation between unequal sets.
- Many measures among them do not show a perfect correlation between equal sets.
- They also repeat their results for many distinct fuzzy sets.
- These measures do not give accurate and correct output.

As a result, it is evident that among the recently developed methods for determining $\mathbb{CC}$ between $\mathbb{SFS}_\mathbb{S}$ this new one is the most trustworthy methodology currently in use because it complies with the fundamental concept of $\mathbb{CC}$. It performs at an exceptionally high rate. Its benefits are demonstrated by examples, and it has the best performance rate of all when the refusal index is taken into account.

Table 1: Comparative analysis

Examples	Result by proposed $\mathbb{CC}$	Result by existing measures
Example-4.1	$\tau_J(A, A_1) = 0.8862$	$\tau_1(A, A_1) = \mathbf{1}$
Example-4.2	$\tau_J(A, A_1) = 0.7719$	$\tau_3(A, A_1) = \mathbf{0.9650} = \tau_3(A, A_2)$
	$\tau_J(A, A_2) = 0.8249$	$\tau_5(A, A_1) = \mathbf{0.7678} = \tau_5(A, A_2)$
		$\tau_{26}(A, A_1) = \mathbf{0.8333} = \tau_{26}(A, A_2)$
Example-4.3	$\tau_J(A, A_1) = 0.8298$	$\tau_2(A, A_1) = \mathbf{0.8886} = \tau_2(A, A_2)$
	$\tau_J(A, A_2) = 0.8310$	$\tau_4(A, A_1) = \mathbf{0.6381} = \tau_4(A, A_2)$
		$\tau_{27}(A, A_1) = \mathbf{0.8583} = \tau_{27}(A, A_2)$
		$\tau_{28}(A, A_1) = \mathbf{0.8865} = \tau_{28}(A, A_2)$
Example-4.4	$\tau_J(A, A_1) = 0.8216$	$\tau_6(A, A_1) = \mathbf{0.8896} = \tau_6(A, A_2)$
	$\tau_J(A, A_2) = 0.8205$	$\tau_8(A, A_1) = \mathbf{0.6425} = \tau_8(A, A_2)$
Example-4.5	$\tau_J(A, A_1) = 0.8219$	$\tau_7(A, A_1) = \mathbf{0.8740} = \tau_7(A, A_2)$
	$\tau_J(A, A_2) = 0.8093$	$\tau_9(A, A_1) = \mathbf{0.5971} = \tau_9(A, A_2)$
Example-4.6	$\tau_J(A, A_1) = 0.8415$	$\tau_{11}(A, A_1) = \mathbf{0}$
		$\tau_{12}(A, A_1) = \mathbf{0}$
		$\tau_{13}(A, A_1) = \mathbf{0}$
		$\tau_{14}(A, A_1) = \mathbf{0}$
Example-4.7	$\tau_J(A, A_1) = 0.5$	$\tau_{10}(A, A_1) = \mathbf{0}$
		$\tau_{15}(A, A_1) = \mathbf{0}$
		$\tau_{16}(A, A_1) = \mathbf{0}$
Example-4.8	$\tau_J(A, A_1) = 0.8415$	$\tau_{17}(A, A_1) = \mathbf{0.5000} = \tau_{17}(A, A_2)$
	$\tau_J(A, A_2) = 0.5000$	$\tau_{18}(A, A_1) = \mathbf{0.7500} = \tau_{18}(A, A_2)$
		$\tau_{22}(A, A_1) = \mathbf{0.4140} = \tau_{22}(A, A_2)$
Example-4.9	$\tau_J(A, A_1) = 0.9619$	$\tau_{19}(A, A_1) = \mathbf{0} = \tau_{19}(A, A_2)$
	$\tau_J(A, A_2) = 0.9705$	$\tau_{23}(A, A_1) = \mathbf{0.8660} = \tau_{23}(A, A_2)$
		$\tau_{24}(A, A_1) = \mathbf{0.8660} = \tau_{24}(A, A_2)$
Example-4.10	$\tau_J(A, A_1) = 0.3918$	$\tau_{20}(A, A_1) = \mathbf{0.1770} = \tau_{20}(A, A_2)$
	$\tau_J(A, A_2) = 0.6027$	$\tau_{22}(A, A_1) = \mathbf{0.1890} = \tau_{22}(A, A_2)$
		$\tau_{25}(A, A_1) = \mathbf{0.2360} = \tau_{25}(A, A_2)$

5 Application in Pattern Detection

Here, in this section, we applied the proposed $\mathbb{CC}$ to detect different patterns. We applied existing approaches and the proposed one to the example discussed in [108] to show the feasibility of our new approach.

Methodology to solve the problem

Let us suppose that $\ddot{S}(\neq \phi)$ be a set of objects. Let 'b' no. of patterns $A_i, (i = 1, 2, \ldots, b)$ are represented in the form of $\mathbb{SFS_S}$. Let A be any pattern that needs to be detected into the given patterns $A_i, (i = 1, 2, \ldots, b)$. In order to solve such problems, the following method is followed:

- **Step-1** Calculate the degree of $\mathbb{CC}$ by using some measure $\tau(A_i, A)$ between A_i $(i = 1, 2, \ldots, b)$ and A.

- **Step-2** Then the pattern A fit into pattern A_i; if
$\tau(A_i, A) = \max_{i=1,2,\ldots,b} \{\tau(A_i, A)\}$

Now we look at some examples to detect the unknown pattern, in literature many researchers have applied different measures such as DM and SM to deal with this problem. In order to handle this problem we apply $\mathbb{CC}$ and show a comparison with some existing measures. We also calculate the degree of confidence (DoC) of each measure used in the example, this formula is taken from [109]. It measures the confidence of each measure for identifying the class of unknown pattern.

$$DoC = \sum_{i=1, i\neq j}^{n} |\tau(A_i, A) - \tau(A_j, A)|, \tag{5.1}$$

where τ is any measure. The higher the value of DoC, the more confident the measure is.

Example 5.1. Let us suppose that we have a fixed set $\ddot{S} = \{s_1, s_2, s_3, s_4, s_5\}$. Five known patterns A_i $(i = 1, 2, 3, 4, 5)$ are defined on $\ddot{S}$. We have to fit the unknown pattern A into any of the given patterns.

$A_1 = \{(s_1, 0.33, 0.51, 0.12), (s_2, 0.42, 0.35, 0.18), (s_3, 0.53, 0.33, 0.09), (s_4, 0.17, 0.53, 0.13),$
$(s_5, 0.08, 0.89, 0.02), (s_6, 0.89, 0.08, 0.03)\}$
$A_2 = \{(s_1, 0.15, 0.76, 0.07), (s_2, 0.04, 0.85, 0.10), (s_3, 0.91, 0.03, 0.02),$
$(s_4, 0.31, 0.39, 0.25), (s_5, 0.68, 0.26, 0.06), (s_6, 0.07, 0.09, 0.05)\}$
$A_3 = \{(s_1, 0.15, 0.73, 0.08), (s_2, 0.05, 0.87, 0.06), (s_3, 0.90, 0.05, 0.02), (s_4, 0.91, 0.03, 0.05),$
$(s_5, 0.13, 0.75, 0.09), (s_6, 0.68, 0.08, 0.21)\}$
$A_4 = \{(s_1, 0.52, 0.31, 0.16), (s_2, 0.03, 0.82, 0.13), (s_3, 0.73, 0.12, 0.08), (s_4, 0.51, 0.24, 0.21),$
$(s_5, 0.73, 0.15, 0.08), (s_6, 0.13, 0.64, 0.21)\}$
$A_5 = \{(s_1, 0.16, 0.71, 0.05), (s_2, 0.02, 0.89, 0.05), (s_3, 0.85, 0.09, 0.05), (s_4, 0.81, 0.15, 0.09),$
$(s_5, 0.08, 0.84, 0.06), (s_6, 0.74, 0.16, 0.10)\}$
$A = \{(s_1, 0.52, 0.31, 0.05), (s_2, 0.42, 0.35, 0.05), (s_3, 0.91, 0.03, 0.02), (s_4, 0.91, 0.03, 0.05),$
$(s_5, 0.73, 0.15, 0.02), (s_6, 0.89, 0.08, 0.03)\}$
Element weight is $\{0.20, 0.09, 0.12, 0.18, 0.16, 0.25\}$

Table 2: Results obtained for pattern detection by using weighted measures

$Different\ Approaches$	(A_1, A)	(A_2, A)	(A_3, A)	(A_4, A)	(A_5, A)	$Classification$
$\tau_1(A_i, A)$	0.1020	0.1038	0.1144	0.1194	0.1142	A_4
$\tau_2(A_i, A)$	0.1206	0.1054	0.1377	0.1208	0.1341	A_3
$\tau_3(A_i, A)$	0.1397	0.1428	0.1484	0.1418	0.1465	A_3
$\tau_4(A_i, A)$	0.0987	0.0765	0.1039	0.0944	0.0953	A_3
$\tau_5(A_i, A)$	0.1121	0.1048	0.1173	0.1134	0.1127	A_3
$\tau_6(A_i, A)$	0.1206	0.1054	0.1377	0.1208	0.1341	A_3
$\tau_7(A_i, A)$	0.1205	0.1052	0.1373	0.1205	0.1340	A_3
$\tau_8(A_i, A)$	0.0987	0.0765	0.1039	0.0944	0.0953	A_3
$\tau_9(A_i, A)$	0.0982	0.0763	0.1036	0.0942	0.0952	A_3
$\tau_J{}^*(A_i, A)$	0.7547	0.6937	0.7910	0.7361	0.7794	A_3

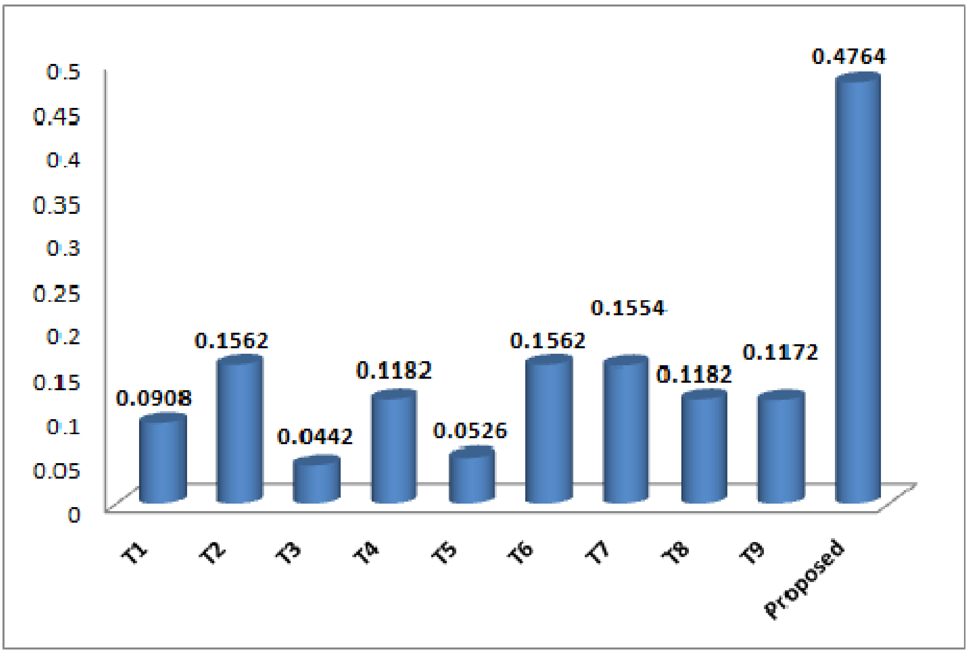

Figure 1: Comparison between DoC of measures

Discussion

In this example, we are attempting to classify the unknown pattern A among the provided patterns. We came up with a solution by combining a few suggested weighted measures with ones already in place. Table-2 displays the output that was obtained. This table makes it evident that the unknown pattern belongs to the pattern A_3 class; the results of all the measures, including the recommended measure, are identical. However, this result was disputed by τ_1. For this reason, the suggested strategy is the best option to handle these kinds of issues. It is able to detect the right pattern for the unknown pattern with success. By using equation (5.1), we find the DoC of each measure. We observe from Figure-1 that all the existing measures that are used in this example have very low DoC as compared to the DoC of the proposed measure. These are the major limitations of the existing measures. Therefore, the proposed measure yields more reliable and accurate results in detecting the class of the known pattern.

6 Application in Clustering of $\mathbb{SFS}_{\mathbb{S}}$

In this section we discuss the clustering problem. For this purpose, the problem has been divided into three levels- Initial Level, Processing Level, and Terminal Level. In the initial level of the problem, the data is collected and converted into $\mathbb{SFS}_{\mathbb{S}}$ by using suitable formula. Furthermore, in processing level, the $\mathbb{CC}$ between obtained $\mathbb{SFS}_{\mathbb{S}}$ is calculated and a correlation matrix is created. After that composition matrix, and $\beta-$cutting matrix for different values of $\beta \in [0,1]$ is formulated. Finally in the terminal level, the clusters based on different values of β are generated. Before understanding the process,we need to understand some terms. This process is shown in the Figure-3 and explained as follows:

Definition 6.1. [110] Let $\check{Q}_i \in PFS(\ddot{S})$, $i = 1,2,\ldots,t$, then $\tilde{Y} = [y_{ij}]_{t\times t}$ is said to be an association matrix, where $y_{ij} = \tau_J(\check{Q}_i, \check{Q}_j)$, $(i,j = 1,2,\ldots,t)$ is the CC of $\check{Q}_i$ and $\check{Q}_j$, having following properties:

i) $0 \leq y_{ij} \leq 1$ $(i,j = 1,2,\ldots,t)$;

ii) $y_{ij} = y_{ji}$ $(i,j = 1,2,\ldots,t)$;

iii) $y_{ij} = 1$ if and only if $\check{Q}_i = \check{Q}_j$.

Definition 6.2. [110] Let $\tilde{Y} = [y_{ij}]_{t\times t}$ be a AM, then $\tilde{Y}^2 = \tilde{Y} \circ \tilde{Y} = [\bar{y_{ij}}]$ is considered as composition matrix of $\tilde{Y}$, where $\bar{y_{ij}} = \max_k\{\min\{y_{ik}, y_{kj}\}\}$ and $(i,j = 1,2,\ldots,t)$.

Definition 6.3. [110] Let $\tilde{Y} = [y_{ij}]_{t\times t}$ be an equivalent AM, then $\tilde{Y}_\beta = [{}_\beta y_{ij}]_{t\times t}$ is considered as β- cutting matrix of $\tilde{Y}$,where ${}_\beta y_{ij} = 0$ for $y_{ij} \leq \beta$; and ${}_\beta y_{ij} = 1$ for $y_{ij} \geq \beta$ $(\beta \in [0,1])$.

- **INITIAL LEVEL** We gather the crisp data in the form of criteria and alternatives in the first stage of clustering. This data is gathered in the form of a matrix, where the rows stand for alternatives and the columns for criteria. Convert the data into $\mathbb{SFS}_{\mathbb{S}}$ so that it may be utilized to solve the problem, as it cannot be used directly.
- **PROCESSING LEVEL** The problem's processing level is where the entire calculating process occurs. Once the original data has been transformed into $\mathbb{SFS}_{\mathbb{S}}$, compute the $\mathbb{CC}$ between the resulting FSs to determine the members of the association matrix $\tilde{T}$. This matrix is symmetric. Create a composition matrix now, and keep going until the matrix begins to repeat itself. Create a $\beta-$cutting matrix for each non-zero value in the composition matrix.
- **TERMINAL LEVEL** The final result is obtained at the problem's terminal level. Group every $\mathbb{SFS}_{\mathbb{S}}$ in the $\beta-$cutting matrix into a class where the i^{th} and j^{th} columns are equal. By employing various $\beta-$cutting matrices, all potential clusters are produced for a range of β values.

Area of case study

In this case study, we outlined the damage that India's floods from 2012 to 2021 caused each state. Rainy weather makes things worse, which leads to extremely gloomy conditions throughout every stage of the day. Following that, there is only destruction everywhere, whether it takes the shape of harm to people, property, wildlife, or any other kind of living thing.We also collected statistics to illustrate the number of Indian areas that may be severely impacted by excessive rainfall in 2022. There will likely be heavy rains on July 14–28, 2022; August 12–28, 2022; and September 12–14, 2022. The Godavari and Sabari rivers' water levels will also rise. During this time, Andhra Pradesh experienced severe damage to seven districts. In Assam, 35 districts were impacted by floods that happened between May 18 and May 26, as well as between June 16 and July 17, 2022. Bihar experienced floods from July 25, 2022, to October 20, 2022, as a result of rising water levels in the Ganga, Kosi, Adhwara group, and Bagmati rivers. These floods affected over 33 districts. In the final week of July 2022, Karnataka state recorded heavy rainfall that severely affected 17 districts. The final week of July 2022 saw reports of heavy rain in the state of Karnataka. In the state of MP, flooding was mapped twice in August and once in September of 2022. In July and August of 2022, there were reports of heavy, continuous rainfall in some districts of Maharashtra. In Meghalaya, severe, continuous rains during the second week of June 2022 were said to have caused flooding. In August 2022, the Mahanadi river basin saw intense rainfall and runoff that flooded numerous villages across 13 districts in the coastal region of Odisha. Cuttack, Khordha, Puri, Kendrapara, Jagatsinghpur, Boudh, Sambalpur, and Angul are among the districts affected by flooding. During the first week of August 2022, there were reports of heavy, continuous rainfall in certain areas of Rajasthan. A large number of low-lying locations along the Chambal River were submerged. In Telangana, there were reports of heavy rainfall in the second week of July 2022. The Godavari River and its tributaries were overflowing in numerous areas of Telangana State, and floods were engulfing the communities that were close to the river. Data from satellites was examined during July 14–27, 2022. In the state of Tripura, heavy, nonstop rains during the third week of June 2022 caused floods that flooded several low-lying sections of the West and South Tripura portions of the state from June 19–21, 2022. In Uttar Pradesh, there have been four floods, with the most number of districts (54), hit in 2022. This information was provided by the Indian Space Research Organization's (ISRO) Department of Space and the National Disaster Management Authority of the Ministry of Home Affairs, Government of India ($https: //www.nrsc.gov.in/sites/default/files/pdf/DMSP/FloodAffectedAreaAtlas_Digital.pdf$). Since severe floods destroy almost every state in India, actual data was used in this article.

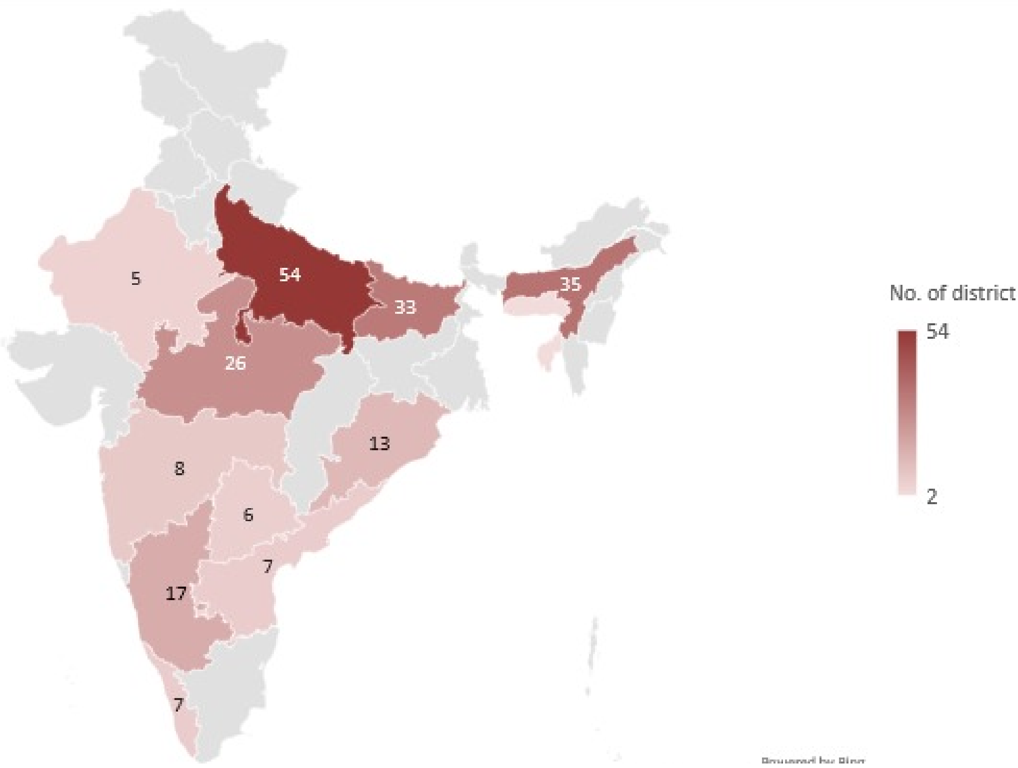

Figure 2: No. of district affected due to floods in India during 2022

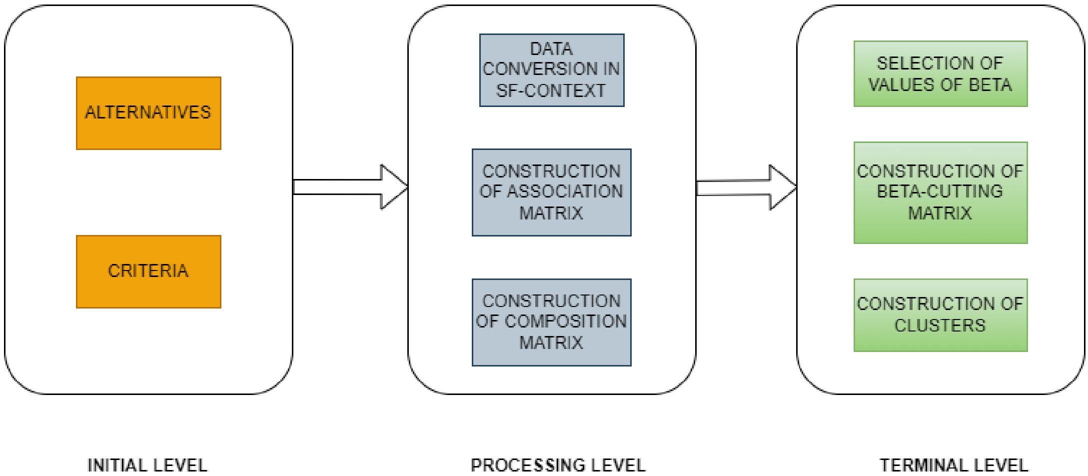

Figure 3: Algorithm to solve clustering problem of real data

Example 6.4. (Case Study) Here in this case study, data of the damage caused by heavy rain and flood during 2012 to 2021 (in India) is collected. These ten years are considered as 10 alternatives: $A_1 = 2012$, $A_2 = 2013$, $A_3 - 2014$, $A_4 = 2015$, $A_5 = 2016$, $A_6 = 2017$, $A_7 = 2018$, $A_8 = 2019$, $A_9 = 2020$, $A_{10} = 2021$. A lot of damage to human lives, houses, crops, cattle has been caused due to this major problem of heavy rain and flood.These are considered as seven attributes: $J_1 = Human\, life\, lost$(in no. $\times 10^3$), $J_2 = damage\, to\, houses$(in no. $\times 10^5$), $J_3 = population\, affected$(in millions $\times 10$), $J_4 = damage\, to\, public\, utility$(in Rs. crore $\times 10^4$), $J_5 = area\, affected$(in m.ha.), $J_6 = damage\, to\, crops$(area in m.ha.), $J_7 = cattle\, lost$(in no. $\times 10^4$).

Step 1 (Data Collection and Conversion) Table-3 displays actual damage statistics from floods and strong rains. In this scenario, columns indicate the criteria, while rows represent the possibilities. This data cannot be utilized directly and must be converted to $\mathbb{SFS}_{\mathbb{S}}$ first. The following equation is used to carry out this conversion [111]:

$$\left.\begin{aligned} \breve{m}(z_{ij}) = \epsilon m(z_{ij}),\ \breve{a}(z_{ij}) = (1-(m(z_{ij})^{\epsilon}))^{\frac{1}{\epsilon}}, \\ \breve{n} = 1-(1-(m(z_{ij}))^{\epsilon},\ 0<\epsilon\leq 1,\ and \\ (m(z_{ij}) = 1-exp\left(-\frac{z_{ij}-min(z_{ij})}{max(z_{ij})-min(z_{ij})}\right) \end{aligned}\right\} \tag{6.1}$$

The above equation is used to convert the data into $\mathbb{SFS}_{\mathbb{S}}$ by setting ϵ=0.5. Table-4 represents this transformed data.

Step 2 (Creation of association matrix) The definition-6.1 is now used to build the association matrix $\tilde{T} = [t_{ij}]_{t\times t}$. Here, we determine each alternative's $\mathbb{CC}$ degree in relation to the other alternatives. The $\mathbb{CC}$ between each row (alternative) of Table-4 is the element t_{ij} in the matrix $\tilde{T}$. Equations (3.1),(3.2),(3.3) are used to calculate $\mathbb{CC}$. A symmetric matrix, $\tilde{T}$, is the resultant matrix.

$$\tilde{T} = \begin{pmatrix} 1 & 0.4843 & 0.5273 & 0.5190 & 0.4722 & 0.5340 & 0.5620 & 0.5042 & 0.5819 & 0.5183 \\ 0.4843 & 1 & 0.8561 & 0.8208 & 0.7322 & 0.8387 & 0.8512 & 0.8107 & 0.8496 & 0.8654 \\ 0.5273 & 0.8561 & 1 & 0.8079 & 0.7984 & 0.8426 & 0.8593 & 0.8766 & 0.9206 & 0.8952 \\ 0.5190 & 0.8208 & 0.8079 & 1 & 0.6907 & 0.8466 & 0.8524 & 0.7834 & 0.7874 & 0.7714 \\ 0.4722 & 0.7322 & 0.7984 & 0.6907 & 1 & 0.7534 & 0.7123 & 0.7990 & 0.8108 & 0.7103 \\ 0.5340 & 0.8387 & 0.8426 & 0.8466 & 0.7534 & 1 & 0.8966 & 0.8808 & 0.8469 & 0.8397 \\ 0.5620 & 0.8512 & 0.8593 & 0.8524 & 0.7123 & 0.8966 & 1 & 0.8311 & 0.8625 & 0.8679 \\ 0.5042 & 0.8107 & 0.8766 & 0.7834 & 0.7990 & 0.8808 & 0.8311 & 1 & 0.8383 & 0.8388 \\ 0.5819 & 0.8496 & 0.9206 & 0.7874 & 0.8108 & 0.8469 & 0.8625 & 0.8383 & 1 & 0.8744 \\ 0.5183 & 0.8654 & 0.8952 & 0.7714 & 0.7103 & 0.8397 & 0.8679 & 0.8388 & 0.8744 & 1 \end{pmatrix}$$

Table 3: Real data of damage caused due to floods and heavy rain

Year	*Human live lost*	*Damage to houses*	*Population affected*	*Damage to public utility*	*Area affected*	*Damage to crops*	*Cattle lost*
2012	0.9330	1.7453	1.4690	0.9170	2.1400	1.9500	3.1558
2013	2.1800	6.9953	2.5930	3.8938	7.5500	7.4800	16.3958
2014	1.9680	3.1132	2.6510	0.7711	12.7800	8.0100	6.0196
2015	1.4200	39.5919	3.3200	3.2200	4.4800	3.3700	4.5597
2016	1.4200	2.7824	2.6550	0.1508	7.0600	6.6600	2.2367
2017	2.0630	12.5291	4.7340	1.2330	6.0800	4.9700	2.6673
2018	1.8390	9.1341	3.7400	1.2133	7.7200	2.5100	6.0279
2019	2.7540	6.5659	4.6350	0.4498	11.6000	10.6900	2.5852
2020	1.4740	2.3954	2.6790	0.5458	6.9000	6.5500	4.6911
2021	1.3710	4.6120	3.8560	2.5245	16.7500	7.7900	6.4880

Table 4: Converted data of the damage in the form of $\mathbb{SFS}_{\mathbb{S}}$ (A).

	Year	*Human live lost*	*Damage to houses*	*Population affected*	*Damage to public utility*
b_1	2012	(0,0,1)	(0,0,1)	(0,0,1)	(0.0925,0.0972,0.3246)
b_2	2013	(0.479,0.2899,0.0874)	(0.0647,0.0669,0.4097)	(0.1456,0.1580,0.2118)	(0.3160,0.3934,0.0419)
b_3	2014	(0.2841,0.3415,0.0612)	(0.0177,0.0178,0.6590)	(0.1518,0.1655,0.2015)	(0.0763,0.0794,0.3713)
b_4	2015	(0.3826,0.5155,0.0156)	(0.3160,0.3934,0.0419)	(0.2163,0.2468,0.1171)	(0.2797,0.3361,0.0635)
b_5	2016	(0.3826,0.5155,0.0156)	(0.0135,0.0135,0.6982)	(0.1522,0.1660,0.2007)	(0,0,1)
b_6	2017	(0.2312,0.2667,0.1024)	(0.1239,0.1327,0.2521)	(0.3160,0.3934,0.0419)	(0.1255,0.1345,0.2490)
b_7	2018	(0.1960,0.2202,0.1398)	(0.0886,0.0929,0.3351)	(0.2505,0.2936,0.0853)	(0.1235,0.1322,0.2530)
b_8	2019	(0.3160,0.3934,0.0419)	(0.0597,0.0616,0.4281)	(0.3103,0.3841,0.0449)	(0.0383,0.0390,0.5230)
b_9	2020	(0.1285,0.1380,0.2430)	(0.0084,0.0085,0.7569)	(0.1548,0.1690,0.1966)	(0.0500,0.0513,0.4673)
b_{10}	2021	(0.1069,0.1133,0.2890)	(0.0364,0.0371,0.5329)	(0.2592,0.3060,0.0783)	(0.2347,0.2716,0.0990)

Table 5: Converted data of the damage in the form of $\mathbb{SFS}_{\mathbb{S}}$ (B).

	Year	*Area affected*	*Damage to crops*	*Cattle lost*
b_1	2012	(0,0,1)	(0,0,1)	(0.0314,0.0319,0.5616)
b_2	2013	(0.1530,0.1669,0.1995)	(0.2344,0.2711,0.0994)	(0.3160,0.3934,0.0419)
b_3	2014	(0.2586,0.3051,0.0788)	(0.2500,0.2928,0.0857)	(0.1172,0.1250,0.2661)
b_4	2015	(0.0739,0.0769,0.3787)	(0.0749,0.0779,0.3757)	(0.0756,0.0786,0.3735)
b_5	2016	(0.1429,0.1548,0.2165)	(0.2083,0.2361,0.1257)	(0,0,1)
b_6	2017	(0.1181,0.1261,0.2640)	(0.1460,0.1586,0.2111)	(0.0149,0.1500,0.6840)
b_7	2018	(0.1587,0.1738,0.1906)	(0.0309,0.0314,0.5643)	(0.1174,0.1252,0.2656)
b_8	2019	(0.2383,0.2765,0.0958)	(0.3160,0.3934,0.0419)	(0.0121,0.0121,0.7130)
b_9	2020	(0.1390,0.1502,0.2234)	(0.2046,0.2313,0.1298)	(0.0795,0.0829,0.3613)
b_{10}	2021	(0.3160,0.3934,0.0419)	(0.2436,0.2839,0.0911)	(0.1286,0.1381,0.2429)

Step 3 (Creation of composition matrix matrix) By using definition-6.2 , we will create composition matrix and the process of creating this matrix will be stopped until the matrix repeat

itself. After calculations, we observe that $\tilde{T^8}$=$\tilde{T^4}$. Therefore, $\tilde{T^8}$ will be the final composition matrix.

$$\tilde{T^2} = \tilde{T} o \tilde{T} = \begin{pmatrix} 1 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 \\ 0.5819 & 1 & 0.8654 & 0.8512 & 0.8108 & 0.8512 & 0.8654 & 0.8561 & 0.8654 & 0.8654 \\ 0.5819 & 0.8654 & 1 & 0.8524 & 0.8108 & 0.8766 & 0.8679 & 0.8766 & 0.9206 & 0.8952 \\ 0.5819 & 0.8512 & 0.8524 & 1 & 0.7984 & 0.8524 & 0.8524 & 0.8466 & 0.8524 & 0.8524 \\ 0.5819 & 0.8108 & 0.8108 & 0.7984 & 1 & 0.8108 & 0.8108 & 0.8108 & 0.8108 & 0.8108 \\ 0.5819 & 0.8512 & 0.8766 & 0.8524 & 0.8108 & 1 & 0.8966 & 0.8808 & 0.8625 & 0.8679 \\ 0.5819 & 0.8654 & 0.8679 & 0.8524 & 0.8108 & 0.8966 & 1 & 0.8808 & 0.8679 & 0.8679 \\ 0.5819 & 0.8561 & 0.8766 & 0.8466 & 0.8108 & 0.8808 & 0.8808 & 1 & 0.8766 & 0.8766 \\ 0.5819 & 0.8654 & 0.9206 & 0.8524 & 0.8108 & 0.8625 & 0.8679 & 0.8766 & 1 & 0.8952 \\ 0.5819 & 0.8654 & 0.8952 & 0.8524 & 0.8108 & 0.8679 & 0.8679 & 0.8766 & 0.8952 & 1 \end{pmatrix}$$

$$\tilde{T^4} = \tilde{T^2} o \tilde{T^2} = \begin{pmatrix} 1 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 \\ 0.5819 & 1 & 0.8654 & 0.8524 & 0.8108 & 0.8654 & 0.8654 & 0.8654 & 0.8654 & 0.8654 \\ 0.5819 & 0.8654 & 1 & 0.8524 & 0.8108 & 0.8766 & 0.8766 & 0.8766 & 0.9206 & 0.8952 \\ 0.5819 & 0.8524 & 0.8524 & 1 & 0.8108 & 0.8524 & 0.8524 & 0.8524 & 0.8524 & 0.8524 \\ 0.5819 & 0.8108 & 0.8108 & 0.8108 & 1 & 0.8108 & 0.8108 & 0.8108 & 0.8108 & 0.8108 \\ 0.5819 & 0.8654 & 0.8766 & 0.8524 & 0.8108 & 1 & 0.8966 & 0.8808 & 0.8766 & 0.8766 \\ 0.5819 & 0.8654 & 0.8766 & 0.8524 & 0.8108 & 0.8966 & 1 & 0.8808 & 0.8766 & 0.8766 \\ 0.5819 & 0.8654 & 0.8766 & 0.8524 & 0.8108 & 0.8808 & 0.8808 & 1 & 0.8766 & 0.8766 \\ 0.5819 & 0.8654 & 0.9206 & 0.8524 & 0.8108 & 0.8766 & 0.8766 & 0.8766 & 1 & 0.8952 \\ 0.5819 & 0.8654 & 0.8952 & 0.8524 & 0.8108 & 0.8766 & 0.8766 & 0.8766 & 0.8952 & 1 \end{pmatrix}$$

$$\tilde{T^8} = \tilde{T^4} o \tilde{T^4} = \begin{pmatrix} 1 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 & 0.5819 \\ 0.5819 & 1 & 0.8654 & 0.8524 & 0.8108 & 0.8654 & 0.8654 & 0.8654 & 0.8654 & 0.8654 \\ 0.5819 & 0.8654 & 1 & 0.8524 & 0.8108 & 0.8766 & 0.8766 & 0.8766 & 0.9206 & 0.8952 \\ 0.5819 & 0.8524 & 0.8524 & 1 & 0.8108 & 0.8524 & 0.8524 & 0.8524 & 0.8524 & 0.8524 \\ 0.5819 & 0.8108 & 0.8108 & 0.8108 & 1 & 0.8108 & 0.8108 & 0.8108 & 0.8108 & 0.8108 \\ 0.5819 & 0.8654 & 0.8766 & 0.8524 & 0.8108 & 1 & 0.8966 & 0.8808 & 0.8766 & 0.8766 \\ 0.5819 & 0.8654 & 0.8766 & 0.8524 & 0.8108 & 0.8966 & 1 & 0.8808 & 0.8766 & 0.8766 \\ 0.5819 & 0.8654 & 0.8766 & 0.8524 & 0.8108 & 0.8808 & 0.8808 & 1 & 0.8766 & 0.8766 \\ 0.5819 & 0.8654 & 0.9206 & 0.8524 & 0.8108 & 0.8766 & 0.8766 & 0.8766 & 1 & 0.8952 \\ 0.5819 & 0.8654 & 0.8952 & 0.8524 & 0.8108 & 0.8766 & 0.8766 & 0.8766 & 0.8952 & 1 \end{pmatrix}$$

Step 4 (Construct β-cutting matrices) Now, extract every non-zero value found in $\tilde{T^8}$, the final composition matrix. These values can be defined as β. Using these, create intervals such that each interval falls inside the interval [0,1]. Confidence levels are the term given to these new intervals. Create a β-cutting matrix ($\tilde{T}_\beta$) for every confidence level using the definition-6.3.

(i) If $0 \leq \beta \leq 0.5819$, then

$$\tilde{T}_\beta = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(ii) If $0.5819 < \beta \leq 0.8108$, then

$$\tilde{T}_\beta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(iii) If $0.8108 < \beta \leq 0.8524$, then

$$\tilde{T}_\beta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(iv) If $0.8524 < \beta \leq 0.8654$, then

$$\tilde{T}_\beta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(v) If $0.8654 < \beta \leq 0.8766$, then

$$\tilde{T}_\beta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(vi) If $0.8766 < \beta \leq 0.8808$, then

$$\tilde{T}_\beta = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&1&1\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&1&1&0&0\\ 0&0&0&0&0&1&1&1&0&0\\ 0&0&0&0&0&1&1&1&0&0\\ 0&0&1&0&0&0&0&0&1&1\\ 0&0&1&0&0&0&0&0&1&1 \end{pmatrix}$$

(vii) If $0.8808 < \beta \leq 0.8952$, then

$$\tilde{T}_\beta = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&1&1\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&1&0&0&0\\ 0&0&0&0&0&1&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0&1&1\\ 0&0&1&0&0&0&0&0&1&1 \end{pmatrix}$$

(viii) If $0.8952 < \beta \leq 0.8966$, then

$$\tilde{T}_\beta = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&1&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&1&0&0&0\\ 0&0&0&0&0&1&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

(ix) If $0.8966 < \beta \leq 0.9206$, then

$$\tilde{T}_\beta = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&1&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

(x) If $0.9206 < \beta \leq 1.0000$, then

$$\tilde{T}_\beta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Step 5 (Create clusters) We now build clusters using the corresponding β-cutting matrix for each confidence level. Both sets will belong to the same clusters if the corresponding items in the i^{th} row/column match those in the j^{th} row/column. Create all of the clusters corresponding to each β-cutting matrix in a similar manner. The desired output is obtained in the Table-6 .

Table 6: Clusters for different values of β by using proposed $\mathbb{CC}$ τ_J

Class	**Level of confidence**	**Clusters**
1	$0 < \beta \leq 0.5819$	$\{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
2	$0.5819 < \beta \leq 0.8108$	$\{\wp_1\}, \{\wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
3	$0.8108 < \beta \leq 0.8524$	$\{\wp_1\}, \{\wp_5\}, \{\wp_2, \wp_3, \wp_4, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
4	$0.8524 < \beta \leq 0.8654$	$\{\wp_1\}, \{\wp_4\}, \{\wp_5\}, \{\wp_2, \wp_3, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
5	$0.8654 < \beta \leq 0.8766$	$\{\wp_1\}, \{\wp_2\}, \{\wp_4\}, \{\wp_5\}, \{\wp_3, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
6	$0.8766 < \beta \leq 0.8808$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3, \wp_9, \wp_{10}\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6, \wp_7, \wp_8\}$
7	$0.8808 < \beta \leq 0.8952$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3, \wp_9, \wp_{10}\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6, \wp_7\}, \{\wp_8\}$
8	$0.8952 < \beta \leq 0.8966$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3, \wp_9\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6, \wp_7\}, \{\wp_8\}, \{\wp_{10}\}$
9	$0.8966 < \beta \leq 0.9206$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3, \wp_9\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6\}, \{\wp_7\}, \{\wp_8\}, \{\wp_{10}\}$
10	$0.9206 < \beta \leq 1.0000$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6\}, \{\wp_7\}, \{\wp_8\}, \{\wp_9\}, \{\wp_{10}\}$

Table 7: Clusters for different values of β by using existing measure τ_{18}

Class	**Level of confidence**	**Clusters**
1	$0 < \beta \leq 0.9240$	$\{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
4	$0.9240 < \beta \leq 0.9557$	$\{\wp_1\}, \{\wp_8\}, \{\wp_9\}, \{\wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_{10}\}$
5	$0.9557 < \beta \leq 0.9717$	$\{\wp_1\}, \{\wp_5\}, \{\wp_8\}, \{\wp_9\}, \{\wp_2, \wp_3, \wp_4, \wp_6, \wp_7, \wp_{10}\}$
6	$0.9717 < \beta \leq 0.9822$	$\{\wp_1\}, \{\wp_2, \wp_4\}, \{\wp_5\}, \{\wp_8\}, \{\wp_9\}, \{\wp_3, \wp_6, \wp_7, \wp_{10}\}$
7	$0.9822 < \beta \leq 0.9879$	$\{\wp_1\}, \{\wp_2, \wp_4\}, \{\wp_5\}, \{\wp_6\}, \{\wp_8\}, \{\wp_9\}, \{\wp_3, \wp_7, \wp_{10}\}$
8	$0.9879 < \beta \leq 0.9951$	$\{\wp_1\}, \{\wp_2, \wp_4\}, \{\wp_3\}, \{\wp_5\}, \{\wp_6\}, \{\wp_8\}, \{\wp_9\}, \{\wp_7, \wp_{10}\}$
10	$0.9951 < \beta \leq 1.0000$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6\}, \{\wp_7\}, \{\wp_8\}, \{\wp_9\}, \{\wp_{10}\}$

After analysis of the Table-6,7,8,9,10, it can be clearly identified that the new method of finding $\mathbb{CC}$ can successfully solve the clustering problem. We obtain ten classes by using new $\mathbb{CC}$. On the other hand, by using existing measures τ_{18}, τ_{19}, τ_{22}, and τ_{23} we get 7, 6, 8, and 8 possibilities respectively. Therefore, the proposed $\mathbb{CC}$ has a great clustering accuracy.

Table 8: Clusters for different values of β by using existing measure τ_{19}

Class	Level of confidence	Clusters
1	$0 < \beta \leq 0.7581$	$\{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
2	$0.7581 < \beta \leq 0.7986$	$\{\wp_1, \wp_2, \wp_3, \wp_8\}, \{\wp_4, \wp_5, \wp_6, \wp_7, \wp_9, \wp_{10}\}$
4	$0.7986 < \beta \leq 0.8202$	$\{\wp_1, \wp_2\}, \{\wp_3, \wp_8\}, \{\wp_4, \wp_5\}, \{\wp_6, \wp_7, \wp_9, \wp_{10}\}$
5	$0.8202 < \beta \leq 0.8564$	$\{\wp_1, \wp_2\}, \{\wp_3, \wp_8\}, \{\wp_4, \wp_5\}, \{\wp_6\}, \{\wp_7, \wp_9, \wp_{10}\}$
9	$0.8564 < \beta \leq 0.9119$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6\}, \{\wp_7\}, \{\wp_8\}, \{\wp_9, \wp_{10}\}$
10	$0.9119 < \beta \leq 1.0000$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6\}, \{\wp_7\}, \{\wp_8\}, \{\wp_9\}, \{\wp_{10}\}$

Table 9: Clusters for different values of β by using existing measure τ_{22}

Class	Level of confidence	Clusters
1	$0 < \beta \leq 0.7783$	$\{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
2	$0.7783 < \beta \leq 0.8216$	$\{\wp_1\}, \{\wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
3	$0.8216 < \beta \leq 0.8538$	$\{\wp_1\}, \{\wp_4\}, \{\wp_2, \wp_3, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
4	$0.8538 < \beta \leq 0.8911$	$\{\wp_1\}, \{\wp_4\}, \{\wp_7\}, \{\wp_2, \wp_3, \wp_5, \wp_6, \wp_8, \wp_9, \wp_{10}\}$
5	$0.8911 < \beta \leq 0.9206$	$\{\wp_1\}, \{\wp_4\}, \{\wp_5, \wp_9\}, \{\wp_7\}, \{\wp_2, \wp_3, \wp_6, \wp_8, \wp_{10}\}$
6	$0.9206 < \beta \leq 0.9444$	$\{\wp_1\}, \{\wp_2, \wp_3\}, \{\wp_4\}, \{\wp_5, \wp_9\}, \{\wp_7\}, \{\wp_6, \wp_8, \wp_{10}\}$
7	$0.9444 < \beta \leq 0.9715$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4\}, \{\wp_5\}, \{\wp_7\}, \{\wp_9\}, \{\wp_6, \wp_8, \wp_{10}\}$
10	$0.9715 < \beta \leq 1.0000$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6\}, \{\wp_7\}, \{\wp_8\}, \{\wp_9\}, \{\wp_{10}\}$

Table 10: Clusters for different values of β by using existing measure τ_{23}

Class	Level of confidence	Clusters
1	$0 < \beta \leq 0.6541$	$\{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
2	$0.6541 < \beta \leq 0.6987$	$\{\wp_4\}, \{\wp_1, \wp_2, \wp_3, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
3	$0.6987 < \beta \leq 0.7445$	$\{\wp_4\}, \{\wp_6\}, \{\wp_1, \wp_2, \wp_3, \wp_5, \wp_7, \wp_8, \wp_9, \wp_{10}\}$
4	$0.7445 < \beta \leq 0.7934$	$\{\wp_4, \wp_5\}, \{\wp_6\}, \{\wp_7\}, \{\wp_1, \wp_2, \wp_3, \wp_8, \wp_9, \wp_{10}\}$
5	$0.7934 < \beta \leq 0.8232$	$\{\wp_4, \wp_5\}, \{\wp_6\}, \{\wp_7, \wp_1\}, \{\wp_3\}, \{\wp_2, \wp_8, \wp_9, \wp_{10}\}$
6	$0.8232 < \beta \leq 0.8679$	$\{\wp_4, \wp_5\}, \{\wp_6\}, \{\wp_7, \wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_8, \wp_9, \wp_{10}\}$
8	$0.8679 < \beta \leq 0.9335$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4, \wp_5\}, \{\wp_6, \wp_9\}, \{\wp_7\}, \{\wp_8\}, \{\wp_{10}\}$
10	$0.9335 < \beta \leq 1.0000$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6\}, \{\wp_7\}, \{\wp_8\}, \{\wp_9\}, \{\wp_{10}\}$

Cluster Validation Index (CVI)

The evaluation of an algorithm's performance is a key component of the cluster analysis problem. There are several perspectives from which one can evaluate an algorithm. The key elements include determining the right number of clusters to best fit the underlying structure of the item or data, the algorithm's complexity, and the degree of confidence in the data comparison. Cluster validity indices (CVI) are used to quickly and easily calculate the appropriate number of clusters in a set of data. A CVI introduced in [112] is relevant to our situation. The key terms and CVI computation mechanism associated with [112] are presented below.

- **Local Density:**
  The local density of a component, $\wp_i$ in the feature space S is defined as the distance to the closest neighbor, $\wp_j$, belonging to the same cluster, stated as:

$$\tilde{L}(\wp_i)_{\wp_i \in G_t} = max\ \tau(\wp_i, \wp_j),\ \forall \wp_j \in G_t, i \neq j, \tag{6.2}$$

where τ is any SM or $\mathbb{CC}$, and G_t is t^{th} cluster.

- **Density of a cluster**
  A cluster's average density is computed as the average of all of its local densities.

$$\tilde{A}(G_t) = \frac{\sum_{i=1}^{n} L(\tilde{b}_i)}{\check{n}}, \ \forall b_i \in G_t, \tag{6.3}$$

  where $\check{n}$ is the total number of elements in the cluster.

- **Uniformity**
  It determines the extent of local density fluctuation within a cluster. The uniformity of a cluster G_t can be expressed as

$$\begin{cases} \tilde{U}(G_t) = \dfrac{\sum_{i=1}^{\check{n}} |\tilde{L}(\wp_i) - \tilde{A}(G_t)|}{\tilde{A}(G_t)}, \ if \ \wp_i \in G_t, \ \check{n} > 1 \\ \hfill 0, \ if \ \check{n} = 1 \end{cases} \tag{6.4}$$

- **Cluster Validity Index** (CVI)
  For given partition $\tilde{R}_t = \{G_1, G_2, \ldots, G_t\}$ of a dataset S, with t clusters,the value of CVI is defined as

$$CVI(\tilde{R}_t) = \frac{\sum_{i=1}^{t} \check{n}_i \tilde{U}(G_i)}{\check{N}}, \tag{6.5}$$

  where $\check{n}_i$ is the number of elements contained in cluster G_i, and $\check{N}$ is the total number of elements in the set S.
  The purpose of this index is to determine the cluster with the greatest degree of homogeneity in density. As a result, this index must be decreased.

Calculation of CVI

Next, we'll proceed to calculate CVI. Once the important terms become known, the CVI minimum is calculated as follows:

- **Step 1**
  The set of distinct partitions produced by any clustering method applied to the dataset S in Table-6 is denoted by $Y = \{\tilde{R}_1, \tilde{R}_2, \ldots, \tilde{R}_t\}$. The amount of clusters that each one has determines how they are arranged. We compute the CVI for every one of these partitions. Starting with $CVI(\tilde{R}_2)$, the smallest solution containing at least two clusters, the process of determining the minimal index value continues as long as $CVI(\tilde{R}_i) > CVI(\tilde{R}_{i+1})$, $1 \leq i \leq m$. The process ends when $CVI(\tilde{R}_i) \geq CVI(\tilde{R}_{i+1})$, and the first i^{th} partitions are chosen for additional analysis.

- **Step 2** The subsequent step involves using the following factor to calculate the relative improvement between two successive partitions, s and $s-1$.

$$CVI_f{}^s = \frac{CVI(\tilde{R}_s)}{CVI(\tilde{R}_{s-1})}, \ s = 2, 3, \ldots, t.$$

- **Step 3**
  In order to determine the optimal partition, $j : CVI_f{}^s = \min_t\{CVI_f{}^2, \ldots, CVI_f{}^t\}$ is finally selected as the j^{th} partition with the highest improvement.

Calculation Process:

Here we use Table-4 for calculation of CVI, we have 10 partitions $\{\tilde{R}_1, \tilde{R}_2, \tilde{R}_3, \tilde{R}_4, \tilde{R}_5, \tilde{R}_6, \tilde{R}_7, \tilde{R}_8, \tilde{R}_9, \tilde{R}_{10}\}$ in this example and each partition has some clusters $\{G_1, G_2, \ldots, G_t\}$, where $1 \leq t \leq 10$. This data is presented below:

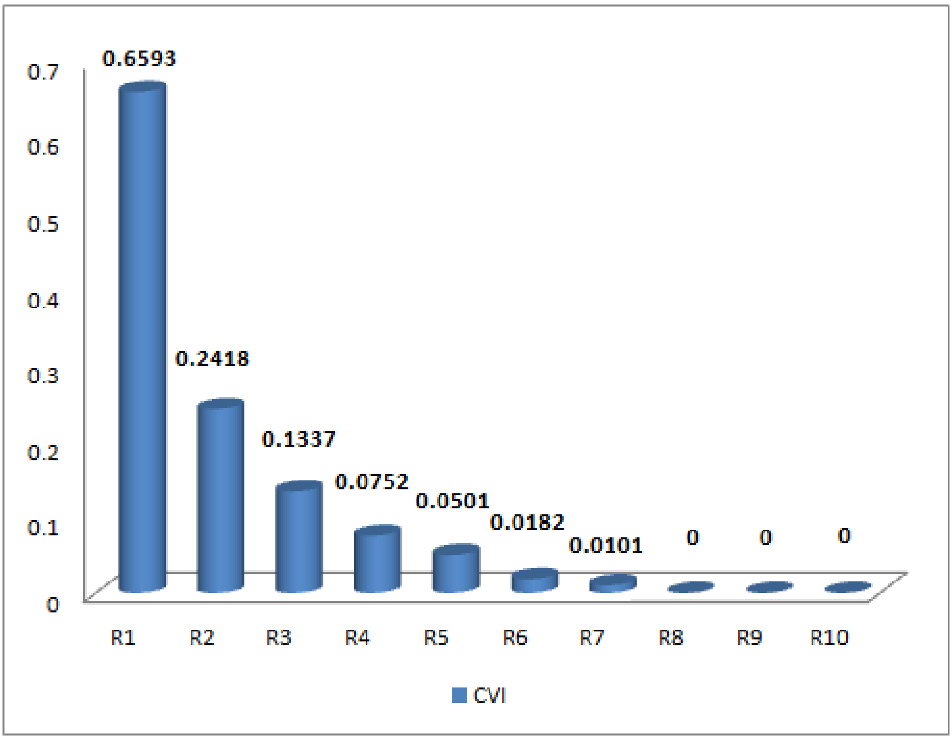

Figure 4: CVI by using proposed $\mathbb{CC}$

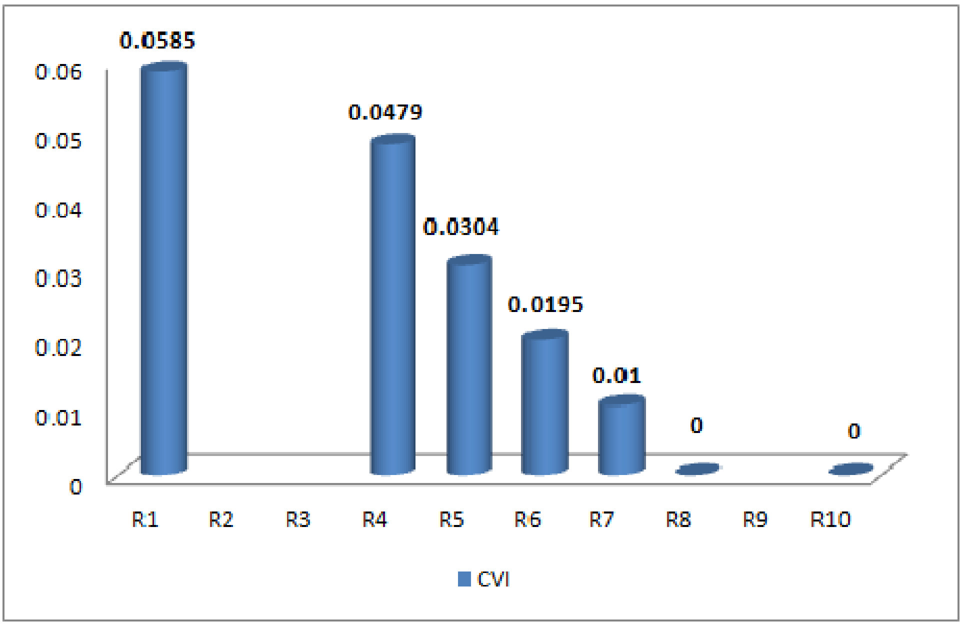

Figure 5: CVI by using existing measure τ_{18}

$\tilde{R}_1 = \{G_1\}, where\ G_1 = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\},$
$\tilde{R}_2 = \{G_1, G_2\}, where\ G_1 = \{\wp_1\}, G_2 = \{\wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\},$
$\tilde{R}_3 = \{G_1, G_2, G_3\}, where\ G_1 = \{\wp_1\}, G_2 = \{\wp_5\}, G_3 = \{\wp_2, \wp_3, \wp_4, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\},$
$\tilde{R}_4 = \{G_1, G_2, G_3, G_4\}, where\ G_1 = \{\wp_1\}, G_2 = \{\wp_4\}, G_3 = \{\wp_5\},$
$G_4 = \{\wp_2, \wp_3, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\},$
$\tilde{R}_5 = \{G_1, G_2, G_3, G_4, G_5\}, where\ G_1 = \{\wp_1\}, G_2 = \{\wp_2\}, G_3 = \{\wp_4\},$
$G_4 = \{\wp_5\}, G_5 = \{\wp_3, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\},$

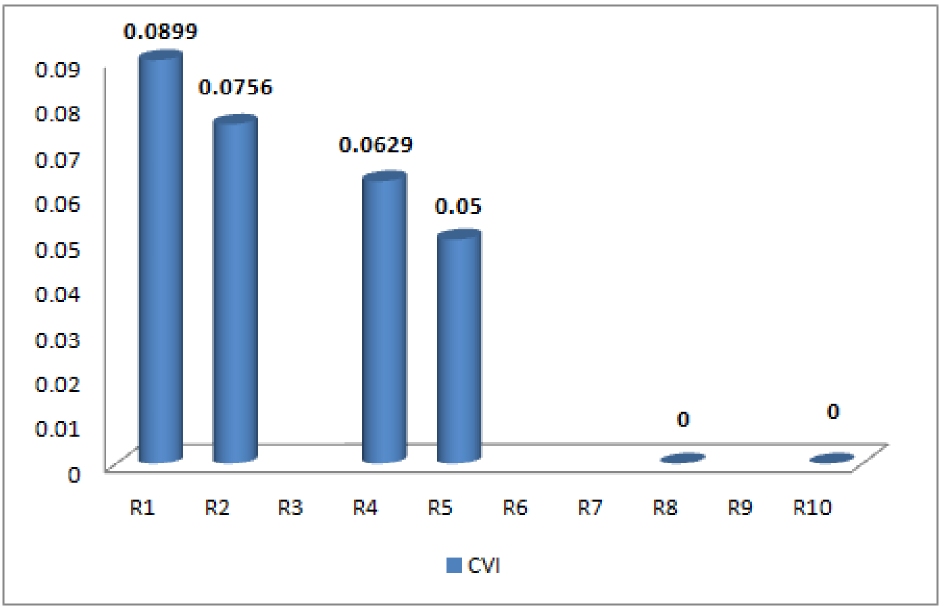

Figure 6: CVI by using existing measure τ_{19}

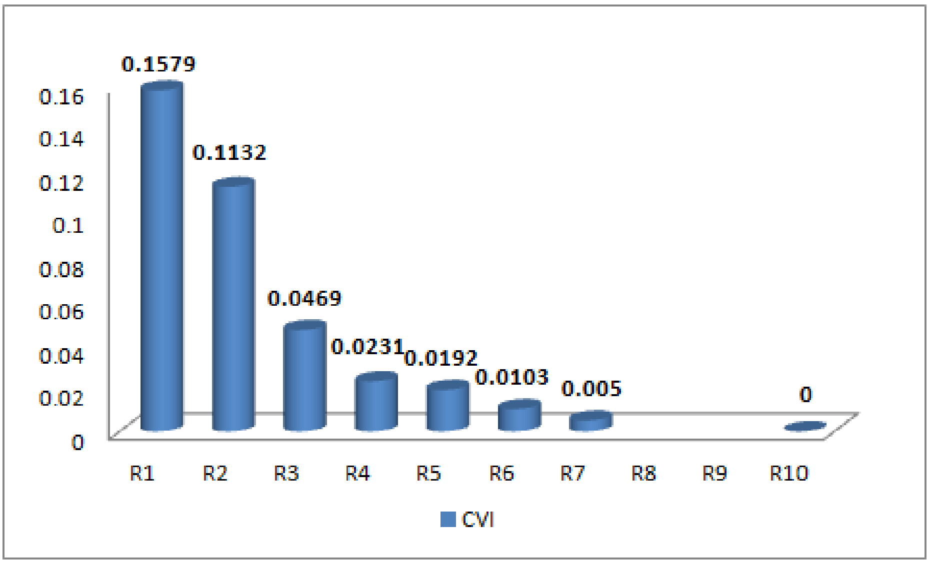

Figure 7: CVI by using existing measure τ_{22}

$\tilde{R}_6 = \{G_1, G_2, G_3, G_4, G_5, G_6\}, where\ G_1 = \{\wp_1\}, G_2 = \{\wp_2\}, G_3 = \{\wp_3, \wp_9, \wp_{10}\},$
$G_4 = \{\wp_4\}, G_5 = \{\wp_5\}, G_6 = \{\wp_6, \wp_7, \wp_8\},$
$\tilde{R}_7 = \{G_1, G_2, G_3, G_4, G_5, G_6, G_7\}, where\ G_1 = \{\wp_1\}, G_2 = \{\wp_2\}, G_3 = \{\wp_3, \wp_9, \wp_{10}\},$
$G_4 = \{\wp_4\}, G_5 = \{\wp_5\}, G_6 = \{\wp_6, \wp_7\}, G_7 = \{\wp_8\},$
$\tilde{R}_8 = \{G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8\}, where\ G_1 = \{\wp_1\}, G_2 = \{\wp_2\}, G_3 = \{\wp_3, \wp_9\},$
$G_4 = \{\wp_4\}, G_5 = \{\wp_5\}, G_6 = \{\wp_6, \wp_7\}, G_7 = \{b_8\}, G_8 = \{\wp_{10}\},$
$\tilde{R}_9 = \{G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9\}, where\ G_1 = \{\wp_1\}, G_2 = \{\wp_2\}, G_3 = \{\wp_3, \wp_9\},$
$G_4 = \{\wp_4\}, G_5 = \{\wp_5\}, G_6 = \{\wp_6\}, G_7 = \{\wp_7\}, G_8 = \{\wp_8\}, G_9 = \{\wp_{10}\},$
$\tilde{R}_{10} = \{G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9, G_{10}\}, where\ G_1 = \{\wp_1\}, G_2 = \{\wp_2\}, G_3 = \{\wp_3\},$
$G_4 = \{\wp_4\}, G_5 = \{\wp_5\}, G_6 = \{\wp_6\}, G_7 = \{\wp_7\}, G_8 = \{\wp_8\}, G_9 = \{\wp_9\}, G_{10} = \{\wp_{10}\}.$
Now, in order to calculate the local density , average density, and uniform density we use association matrix $\tilde{T}$. It is well known that we must use local density to determine each element's

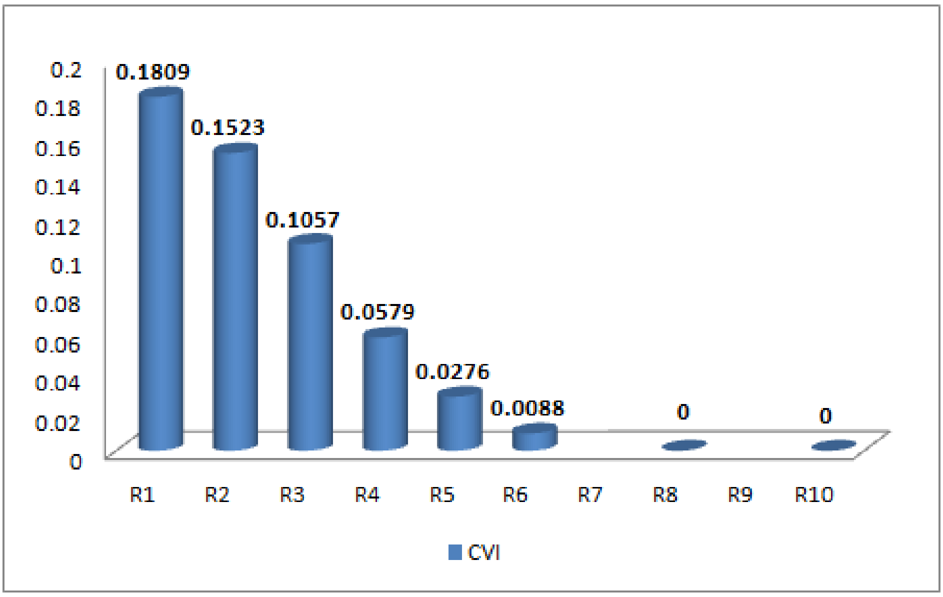

Figure 8: CVI by using existing measure τ_{23}

closest neighbor within the same cluster. Thus, there won't be any local density, average density, or uniformity in the occurrence of a singleton set of clusters.

- For partition $\tilde{R}_1$ we have $\tilde{L}(\wp_1)_{\wp_1\in G_1} = 0.5819$, $\tilde{L}(\wp_2)_{\wp_2\in G_1} = 0.8654$, $\tilde{L}(\wp_3)_{\wp_3\in G_1} = 0.9206 = \tilde{L}(\wp_9)_{\wp_9\in G_1}$, $\tilde{L}(\wp_4)_{\wp_4\in G_1} = 0.8524$, $\tilde{L}(\wp_5)_{\wp_5\in G_1} = 0.8108$, $\tilde{L}(\wp_6)_{\wp_6\in G_1} = 0.8966 = \tilde{L}(\wp_7)_{\wp_7\in G_1}$, $\tilde{L}(\wp_8)_{\wp_8\in G_1} = 0.8808$, $\tilde{L}(\wp_{10})_{\wp_{10}\in G_1} = 0.8952$. Also $\tilde{A}(G_1) = 0.8520$, and $\tilde{U}(G_1) = 0.6593$.
- For partition $\tilde{R}_2$ we have $\tilde{L}(\wp_2)_{\wp_2\in G_2} = 0.8654$, $\tilde{L}(\wp_3)_{\wp_3\in G_2} = 0.9206 = \tilde{L}(\wp_9)_{\wp_9\in G_2}$, $\tilde{L}(\wp_4)_{\wp_4\in G_2} = 0.8524$, $\tilde{L}(\wp_5)_{\wp_5\in G_2} = 0.8108$, $\tilde{L}(\wp_6)_{\wp_6\in G_2} = 0.8966 = \tilde{L}(\wp_7)_{\wp_7\in G_2}$, $\tilde{L}(\wp_8)_{\wp_8\in G_2} = 0.8808$, $\tilde{L}(\wp_{10})_{\wp_{10}\in G_2} = 0.8952$. Also $\tilde{A}(G_2) = 0.8829$, and $\tilde{U}(G_2) = 0.2687$.
- For partition $\tilde{R}_3$ we have $\tilde{L}(\wp_2)_{\wp_2\in G_3} = 0.8654$, $\tilde{L}(\wp_3)_{\wp_3\in G_3} = 0.9206 = \tilde{L}(\wp_9)_{\wp_9\in G_3}$, $\tilde{L}(\wp_4)_{\wp_4\in G_3} = 0.8524$, $\tilde{L}(\wp_6)_{\wp_6\in G_3} = 0.8966 = \tilde{L}(\wp_7)_{\wp_7\in G_3}$, $\tilde{L}(\wp_8)_{\wp_8\in G_3} = 0.8808$, $\tilde{L}(\wp_{10})_{\wp_{10}\in G_3} = 0.8952$. Also $\tilde{A}(G_3) = 0.8910$, and $\tilde{U}(G_3) = 0.1671$.
- For partition $\tilde{R}_4$ we have $\tilde{L}(\wp_2)_{\wp_2\in G_4} = 0.8654$, $\tilde{L}(\wp_3)_{\wp_3\in G_4} = 0.9206 = \tilde{L}(\wp_9)_{\wp_9\in G_4}$, $\tilde{L}(\wp_6)_{\wp_6\in G_4} = 0.8966 = \tilde{L}(\wp_7)_{\wp_7\in G_4}$, $\tilde{L}(\wp_8)_{\wp_8\in G_4} = 0.8808$, $\tilde{L}(\wp_{10})_{\wp_{10}\in G_4} = 0.8952$. Also $\tilde{A}(G_4) = 0.8965$, and $\tilde{U}(G_4) = 0.1075$.
- For partition $\tilde{R}_5$ we have $\tilde{L}(b_3)_{\wp_3\in G_5} = 0.9206 = \tilde{L}(\wp_9)_{\wp_9\in G_5}$, $\tilde{L}(\wp_6)_{\wp_6\in G_5} = 0.8966 = \tilde{L}(\wp_7)_{\wp_7\in G_5}$, $\tilde{L}(\wp_8)_{\wp_8\in G_5} = 0.8808$, $\tilde{L}(\wp_{10})_{\wp_{10}\in G_5} = 0.8952$. Also $\tilde{A}(G_5) = 0.9017$, and $\tilde{U}(G_5) = 0.0836$.
- For partition $\tilde{R}_6$ we have $\tilde{L}(\wp_3)_{\wp_3\in G_3} = 0.9206 = \tilde{L}(\wp_9)_{\wp_9\in G_3}$, $\tilde{L}(\wp_6)_{\wp_6\in G_6} = 0.8966 = \tilde{L}(\wp_7)_{\wp_7\in G_6}$, $\tilde{L}(\wp_8)_{\wp_8\in G_6} = 0.8808$, $\tilde{L}(\wp_{10})_{\wp_{10}\in G_3} = 0.8952$. Also $\tilde{A}(G_3) = 0.9121$, $\tilde{A}(G_6) = 0.8913$, $\tilde{U}(G_3) = 0.0371$, and $\tilde{U}(G_6) = 0.0236$.
- For partition $\tilde{R}_7$ we have $\tilde{L}(\wp_3)_{\wp_3\in G_3} = 0.9206 = \tilde{L}(\wp_9)_{\wp_9\in G_3}$, $\tilde{L}(\wp_6)_{\wp_6\in G_6} = 0.8966 = \tilde{L}(\wp_7)_{\wp_7\in G_6}$, $\tilde{L}(\wp_{10})_{\wp_{10}\in G_3} = 0.8952$. Also $\tilde{A}(G_3) = 0.9121$, $\tilde{A}(G_6) = 0.8966$, $\tilde{U}(G_3) = 0.0371$, and $\tilde{U}(G_6) = 0$.
- For partition $\tilde{R}_8$ we have $\tilde{L}(\wp_3)_{\wp_3\in G_3} = 0.9206 = \tilde{L}(\wp_9)_{\wp_9\in G_3}$, $\tilde{L}(\wp_6)_{\wp_6\in G_6} = 0.8966 = \tilde{L}(\wp_7)_{\wp_7\in G_6}$. Also $\tilde{A}(G_3) = 0.9206$, $\tilde{A}(G_6) = 0.8966$, and $\tilde{U}(G_3) = 0 = \tilde{U}(G_6)$.
- For partition $\tilde{R}_9$ we have $\tilde{L}(\wp_3)_{\wp_3\in G_3} = 0.9206 = \tilde{L}(\wp_9)_{\wp_9\in G_3}$. Also $\tilde{A}(G_3) = 0.9206$, and $\tilde{U}(G_5) = 0$.

Table 11: CVI by using different measures(blank spaces show that there is no partition $\tilde{R}_t$ with t clusters.)

	$Cluster\ Validity\ Indices\ by\ using\ different\ measures$									
$Partition$	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$	$\tilde{R}_4$	$\tilde{R}_5$	$\tilde{R}_6$	$\tilde{R}_7$	$\tilde{R}_8$	$\tilde{R}_9$	$\tilde{R}_{10}$
τ_J	0.6593	0.2418	0.1337	0.0752	0.0501	0.0182	0.0101	0	0	0
τ_{18}	0.0585			0.0479	0.0304	0.0195	0.0100	0		0
τ_{19}	0.0899	0.0756		0.0629	0.05			0		0
τ_{22}	0.1579	0.1132	0.0469	0.0231	0.0192	0.0103	0.005			0
τ_{23}	0.1809	0.1523	0.1057	0.0579	0.0276	0.0088		0		0

- In order to find the optimal partition, we calculate
  $CVI_f{}^2 = \frac{CVI(\tilde{R}_2)}{CVI(\tilde{R}_1)} = 0.3667,$
  $CVI_f{}^3 = \frac{CVI(\tilde{R}_3)}{CVI(\tilde{R}_2)} = 0.5529,$
  $CVI_f{}^4 = \frac{CVI(\tilde{R}_4)}{CVI(\tilde{R}_3)} = 0.5624,$
  $CVI_f{}^5 = \frac{CVI(\tilde{R}_5)}{CVI(\tilde{R}_4)} = 0.6662,$
  $CVI_f{}^6 = \frac{CVI(\tilde{R}_6)}{CVI(\tilde{R}_5)} = 0.3632,$
  $CVI_f{}^7 = \frac{CVI(\tilde{R}_7)}{CVI(\tilde{R}_6)} = 0.5449,$
  $CVI_f{}^8 = \frac{CVI(\tilde{R}_8)}{CVI(\tilde{R}_7)} = 0.$
  Since $CVI_f{}^8 = 0 = min\ \{CVI_f{}^2, \ldots, CVI_f{}^t\}$. Therefore, optimal partition is $\tilde{R}_8$.

Table 12: CVI by using different measures(blank spaces show that there is no partition $\tilde{R}_t$ with t clusters.)

$Measures$	$Confidence\ level$	$Valid\ Clusters$
Proposed τ_J	$0.8952 < \beta \leq 0.8966$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3, \wp_9\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6, \wp_7\}, \{\wp_8\}, \{\wp_{10}\}$
τ_{18}	$0.9879 < \beta \leq 0.9951$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3, \wp_9\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6, \{\wp_8\}, \{\wp_7, \wp_{10}\}$
τ_{19}	$0.8564 < \beta \leq 0.9119$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6\}, \{\wp_8\}, \{\wp_9, \wp_{10}\}$
τ_{22}	$0.9715 < \beta \leq 1.0000$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4\}, \{\wp_5\}, \{\wp_6\}, \{\wp_7\}, \{\wp_8\}, \{\wp_9\}, \{\wp_{10}\}$
τ_{23}	$0.8679 < \beta \leq 0.9335$	$\{\wp_1\}, \{\wp_2\}, \{\wp_3\}, \{\wp_4, \wp_5\}, \{\wp_6, \wp_9\}, \{\wp_7\}, \{\wp_8\}, \{\wp_{10}\}$

Discussion

In this example, we calculated CVI by using proposed $\mathbb{CC}$ as well as some of the existing measures to see the effectiveness of the $\mathbb{CC}$. We know that the highest CVI yields the worst cluster and the lowest CVI gives the best cluster. From Figure 4 we observe that we get the first lowest CVI for the partition R_8, in other words the process terminated for 8 clusters. Therefore we obtain 8 clusters. The valid clusters (by using proposed $\mathbb{CC}$) are obtained in 8^{th} partition, $R_8 = \{\{\wp_1\}, \{\wp_2\}, \{\wp_4\}, \{\wp_5\}, \{\wp_8\}, \{\wp_{10}\}, \{\wp_3, \wp_9\}, \{\wp_6, \wp_7\}\}$, we get it at $0.8952 < \beta \leq 0.8966$ level of confidence. From Table-12 and figures-5,6,8 we observe that by using τ_{18} [62], τ_{19} [62], and τ_{23} [65] we get 8 valid clusters at $0.9879 < \beta \leq 0.9951$, $0.8564 < \beta \leq 0.9119$, and $0.8679 < \beta \leq 0.9335$ levels of confidence respectively. By using τ_{22} [65] we get 10 valid clusters at $0.9715 < \beta \leq 1.0000$ level of confidence. We get valid clusters at low level of confidence as compared to that of other measures. Therefore, the proposed $\mathbb{CC}$ can be considered as an effective $\mathbb{CC}$.

7 Application in Green Supply Chain Management

An innovative approach to management that addresses environmental concerns is called "green supply chain management ($GSCM$)." Green supply chain strategies support the competitiveness of the corporate market, foster consumer loyalty, enhance brand perception, and reduce adverse effects, which are discussed by Kaur et. al ([113]). It encompasses a wide range of tasks, such as packaging, marketing, production, and supplier evaluation in an environmentally friendly manner. Choosing the right supplier in terms of social and environmental factors is crucial for sustainable growth([114]). The fundamental $GSCM$ stage that directly affects environmental protection is green supplier selection GSS. The selection of green suppliers is an MCDM challenge with numerous conflicting evaluation criteria. One of the main uses of $\mathbb{CC}$ is in MCDM. In this case, a GSS problem is resolved using $\mathbb{CC}$. Initially, a set of criteria is used to pick some alternatives. Next, a group of experts assess each criteria of each option using a pre-prepared set of criteria for the evaluation process. A reference index is then chosen as the ideal alternative based on this set of criteria. Currently, a combined decision matrix is created using the assessed alternatives as a basis. The best alternative among all of them is determined by calculating the correlation coefficient between each alternative and the best alternative that has been chosen. The alternative with the highest $\mathbb{CC}$ is chosen. The original discussion of this issue was done Banaeian et. al [115], who created triangular fuzzy numbers using the expert's linguistically expressed ratings. Furthermore, Shishvan et. al [116] converts these terms into $\mathbb{SFS}_{\mathbb{S}}$. Following that, various already-existing techniques as well as the TOPSIS and VIKOR methodologies were utilized to address this issue. Now, we apply the proposed technique to resolve this problem in order to demonstrate the efficiency of the proposed method.

- **Algorithm:** Steps to solve $GSCM$ problem are as follows:
- **Step 1** In this phase, the $\mathbb{SF}$ decision matrix will be created. To begin, a sample space will be created comprising each possible value (in the form of $\mathbb{SFN}$) that can be assigned to a product supply factor. Following that, each specialist will assign a value from the sample space to each provider for each factor.
- **Step 2** The overall matrix will be created by taking the average of $\mathbb{SFN}_{\mathbb{S}}$ in each decision matrix for each oil provider. The weight of each criterion, depending on expert review, will be assigned to each oil source using the technique of [116].
- **Step 3** A reference index RI=$\{(\breve{m}_i, \breve{a}_i, \breve{n}_i | 1 \leq i \leq t)\}$ is chosen from the criteria in linguistic terms, which are converted into $\mathbb{SF}$ numbers. It is chosen by using the following methodology:

$$\breve{m} = \max_j \breve{m}_j, \breve{a} = \min_j \breve{a}_j, \breve{n} = \min_j \breve{n}_j, \ (1 \leq j \leq 7) \tag{7.1}$$

- **Step 4** Calculate the degree of $\mathbb{CC}$ between each supplier's ratings for the assessment criteria and the RI using equations ((3.1)-(3.3)).
- **Step 5** Sort the suppliers by $\mathbb{CC}$ in decreasing order. The best supplier is the one with the highest $\mathbb{CC}$.

Example 7.1. (Case Study)
The source of this case study are [115, 116]. Eatable vegetable oils and detergents are produced by a corporation in the agri-food sector. It is an ISO 14000 accredited company. To handle its environmental obligations, it makes use of the relevant guidelines. One way it does this is by pushing its suppliers to continuously enhance their environmental performance and practices. The company employs coconut, sesame, peanut, and avocado oils as its main raw oils. Each raw material provider must be evaluated and chosen. There are thirteen suppliers in total: four for coconut oil ($S_1^c, S_2^c, S_3^c, S_4^c$), three for sesame oil (S_1^s, S_2^s, S_3^s), three for peanut oil (S_1^p, S_2^p, S_3^p), and three for avocado oil (S_1^a, S_2^a, S_3^a). These thirteen vendors were chosen based on their backgrounds and experience as providers in a variety of industries. The finest four providers for each oil will be chosen from this list. Similar to traditional suppliers, but with the environmental factor taken into account, are the assessment criteria for green suppliers.

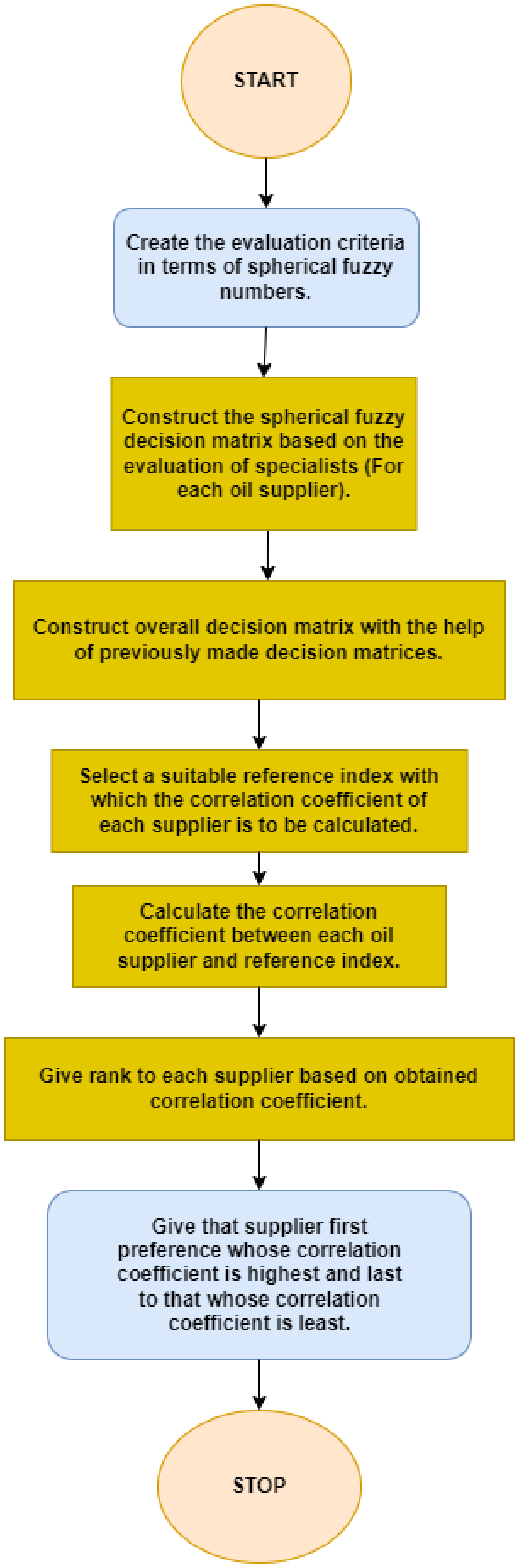

Figure 9: Algorithm to solve green supply chain management problem

The factors include $\{delivery\ time\ (C_1), product\ quality\ (C_2), cost\ (C_3), service\ quality\ (C_4), effect\ onenvironment\ (C_5)\}$. Three specialists have been chosen based on their expertise and industry knowledge. These specialists have been assigned with evaluating the suppliers' ratings for the assessment criteria. These three industry specialists evaluate providers in linguistic terms using the standards that have been established.

Table 13: Evaluation criteria for suppliers

$Linguistic\,Terms$	$SFNs$
Very High Level (VHL)	(0.9,0.1,0.0)
High Level (HL)	(0.7,0.3,0.2)
Slightly High Level (SHL)	(0.6,0.4,0.3)
Moderate Level (ML)	(0.5,0.5,0.5)
Slightly Low Level (SLL)	(0.4,0.6,0.3)
Low Level (LL)	(0.3,0.7,0.2)
Very Low Level (VLL)	(0.1,0.9,0.0)

Step 1 (Construction of Spherical Fuzzy Decision Matrix) In this step the decision matrix for each oil supplier is constructed, on the basis of specialists' evaluation criteria. Table-13 provides all of the various options for evaluating any oil supplier based on factors affecting product supply. Furthermore, the specialists evaluated each oil provider based on the elements influencing supply and options presented in Table 13. Each specialist assigned one choice from Table-13 to each oil provider for every factor. The following tables give the evaluation reports from the specialists for sesame oil, avocado oil, coconut oil, and peanut oil, respectively: Table-14, 15, 16, and 17. Here, weights of all the factors have been taken from [116].

Table 14: Spherical Fuzzy Decision Matrix for sesame oil suppliers

$Industry\ Specialists$	$Suppliers$	C_1	C_2	C_3	C_4	C_5
$\check{I}_1$	S_1^s	(0.3,0.7,0.2)	(0.4,0.6,0.3)	(0.4,0.6,0.3)	(0.3,0.7,0.2)	(0.7,0.3,0.2)
	S_2^s	(0.7,0.3,0.2)	(0.6,0.4,0.3)	(0.6,0.4,0.3)	(0.7,0.3,0.2)	(0.9,0.1,0.0)
	S_3^s	(0.6,0.4,0.3)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.6,0.4,0.3)	(0.4,0.6,0.3)
$\check{I}_2$	S_1^s	(0.3,0.7,0.2)	(0.3,0.7,0.2)	(0.3,0.7,0.2)	(0.3,0.7,0.2)	(0.5,0.5,0.5)
	S_2^s	(0.7,0.3,0.2)	(0.6,0.4,0.3)	(0.6,0.4,0.3)	(0.6,0.4,0.3)	(0.4,0.6,0.3)
	S_3^s	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.7,0.3,0.2)	(0.5,0.5,0.5)	(0.9,0.1,0.0)
$\check{I}_3$	S_1^s	(0.6,0.4,0.3)	(0.5,0.5,0.5)	(0.6,0.4,0.3)	(0.4,0.6,0.3)	(0.6,0.4,0.3)
	S_2^s	(0.9,0.1,0.0)	(0.9,0.1,0.0)	(0.7,0.3,0.2)	(0.7,0.3,0.2)	(0.3,0.7,0.2)
	S_3^s	(0.6,0.4,0.3)	(0.5,0.5,0.5)	(0.7,0.3,0.2)	(0.4,0.6,0.3)	(0.5,0.5,0.5)

Step 2 (Construction of overall decision matrix) In this step we will construct a final decision matrix on the basis of the evaluation provided by the industry specialists. The overall decision matrix is created once all three specialists' reports for each oil supplier have been received. After taking the average of the ith element from Table—14, 15, 16, and 17—the corresponding ith element of the overall decision matrix is produced. The final decision matrix's remaining elements are constructed in a similar manner. This new matrix is presented by the Table-18

Step 3 (Selection of Reference Index) Now the best Reference Index (RI) is generated based on linguistic terms found in Table-13. This index can be considered as the ideal supplier. It is obtained with the help of equation (7.1).

Table 15: Spherical Fuzzy Decision Matrix for avocado oil suppliers

Industry Specialists	*Suppliers*	C_1	C_2	C_3	C_4	C_5
$\check{I}_1$	S_1^a	(0.5,0.5,0.5)	(0.6,0.4,0.3)	(0.4,0.6,0.3)	(0.4,0.6,0.3)	(0.7,0.3,0.2)
	S_2^a	(0.6,0.4,0.3)	(0.5,0.5,0.5)	(0.7,0.3,0.2)	(0.7,0.3,0.2)	(0.9,0.1,0.0)
	S_3^a	(0.6,0.4,0.3)	(0.7,0.3,0.2)	(0.6,0.4,0.3)	(0.7,0.3,0.2)	(0.6,0.4,0.3)
$\check{I}_2$	S_1^a	(0.9,0.1,0.0)	(0.3,0.7,0.2)	(0.3,0.7,0.2)	(0.6,0.4,0.3)	(0.3,0.7,0.2)
	S_2^a	(0.5,0.5,0.5)	(0.7,0.3,0.2)	(0.7,0.3,0.2)	(0.7,0.3,0.2)	(0.6,0.4,0.3)
	S_3^a	(0.6,0.4,0.3)	(0.5,0.5,0.5)	(0.6,0.4,0.3)	(0.3,0.7,0.2)	(0.5,0.5,0.5)
$\check{I}_3$	S_1^a	(0.7,0.3,0.2)	(0.7,0.3,0.2)	(0.6,0.4,0.3)	(0.5,0.5,0.5)	(0.3,0.7,0.2)
	S_2^a	(0.4,0.6,0.3)	(0.3,0.7,0.2)	(0.9,0.1,0.0)	(0.4,0.6,0.3)	(0.6,0.4,0.3)
	S_3^a	(0.9,0.1,0.0)	(0.3,0.7,0.2)	(0.6,0.4,0.3)	(0.9,0.1,0.0)	(0.5,0.5,0.5)

Table 16: Spherical Fuzzy Decision Matrix for coconut oil suppliers

Industry Specialists	*Suppliers*	C_1	C_2	C_3	C_4	C_5
$\check{I}_1$	S_1^c	(0.7,0.3,0.2)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.6,0.4,0.3)	(0.3,0.7,0.2)
	S_2^c	(0.3,0.7,0.2)	(0.1,0.9,0.0)	(0.3,0.7,0.2)	(0.4,0.6,0.3)	(0.1,0.9,0.0)
	S_3^c	(0.5,0.5,0.5)	(0.4,0.6,0.3)	(0.4,0.6,0.3)	(0.9,0.1,0.0)	(0.6,0.4,0.3)
	S_4^c	(0.6,0.4,0.3)	(0.9,0.1,0.0)	(0.6,0.4,0.3)	(0.7,0.3,0.2)	(0.7,0.3,0.2)
$\check{I}_2$	S_1^c	(0.5,0.5,0.5)	(0.7,0.3,0.2)	(0.6,0.4,0.3)	(0.5,0.5,0.5)	(0.7,0.3,0.2)
	S_2^c	(0.4,0.6,0.3)	(0.6,0.4,0.3)	(0.5,0.5,0.5)	(0.6,0.4,0.3)	(0.5,0.5,0.5)
	S_3^c	(0.3,0.7,0.2)	(0.7,0.3,0.2)	(0.4,0.6,0.3)	(0.5,0.5,0.5)	(0.9,0.1,0.0)
	S_4^c	(0.9,0.1,0.0)	(0.4,0.6,0.3)	(0.9,0.1,0.0)	(0.9,0.1,0.0)	(0.6,0.4,0.3)
$\check{I}_3$	S_1^c	(0.9,0.1,0.0)	(0.3,0.7,0.2)	(0.7,0.3,0.2)	(0.7,0.3,0.2)	(0.6,0.4,0.3)
	S_2^c	(0.5,0.5,0.5)	(0.3,0.7,0.2)	(0.7,0.3,0.2)	(0.7,0.3,0.2)	(0.6,0.4,0.3)
	S_3^c	(0.4,0.6,0.3)	(0.5,0.5,0.5)	(0.7,0.3,0.2)	(0.6,0.4,0.3)	(0.3,0.7,0.2)
	S_4^c	(0.3,0.7,0.2)	(0.7,0.3,0.2)	(0.7,0.3,0.2)	(0.7,0.3,0.2)	(0.7,0.3,0.2)

Table 17: Spherical Fuzzy Decision Matrix for peanut oil suppliers

Industry Specialists	*Suppliers*	C_1	C_2	C_3	C_4	C_5
$\check{I}_1$	S_1^p	(0.6,0.4,0.3)	(0.6,0.4,0.3)	(0.6,0.4,0.3)	(0.6,0.4,0.3)	(0.7,0.3,0.2)
	S_2^p	(0.9,0.1,0.0)	(0.5,0.5,0.5)	(0.9,0.1,0.0)	(0.5,0.5,0.5)	(0.4,0.6,0.3)
	S_3^p	(0.3,0.7,0.2)	(0.9,0.1,0.0)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.3,0.7,0.2)
$\check{I}_2$	S_1^p	(0.4,0.6,0.3)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.1,0.9,0.0)
	S_2^p	(0.7,0.3,0.2)	(0.4,0.6,0.3)	(0.7,0.3,0.2)	(0.5,0.5,0.5)	(0.1,0.9,0.0)
	S_3^p	(0.3,0.7,0.2)	(0.4,0.6,0.3)	(0.4,0.6,0.3)	(0.5,0.5,0.5)	(0.6,0.4,0.3)
$\check{I}_3$	S_1^p	(0.6,0.4,0.3)	(0.4,0.6,0.3)	(0.1,0.9,0.0)	(0.1,0.9,0.0)	(0.3,0.7,0.2)
	S_2^p	(0.7,0.3,0.2)	(0.7,0.3,0.2)	(0.6,0.4,0.3)	(0.6,0.4,0.3)	(0.5,0.5,0.5)
	S_3^p	(0.7,0.3,0.2)	(0.3,0.7,0.2)	(0.6,0.4,0.3)	(0.6,0.4,0.3)	(0.9,0.1,0.0)

Table 19: Reference Index for each suppliers

	C_1	C_2	C_3	C_4	C_5
Reference Index (RI)	(0.9,0.1,0)	(0.9,0.1,0)	(0.9,0.1,0)	(0.9,0.1,0)	(0.9,0.1,0)

Table 18: Overall decision matrix for $GSCM$ problem

$Different\ Oils$	$Suppliers$	$Weight$	C_1	C_2	C_3	C_4	C_5
$Sesame\ oil$	S_1^s	$weight$	0.1 (0.41,0.59,0.38)	0.1 (0.34,0.66,0.24)	0.15 (0.44,0.58,0.25)	0.3 (0.46,0.55,0.28)	0.35 (0.61,0.39,0.35)
	S_2^s	$weight$	0.3 (0.76,0.25,0.2)	0.1 (0.67,0.33,0.23)	0.35 (0.8,0.21,0.41)	0.05 (0.64,0.36,0.27)	0.2 (0.69,0.35,0.16)
	S_3^s	$weight$	0.1 (0.5,0.5,0.5)	0.1 (0.51,0.49,0.38)	0.15 (0.57,0.43,0.38)	0.3 (0.65,0.36,0.32)	0.35 (0.71,0.31,0.28)
$Avocado\ oil$	S_1^a	$weight$	0.05 (0.58,0.44,0.24)	0.35 (0.51,0.49,0.38)	0.05 (0.76,0.25,0.25)	0.3 (0.46,0.55,0.28)	0.25 (0.5,0.53,0.2)
	S_2^a	$weight$	0.1 (0.54,0.47,0.34)	0.2 (0.63,0.38,0.23)	0.1 (0.44,0.56,0.39)	0.4 (0.8,0.21,0.14)	0.2 (0.76,0.25,0.2)
	S_3^a	$weight$	0.3 (0.38,0.63,0.36)	0.1 (0.74,0.28,0.14)	0.35 (0.76,0.25,0.2)	0.05 (0.6,0.4,0.3)	0.2 (0.54,0.46,0.44)
$Coconut\ oil$	S_1^c	$weight$	0.1 (0.54,0.47,0.34)	0.2 (0.61,0.39,0.35)	0.4 (0.76,0.25,0.25)	0.2 (0.61,0.39,0.35)	0.1 (0.54,0.47,0.34)
	S_2^c	$weight$	0.05 (0.41,0.63,0.23)	0.35 (0.59,0.42,0.27)	0.05 (0.41,0.59,0.38)	0.3 (0.54,0.47,0.34)	0.25 (0.47,0.56,0.36)
	S_3^c	$weight$	0.15 (0.56,0.45,0.35)	0.35 (0.74,0.27,0.28)	0.05 (0.41,0.59,0.38)	0.15 (0.54,0.48,0.26)	0.3 (0.72,0.3,0.17)
	S_4^c	$weight$	0.2 (0.75,0.26,0.17)	0.3 (0.8,0.21,0.14)	0.15 (0.72,0.3,0.17)	0.3 (0.78,0.23,0.17)	0.05 (0.67,0.33,0.35)
$Peanut\ oil$	S_1^p	$weight$	0.2 (0.51,0.49,0.38)	0.3 (0.47,0.56,0.36)	0.15 (0.55,0.46,0.3)	0.3 (0.47,0.56,0.3)	0.05 (0.48,0.57,0.18)
	S_2^p	$weight$	0.15 (0.56,0.45,0.35)	0.15 (0.54,0.46,0.44)	0.3 (0.8,0.21,0.14)	0.25 (0.78,0.23,0.17)	0.15 (0.38,0.65,0.37)
	S_3^p	$weight$	0.3 (0.69,0.35,0.16)	0.15 (0.54,0.46,0.44)	0.1 (0.5,0.53,0.2)	0.1 (0.51,0.49,0.38)	0.35 (0.72,0.3,0.17)

Step 4 (Calculation of $\mathbb{CC}$) We now compute the $\mathbb{CC}$ between the reference index and rating of each oil supplier by utilizing a few of the currently in use methodologies as well as the proposed approach (3.2-3.4). This $\mathbb{CC}$ will be used to determine which oil source is the best fit for each oil. The output obtained by the proposed approach is presented in the Table-20. The supplier with the highest $\mathbb{CC}$ is given the first rank and so on. The result yielded by different approaches is presented by the Table-21.

Step 5 (Ranking of suppliers) Now, each supplier is given a rank for the corresponding oil based on the results shown in Tables 20 and 21. The $\mathbb{CC}_{\mathbb{S}}$ between each oil supply and the chosen ideal supplier are shown in these two tables, which were created using both the suggested and some current methods. The supplier with the highest $\mathbb{CC}$ receives first rank, and so on. Based on the ranks that are given to suppliers, a preference order is created for them. This preference order is shown in the Table-22.

- **Discussion**
  The outcomes acquired with the suggested methodology are showcased in Table-20. The table shows that among all the providers of the comparable oils, the second supplier S_2^s of sesame oil, the second supplier S_2^a of avocado oil, the fourth supplier S_4^c of coconut oil, and the second supplier S_2^p of peanut oil received top rank. These outcomes are contrasted with some of the current techniques. The values that were acquired are shown in Table-21, and Table-22 displays the preference order determined by these techniques. With the use of a distance metric, TOPSIS and VIKOR are distance-based techniques that determine how distant a solution is from perfection. Table-22 shows that the suggested technique's ranking matches with ranking lists of the TOPSIS, VIKOR, τ_1, S_{J1}, S_{ed}, S_{ed}, S_{sqc}, and S_{CoI} for avocado and sesame oil, respectively. They all indicate that S_2^s is the best source of sesame oil and S_1^s is the worst. It is unanimous among them that the best provider for avocado oil is S_2^a, whereas the worst supplier is S_1^a. For avocado oil, the only result that differs from

Table 20: Results obtained by the proposed approach for $GSCM$ problem

$Different\ oils$	$Suppliers$	CC	$Ranking$
$Sesame\ oil$	S_1^s	0.6486	III
	S_2^s	0.7981	I
	S_3^s	0.7299	II
$Avocado\ oil$	S_1^a	0.6238	III
	S_2^a	0.7820	I
	S_3^a	0.7025	II
$Coconut\ oil$	S_1^c	0.7406	III
	S_2^c	0.6669	IV
	S_3^c	0.7510	II
	S_4^c	0.8426	I
$Peanut\ oil$	S_1^p	0.6405	III
	S_2^p	0.7631	I
	S_3^p	0.7441	II

Table 21: Result obtained by different approaches for $GSCM$ problem

Oil	$Oil\ Suppliers$	τ_1	S_{J1}	S_{ed}	$S_{ed'}$	S_{sqc}	S_{CoI}
$Sesame\ oil$	S_1^s	0.73	0.35	0.33	0.52	0.78	0.4372
	S_2^s	0.95	0.85	0.61	0.75	0.99	0.7081
	S_3^s	0.86	0.62	0.42	0.60	0.93	0.5176
$Avocado\ oil$	S_1^a	0.75	0.37	0.34	0.53	0.82	0.4560
	S_2^a	0.90	0.75	0.56	0.70	0.95	0.6527
	S_3^a	0.80	0.56	0.43	0.60	0.84	0.5277
$Coconut\ oil$	S_1^c	0.89	0.67	0.46	0.63	0.95	0.5557
	S_2^c	0.77	0.41	0.35	0.54	0.84	0.4560
	S_3^c	0.88	0.69	0.48	0.65	0.94	0.5828
	S_4^c	0.96	0.89	0.64	0.77	1.0	0.7360
$Peanut\ oil$	S_1^p	0.72	0.32	0.30	0.50	0.78	0.4140
	S_2^p	0.85	0.64	0.47	0.64	0.89	0.5999
	S_3^p	0.87	0.67	0.51	0.66	0.93	0.5910

the others' is S_{J1}; using this technique, the top ranker remains the same. According to every method, S_4^c, the fourth supplier, is the best source for coconut oil, whereas S_2^c is the worst. According to all the methods, S_3^p, the fourth provider, is the best supplier for peanut oil, whereas S_1^p is the worst. However, for peanut oil, the fourth supplier, S_2^p, is the best supplier and the worst provider, S_1^p, according to the suggested technique, S_{CoI}, as reported in [115]. It is graphically represented by the Figure-10 As a result, every outcome of the suggested strategy agrees with the outcome of [115]. As a result, the suggested strategy produces ideal, dependable, and successful results. It can definitely be used to the $GSCM$ situation.

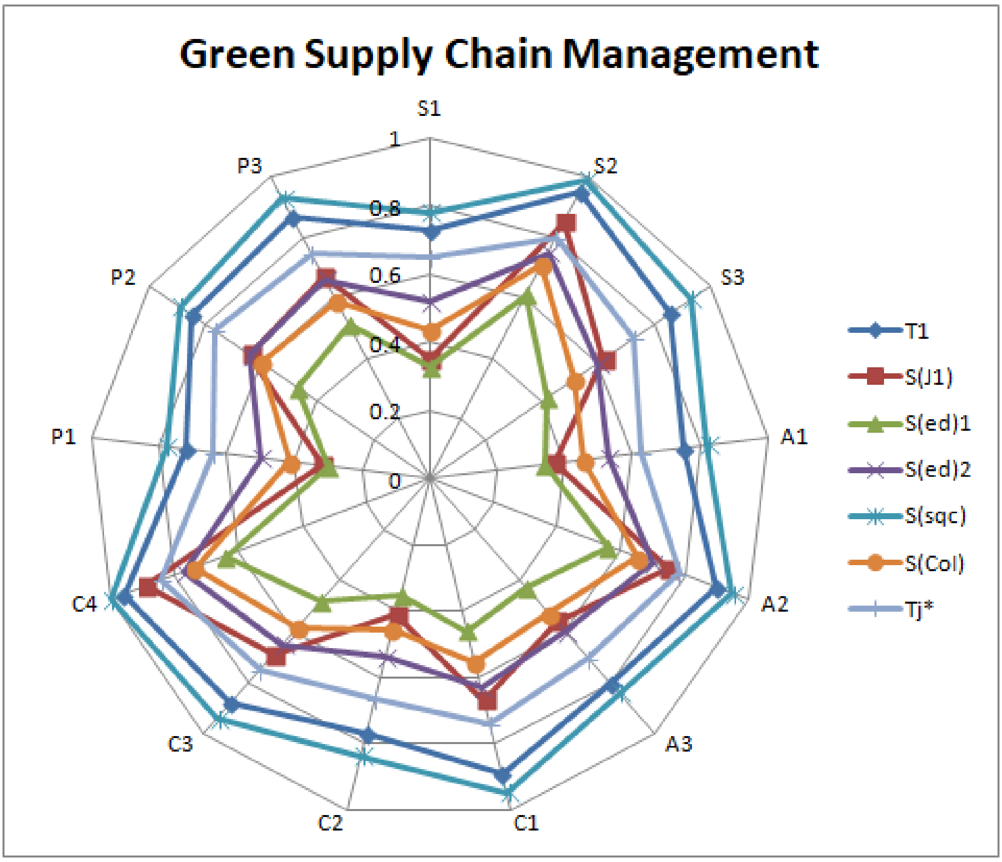

Figure 10: Final result of Green Supply Chain Management

Table 22: Preference order for suppliers of oils by different approaches for $GSCM$ problem

$Methods$	$Preference\ order$	$For\ suppliers$		
	$Sesame\ oil$	$Avocado\ oil$	$Coconut\ oil$	$Peanut\ oil$
$VIKOR$	$S_1^s \prec S_3^s \prec S_2^s$	$S_1^a \prec S_3^a \prec S_2^a$	$S_2^c \prec S_3^c \prec S_1^c \prec S_4^c$	$S_1^p \prec S_2^p \prec S_3^p$
$TOPSIS$	$S_1^s \prec S_3^s \prec S_2^s$	$S_1^a \prec S_3^a \prec S_2^a$	$S_2^c \prec S_3^c \prec S_1^c \prec S_4^c$	$S_1^p \prec S_2^p \prec S_3^p$
τ_1 [84]	$S_1^s \prec S_3^s \prec S_2^s$	$S_1^a \prec S_3^a \prec S_2^a$	$S_2^c \prec S_3^c \prec S_1^c \prec S_4^c$	$S_1^p \prec S_2^p \prec S_3^p$
S_{J1} [116]	$S_1^s \prec S_3^s \prec S_2^s$	$S_3^a \prec S_1^a \prec S_2^a$	$S_2^c \prec S_3^c \prec S_1^c \prec S_4^c$	$S_1^p \prec S_2^p \prec S_3^p$
S_{ed} [116]	$S_1^s \prec S_3^s \prec S_2^s$	$S_1^a \prec S_3^a \prec S_2^a$	$S_2^c \prec S_1^c \prec S_3^c \prec S_4^c$	$S_1^p \prec S_2^p \prec S_3^p$
$S_{ed'}$ [116]	$S_1^s \prec S_3^s \prec S_2^s$	$S_1^a \prec S_3^a \prec S_2^a$	$S_2^c \prec S_1^c \prec S_3^c \prec S_4^c$	$S_1^p \prec S_2^p \prec S_3^p$
S_{sqc} [116]	$S_1^s \prec S_3^s \prec S_2^s$	$S_3^a \prec S_1^a \prec S_2^a$	$S_2^c \prec S_1^c \prec S_3^c \prec S_4^c$	$S_1^p \prec S_2^p \prec S_3^p$
S_{CoI} [91]	$S_1^s \prec S_3^s \prec S_2^s$	$S_1^a \prec S_3^a \prec S_2^a$	$S_2^c \prec S_1^c \prec S_3^c \prec S_4^c$	$S_1^p \prec S_3^p \prec S_2^p$
$Proposed$	$S_1^s \prec S_3^s \prec S_2^s$	$S_1^a \prec S_3^a \prec S_2^a$	$S_2^c \prec S_1^c \prec S_3^c \prec S_4^c$	$S_1^p \prec S_3^p \prec S_2^p$

8 Conclusion

This work proposes a new technique to construct $\mathbb{CC}$ for $\mathbb{SFS}_\mathbb{S}$ that satisfies all of the fundamental properties of a $\mathbb{CC}$. It is capable of overcoming the limitations and drawbacks of present procedures. A comparison analysis is carried out to show how this novel concept outperforms the other approaches. Its effectiveness in solving pattern identification problems indicates its validity, dependability, and utility. An effective case study of an agri-food company to address Green Supply Chain Management is conducted. Finally, it was used to examine the $\mathbb{SFS}_\mathbb{S}$ clustering by obtaining real data on the damage caused by floods in different states of INDIA between 2012 and 2021. The outcomes of each application are compared to those of many other approaches now in use. The only drawback to this strategy is that the computation process is somewhat complicated. It has the potential to be successfully applied in the future to a wide range of multi-criteria decision-making difficulties. It can also be applied to a wide range of other challenges, including waste management, disaster management, image processing, texture analysis, and data mining. With the appropriate adaptations, this method can be used in a variety of scenarios, including Fermatean, q-rung orthopair, hyperspherical neutrosophic numbers, T-spherical fuzzy sets, and many more.

Declarations

Conflict of Interest The authors declare no competing interests.

Data Availability Data will be made available on reasonable request.

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Ethics This article does not contain any studies with human participants or animals performed by the authors.

Authors' Contributions
Jyoti Bajaj: Conceptualization, Data curation, Formal analysis, Investigation, Project administration, Software, Validation, Visualization, Writing- original draft. **Satish Kumar:** Conceptualization,Data curation, Methodology, Resources, Supervision, Validation, Writing- review and editing.

A Appendix

A.1 Some existing measures

Table 23: Details of some existing measures for $\mathbb{SFS}$

Sr. No.	Some Existing measures	Corresponding Formula for A_1, A_2
1	τ_1 [84]	$\frac{1}{N}\sum_{i=1}^{N}\frac{\breve{m}^2_{A_1}(s_i)\breve{m}^2_{A_2}(s_i)+\breve{a}^2_{A_1}(s_i)\breve{a}^2_{A_2}(s_i)+\breve{n}^2_{A_1}(s_i)\breve{n}^2_{A_2}(s_i)}{\sqrt{\breve{m}^2_{A_1}(s_i)+\breve{a}^2_{A_1}(s_i)+\breve{n}^2_{A_1}(s_i)}\sqrt{\breve{m}^2_{A_2}(s_i)+\breve{a}^2_{A_2}(s_i)+\breve{n}^2_{A_2}(s_i)}}$
2	τ_2 [84]	$\frac{1}{N}\sum_{i=1}^{N} cos\left\{\frac{\pi}{2}\max[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert, \lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert, \lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert]\right\}$
3	τ_3 [84]	$\frac{1}{N}\sum_{i=1}^{N} cos\left\{\frac{\pi}{2}\max[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert, \lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert, \lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert]\right\}$
4	τ_4 [84]	$\frac{1}{N}\sum_{i=1}^{N} cot\left\{\frac{\pi}{4}+\frac{\pi}{4}\max[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert, \lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert, \lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert]\right\}$
5	τ_5 [84]	$\frac{1}{N}\sum_{i=1}^{N} cot\left\{\frac{\pi}{4}+\frac{\pi}{4}[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert+\lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert+\lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert]\right\}$
6	τ_6 [84]	$\frac{1}{N}\sum_{i=1}^{N} cos\left\{\frac{\pi}{2}\max[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert, \lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert, \lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert, \lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert]\right\}$
7	τ_7 [84]	$\frac{1}{N}\sum_{i=1}^{N} cos\left\{\frac{\pi}{2}\max[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert, \lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert, \lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert, \lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert]\right\}$
8	τ_8 [84]	$\frac{1}{N}\sum_{i=1}^{N} cot\left\{\frac{\pi}{4}+\frac{\pi}{4}\max[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert, \lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert, \lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert, \lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert]\right\}$
9	τ_9 [84]	$\frac{1}{N}\sum_{i=1}^{N} cot\left\{\frac{\pi}{4}+\frac{\pi}{8}[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert+\lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert+\lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert+\lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert]\right\}$
10	τ_{10} [60]	$\frac{1}{N}\sum_{i=1}^{N} cos\left\{\frac{\pi}{2}\max[\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert, \lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert, \lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert]\right\}$
11	τ_{11} [60]	$\frac{1}{N}\sum_{i=1}^{N} cos\left\{\frac{\pi}{2}\max[\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert, \lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert, \lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert, \lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert]\right\}$
12	τ_{12} [60]	$\frac{1}{N}\sum_{i=1}^{N} cos\left\{\frac{\pi}{4}[\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert+\lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert+\lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert+\lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert]\right\}$
13	τ_{13} [60]	$\frac{1}{N}\sum_{i=1}^{N} cot\left\{\frac{\pi}{4}+\frac{\pi}{4}\max[\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert, \lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert, \lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert, \lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert]\right\}$
14	τ_{14} [61]	$\frac{1}{N}\sum_{i=1}^{N}\frac{\breve{m}_{A_1}(s_i)\breve{m}_{A_2}(s_i)+\breve{a}_{A_1}(s_i)\breve{a}_{A_2}(s_i)+\breve{n}_{A_1}(s_i)\breve{n}_{A_2}(s_i)}{\max\{\left(\breve{m}^2_{A_1}(s_i)+\breve{a}^2_{A_1}(s_i)+\breve{n}^2_{A_1}(s_i)\right),\left(\breve{m}^2_{A_2}(s_i)+\breve{a}^2_{A_2}(s_i)+\breve{n}^2_{A_2}(s_i)\right)\}}$
15	τ_{15} [64]	$\frac{1}{N}\sum_{i=1}^{N}\frac{2\left(\breve{m}_{A_1}(s_i)\breve{m}_{A_2}(s_i)+\breve{a}_{A_1}(s_i)\breve{a}_{A_2}(s_i)+\breve{n}_{A_1}(s_i)\breve{n}_{A_2}(s_i)\right)}{\{\left(\breve{m}^2_{A_1}(s_i)+\breve{a}^2_{A_1}(s_i)+\breve{n}^2_{A_1}(s_i)\right)+\left(\breve{m}^2_{A_2}(s_i)+\breve{a}^2_{A_2}(s_i)+\breve{n}^2_{A_2}(s_i)\right)\}}$
16	τ_{16} [64]	$\frac{1}{N}\sum_{i=1}^{N}\frac{2\left(\breve{m}_{A_1}(s_i)\breve{m}_{A_2}(s_i)+\breve{a}_{A_1}(s_i)\breve{a}_{A_2}(s_i)+\breve{n}_{A_1}(s_i)\breve{n}_{A_2}(s_i)+\breve{\rho}_{A_1}(s_i)\breve{\rho}_{A_2}(s_i)\right)}{\{\left(\breve{m}^2_{A_1}(s_i)+\breve{a}^2_{A_1}(s_i)+\breve{n}^2_{A_1}(s_i)+\breve{\rho}^2_{A_1}(s_i)\right)+\left(\breve{m}^2_{A_2}(s_i)+\breve{a}^2_{A_2}(s_i)+\breve{n}^2_{A_2}(s_i)+\breve{\rho}^2_{A_2}(s_i)\right)\}}$
17	τ_{17} [62]	$1-\frac{1}{4N}\sum_{i=1}^{N}\left(\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert+\lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert+\lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert+\lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert\right)$

Table 24: Details of some existing measures for $\mathbb{SFS}$ 2

Sr. No.	Some Existing measures	Corresponding Formula for A_1, A_2
18	τ_{18} [62]	$1-\frac{1}{4N}\sum_{i=1}^{N}\max\left\{\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert,\lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert,\lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert,\lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert\right\}$
19	τ_{19} [62]	$\frac{1}{4N}\sum_{i=1}^{N}\frac{\min\left\{\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert,\lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert,\lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert,\lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert\right\}}{\max\left\{\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert,\lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert,\lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert,\lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert\right\}}$
20	τ_{20} [63]	$\frac{1}{4N}\sum_{i=1}^{N}\Big[3\sqrt{\breve{m}_{A_1}(s_i)\breve{m}_{A_2}(s_i)}+3\sqrt{\breve{a}_{A_1}(s_i)\breve{a}_{A_2}(s_i)}+3\sqrt{\breve{n}_{A_1}(s_i)\breve{n}_{A_2}(s_i)}+\sqrt{\breve{\rho}_{A_1}(s_i)\breve{\rho}_{A_2}(s_i)}$ $+\sqrt{\big(1-\breve{m}_{A_1}(s_i)-\breve{a}_{A_1}(s_i)\big)\big(1-\breve{m}_{A_2}(s_i)-\breve{a}_{A_2}(s_i)\big)}+\sqrt{\big(1-\breve{m}_{A_1}(s_i)-\breve{n}_{A_1}(s_i)\big)\big(1-\breve{m}_{A_2}(s_i)-\breve{n}_{A_2}(s_i)\big)}$ $+\sqrt{\big(1-\breve{a}_{A_1}(s_i)-\breve{n}_{A_1}(s_i)\big)\big(1-\breve{a}_{A_2}(s_i)-\breve{n}_{A_2}(s_i)\big)}\Big]$
21	τ_{21} [65]	$\frac{1}{N}\sum_{i=1}^{N}\left\{2^{[1-\max\{\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert,\lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert,\lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert\}]}-1\right\}$
22	τ_{22} [65]	$\frac{1}{N}\sum_{i=1}^{N}\left\{2^{\left[1-\frac{1}{2}\left(\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert+\lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert+\lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert\right)\right]}-1\right\}$
23	τ_{23} [65]	$\frac{1}{N}\sum_{i=1}^{N}\left\{2^{[1-\max\{\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert,\lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert,\lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert,\lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert\}]}-1\right\}$
24	τ_{24} [65]	$\frac{1}{N}\sum_{i=1}^{N}\left\{2^{\left[1-\frac{1}{2}\left(\lvert\breve{m}_{A_1}(s_i)-\breve{m}_{A_2}(s_i)\rvert+\lvert\breve{a}_{A_1}(s_i)-\breve{a}_{A_2}(s_i)\rvert+\lvert\breve{n}_{A_1}(s_i)-\breve{n}_{A_2}(s_i)\rvert+\lvert\breve{\rho}_{A_1}(s_i)-\breve{\rho}_{A_2}(s_i)\rvert\right)\right]}-1\right\}$
25	τ_{25} [107]	$\frac{1}{3N}\sum_{i=1}^{N}\Big[2\sqrt{\breve{m}_{A_1}(s_i)\breve{m}_{A_2}(s_i)}+2\sqrt{\breve{a}_{A_1}(s_i)\breve{a}_{A_2}(s_i)}+2\sqrt{\breve{n}_{A_1}(s_i)\breve{n}_{A_2}(s_i)}$ $+\sqrt{\big(1-\breve{m}_{A_1}(s_i)-\breve{a}_{A_1}(s_i)\big)\big(1-\breve{m}_{A_2}(s_i)-\breve{a}_{A_2}(s_i)\big)}+\sqrt{\big(1-\breve{m}_{A_1}(s_i)-\breve{n}_{A_1}(s_i)\big)\big(1-\breve{m}_{A_2}(s_i)-\breve{n}_{A_2}(s_i)\big)}$ $+\sqrt{\big(1-\breve{a}_{A_1}(s_i)-\breve{n}_{A_1}(s_i)\big)\big(1-\breve{a}_{A_2}(s_i)-\breve{n}_{A_2}(s_i)\big)}\Big]$
26	τ_{26} [117]	$1-\frac{1}{2N}\sum_{i=1}^{N}\left[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert+\lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert+\lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert\right]$
27	τ_{27} [117]	$1-\frac{1}{2N}\sum_{i=1}^{N}\max\left[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert,\lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert,\lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert\right]$
28	τ_{28} [117]	$1-\sqrt{\frac{1}{2N}\sum_{i=1}^{N}\max\left[\lvert\breve{m}^2_{A_1}(s_i)-\breve{m}^2_{A_2}(s_i)\rvert,\lvert\breve{a}^2_{A_1}(s_i)-\breve{a}^2_{A_2}(s_i)\rvert,\lvert\breve{n}^2_{A_1}(s_i)-\breve{n}^2_{A_2}(s_i)\rvert\right]}$

A.2 Proof of Theorem-3.1

Proof. (i) 1. The novel method of calculating $\mathbb{CC}$ is

$$\tau_J(A_1, A_2) = \frac{1}{4N}\sum_{i=1}^{N}\big(\phi_i(1-\Delta\breve{m}_i) + \varphi_i(1-\Delta\breve{a}_i) + \psi_i(1-\Delta\breve{n}_i) + \theta_i(1-\Delta\breve{\rho}_i)\big), \text{where}$$

$$\left.\begin{aligned}\phi_i &= \frac{\alpha - \Delta\breve{m}_i - \Delta\breve{m}_{max}}{\alpha - \Delta\breve{m}_{min} - \Delta\breve{m}_{max}},\\ \varphi_i &= \frac{\alpha - \Delta\breve{a}_i - \Delta\breve{a}_{max}}{\alpha - \Delta\breve{a}_{min} - \Delta\breve{a}_{max}},\\ \psi_i &= \frac{\alpha - \Delta\breve{n}_i - \Delta\breve{n}_{max}}{\alpha - \Delta\breve{n}_{min} - \Delta\breve{n}_{max}},\\ \theta_i &= \frac{\alpha - \Delta\breve{\rho}_i - \Delta\breve{\rho}_{max}}{\alpha - \Delta\breve{\rho}_{min} - \Delta\breve{\rho}_{max}},\ \alpha > 2 \text{ and finite;}\end{aligned}\right\}$$

$$\left.\begin{aligned}\Delta\breve{m}_i &= |\breve{m}_{A_1}(s_i) - \breve{m}_{A_2}(s_i)|,\ \Delta\breve{a}_i = |\breve{a}_{A_1}(s_i) - \breve{a}_{A_2}(s_i)|,\\ \Delta\breve{n}_i &= |\breve{n}_{A_1}(s_i) - \breve{n}_{A_2}(s_i)|,\ \Delta\breve{\rho}_i = |\breve{\rho}_{A_1}(s_i) - \breve{\rho}_{A_2}(s_i)|;\\ \Delta\breve{m}_{min} &= \min_i(\Delta\breve{m}_i),\ \Delta\breve{a}_{min} = \min_i(\Delta\breve{a}_i),\\ \Delta\breve{n}_{min} &= \min_i(\Delta\breve{n}_i),\ \Delta\breve{\rho}_{min} = \min_i(\Delta\breve{\rho}_i);\\ \Delta\breve{m}_{max} &= \max_i(\Delta\breve{m}_i),\ \Delta\breve{a}_{max} = \max_i(\Delta\breve{a}_i),\\ \Delta\breve{n}_{max} &= \max_i(\Delta\breve{n}_i),\ \Delta\breve{\rho}_{max} = \max_i(\Delta\breve{\rho}_i),\\ \breve{\rho}_{A_1}(s_i) &= \sqrt{1 - \breve{m}^2_{A_1}(s_i) - \breve{a}^2_{A_1}(s_i) - \breve{n}^2_{A_1}(s_i)}.\end{aligned}\right\}$$

Now

$$\tau_J(A_1, A_2) = \frac{1}{4N}\sum_{i=1}^{N}\Big(\phi_i(1 - |\breve{m}_{A_1}(s_i) - \breve{m}_{A_2}(s_i)|) + \varphi_i(1 - |\breve{a}_{A_1}(s_i) - \breve{a}_{A_2}(s_i)|) + \psi_i(1 - |\breve{n}_{A_1}(s_i) - \breve{n}_{A_2}(s_i)|) + \theta_i(1 - |\breve{\rho}_{A_1}(s_i) - \breve{\rho}_{A_2}(s_i)|+)\Big)$$

$$= \frac{1}{4N}\sum_{i=1}^{N}\Big(\phi_i(1 - |-1||\breve{m}_{A_2}(s_i) - \breve{m}_{A_1}(s_i)|) + \varphi_i(1 - |-1||\breve{a}_{A_2}(s_i) - \breve{a}_{A_1}(s_i)|) + \psi_i(1 - |-1||\breve{n}_{A_2}(s_i) - \breve{n}_{A_1}(s_i)|) + \theta_i(1 - |-1||\breve{\rho}_{A_2}(s_i) - \breve{\rho}_{A_1}(s_i)|+)\Big)$$

$$= \frac{1}{4N}\sum_{i=1}^{N}\Big(\phi_i(1 - |\breve{m}_{A_1}(s_i) - \breve{m}_{A_2}(s_i)|) + \varphi_i(1 - |\breve{a}_{A_1}(s_i) - \breve{a}_{A_2}(s_i)|)+ \psi_i(1 - |\breve{n}_{A_1}(s_i) - \breve{n}_{A_2}(s_i)|) + \theta_i(1 - |\breve{\rho}_{A_1}(s_i) - \breve{\rho}_{A_2}(s_i)|+)\Big)$$

$$= \tau_J(A_2, A_1)$$

Therefore, $\tau_J(A_1, A_2) = \tau_J(A_2, A_1)$.

(ii) Clearly

$$\Delta\breve{m}_i = |\breve{m}_{A_1}(s_i) - \breve{m}_{A_2}(s_i)| \geq 0,\ \Delta\breve{m}_i = |\breve{a}_{A_1}(s_i) - \breve{a}_{A_2}(s_i)| \geq 0,$$
$$\Delta\breve{n}_i = |\breve{n}_{A_1}(s_i) - \breve{n}_{A_2}(s_i)| \geq 0,\ \Delta\breve{\rho}_i = |\breve{\rho}_{A_1}(s_i) - \breve{\rho}_{A_2}(s_i)| \geq 0;$$

$$\Delta\breve{m}_{min} = \min_i(\Delta\breve{m}_i) \geq 0,\ \Delta\breve{a}_{min} = \min_i(\Delta\breve{a}_i) \geq 0,\ \Delta\breve{n}_{min} = \min_i(\Delta\breve{n}_i) \geq 0,$$
$$\Delta\breve{\rho}_{min} = \min_i(\Delta\breve{\rho}_i) \geq 0; \Delta\breve{m}_{max} = \max_i(\Delta\breve{m}_i) \geq 0,\ \Delta\breve{a}_{max} = \max_i(\Delta\breve{a}_i) \geq 0,$$
$$\Delta\breve{n}_{max} = \max_i(\Delta\breve{n}_i) \geq 0,\ \Delta\breve{\rho}_{max} = \max_i(\Delta\breve{\rho}_i) \geq 0,$$

Similarly $\phi_i \geq 0,\ \varphi_i \geq 0,\ \psi_i \geq 0,\ \theta_i \geq 0.$

$Also,\ \Delta\breve{m}_i \leq 1,\ \Delta\breve{a}_i \leq 1,\ \Delta\breve{n}_i \leq 1,\ \Delta\breve{\rho}_i \leq 1.$

Therefore,

$$\tau_J(A_1, A_2) = \frac{1}{4N}\sum_{i=1}^{N}\big(\phi_i(1-\Delta\breve{m}_i) + \varphi_i(1-\Delta\breve{a}_i) + \psi_i(1-\Delta\breve{n}_i) + \theta_i(1-\Delta\breve{\rho}_i)\big) \geq 0$$

Let

$$\sum_{i=1}^{N}\phi_i(1-\Delta\breve{m}_i) = \epsilon_1,\ \sum_{i=1}^{N}\varphi_i(1-\Delta\breve{a}_i) = \epsilon_2,\ \sum_{i=1}^{N}\psi_i(1-\Delta\breve{n}_i) = \epsilon_3,\ \sum_{i=1}^{N}\psi_i(1-\Delta\breve{\rho}_i) = \epsilon_4.$$

By making use of Cauchy-Schwarz Inequality, we have

$$\tau_J(A_1, A_2) = \frac{1}{4N}\sum_{i=1}^{N}\Bigg(\phi_i(1-|\breve{m}_{A_1}(s_i) - \breve{m}_{A_2}(s_i)|) + \varphi_i(1-|\breve{a}_{A_1}(s_i) - \breve{a}_{A_2}(s_i)|) +$$
$$\psi_i(1-|\breve{n}_{A_1}(s_i) - \breve{n}_{A_2}(s_i)|) + \theta_i(1-|\breve{\rho}_{A_1}(s_i) - \breve{\rho}_{A_2}(s_i)|)\Bigg)$$

$$\leq \frac{\sum_{i=1}^{N}\phi_i(1-\Delta\breve{m}_i) + \sum_{i=1}^{N}\varphi_i(1-\Delta\breve{a}_i) + \sum_{i=1}^{N}\psi_i(1-\Delta\breve{n}_i) + \sum_{i=1}^{N}\theta_i(1-\Delta\breve{\rho}_i)}{4N}$$

$$= \frac{\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4}{4N}$$

Therefore,

$$\tau_J(A_1, A_2) - 1 = \frac{\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4}{4N} - 1 = \frac{\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 - 4N}{4N}$$
$$= -\frac{(4N - \epsilon_1 - \epsilon_2 - \epsilon_3) - \epsilon_4}{4N} \leq 0,$$

Therefore, we have $\tau_J(A_1, A_2) \leq 1$. Hence, $\tau_J(A_1, A_2) \in [0, 1]$

(iii) Let us assume that $A_1 = A_2$. Then, we have

$$|\breve{n}_{A_1}(s_i) - \breve{n}_{A_2}(s_i)| = |\breve{a}_{A_1}(s_i) - \breve{a}_{A_2}(s_i)| = |\breve{m}_{A_1}(s_i) - \breve{m}_{A_2}(s_i)| = |\breve{\rho}_{A_1}(s_i) - \breve{\rho}_{A_2}(s_i)| = 0\ \forall i.$$

So, we get

$$\Delta\breve{m}_i = \Delta\breve{a}_i = \Delta\breve{n}_i = \Delta\breve{\rho}_i = 0;\ \Delta\breve{m}_{min} = \Delta\breve{a}_{min} = \Delta\breve{n}_{min} = \Delta\breve{\rho}_{min}$$

$= \Delta\breve{m}_{max} = \Delta\breve{a}_{max} = \Delta\breve{n}_{max} = \Delta\breve{\rho}_{max} = 0;$

Also $\phi_i = \varphi_i = \psi_i = \theta_i = 1\ \forall i.$

Hence $\tau_J(A_1, A_2) = \dfrac{1}{4N}\sum_{i=1}^{N} 4 = 1.$

Conversely, if $\tau_J(A_1, A_2) = 1$, then $A_1 = A_2$.

(iv) Here $A_1 = (\delta_1, \delta_2, \delta_3)$, then $A_1{}^c = (\delta_3, \delta_2, \delta_1)$. Therefore, we have

$$\tau_J(A_1, A_1{}^c) = \frac{1}{4N}\sum_{i=1}^{N}\Big(\phi_i(1 - |\breve{m}_{A_1}(s_i) - \breve{m}_{A_1{}^c}(s_i)|) + \varphi_i(1 - |\breve{a}_{A_1}(s_i) - \breve{a}_{A_1{}^c}(s_i)|) + \psi_i(1 - |\breve{n}_{A_1}(s_i) - \breve{n}_{A_1{}^c}(s_i)|) + \theta_i(1 - |\breve{\rho}_{A_1}(s_i) - \breve{\rho}_{A_1{}^c}(s_i)|)\Big)$$

$$= \frac{1}{4}\Big((1 - |\delta_1 - \delta_3|) + (1 - |\delta_2 - \delta_2|) + (1 - |\delta_3 - \delta_1|) + (1 - \Big|\sqrt{1 - \delta_1{}^2 - \delta_2{}^2 - \delta_3{}^2} - \sqrt{1 - \delta_2{}^2 - \delta_1{}^2 - \delta_3{}^2}\Big|)\Big)$$

$$= \frac{4 - 2|\delta_1 - \delta_3|}{4}.$$

(v) Let $A_1 = (\delta_1, \delta_2, \delta_3)$ and $A_2 = (\delta_3, \delta_4, \delta_5)$, then $A_1{}^c = (\delta_3, \delta_2, \delta_1)$ and $A_2{}^c = (\delta_5, \delta_4, \delta_3)$. So, we have

$$\tau_J(A_1, A_2) = \frac{1}{4N}\sum_{i=1}^{N}\Big(\phi_i(1 - |\breve{m}_{A_1}(s_i) - \breve{m}_{A_2}(s_i)|) + \varphi_i(1 - |\breve{a}_{A_1}(s_i) - \breve{a}_{A_2}(s_i)|) + \psi_i(1 - |\breve{n}_{A_1}(s_i) - \breve{n}_{A_2}(s_i)|) + \theta_i(1 - |\breve{\rho}_{A_1}(s_i) - \breve{\rho}_{A_2}(s_i)|)\Big)$$

$$= \frac{1}{4}\Big((1 - |\delta_1 - \delta_3|) + (1 - |\delta_2 - \delta_4|) + (1 - |\delta_3 - \delta_5|) + (1 - \Big|\sqrt{1 - \delta_1{}^2 - \delta_2{}^2 - \delta_3{}^2} - \sqrt{1 - \delta_3{}^2 - \delta_4{}^2 - \delta_5{}^2}\Big|)\Big)$$

Similarly,

$$\tau_J(A_1{}^c, A_2{}^c) = \frac{1}{4N}\sum_{i=1}^{N}\Big(\phi_i(1 - |\breve{m}^c_{A_1}(s_i) - \breve{m}^c_{A_2}(s_i)|) + \varphi_i(1 - |\breve{a}^c_{A_1}(s_i) - \breve{a}^c_{A_2}(s_i)|) + \psi_i(1 - |\breve{n}^c_{A_1}(s_i) - \breve{n}^c_{A_2}(s_i)|) + \theta_i(1 - |\breve{\rho}^c_{A_1}(s_i) - \breve{\rho}^c_{A_2}(s_i)|)\Big)$$

$$= \frac{1}{4}\Big((1 - |\delta_3 - \delta_5|) + (1 - |\delta_2 - \delta_4|) + (1 - |\delta_1 - \delta_3|) + (1 - \Big|\sqrt{1 - \delta_1{}^2 - \delta_2{}^2 - \delta_3{}^2} - \sqrt{1 - \delta_3{}^2 - \delta_4{}^2 - \delta_5{}^2}\Big|)\Big)$$

$$= \frac{1}{4}\Bigg((1 - |\delta_1 - \delta_3|) + (1 - |\delta_2 - \delta_4|) + (1 - |\delta_3 - \delta_5|) + (1 - \Bigg|\sqrt{1 - {\delta_1}^2 - {\delta_2}^2 - {\delta_3}^2} - \sqrt{1 - {\delta_3}^2 - {\delta_4}^2 - {\delta_5}^2}\Bigg|)\Bigg)$$

$Therefore,\ \tau_J(A_1, A_2) = \tau_J({A_1}^c, {A_2}^c).$

(vi) Proof is straight forward. In the similar way, all these properties are also satisfied by weighted $\mathbb{CC}$ τ_J^* represented by equation (3.4).

□

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