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Fuzzy Knowledge and Accuracy Measures with its Applications in E-Commerce Websites Decision Making

Meenu Goel and Shiv Narain

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Corresponding Author: Meenu Goel

Abstract Entropy measures the uncertainty of a random variable, while the knowledge measure represents the fuzzy information inherited by a fuzzy set. This work introduces a new fuzzy knowledge measure and employs the classic fuzzy VIKOR approach to demonstrate its theoretical and practical value using numerous numerical examples. This paper presents a comprehensive study of all characteristics of this measure. The main goal of the study is to discuss advantages of this fuzzy measure from a variety of perspectives, including the calculation of criteria weights, ambiguity, and the appropriate treatment of structured language variables. In addition, induced measures such as the Accuracy measure and the Information measure are deduced in a fuzzy environment from the proposed knowledge measure. We apply the proposed fuzzy accuracy measure in the VIKOR approach in place of the distance measure.We validate the proposed approach for the selection of the best E-Commerce website. On the basis of numerical results obtained, the present study validates the capability and effectiveness of the proposed approach as compared to those of the existing approaches

1 Introduction

A fuzzy set FS is the most useful concept to extract information from fuzzy terms such as "weak," "fast," "more," and so on. Zadeh [1] was the first who described the uncertainty associated with the fuzzy terms. To introduce the idea of Fuzzy Information Measure (FIM), Luca and Termini [2] used Shannon entropy given by Shannon [3] which was improved by Atanassov [4] and Nguyen [5]. FIM has been defined differently by numerous studies. Let A be a fuzzy set. Then $\mu_A : A \longrightarrow [0, 1]$ denotes its membership function. To distinguish membership function μ_A of a fuzzy set A and its closest crisp set, Kaufmann [6] proposed a methodology. To differentiate between μ_A and its complement Yager [7] provided a measure Szmidt et al. [8] introduced a FIM to distinguish between the most FS and FS. Using generalized fuzzy numbers based on left and right apex angles, Rezvani [9] proposed a new similarity measure. Using a dynamic approach to logarithmic similarity measure for pythagorean fuzzy sets, Arora and Naithani [10] presented a case study of customer preferences of airlines. Contrary to entropy, knowledge present in a fuzzy set can be estimated through knowledge measure (KM). Montes et al. [11] described knowledge measure as entropy complement. After much research, several authors came to the opinion that the KM can replace the entropy measure as the complementing notion of the total uncertainty measure. It suggests that a lower amount of information corresponds to a higher total uncertainty value and vice versa. For multi-criteria ranking, Hasan et al. [12] proposed a knowledge discovery framework using fuzzy and wavelet methods. Using various types of fuzzy numbers, many researchers [13, 14, 15, 16] discussed multicriterion decision making with their industrial applications. Wei et al. [17] modified the work of Szmidt & Kacprzyk [18] for FIM on an IFS. Szmidt et al. [19] suggested a KM for an IFS. FS theory has many applications in various fields. Various authors [14, 20, 21, 22] enriched the field with their contribution in this field and discussed real life applications using different types of fuzzy sets. Kumari et al. [23] used Gaussian fuzzy Numbers to carry out a fuzzy Analysis of a retrial machine repair problem, Radhakrishnan and Saikeerthana [24] proposed a new method for solving fuzzy sequencing problem. Applications of FS extension include pattern recognition by [25]; fuzzy regression and clustering by [26], image processing by [27] etc. Numerous extensions of fuzzy set theory exist, including Intutionistic Fuzzy Set (IFS) [4], Hesitant Fuzzy set (HFS) [28], Neutrosophic Fuzzy set (NFS) [29], and Picture Fuzzy set (PFS) [30]. Besides this many researchers worked on fuzzy set based diophantine equations([31, 32]). We can apply any metric created for FS to extensions of underlying FS. The FIM measures the level of fuzziness for FS's. Many issues in real life do not lend themselves to FIM. Thus a KM is better for these difficulties than FIM. Therefore we use the idea of a fuzzy knowledge measure (FKM) there as well. We suggest an efficient FKM from this perspective. The suggested knowledge measure addresses ambiguity computation, criteria weight calculations, and linguistic comparisons while overcoming all the drawbacks of some measures mentioned in the literature and providing reliable findings. The layout of the present study is:

- We provide a new FKM and its characteristics.
- We provide numerical examples to show how the proposed FKM surpasses all of the current FIMs' and FKMs' drawbacks.
- Measures like FAM and FIM, were derived from the proposed FKM.
- The proposed FAM is utilized in the VIKOR approach to solve the MCDM problem instead of a distance measure.
- Some numerical examples are presented to support the study's assumptions and results.
- We also show how useful it is for validating the top e-commerce websites among a given set of alternatives in the MCDM problem. The conclusion also includes a comparative analysis.

The primary contents of this study are: primary goal and the literature is covered in Section 1. Some of the fundamental definitions are provided in Section 2. Section 3 examines a novel FKM. Its validity and some of its features are examined. We compare the suggested FKM with other measures. Section 4 exhibits the application of the suggested FKM using the VIKOR method on an MCDM problem to find the best e-commerce website among available alternatives. The conclusion of the study and future scope are given in Section 5. Notations used are standard. Throughout the paper, $\mathscr{F}(S)$ is the collection of all fuzzy sets defined on S.

2 Some Definitions

Definition 2.1. A fuzzy set T associated with a finite set S($\neq \phi$) is given by

$$T = \{\langle s_i, \mu_T(s_i)\rangle : s_i \in S\}\,; \tag{2.1}$$

where the membership of an element $s_i \in S$ is given by μ_T : S $\rightarrow [0,1]$.

Definition 2.2. For $T_1, T_2 \in \mathscr{F}(S)$, some basic set theoretic operations are defined as follows:

$$T_1 \cup T_2 = \{\langle s_i, max\left(\mu_{T_1}\left(s_i\right), \mu_{T_2}\left(s_i\right)\right)\rangle : s_i \in S\}\,. \tag{2.2}$$

$$T_1 \cap T_2 = \{\langle s_i, min\left(\mu_{T_1}\left(s_i\right), \mu_{T_2}\left(s_i\right)\right)\rangle : s_i \in S\}\,. \tag{2.3}$$

$$T^c = \{\langle s_i, 1 - \mu_T\left(s_i\right)\rangle : s_i \in S\}\,. \tag{2.4}$$

$$T_1 \subseteq T_2 \Leftrightarrow \mu_{T_1}\left(s_i\right) \leq \mu_{T_2}\left(s_i\right), \forall s_i \in S. \tag{2.5}$$

Definition 2.3. Let $T \in \mathscr{F}(S)$. Then T induces a fuzzy set, $\hat{T}$ such that

$$\begin{cases} \mu_T(s_i) \geq \mu_{\hat{T}}(s_i) & if \quad \mu_T(s_i) \leq \frac{1}{2} \\ \mu_T(s_i) \leq \mu_{\hat{T}}(s_i) & if \quad \mu_T(s_i) \geq \frac{1}{2} \end{cases}. \tag{2.6}$$

$\hat{T}$ is called a sharpened version of T.

Definition 2.4. $T \in \mathscr{F}(S)$ is called most fuzzy set (MFS) if $\mu_T(s_i)$ =0.5 $\forall$ $s_i \in S$ and T is called a crisp set if $\mu_T(s_i)$ ={0,1} $\forall$ $s_i \in S$

Definition 2.5. An information measure [2], is a mapping $\eta : \mathscr{F}(S) \rightarrow [0,1]$ if following properties holds :

P1. $\eta(T)$ has maximum value iff T is MFS

P2. $\eta(T) = 0 \Leftrightarrow \mu_T(s_i) \in \{0,1\}$ $\forall$ $s_i \in S$.

P3. $\eta(T) \geq \eta(\hat{T})$, here $\hat{T}$ is sharpened version of T.

P4. $\eta(T) = \eta(T^c)$.

It is well established that fuzzy entropy refers to the fuzziness of a fuzzy set. On the other hand, the amount of knowledge carried by a fuzzy set is described as a fuzzy knowledge measure. It is asserted that these two terms are dual. Now we present a mathematical definition of a fuzzy knowledge measure(FKM) given by [27].

Definition 2.6. A function $\kappa : \mathscr{F}(S) \rightarrow [0,1]$ is an FKM if following properties holds for κ

1. $\kappa(T)$ has maximum value iff $\mu_T(s_i) \in \{0,1\}$
2. $\kappa(T) = 0$ iff T is MFS.
3. $\kappa(T) \leq \kappa(\hat{T})$.
4. $\kappa(T) = \kappa(T^c)$.

Definition 2.7. An Accuracy measure is a mapping $\mathcal{I} : \mathscr{F}(S) \times \mathscr{F}(S) \rightarrow [0,1]$ if following properties holds :

P1. $\mathcal{I} \in [0,1]$

P2. $\mathcal{I}(T_1,T_2) = 0$ iff $T_1 = T_2$ and both are MFS.

P3. $\mathcal{I}(T_1,T_2) = 1$ iff $T_1 = T_2$ and both are crisp sets.

P4. $\mathcal{I}(T_1,T_2) = \kappa(T_1)$ if $T_1 = T_2$. where κ denotes the proposed knowledge measure.

Definition 2.8. For two fuzzy sets T_1 and T_2, the basic distance measures are given as follows

$$d_H(T_1,T_2) = \frac{1}{\ell}\sum_{i=1}^{\ell}|\mu_{T_1}(s_i) - \mu_{T_2}(s_i)| \qquad (\textbf{Hamming Distance}) \tag{2.7}$$

where $s_i \in S$.

$$d_E(T_1,T_2) = \sqrt{\frac{1}{\ell}\sum_{i=1}^{\ell}(\mu_{T_1}(s_i) - \mu_{T_2}(s_i))^2} \qquad (\textbf{Euclidean Distance}) \tag{2.8}$$

where $s_i \in S$.

3 Parametric Entropy Based Knowledge Measure

Using the full set of probability distributions, Shannon [3] defined the entropy measure as follows:
Let the set

$$\vartheta_\ell = \left\{ E = (\eta_1,\eta_2,\eta_3,\ldots,\eta_\ell) \,|\, \sum_{i=1}^{\ell}\eta_i = 1;\ 0 \leq \eta_i \leq 1;\ \ 1 \leq i \leq \ell \right\}, \tag{3.1}$$

represents all probability distributions for $\ell \geq 2$. Then, the measure for Shannon entropy is given by

$$H(E) = -\sum_{i=1}^{\ell}\eta_i \log_D \eta_i;\ for\ some\ E \in \vartheta_\ell. \tag{3.2}$$

Many researchers proposed generalized entropies based on various tools viz. Boekee et al. [33] discussed R-norm based information measure, Renyil [34] studied generalized entropies, Tsallis [35] proposed Tsallis entropy, Havrda-Charvat entropy by [36]. Below we have shown a generalized version of Shannon entropy induced by the p-norm

$$H_\alpha(E) = \frac{\alpha}{\alpha - 1}\left[1 - \left(\sum_{i=1}^{\ell} \eta_i^\alpha\right)^{\frac{1}{\alpha}}\right]; \alpha \in (0, \infty) - \{1\}. \tag{3.3}$$

This entropy was due to Boekee et al. [33]. Note that $H(E) = \lim_{\alpha \to 1} H_\alpha(E)$.
Further, Markechova et al. [37] and Hooda et al. [38] proposed information measures based on R-norm.

The study of fuzzy information to find the fuzziness of a fuzzy set has found attraction from many researchers. Zadeh [1] was first to have an insight about fuzziness inherited by a fuzzy set. Some more general information measures were proposed by many authors [2, 39, 40]. Since fuzziness in an FS is accounted for by FIM and knowledge possessed by an FS is determined by FKM, FIM, and FKM can be treated as complements to each other. Many researchers [41, 42, 43, 44, 45, 46] gave their contribution in this field.

3.1 A New Knowledge Measure

Let $\kappa : \mathscr{F}(S) \longrightarrow [0, 1]$ be defined as

$$\kappa^A(T) = \frac{1}{0.864n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_A(s_i))^{10} + (1 - \mu_A(s_i))^{10}\right)} - 1\right]; \tag{3.4}$$

where T $\in \mathscr{F}(S)$. The validity of κ^A is established in the following theorem.

Theorem 3.1. *Let $T = \{\langle s_i, \mu_T(s_i)\rangle : s_i \in S\} \in \mathscr{F}(S)$; where $\phi \neq S$ be a finite set.Then the following properties hold for κ^A.*

1. *κ^A(T) has maximum value iff the set T is crisp.*
2. *If T is a MFS, then $\kappa^A(T) = 0$ and conversely.*
3. *$\kappa^A(T) \leq \kappa^A(\hat{T})$.*
4. *$\kappa^A(T^c) = \kappa^A(T)$.*

Proof. ***(K1)***.For a crisp set T, μ_T is either 0 or 1. Eq. (3.4) now becomes

$$\kappa^A(T) = \frac{n\left(\sqrt[10]{512} - 1\right)^{-1}\left(\sqrt[10]{512} - 1\right)}{n} = 1.$$

For the converse part, if κ^A(T)=1, then using Eq. (3.4),

$$\frac{1}{0.864n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_T(s_i))^{10} + (1 - \mu_T(s_i))^{10}\right)} - 1\right] = 1. \tag{3.5}$$

On simplifying (3.5), we get

$$\sqrt[10]{512\left((\mu_T(s_i))^{10} + (1 - \mu_T(s_i))^{10}\right)} = \sqrt[10]{512}, \quad \forall s_i \in S; \tag{3.6}$$

which gives

$$\left((\mu_T(s_i))^{10} + (1 - \mu_T(s_i))^{10}\right) = 1, \quad \forall s_i \in S; \tag{3.7}$$

which holds only if μ_T is either 0 or 1. Thus *K1* holds.

(K2). If T is a most FS, then $\mu_T(s_i)$=$\frac{1}{2}$ $\forall$ $s_i \in S$. Now from Eq. (3.4), we have κ^A(T)=0.

On the other hand if κ^A(T)=0, then Eq. (3.4) implies

$$\frac{1}{0.864\ n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_T(s_i))^{10}+(1-\mu_T(s_i))^{10}\right)}-1\right]=0. \tag{3.8}$$

It gives

$$\sqrt[10]{512\left((\mu_T(s_i))^{10}+(1-\mu_T(s_i))^{10}\right)}=1,\ \ \forall s_i\in S;$$

which holds true only if $\mu_T(s_i)$=0.5 $\forall$ $s_i \in S$. This establishes *K2*.

(K3). Define a function by

$$I(\mu_T(s_i))=\sqrt[10]{512\left((\mu_T(s_i))^{10}+(1-\mu_T(s_i))^{10}\right)}. \tag{3.9}$$

Differentiating $I(\mu_T(s_i))$ w.r.t. $\mu_T(s_i)$, we have

$$\frac{dI(\mu_T(s_i))}{d\mu_T(s_i)}=\frac{\sqrt[10]{512}\left[(\mu_T(s_i))^9-(1-(\mu_T(s_i))^9)\right]}{\sqrt[10]{\left((\mu_T(s_i))^{10}+(1-\mu_T(s_i))^{10}\right)^9}}. \tag{3.10}$$

The above equation has a positive denominator whereas the numerator of the same depends on $\left[(\mu_T(s_i))^9-(1-(\mu_T(s_i))^9)\right]$.

Now,

$$\begin{cases}\left[(\mu_T(s_i))^9-(1-(\mu_T(s_i))^9)\right]>0 & if \quad \mu_T(s_i)\in\left(\frac{1}{2},1\right]\\ \left[(\mu_T(s_i))^9-(1-(\mu_T(s_i))^9)\right]<0 & if \quad \mu_T(s_i)\in\left[0,\frac{1}{2}\right)\end{cases}.$$

Let $\tilde{T}$ be the sharpened version of T. Since, I is an increasing function in (0.5,1], therefore, $I(\mu_T(s_i)) \geq$ I($\mu_{\tilde{T}}(s_i)$). Hence $\kappa^A(\tilde{T})$≥κ^A(T). Again, I is a decreasing function in [0,0.5), $I(\mu_T(s_i)) \leq$ I($\mu_{\tilde{T}}(s_i)$). It gives $\kappa^A(\tilde{T})$≥κ^A(T). This shows that *(P3)*. holds

(K4). Replacing T with T^C in Eq.(3.4), we have

$$\begin{aligned}\kappa^A(T^c)&=\frac{1}{0.864\ n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_{T^c}(s_i))^{10}+(1-\mu_{T^c}(s_i))^{10}\right)}-1\right]\\&=\frac{1}{0.864\ n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((1-\mu_T(s_i))^{10}+(\mu_T(s_i))^{10}\right)}-1\right]\\&=\kappa^A(T)\end{aligned} \tag{3.11}$$

Axiom *(4)* holds for $\kappa^A(T)$.

This establishes the validity of κ^A(T) as FKM.

3.2 Some Properties of New Knowledge Measure

This section deals with some basic properties of κ^A(T), the suggested KM.

Theorem 3.2. *The proposed measure satisfy the following properties:*

(1). $\kappa^A(T^c)=\kappa^A(T)$.

(2). κ^A*(T)* $\in$*[0,1], for any fuzzy set T.*

(3). ***Exclusion-inclusion principle***
$\kappa^A(T_1\cup T_2)$ = $\kappa^A(T_1)$ + $\kappa^A(T_2)$ - $\kappa^A(T_1\cap T_2)$*, for any two fuzzy sets* T_1, T_2.

Proof. *(1).* Proof follows directly from *(P4)*.

(2). We know $\mu_T(s_i)$,$(1-\mu_T(s_i)) \in [0,1]$ for each $s_i \in S$,

Therefore, $0 \leq (\mu_T(s_i))^{10} \leq 1$ and $0 \leq (1-\mu_T(s_i))^{10} \leq 1$ $\forall s_i \in S$.

$\Rightarrow 0 \leq \left((\mu_T(s_i))^{10}+(1-\mu_T(s_i))^{10}\right) \leq 1$;

$\Rightarrow 0 \leq 512\left((\mu_T(s_i))^{10} + (1-\mu_T(s_i))^{10}\right) \leq 512;$
$\Rightarrow 0 \leq \sqrt[10]{512\left((\mu_T(s_i))^{10} + (1-\mu_T(s_i))^{10}\right)} \leq \sqrt[10]{512};$
$\Rightarrow 0 \leq \sqrt[10]{512\left((\mu_T(s_i))^{10} + (1-\mu_T(s_i))^{10}\right)} - 1 \leq \sqrt[10]{512} - 1;$
$\Rightarrow 0 \leq \frac{1}{0.864\, n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_T(s_i))^{10} + (1-\mu_T(s_i))^{10}\right)} - 1\right] \leq 1;$
$\Rightarrow 0 \leq \kappa^A(T) \leq 1.$
(3). Let $T_1, T_2 \in \mathscr{F}(S)$ and

$$\begin{aligned} S_1 &= \{s_i \in S | \mu_{T_1}(s_i) \geq \mu_{T_2}(s_i)\} \\ S_2 &= \{s_i \in S | \mu_{T_1}(s_i) < \mu_{T_2}(s_i)\} \end{aligned} \tag{3.12}$$

where membership functions T_1 and T_2 are $\mu_{T_1}(s_i)$ and $\mu_{T_2}(s_i)$, respectively.
For $s_i \in S_1$, we have

$$\begin{aligned} \mu_{T_1 \cup T_2}(s_i) &= max\{\mu_{T_1}(s_i), \mu_{T_2}(s_i)\} = \mu_{T_1}(s_i) \\ \mu_{T_1 \cap T_2}(s_i) &= min\{\mu_{T_1}(s_i), \mu_{T_2}(s_i)\} = \mu_{T_2}(s_i) \end{aligned} \tag{3.13}$$

and if $s_i \in S_2$, then

$$\begin{aligned} \mu_{T_1 \cup T_2}(s_i) &= max\{\mu_{T_1}(s_i), \mu_{T_2}(s_i)\} = \mu_{T_2}(s_i) \\ \mu_{T_1 \cap T_2}(s_i) &= min\{\mu_{T_1}(s_i), \mu_{T_2}(s_i)\} = \mu_{T_1}(s_i) \end{aligned} \tag{3.14}$$

Now, $\forall s_i \in S$,

$$\begin{aligned} \kappa^A(T_1 \cup T_2) + \kappa^A(T_1 \cap T_2) = {} & \frac{1}{0.864\, n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_{T_1\cup T_2}(s_i))^{10} + (1-\mu_{T_1\cup T_2}(s_i))^{10}\right)} - 1\right] \\ & + \frac{1}{0.864\, n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_{T_1\cap T_2}(s_i))^{10} + (1-\mu_{T_1\cap T_2}(s_i))^{10}\right)} - 1\right]. \end{aligned}$$

It gives

$$\begin{aligned} \kappa^A(T_1 \cup T_2) + \kappa^A(T_1 \cap T_2) = {} & \frac{1}{0.864\, n}\sum_{s_1}\left[\sqrt[10]{512\left((\mu_{T_1}(s_i))^{10} + (1-\mu_{T_1}(s_i))^{10}\right)} - 1\right] \\ & + \frac{1}{0.864\, n}\sum_{s_1}\left[\sqrt[10]{512\left((\mu_{T_2}(s_i))^{10} + (1-\mu_{T_2}(s_i))^{10}\right)} - 1\right] \\ & + \frac{1}{0.864\, n}\sum_{s_2}\left[\sqrt[10]{512\left((\mu_{T_1}(s_i))^{10} + (1-\mu_{T_1}(s_i))^{10}\right)} - 1\right] \\ & + \frac{1}{0.864\, n}\sum_{s_2}\left[\sqrt[10]{512\left((\mu_{T_2}(s_i))^{10} + (1-\mu_{T_2}(s_i))^{10}\right)} - 1\right]; \end{aligned} \tag{3.15}$$

On solving, we have

$$\kappa^A(T_1 \cup T_2) = \kappa^A(T_1) + \kappa^A(T_2) - \kappa^A(T_1 \cap T_2) \tag{3.16}$$

3.3 Comparison With Some Existing Measures

Now we present a comparative analysis of suggested FKM with available FIMs & FKMs. We examine the advantages of κ^A over other FIMs & FKMs in determining ambiguity or uncertainty in an FS as well as to determine attribute weights related to MCDM problems and structured language variables. For this purpose, we carry out a comparative study of the proposed FKM over the following measures which are documented in the literature.

$$H_K(T) = \frac{d_p(T, T_{near})}{d_p(T, T_{far})}; \quad \text{(Kosko, [47]),}$$
$$\mu_{T_{near}}(s_i) = \begin{cases} 1 & \text{if } \mu_T(s_i) \geq \frac{1}{2} \\ 0 & \text{if } \mu_T(s_i) < \frac{1}{2} \end{cases} \text{ and } \mu_{T_{far}}(s_i) = \begin{cases} 1 & \text{if } \mu_T(s_i) < \frac{1}{2} \\ 0 & \text{if } \mu_T(s_i) \geq \frac{1}{2} \end{cases}. \tag{3.17}$$

$$H_P(T) = \frac{1}{k} \sum_{i=1}^{k} \left[\mu_T(s_i) e^{1-\mu_T(s_i)} + (1 - \mu_T(s_i))\, e^{\mu_T(s_i)} \right]; \quad \text{(Pal and Pal, [48]).} \tag{3.18}$$

$$K_S(T) = \frac{1}{k} \sum_{i=1}^{k} 2 \left[(\mu_T(s_i))^2 + (1 - \mu_T(s_i))^2 \right] - 1; \quad \text{(Singh et al., [27]).} \tag{3.19}$$

$$K_S^{\alpha}(T) = \frac{1}{k} \sum_{i=1}^{k} 2 \left[(\mu_T(s_i))^{\alpha} + (1 - \mu_T(s_i))^{\alpha} \right] - 1; \quad \alpha > 1; \quad \text{(Singh et al., [49]).} \tag{3.20}$$

$$K_{VK}(T) = \log_2 \left[\frac{2}{r} \sum_{i=1}^{r} \left((\mu_T(s_i))^2 + (1 - \mu_T(s_i))^2 \right) \right]; \quad \text{(Arya and Kumar, [50]).} \tag{3.21}$$

$$K_S^{\alpha,\beta}(T) = \frac{1}{k} \sum_{i=1}^{k} 2 \left[(\mu_T(s_i))^{\alpha} + (1 - \mu_T(s_i))^{\alpha} \right]^{\frac{\alpha-1}{\beta-1}} - 1; \quad \alpha \in (1, 2], \beta \geq \alpha, \tag{3.22}$$

(Singh and Ganie, [51].

$$H_Y(T) = 1 - \frac{d_q(T^c, T)}{r^{\frac{1}{q}}}; \; where \; d_q(T_1, T_2) = \left[\sum_{i=1}^{k} |\mu_{T_1}(s_i) - \mu_{T_2}(s_i)|^q \right]^{\frac{1}{q}}. \tag{3.23}$$

(Yager, [7]),

$$\kappa^A(T) = \frac{1}{0.864n} \sum_{i=1}^{n} \left[\sqrt[10]{512 \left((\mu_A(s_i))^{10} + (1 - \mu_A(s_i))^{10} \right)} - 1 \right]$$

(Suggested Knowledge Measure) (3.24)

Computation of Uncertainty in a Fuzzy Set

It is well known that two fuzzy sets may differ in the ambiguity associated with them. However, some FIM/FKM provide the same ambiguity values for distinct FS. Therefore, a new fuzzy measure is ever essential to generalize existing fuzzy measures. The following example exhibits the performance of the suggested measure

Example 3.3. Let $\Phi = \{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5\}$ and T_i (for i=1,2,3,4) be four FS's defined on S given as below

$$T_1 = \{(\phi_1, 0.037), (\phi_2, 0.579), (\phi_3, 0.415), (\phi_4, 0.647), (\phi_5, 0.593)\}.$$
$$T_2 = \{(\phi_1, 0.066), (\phi_2, 0.142), (\phi_3, 0.5), (\phi_4, 0.525), (\phi_5, 0.543)\};$$
$$T_3 = \{(\phi_1, 0.582), (\phi_2, 0.658), (\phi_3, 0.037), (\phi_4, 0.591), (\phi_5, 0.434)\};$$
$$T_4 = \{(\phi_1, 0.541), (\phi_2, 0.482), (\phi_3, 0.708), (\phi_4, 0.04), (\phi_5, 0.534)\};$$

The above table shows the computed ambiguity contents of Fuzzy sets (shown in Example 1) computed by existing measures and κ^A. It is evident that κ^A fairly differentiates between these FS. However, some existing measures give the same ambiguity content for different FSs.

Table 1. Computation of κ^A and some other measures on T_i (for i=1,2,3,4) as described in Example 3.3.

Information or Knowledge Measures	T_1	T_2	T_3	T_4
$H_Y(T)$	**0.2568**	0.2272	0.2589	**0.2568**
$H_K^A(T)$	**0.2552**	0.2327	0.2567	**0.2552**
$H_P(T)$	0.2536	**0.2538**	**0.2538**	0.2387
$K_S^\alpha(T)$	0.2310	**0.2302**	**0.2302**	0.3085
$K_{VK}(T)$	0.2443	**0.2440**	**0.2440**	0.2675
$K_S^{\alpha,\beta}(T)$	0.2458	**0.2456**	**0.2456**	0.26287
$\kappa^A(T)$	0.2365	0.2358	0.2359	0.2916

Here α=2.1 for K_S^α(T); and α=2.1, β=3 for $K_S^{\alpha,\beta}$(T).

Computation of Attribute weights

Any MCDM problem has a vital use of Attribute weights. In the following example, attribute weights specified by existing measures and proposed measures are presented

Example 3.4. Consider an MCDM problem have set of alternatives $\{W_1, W_2, W_3, W_4, W_5\}$ and a set of attributes $\{C_1, C_2, C_3, C_4\}$. Let D be the decision matrix given by

$$D = \begin{bmatrix} 0.066 & 0.037 & 0.582 & 0.541 \\ 0.142 & 0.579 & 0.658 & 0.482 \\ 0.500 & 0.415 & 0.037 & 0.708 \\ 0.525 & 0.647 & 0.591 & 0.040 \\ 0.543 & 0.593 & 0.434 & 0.534 \end{bmatrix}$$

Now one can compute attribute weights by following two ways

(1). Entropy dependent technique- In this method, weights corresponding to different attributes can be computed by the formula

$$w_\ell = \frac{E(C_\ell) - 1}{\sum_{\ell=1}^{n} E(C_\ell) - n}, \ell = 1, 2, 3, \ldots, n;$$

where E is the fuzzy information measure.

(2). Knowledge dependent technique -In this method, weights corresponding to different attributes can be computed by the expression

$$w_\ell = \frac{K(C_\ell)}{\sum_{\ell=1}^{n} K(C_\ell)}, \ell = 1, 2, 3, \ldots, n;$$

where K is a FKM.

Weights of attributes computed by the above techniques are included in the Table 2. Note that attribute weights derived using pre-defined FIMs/FKMs are inconsistent. However, in certain instances, weights related to different attributes may not be equal. However, κ^A assigns distinct weights to different attributes. As a result, new measures must be implemented.

Comparative Study for Structured linguistic terms

Tahani [52] developed a framework to address fuzzy queries related to fuzzy sets. For database queries, Kacprzyk [53] introduced linguistic quantifiers in a fuzzy sense. Fuzzy database with its principles and applications was considered by Petry's book [54]. Linguistic terms like "MORE", "FEW", "VERY", "SLIGHTLY", and "LESS" include linguistic variables. These linguistic variables can be represented by fuzzy sets whereas linguistic terms can be identified as fuzzy operations performed on an FS. We investigate these fuzzy operations and the performance of κ^A over

Table 2. Attribute's Weights for Example 3.4.

FKM/FIM	w_1	w_2	w_3	w_4
$H_Y(T)$	0.2273	**0.2569**	0.2590	**0.2569**
$H_K(T)$	0.2327	**0.2553**	0.2568	**0.2553**
$H_P(T)$	**0.2538**	0.2536	**0.2538**	0.2388
$K_S^{\alpha}(T)$	**0.2397**	0.2401	**0.2397**	0.2805
$K_{VK}(T)$	**0.2521**	**0.2521**	0.2516	0.2442
$K_S^{\alpha,\beta}(T)$	**0.2456**	0.2458	**0.2456**	0.2629
$\kappa^A(T)$	0.2339	0.2371	0.2368	0.2921

Here α=1.89 for K_S^{α}(T); and for $K_S^{\alpha,\beta}$(T)α=1.89, β=2.09.

other measures.
Let a fuzzy set FS $T = \{\langle s_i, \mu_T(s_i)\rangle : s_i \in S\}$ on S is regarded as "Huge" on S. Then modifier of T on S is given as

$$T^n = \left\{\langle s_i, (\mu_T(s_i))^n\rangle : s_i \in S\right\}. \tag{3.25}$$

Hung and Hwang [55, 56] defined dilatation (DIL) and concentration (CON) of an FS T as

$$DIL(T) = T^{0.5} \text{ and } CON(T) = T^2. \tag{3.26}$$

There may be some more modifiers like Dilation and concentration. For instance, H denotes HUGE, VH denotes VERY HUGE, MLH denotes MORE/LESS HUGE, QVH denotes QUITE VERY HUGE, and VVH represents VERY VERY HUGE. Hedges of an FS T now can be characterized as follows:

$$\begin{cases} VVH & \text{by } T^4 \\ QVH & \text{by } T^3 \\ VH & \text{by } T^2 \\ H & \text{by } T \\ MLH & \text{by } T^{0.5} \end{cases} \tag{3.27}$$

Linguistic terms identified as variables are used broadly to test the performance of various measures. Many authors [55, 56, 57, 58] have utilized these linguistic variables to compare various FIM. For optimal performance, a FIM ξ should follow the sequence below:

$$\xi(MLH) > \xi(H) > \xi(VH) > \xi(QVH) > \xi(VVH) \tag{3.28}$$

where $\xi(T)$ is FIM for $T \in \mathscr{F}(S)$

On the other hand, if η is any FKM, the following sequence should be followed (see. [27]):

$$\eta(MLH) < \eta(H) < \eta(VH) < \eta(QVH) < \eta(VVH) \tag{3.29}$$

We present an example to investigate the efficacy of κ^A(T),

Example 3.5. Let T $\in \mathscr{F}(S)$ defined on S= $\{s_i; 1 \le i \le 5\}$ such that

$$T = \{(s_1, 0.2), (s_2, 0.4), (s_3, 0.5), (s_4, 0.9), (s_5, 1)\}. \tag{3.30}$$

Now choose linguistic variables as in Eq.(3.27) and consider T as "Huge" on S. Following fuzzy sets can now be computed

$$\begin{aligned} T^{0.5} &= \{(s_1, 0.4472), (s_2, 0.6324), (s_3, 0.7071), (s_4, 0.9486), (s_5, 1)\}; \\ T &= \{(s_1, 0.2), (s_2, 0.4), (s_3, 0.5), (s_4, 0.9), (s_5, 1)\}; \\ T^2 &= \{(s_1, 0.04), (s_2, 0.16), (s_3, 0.25), (s_4, 0.81), (s_5, 1)\}; \\ T^3 &= \{(s_1, 0.008), (s_2, 0.064), (s_3, 0.125), (s_4, 0.729), (s_5, 1)\}; \\ T^4 &= \{(s_1, 0.0016), (s_2, 0.0256), (s_3, 0.0625), (s_4, 0.6561), (s_5, 1)\}. \end{aligned} \tag{3.31}$$

Table 3. Various measures (Eq.(3.17)-Eq.(3.24)) on linguistic variables

FS	$H_Y(T)$	$H_K(T)$	$H_P(T)$	$K_S(T)$	$K_S^{\alpha}(T)$	$K_{VK}(T)$	$K_S^{\alpha,\beta}(T)$	$\kappa^A(T)$
MLH	0.3584	0.4423	1.3840	0.4115	0.3033	0.4972	0.7690	1.1081
H	0.3612	0.4981	1.3871	0.4080	0.2998	0.4936	0.7692	1.0656
VH	0.2327	0.2462	1.2717	0.5886	0.5161	0.6677	0.8465	1.0248
QVH	0.1632	0.2624	1.1983	0.7001	0.6477	0.7656	0.8884	1.0089
VVH	0.1331	0.3291	1.1635	0.7513	0.7070	0.8084	0.9050	1.0031

Here α=1.5 for K_S^{α}(T); and α=2.1, β=3.7 for $K_S^{\alpha,\beta}$(T).

Table 4. Comparison of $K_S^{\alpha,\beta}(T)$ and $\kappa^A(T)$, given in Eq.(3.22) and Eq.(3.24).

FS	$K_S^{\alpha,\beta}(T)$	$\kappa^A(T)$
MLH	0.6099	0.7438
H	0.8591	0.7461
VH	0.2335	0.8082
QVH	-0.0494	0.8496
VVH	-0.2716	0.8796

Here α=2.3, β=4.2 for $K_S^{\alpha,\beta}$(T).

Now the effectiveness of $\kappa^A(T)$ and other existing measures is computed in the following Table 3.

Now, the following observations can be made From Table 3

$$\begin{aligned}
&H_Y(MLH) < H_Y(H) > H_Y(VH) > H_Y(QVH) > H_Y(VVH);\\
&H_P(VVH) < H_P(QVH) < H_P(VH) < H_P(H) > H_P(MLH);\\
&K_S(VVH) > K_S(QVH) > K_S(VH) > K_S(H) < K_S(MLH);\\
&K_{VK}(MLH) > K_{VK}(H) < K_{VK}(VVH) < K_{VK}(QVH) < K_{VK}(VVH);\\
&H_K(VVH) > H_K(QVH) > H_K(VH) < H_K(H) > H_K(MLH);\\
&K_S^{\alpha}(VVH) > K_S^{\alpha}(QVH) > K_S^{\alpha}(VH) > K_S^{\alpha}(H) < K_S^{\alpha}(MLH);\\
&K_S^{\alpha,\beta}(VVH) > K_S^{\alpha,\beta}(QVH) > K_S^{\alpha,\beta}(VH) > K_S^{\alpha,\beta}(H) > K_S^{\alpha,\beta}(MLH);\\
&\kappa^A(VVH) < \kappa^A(QVH) < \kappa^A(VH) < \kappa^A(H) < \kappa^A(MLH).
\end{aligned} \tag{3.32}$$

Note that the sequence in Eq. (3.28) is not followed by any FIM/FKM. This indicates that the performance of these measures is not fair enough. FKMs $K_S^{\alpha,\beta}(T)$ and $\kappa^A(T)$ satisfy the sequence but $K_{VK}(T)$ does not adhere the sequence in Eq. (3.29). We investigate FKMs $K_S^{\alpha,\beta}(T)$ and $\kappa^A(T)$ on another FS given by

$$T = \{(s_1, 0.2), (s_2, 0.3), (s_3, 0.4), (s_4, 0.7), (s_5, 0.8)\}. \tag{3.33}$$

Table 4, records the calculated values for these measures on this fuzzy set From Table 4, we get following observations:

$$\begin{aligned}
&K_S^{\alpha,\beta}(VVH) < K_S^{\alpha,\beta}(QVH) < K_S^{\alpha,\beta}(VH) < K_S^{\alpha,\beta}(H) > K_S^{\alpha,\beta}(MLH);\\
&\kappa^A(VVH) > \kappa^A(QVH) > \kappa^A(VH) > \kappa^A(H) > \kappa^A(MLH).
\end{aligned} \tag{3.34}$$

Note that the FKMs $K_{VK}(T)$ and $K_S^{\alpha,\beta}(T)$ do not obey Eq. (3.29). However, the proposed KM follows the desired sequence. Therefore, the effectiveness of KM over other measures is established.

3.4 New Fuzzy Accuracy Measure

Kullback and Leibler were the first to propose the Divergence measure (the difference between two probability distributions of the same length) following [3]. Consider U=$\{u_1, u_2, \ldots, u_n\}$ and V=$\{v_1, v_2, \ldots, v_n\}$ to be two probability distributions in ϑ_n. The divergence and inaccuracy measures denoted by $D_{KL}(U||V)$ and $I_K(U,V)$ respectively(due to [59]) are defined as

$$D_{KL}(U||V) = \sum_{i=1}^{n} u_i \log\left(\frac{u_i}{v_i}\right). \tag{3.35}$$

$$I_K(U,V) = -\sum_{i=1}^{n} u_i \log(v_i). \tag{3.36}$$

$D_{KL}(U||V)$ and $I_K(U,V)$ are connected by the relation

$$I_K(U,V) = D_{KL}(U||V) + H(U). \tag{3.37}$$

The inaccuracy measure $D_{KL}(U||V)$ was used by Verma and Sharma [60] on FS theory to present the following inaccuracy measure

$$I_{VS}(T_1,T_2) = -\frac{1}{n}\sum_{\ell=1}^{n}\left[\mu_{T_1}(s_\ell)\log(\mu_{T_2}(s_\ell)) + (1-\mu_{T_1}(s_\ell))\log(1-\mu_{T_2}(s_\ell))\right], \tag{3.38}$$

for $T_1, T_2 \in \mathscr{F}(S)$. The relationship between measurements $I_{VS}(T_1,T_2)$ and $D(T_1,T_2)$ can be expressed as follows:

$$I_{VS}(T_1,T_2) = H_{LT}(T_1) + D_{KL}(T_1,T_2), \tag{3.39}$$

for $T_1, T_2 \in \mathscr{F}(S)$. Here $D(T_1,T_2)$ is Fuzzy Directed Divergence Measure (FDDM) and $H_{LT}(T)$ is the FIM given by Luca and Termini [2]. Also, for $T_1 = T_2$, we have $I_{VS}(T_1,T_2) = H_{LT}(T_1)$.

We now define a new FAM $K_{accy}(T_1,T_2)$ of FS T_2 w.r.t. FS T_1 corresponding to proposed FKM κ^A(T) as follows

$$\begin{aligned} K_{Accy}(T_1,T_2) &= \frac{1}{1.728n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_{T_1}(s_i))^{10} + (1-\mu_{T_1}(s_i))^{10}\right)} - 1\right] \\ &+ \frac{1}{1.728n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_{T_1}(s_i))^{5}(\mu_{T_2}(s_i))^{5} + (1-\mu_{T_1}(s_i))^{5}(1-\mu_{T_2}(s_i))^{5}\right)} - 1\right] \end{aligned} \tag{3.40}$$

Note that, when $T_1 = T_2 = T$, $K_{Accy}(T_1,T_2) = \kappa^A(T)$. Properties of $K_{Accy}(T_1,T_2)$ are compiled in the following theorem.

Theorem 3.6. *The FAM satisfies the following*

(1). $K_{Accy}(T_1,T_2)$ *is maximum iff* T_1 *and* T_2 *are equal crisp sets i.e.* $\mu_{T_1}(s_i) = 0 = \mu_{T_2}(s_i)$ *or* $\mu_{T_1}(s_i) = 1 = \mu_{T_2}(s_i)$ *in this case,* $K_{Accy}(T_1,T_2) = \kappa^A(T)$

(2). If $T_1, T_2, T_3 \in \mathscr{F}(S)$ *be s.t.* $T_3 \supseteq T_2 \supseteq T_1$, *then*

- *(a).* $K_{Accy}(T_1,T_3) \leq K_{Accy}(T_1,T_2)$ *if* $0 < \mu_{T_1}(s_i) \leq 0.5$.
- *(b).* $K_{Accy}(T_1,T_2) \leq K_{Accy}(T_1,T_3)$ *if* $0.5 \leq \mu_{T_1}(s_i) < 1$.

(3). If $T_1, T_2 \in \mathscr{F}(S)$, *then*

- *(a).* $K_{Accy}(T_1,T_2) = K_{Accy}(T_1^c,T_2^c)$.
- *(b).* $K_{Accy}(T_1,T_1^c) = K_{Accy}(T_1^c,T_1)$.
- *(c).* $K_{Accy}(T_1,T_2^c) = K_{Accy}(T_1^c,T_2)$.
- *(d).* $K_{Accy}(T_1,T_2) + K_{Accy}(T_1^c,T_2) = K_{Accy}(T_1^c,T_2^c) + K_{Accy}(T_1,T_2^c)$.

Proof. *(1).* Let T_1 and T_2 be two crisp FS, it implies

$$\begin{aligned} K_{Accy}(T_1,T_2) &= \frac{1}{1.728n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_{T_1}(s_i))^{10}+(1-\mu_{T_1}(s_i))^{10}\right)}-1\right] \\ &+\frac{1}{1.728n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left((\mu_{T_1}(s_i))^{5}(\mu_{T_2}(s_i))^{5}+(1-\mu_{T_1}(s_i))^{5}(1-\mu_{T_2}(s_i))^{5}\right)}-1\right] \\ &= \frac{n\left(\sqrt[10]{512}-1\right)^{-1}\left[\sqrt[10]{512}-1\right]}{2n}+\frac{n\left(\sqrt[10]{512}-1\right)^{-1}\left[\sqrt[10]{512}-1\right]}{2n} \\ &= 1. \end{aligned} \tag{3.41}$$

From the above discussion we conclude that if $\mu_{T_1}(s_i)=0=\mu_{T_2}(s_i)$ or $\mu_{T_1}(s_i)=1=\mu_{T_2}(s_i)$, then $K_{Accy}(T_1,T_2)$ is maximum. Now from Eq.(3.4) and Eq.(3.40), it follows trivially when $T_1=T_2$. *(2).* To prove this we consider the function

$$g(\alpha,\beta)=\left[\sqrt[10]{512\left(\alpha^{10}+(1-\alpha)^{10}\right)}-1\right]+\left[\sqrt[10]{512\left(\alpha^5\beta^5+(1-\alpha)^5(1-\beta)^5\right)}-1\right] \tag{3.42}$$

On partial differentiation w.r.t. α and β, we get

$$\frac{\partial g(\alpha,\beta)}{\partial\alpha}=C\left[\frac{10\alpha^9-10(1-\alpha)^9}{\sqrt[10]{(\alpha^{10}+(1-\alpha)^{10})^9}}+\frac{5\alpha^4\beta^5-5(1-\alpha)^4(1-\beta)^5}{\sqrt[10]{(\alpha^5\beta^5+(1-\alpha)^5(1-\beta)^5)^9}}\right]. \tag{3.43}$$

$$\frac{\partial f(\alpha,\beta)}{\partial\beta}=C\left[\frac{5\alpha^5\beta^4-5(1-\alpha)^5(1-\beta)^4}{\sqrt[10]{(\alpha^5\beta^5+(1-\alpha)^5(1-\beta)^5)^9}}\right]. \tag{3.44}$$

where C=$\frac{\sqrt[10]{512}}{10}$.

From Eqs. (3.43) and (3.44), we find that

$$\frac{\partial f(\alpha,\beta)}{\partial\alpha}\text{ and }\frac{\partial f(\alpha,\beta)}{\partial\alpha}=\begin{cases} Positive & \text{if } 0.5\le\alpha,\beta\le 1,\\ Negative & \text{if } 0\le\alpha,\beta\le 0.5.\end{cases} \tag{3.45}$$

$g(\alpha,\beta)$ is decreasing in the interval [0,0.5] whereas it is increasing in [0.5,1]. Since, $T_1\subseteq T_2\subseteq T_3$, therefore, it implies that $\mu_{T_1}(s_i)\le\mu_{T_2}(s_i)\le\mu_{T_3}(s_i)$. Hence the result holds. *(3).* Follows from the definition $K_{Accy}(T_1,T_2)$. *(3).* Follows from the definition $K_{Accy}(T_1,T_2)$.

3.5 An FKM Induced By The New Fuzzy Accuracy Measure

Let $T\in\mathscr{F}(S)$ and T_{near} and T_{far} are as defined in Eq. (3.17).Then we define a new fuzzy knowledge measure from the Eq. (3.40), as follows:

$$\bar{K}(T)=K_{Accy}(T,T_{near})/K_{Accy}(T,T_{far}); \tag{3.46}$$

Next theorem shows validity of $\bar{K}(T)$

Theorem 3.7. $\bar{K}(T)$ *is valid fuzzy knowledge measure.*

Proof. *(K1).* If $\bar{K}(T)=1$, then $K_{Accy}(T,T_{near})=K_{Accy}(T,T_{far})$. Thus $T_{near}=T_{far}$ and hence T is a crisp set. For the converse, take $T_1=T$ and $T_2=T_{near}$ in Eq. (3.40). Now we have

$$K_{Accy}\left(T,T_{near}\right)=\frac{\left(\sqrt[10]{512}-1\right)^{-1}}{2n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left(\left(\mu_T(s_i)\right)^{10}+\left(1-\mu_T(s_i)\right)^{10}\right)}-1\right]+$$
$$\frac{\left(\sqrt[10]{512}-1\right)^{-1}}{2n}\sum_{i=1}^{n}\left[\sqrt[10]{512}\left(\left(\mu_T(s_i)^5\right)\left(\mu_{T_{near}}(s_i)\right)^5+\left(1-\mu_T(s_i)\right)^5\left(1-\mu_{T_{near}}(s_i)\right)^5\right)-1\right] \tag{3.47}$$

For $T=T_{near}$, the above equation becomes

$$K_{Accy}\left(T,T_{near}\right)=\frac{\left(\sqrt[10]{512}-1\right)^{-1}}{2n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left(\left(\mu_T(s_i)\right)^{10}+\left(1-\mu_T(s_i)\right)^{10}\right)}-1\right]$$
$$+\frac{\left(\sqrt[10]{512}-1\right)^{-1}}{2n}\sum_{i=1}^{n}\left[\sqrt[10]{512}\left(\begin{array}{c}\left(\mu_T(s_i)^5\right)\left(\mu_{T_{near}}(s_i)\right)^5\\+\left(1-\mu_T(s_i)\right)^5\left(1-\mu_{T_{near}}(s_i)\right)^5\end{array}\right)-1\right] \tag{3.48}$$

For $T=T_{near}$, the above equation becomes

$$K_{Accy}\left(T_{near},T_{near}\right)=\kappa^A(T_{near}) \tag{3.49}$$

Now we deal with the remaining cases:

I. When $\mu_T(s_i)<0.5$, then Eq. (3.17) implies that $\mu_{T_{near}(s_i)}=0$. Since $\kappa^A(T_{near})$=1(from Eq. (3.4)), Eq. (3.46) leads to $K_{Accy}\left(T,T_{near}\right)=1$.

II. If $\mu_T(s_i)\geq 0.5$, then using Eq.(3.17), $\mu_{T_{near}(s_i)}=1$. Now Eq. (3.4) leads to $\kappa^A(T_{near})$=1. In addition, $K_{Accy}\left(T,T_{near}\right)=1$.

Using same argument, $K_{Accy}\left(T,T_{far}\right)=1$. From Eq. (3.46), we deduce that $\bar{K}(T)=1$. *(K2)*. When T is a most fuzzy set, then $\mu_T(s_i)=0.5$. Thus, $\mu_{T_{near}}(s_i)=1$, it implies $K_{Accy}\left(T,T_{near}\right)=0$. For the converse part, if $K_{Accy}\left(T,T_{near}\right)=0$, then $\mu_T(s_i)=\mu_{T_{near}}(s_i)=0.5$. Thus T and T_{near} are the most fuzzy sets. *(K3)*. Since $\kappa^A(T)\leq\kappa(\hat{T})$, $\hat{T}_{near}=T_{near}$ and $T_{far}=\hat{T}_{far}$, it implies that $K_{Accy}(T,T_{near})=K_{Accy}(T,\hat{T}_{near})$ and $K_{Accy}(T,T_{far})=K_{Accy}(T,\hat{T}_{far})$.
Now, we have two cases

- If $\mu_T(s_i)\geq 0.5$, then $T\subseteq\tilde{T}\subseteq\tilde{T}_{near}$. Using theorem 3.6, we have $K_{Accy}(\tilde{T},\tilde{T}_{near})\geq K_{Accy}(T,T_{near})$.
- If $\mu_T(s_i)<0.5$, then $\tilde{T}_{near}\subseteq\tilde{T}\subseteq T$. Again using theorem 3.6, we have $K_{Accy}(\tilde{T},\tilde{T}_{near})\geq K_{Accy}(T,T_{near})$.

Similiarly, $K_{Accy}(\tilde{T},\tilde{T}_{far})\leq K_{Accy}(T,T_{far})$.
Now
$$\frac{K_{Accy}(T,T_{near})}{K_{Accy}(T,T_{far})}\leq\frac{K_{Accy}(\tilde{T},\tilde{T}_{near})}{K_{Accy}(\tilde{T},\tilde{T}_{far})};$$

Hence $\bar{\kappa}(\tilde{T})\geq\bar{\kappa}(T)$. *(K4)*. Using theorem 3.6 (part 3), we have $K_{Accy}(T_1,T_2)=K_{Accy}(T_1^c,T_2^c)$. If we put T_1=T and $T_2=T_{near}$, then we get $K_{Accy}(T,T_{near})=K_{Accy}(T^c,T^c_{near})$. Similarly, If we put T_1=T and $T_2=T_{far}$, then we get $K_{Accy}\left(T,T_{far}\right)=K_{Accy}(T^c,T^c_{far})$. It implies that

$$\begin{aligned}\bar{K}&=\frac{K_{Accy}\left(T,T_{near}\right)}{K_{Accy}\left(T,T_{far}\right)},\\&=\frac{K_{Accy}\left(T^c,T^c_{near}\right)}{K_{Accy}\left(T^c,T^c_{far}\right)},\\&=\bar{K}(T^c).\end{aligned}$$

Thus $\bar{K}(T)$ is valid FKM.

Table 5. Fuzzy Decision matrix $D_{r\times s}$.

$D_{r\times s}$	C_1	C_2	$\dots$	C_s
W_1	d_{11}	d_{12}	$\dots$	d_{1s}
W_2	d_{21}	d_{22}	$\dots$	d_{2s}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
W_r	d_{r1}	d_{r2}	$\dots$	d_{rs}

3.6 Fuzzy Information Measure Induced by the Given Knowledge Measure

Using FKM κ^A(T), we can define the following Fuzzy information Measure $\left(\overline{\kappa^A(T)}\right)$

$$\begin{aligned}\overline{\kappa^A(T)} &= 1-\kappa^A(T),\\ &= 1-\frac{\left(\sqrt[10]{512}-1\right)^{-1}}{n}\sum_{i=1}^{n}\left[\sqrt[10]{512\left(\left(\mu_T(s_i)\right)^{10}+\left(1-\mu_T(s_i)\right)^{10}\right)}-1\right].\end{aligned} \tag{3.50}$$

Now, it is easy to see that the FIM $\overline{\kappa^A(T)}$ is valid.

Theorem 3.8. *FIM $\overline{\kappa^A(T)}$ satisfy the following properties:*

(P1) $\overline{\kappa^A(T)} \in [0,1]$.

(P2) T is most fuzzy set if and only if $\overline{\kappa^A(T)} = 1$.

(P3) T is crisp iff $\overline{\kappa^A(T)} = 0$.

(P4) $\overline{\kappa^A(T^c)} = \overline{\kappa^A(T)}$.

Proof. Proofs of P1-P4 are obvious.

4 Application of FKM and FAM to an MCDM problem

We apply κ^A(T) and $K_{Accy}(T_1, T_2)$ to an MCDM problem in this section. An MCDM problem is a mathematical representation of any real-life problem of choosing the best option/Alternative from given accessible options. Usually, these accessible options are governed by certain criteria. The model's prerequisites are as follows:

(1). A collection of options(alternatives).

(2). A set of decision-making criteria (or qualities).

(3). Weights assigned to criteria and attributes.

(4). Parameters which can affect the preferred order of these options (alternatives).

4.1 A typical layout of an MCDM

Consider an MCDM problem where $X_{AL} = \{W_i | 1 \le i \le r\}$ is the set of all alternatives, whereas $Y_{AT} = \{C_j | 1 \le j \le s\}$ is the set of all attributes. Let $Z_E = \{E_d | 1 \le d \le n\}$ represent the invited experts. Let $FK = \{FK_1, FK_2, \dots, FK_q\}$ be the weight vectors of the attributes C_j such that $\sum_{j=1}^{s} FK_j = 1$. After getting feedback from experts via a questionnaire, we may create a fuzzy decision matrix. Let 'n' be the number of experts to make a decision. Let s_{ij} be the number of experts who support a specific option W_i according to a given criteria C_j. Then d_{ij}, can be computed by

$$d_{ij} = \frac{s_{ij}}{n}; \quad 1 \le i \le r, 1 \le j \le s.$$

4.2 VIKOR approach based on proposed Fuzzy Accuracy Measure

I. To start, collect expert responses to the MCDM problem and develop the fuzzy decision matrix $D_{r\times s}$.

II. Normalize the fuzzy decision matrix, using the formula:

$$dn_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{p}\left(d_{ij}\right)^2}}, \quad 1 \le i \le r, 1 \le j \le s. \tag{4.1}$$

Eq. (3.4) calculates the quantity of knowledge passed by each criterion after generating the normalized fuzzy decision matrix.

III. Criteria weights are crucial in every MCDM problem. Any change in criterion weights could impact the outcomes of the problem. There are two well-known methods to compute criteria weights

(a). We employ Chen and Li's [61] approach when the criteria weights are unknown. In this case, we have

$$FE_j = \frac{1 - E_j}{q - \sum_{j=1}^{q} E_j}, \forall j = 1, 2, \ldots, s; \tag{4.2}$$

where $E_j = \sum_{i=1}^{r} H(W_i, C_j)$ and $H(W_i, C_j)$ represent fuzzy information of i^{th} entity w.r.t. j^{th} criteria. To calculate the criteria weights FK_j, we utilize the following formula:

$$FK_j = \frac{k_{ij}}{\sum_{j=1}^{q} k_{ij}}, \forall j = 1, 2, \ldots, s; \tag{4.3}$$

where $k_{ij} = \sum_{i=1}^{r} \kappa(W_i, C_j)$ and $\kappa(W_i, C_j)$ is the knowledge received from i^{th} entity w.r.t. to j^{th} criteria.

(b). When criteria weights are known partially, we prefer to describe the expert information in intervals. In this case, $\hat{I}$ represents the information provided by experts. Now the complete amount of knowledge is written as

$$k_{ij} = \sum_{i=1}^{r} \kappa(dn_{ij}); \tag{4.4}$$

where

$$\begin{aligned} \kappa(dn_{ij}) &= \kappa(W_i, C_j), \\ &= \frac{\left(\sqrt[10]{512} - 1\right)^{-1}\left[\sqrt[10]{512\left(\left(dn_{ij}\right)^{10} + \left(1 - dn_{ij}\right)^{10}\right)} - 1\right]}{n}, \\ &\qquad \forall\ i \,\&\, j, \quad 1 \le i \le r, \quad 1 \le j \le s. \end{aligned} \tag{4.5}$$

For criteria weights to be optimum, we can write

$$\begin{aligned} max(E) &= \sum_{j=1}^{s} (FK_j)(k_{ij}), \\ &= \sum_{j=1}^{s}\left(FK_j \sum_{i=1}^{r} \kappa(dn_{ij})\right), \\ &= \left(\sqrt[10]{512} - 1\right)^{-1} \sum_{i=1}^{r}\sum_{j=1}^{s}\left[FK_j\left(\frac{\left[512\left(\left(dn_{ij}\right)^{10} + \left(1 - dn_{ij}\right)^{10}\right)\right]^{\frac{1}{10}} - 1}{n}\right)\right]; \end{aligned} \tag{4.6}$$

where $FK_j \in \hat{I}$ and $\sum_{j=1}^{q} FK_j = 1$.
Now using Eq. (4.6), criteria weight vectors can be computed as

$$\arg\min(E) = \left(W_1, W_2, \ldots, W_s\right)'. \tag{4.7}$$

IV. We determine the optimal solution after getting the criteria weight vectors. Let ${}_*\Psi$ and ${}^*\Psi$ indicate the worst and best solutions, respectively. Then we can compute them using the following

(i). **Profit Computation**

$$ {}^*\Psi_j = \max_{\{1\leq i\leq r\}} dn_{ij}, \text{ and } {}_*\Psi_j = \min_{\{1\leq i\leq r\}} dn_{ij}, \quad \forall j = 1,2,3,\ldots,s. \tag{4.8}$$

(ii). **Cost Computation**

$$ {}^*\Psi_j = \min_{\{1\leq i\leq r\}} dn_{ij}, \text{ and } {}_*\Psi_j = \max_{\{1\leq i\leq r\}} dn_{ij}, \quad \forall j = 1,2,3,\ldots,s. \tag{4.9}$$

V. Yu [62] proposed a distance-based method in which distance measurement is replaced by FAM to obtain an optimal solution. The amended formula used in this approach is given below

$$L_{\tau,i} = \left[\sum_{j=1}^{s}\left(FK_j \frac{K_{Accy}({}^*\Psi, W_i)}{K_{Accy}({}^*\Psi, {}_*\Psi)}\right)^{\tau}\right]^{\frac{1}{\tau}}, \forall i = 1,2,\ldots,r; \tag{4.10}$$

where ${}_*\Psi$ ${}^*\Psi$, are the worst and best solution respectively.
The optimum choice is now determined by maximizing collective benefit while minimizing individual regret. To generate these values, use $\tau = 1$ and $\tau = \infty$ in Eq. (4.10). Let $L_{1,i} = \boxplus_i$ and $L_{\infty,i} = \boxtimes_i$. Then

$$\boxplus_i = \sum_{j=1}^{q}\left(FK_j \frac{K_{Accy}({}^*\Psi, W_i)}{K_{Accy}({}^*\Psi, {}_*\Psi)}\right), \forall i = 1,2,\ldots,r, \tag{4.11}$$

and

$$\boxtimes_i = \max_{\{j\}}\left(FK_j \frac{K_{Accy}({}^*\Psi, W_i)}{K_{Accy}({}^*\Psi, {}_*\Psi)}\right), \forall i = 1,2,\ldots,r; \tag{4.12}$$

where FK_j can be determined by Eq. (4.3).

VI. On computing the Max($\boxplus_i$) and Min($\boxtimes_i$), VIKOR indices (VK_i) are calculated using the formula given below. These (VK_i) are then used to assign a rank to each alternative.

$$VK_i = \Psi\left(\frac{\boxplus_i - \min_{\{i\}}\boxplus_i}{\max_{\{i\}}\boxplus_i - \min_{\{i\}}\boxplus_i}\right) + (1-\Psi)\left(\frac{\boxtimes_i - \min_{\{i\}}\boxtimes_i}{\max_{\{i\}}\boxtimes_i - \min_{\{i\}}\boxtimes_i}\right), \forall i = 1,2,\ldots,r; \tag{4.13}$$

where Ψ denotes the weightage assigned to $\boxplus_i$ and $\boxtimes_i$. Usually, Ψ assumes a value of 0.5. Ranking can be done by re-ordering $\boxplus_i$, $\boxtimes_i$, and VK_i in descending order. If VK_i is the maximum for some i, then it is considered the best option. There are two kinds of solutions, as shown below:

(A). **Acceptable Advantage ($\overline{ACAD}$)**: After sorting VK_i, we gain acceptable advantage if

$$VK_1 - VK_2 \geq \frac{1}{s-1};$$

Here VK_1 and VK_2 are first and second positions in the VIKOR list. If the solution obtained satisfies the above condition, then the given approach has an acceptable advantage.

(B). **Acceptable Stability ($\overline{ACST}$)**: Acceptable Stability is the condition where the same alternative is at the first rank in VIKOR and $\boxplus_i$,$\boxtimes_i$ lists.
When these conditions fail, then the remaining cases are:

(i). $\overline{ACAD}$ **does not hold:** In this cases the set of alternatives $\{W_1, W_2, \ldots, W_r\}$ makes a compromised solution

$$VK_1 - VK_P < \frac{1}{q}; \tag{4.14}$$

with some maximum value of P.

(ii). $\overline{ACST}$ **does not hold**:In this case, we declare the set (W_1, W_2) as compromised solution.

VII. We compute the degree of efficiency/utility $\wp_i$ by having a comparison with VIKOR indices of every investigated option to the best one. To present relevant and clear results, a scale of 0 to 100 is used. The best achieve the highest score, while the worst will own the lowest. The formula determines the utility degree/efficiency

$$\wp_i = \frac{VK_i}{VK_{max}} \times 100, i = 1, 2, 3, \ldots, m; \tag{4.15}$$

The VIKOR index VK_i indicates the i^{th} alternative, while VK_{max} represents the highest value among all VIKOR indices. Degrees of efficiency/utility are assembled in descending order i.e., we put the highest degree of utility at the top position followed by the next highest, and so on. The whole algorithm is shown in the form of a flow chart in the figure below

4.3 Case Study: Selection of best E-Commerce Site

Post-pandemic, India is being shaped into an online shopping market steadfastly. The Indian e-commerce market is expected to achieve a high of $ 170 billion by 2025. But still, it is covering 7% of the global market and is far behind China which is covering 50% of the global market. There are many giant players like Amazon, and Walmart along with Indian counterparts such as Jio Mart, Flipkart, etc. which are shaping a future for potential growth in the online market of India. Some main reasons backing this growth are:
1. Ease Of Availability
2. Economical Pricing
3. Variety Of Products
4. Time Saving
5. Door Step Services
6. Attractive Offers
7. Flexibility for Customers
8. Efficient Return Policy E-commerce has inevitably become an important part of our daily life. An increasing number of players in this industry will lead to a tough journey of these. The successful companies of the future will be those that take e-commerce seriously. The present case study addresses the future of some E-commerce websites (given in the table) tested through some pivot technical factors. A concise introduction of these criteria is compiled in the Table 6

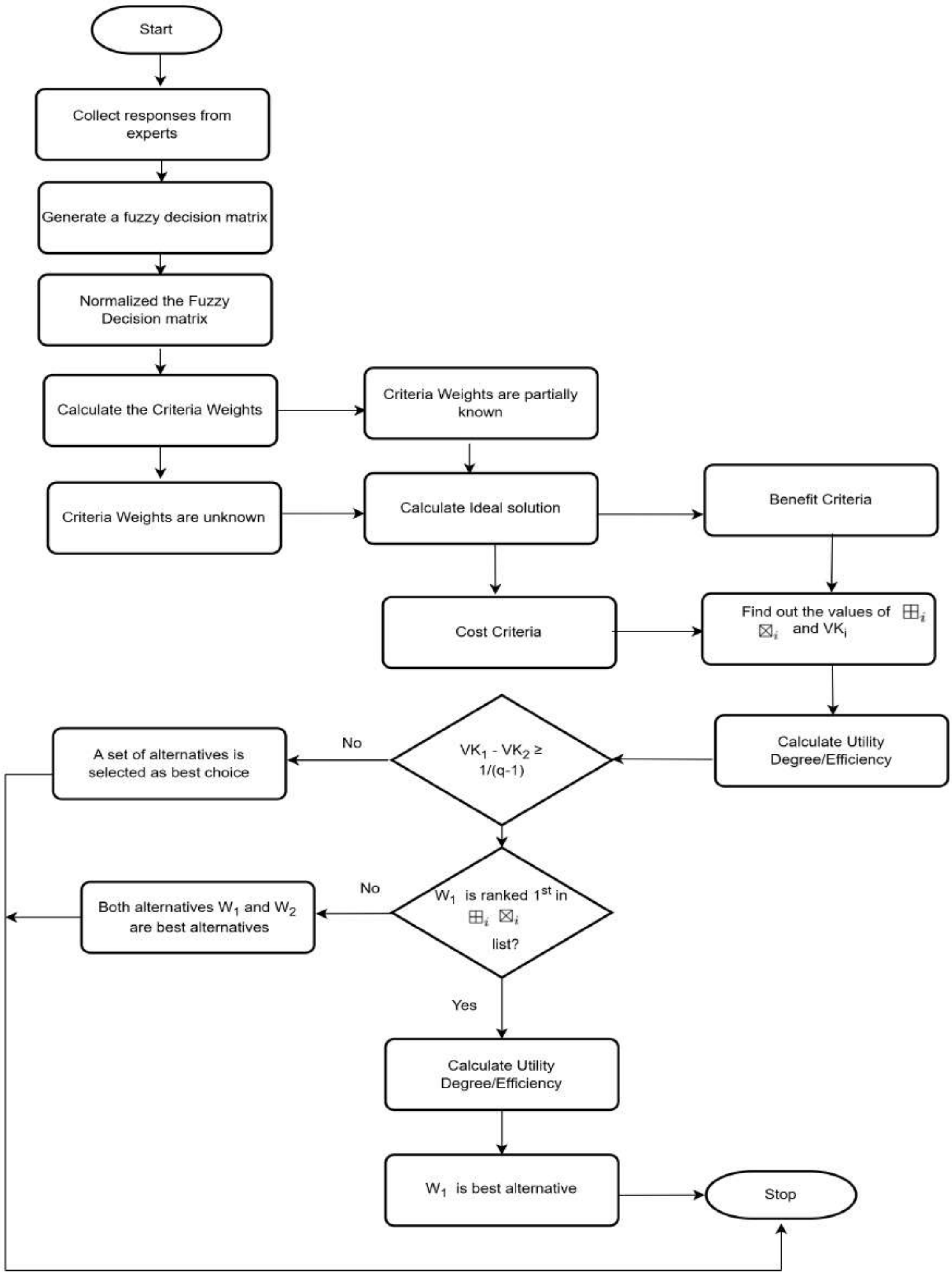

Figure 1. Flow Chart Of An MCDM Problem

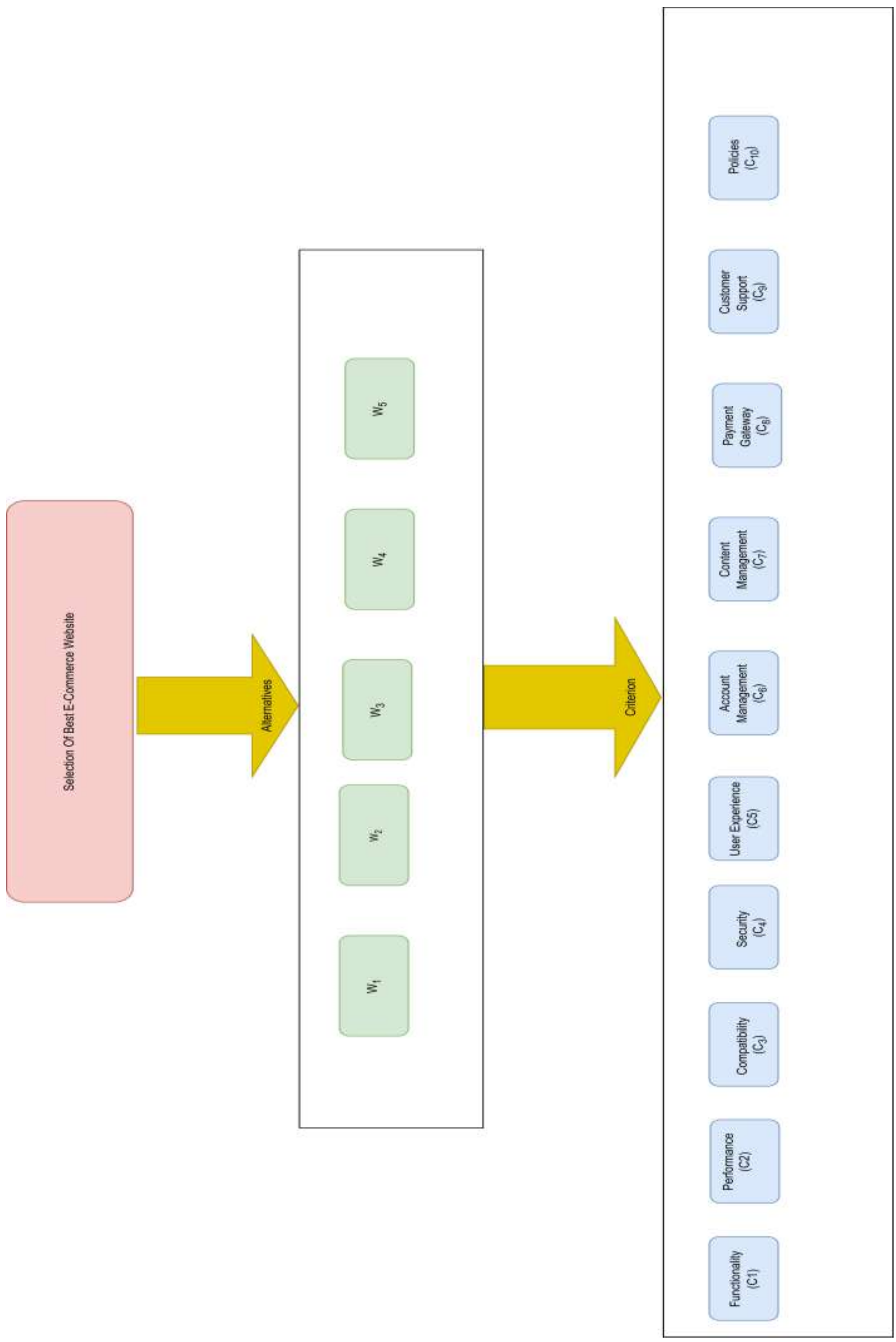

Figure 2. MCDM problem to find the Best E-Commerce Website.

To meet expected growth and consumer satisfaction, an E-Commerce website must satisfy certain criteria. Selecting an ideal website from growth and consumer satisfaction prospects is therefore a challenging task because it has to go through several test criteria. Among a large number of e-commerce websites available in the online market, to select the best one, we include the following criteria:

- Functionality
- Performance
- Compatibility
- Security
- User Experience
- Account Management
- Content Management
- Payment Gateway
- Customer Support
- Policies

Table 6. Description of Criteria.

Criteria	Description
Functionality (C_1)	Ease of Navigation, the accuracy of the search bar, and comfortable handling of a shopping cart are some of the major components that contribute towards the functionality of an e-Commerece website.
Performance (C_2)	There are two major components viz. page load speed during checkout of products and scalability which is the performance in peak hours.
Compatibility (C_3)	Efficiency of e-Commerce website to work on different browsers and operating systems.
Security (C_4)	Use of secure connections, encryption and security of payment transactions and protection of user data.
User's Experience (C_5)	Ease of user interface, and accessibility of the website for differently abled users are components that should be counted toward user experience.
Account Management (C_6)	Testing of user's registration, login, and account management, tracking of order, and verification of promotional features.
Content Management (C_7)	Timely updation of product information and personalized recommendations.
Payment Gateway (C_8)	Proper feasibility and correct functioning of various payment methods.
Customer Support (C_9)	Trust building of customers, proper address to their feedback, and legible disposal of their grievances.
Policies (C_{10})	Effective and favorable execution of privacy, return and refund, shipping and cancellation policy, etc.

To make a comparative decision, we select five e-commerce website alternatives, W_1, W_2, W_3, W_4, and W_5, and ten criteria, C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, and C_{10}. To select the best e-Commerce website among these we seek feedback from ten experts E_1, E_2, $E_3, \ldots, E_{10}$ to

Table 7. Specialization and Experiences of Experts.

Experts	Back history and Experience
E_1	Financial analyst having Experience of 12-15 years in market research and development.
E_2	Financial advisor in a reputed securities company having 32 years of experience.
E_3	Professor of Commerce from a Central Research Institute with 25 years of experience.
E_4	Computer Engineer with 15 years of experience in digital marketing.
E_5	Enterpneur from a well-established non-govt. business venture
E_6	Business analyst in Non-govt industry
E_7	Business analyst in PSU having 20 years of experience.
E_8	CEO at a multinational non-banking finance company
E_9	Male Consumer having experience of one decade in online shopping
E_{10}	Senior consultant/blog writer in a commercial magazine

identify the most desirable alternative from the specified list of five alternatives. Figure 1 depicts the pictorial representation of the algorithm for MCDM problems. Table 6 summarizes the criteria's essential definitions, whereas Table 7 inherits the information about resource persons/experts deployed in the given MCDM problem. The process to find the best website from available options/alternatives is shown in Figure 2. MCDM problem as stated above can be solved using the following steps:

- **Case-A: If the criteria weights are unknown completely**

1. Compile responses from empanelled experts. Subjective information provided by these experts is recorded in the following table where Y denotes the condition when an Expert agrees whereas $\times$ denotes the condition where an expert disagrees with a particular alternative on the given criterion. Table 8 shows the decision matrix (d_{ij}) generated from the decision provided by resource persons/experts. An element d_{ij} of this matrix is the ratio of the number of experts supporting W_i w.r.t. C_j to the number of resource persons.

2 .Normalize the decision matrix (d_{ij}) using Eq. (4.1). Matrix (d_{ij}) and knowledge rendered by a given criterion is shown in the Table 9 depicts the normalized matrix.

3. When criteria weights are partially known or unknown, determine criteria weights using Eq. (4.3). In our case,

$$FK = \left\{ \begin{array}{c} 0.0757 \ , 0.1331 \ , 0.1319 \ , 0.0695 \ , 0.1706 \\ 0.04619 \ , 0.1241 \ , 0.0740 \ , 0.0905 \ , 0.0844 \end{array} \right\}. \tag{4.16}$$

4. Identify the ideal solutions[1]. The best and worst solution can be found by using Eq. (4.8) and Eq. (4.9)

$$\begin{aligned} {}^{*}\Psi &= \{0.5291, 0.6621, 0.5754, 0.5035, 0.6964, 0.4743, 0.6385, 0.5242, 0.5466, 0.5835\}, \\ {}_{*}\Psi &= \{0.3024, 0.3310, 0.2466, 0.2517, 0.2321, 0.3162, 0.3192, 0.2621, 0.3123, 0.2917\}. \end{aligned} \tag{4.17}$$

5. Using Eqs. (4.11), (4.12) and (4.13), we compute ($\boxplus_i$, ($\boxtimes_i$, and VIKOR indices (VK_i, for all i; $1 \leq i \leq r$). Table 10 and Figure 4 display their computed values. Now, we assign rankings to given options/alternatives by rearranging values of $\boxplus_i, \boxtimes_i$, and VK_i in descending fashion. The following table shows these ranks

[1]Only benefit criteria are considered in this example.

Figure 3. Evaluation Of Linguistic Alternatives by Experts

Alternatives	Experts	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
	E_1	Y	×	×	Y	Y	×	Y	Y	×	×
	E_2	×	Y	Y	×	×	Y	×	×	Y	×
	E_3	Y	×	Y	Y	×	Y	Y	×	Y	Y
	E_4	Y	Y	×	×	Y	×	×	Y	×	×
W_1	E_5	×	Y	Y	Y	Y	Y	×	×	×	Y
	E_6	Y	×	×	Y	×	×	Y	×	Y	×
	E_7	×	×	Y	×	×	Y	×	×	Y	×
	E_8	Y	×	×	Y	Y	×	×	Y	×	Y
	E_9	×	Y	×	×	×	Y	Y	×	Y	Y
	E_{10}	×	×	Y	Y	×	Y	×	×	×	×
	E_1	Y	Y	×	Y	Y	×	Y	Y	×	Y
	E_2	×	×	Y	×	×	Y	×	Y	Y	Y
	E_3	Y	×	×	×	Y	Y	×	×	×	×
	E_4	×	Y	Y	×	×	×	×	Y	Y	Y
W_2	E_5	×	×	×	Y	Y	Y	Y	×	×	×
	E_6	Y	Y	Y	×	Y	×	Y	×	×	Y
	E_7	×	×	×	×	×	Y	×	Y	×	Y
	E_8	×	Y	Y	×	Y	×	×	Y	Y	×
	E_9	Y	×	×	Y	×	Y	Y	×	Y	×
	E_{10}	×	Y	×	×	Y	Y	×	×	×	Y
	E_1	Y	×	Y	Y	Y	×	Y	×	Y	Y
	E_2	Y	Y	×	×	Y	Y	Y	Y	×	Y
	E_3	×	Y	Y	Y	Y	Y	Y	Y	Y	Y
	E_4	Y	×	Y	×	Y	×	×	×	Y	Y
W_3	E_5	×	Y	Y	Y	×	×	Y	Y	×	×
	E_6	Y	Y	×	Y	Y	Y	×	×	Y	Y
	E_7	Y	Y	Y	×	Y	Y	Y	Y	Y	Y
	E_8	Y	Y	Y	Y	Y	×	Y	Y	×	×
	E_9	×	Y	Y	×	Y	Y	Y	×	Y	Y
	E_{10}	Y	Y	×	Y	Y	Y	Y	Y	Y	Y
	E_1	×	Y	Y	×	Y	Y	×	Y	Y	×
	E_2	Y	×	×	Y	×	×	Y	×	×	Y
	E_3	Y	Y	×	×	×	Y	Y	Y	Y	Y
	E_4	×	×	Y	Y	×	×	×	Y	×	×
W_4	E_5	Y	Y	×	×	Y	×	Y	×	Y	Y
	E_6	Y	×	×	Y	×	Y	×	Y	×	Y
	E_7	Y	×	×	Y	×	×	Y	×	Y	×
	E_8	×	Y	×	×	Y	×	×	Y	×	Y
	E_9	Y	×	Y	Y	×	Y	Y	×	Y	×
	E_{10}	×	×	×	×	×	×	×	Y	×	Y
	E_1	Y	Y	×	Y	×	×	Y	Y	Y	Y
	E_2	Y	×	Y	×	Y	Y	Y	×	Y	×
	E_3	×	Y	Y	Y	Y	×	×	Y	×	Y
	E_4	Y	×	×	×	×	Y	Y	×	Y	×
W_5	E_5	×	Y	Y	Y	Y	Y	×	Y	Y	Y
	E_6	Y	×	×	Y	×	Y	Y	×	Y	×
	E_7	×	Y	Y	×	Y	×	×	Y	×	Y
	E_8	Y	×	Y	Y	×	Y	×	×	Y	Y
	E_9	Y	Y	Y	×	Y	×	Y	Y	×	Y
	E_{10}	Y	×	Y	Y	×	Y	Y	×	Y	×

]

Table 8. Fuzzy Decision matrix $D_{5\times 10}$.

$D_{5\times 10}$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
W_1	0.5	0.4	0.5	0.6	0.4	0.6	0.4	0.3	0.5	0.4
W_2	0.4	0.5	0.4	0.3	0.6	0.6	0.4	0.5	0.4	0.6
W_3	0.7	0.8	0.7	0.6	0.9	0.6	0.8	0.6	0.7	0.8
W_4	0.6	0.4	0.3	0.5	0.3	0.4	0.5	0.6	0.5	0.6
W_5	0.7	0.5	0.7	0.6	0.5	0.6	0.6	0.5	0.7	0.6

Table 9. Normalized Fuzzy decision matrix $DC_{5\times 10}$.

$DC_{5\times 10}$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
W_1	0.3780	0.3310	0.4110	0.5035	0.3095	0.4743	0.3192	0.2621	0.3904	0.2917
W_2	0.3024	0.4138	0.3288	0.2518	0.4643	0.4743	0.3192	0.4368	0.3123	0.4376
W_3	0.5291	0.6621	0.5754	0.5035	0.6964	0.4743	0.6385	0.5242	0.5466	0.5834
W_4	0.4535	0.3310	0.2466	0.4196	0.2321	0.3162	0.3990	0.5242	0.3904	0.4376
W_5	0.5291	0.4138	0.5754	0.5035	0.3869	0.4743	0.4788	0.4368	0.5466	0.4376
$K^A(T)$	0.0848	0.0635	0.1268	0.0706	0.0642	0.1217	0.0783	0.1139	0.1350	0.1359

Table 10. Listing of $\boxplus_i, \boxtimes_i$ and VK_i.

Alternatives	$\boxplus_i$	$\boxtimes_i$	VK_i
W_1	0.7036	0.1407	0
W_2	0.9337	0.1867	0.3121
W_3	1.4408	0.2882	1
W_4	0.7498	0.1499	0.0627
W_5	0.8333	0.1666	0.176

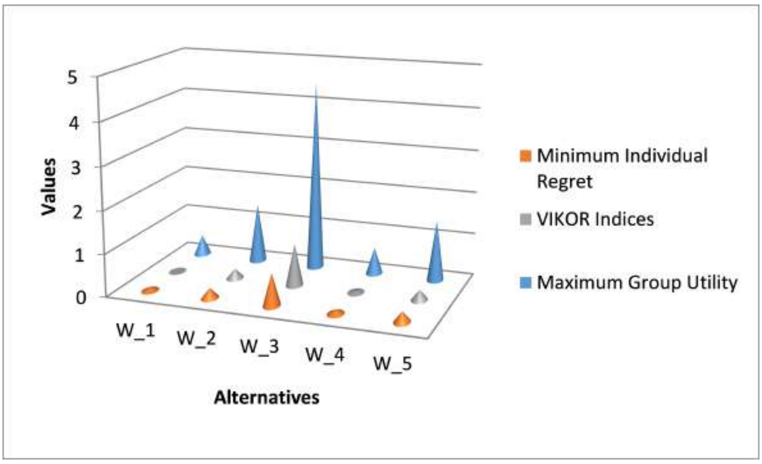

Figure 4. VIKOR indices, Minimum individual regret, and Maximum group utility

Table 11. Ranks of $\boxplus_i, \boxtimes_i$ and VK_i.

	W_1	W_2	W_3	W_4	W_5
$\boxplus_i$	5	2	1	4	3
$\boxtimes_i$	5	2	1	4	3
VK_i	5	2	1	4	3

Table 12. Degree Of Utility for given alternatives.

Alternatives	VK_i	$\wp$	Rank
W_1	0	0 %	5
W_2	0.3121	31.21 %	2
W_3	1	100 %	1
W_4	0.0627	6.2 %	4
W_5	0.176	17.6 %	3

6. To estimate the most suitable solution, we need to check for Acceptable Advantage ($\overline{ACAD}$) and Acceptable Stability ($\overline{ACST}$). Table 11 shows that W_3 stands 1^{st}, while W_2 is declared 2^{nd} in VIKOR indices. Thus,

$$\begin{aligned} VK(W_3) - VK(W_2) &= 1 - 0.3121, \\ &= 0.6879, \\ &> \frac{1}{10-1}, \\ &= 0.1111. \end{aligned}$$

$\overline{ACAD}$ is satisfied. Additionally, the alternative W_3 fetches 1^{st} rank in list of $\boxplus_i$ & $\boxtimes_i$, and the condition $\overline{ACST}$ is met. As a result, the alternative W_3 is the preferred option. The preference order of the choices is determined by

$$W_3 \succ W_2 \succ W_5 \succ W_4 \succ W_1. \tag{4.18}$$

7. Calculate the degree of utility by applying the formula given in Eq. (4.15). These results are shown in the Table 12 and Figure 5. As per Table 12, W_3 is placed 1^{st} as it has 100 % efficiency whereas W_2 is placed 2^{nd} and so on. We have used a fixed weightage value (Ψ) of $\frac{1}{2}$ in above example. We now take distinct weightage values for ($\boxplus_i$) and ($\boxtimes_i$). Changing the weightage (Ψ) from 0 to 1 results in the same series of choices as shown above, with W_3 being the best option.

- **Case-B:When criteria weights are partially known**

  Several real-life practical problems are there in which criteria weights can not be assigned in the form of numbers. Criteria weights in the form of intervals are suitable for these situations. To describe this better, suppose the MCDM problem given above has partially known criteria weights as shown below

$$\hat{I} = \begin{cases} 0.07 \le FK_1 \le 0.1, 0.08 \le FK_2 \le 0.09, 0.06 \le FK_3 \le 0.2, \\ 0.05 \le FK_4 \le 0.09, 0.04 \le FK_5 \le 0.8, 0.1 \le FK_6 \le 0.3, \\ 0.04 \le FK_7 \le 0.11, 0.09 \le FK_8 \le 0.2, 0.08 \le FK_9 \le 0.25, \\ 0.08 \le FK_{10} \le 0.3. \end{cases} \tag{4.19}$$

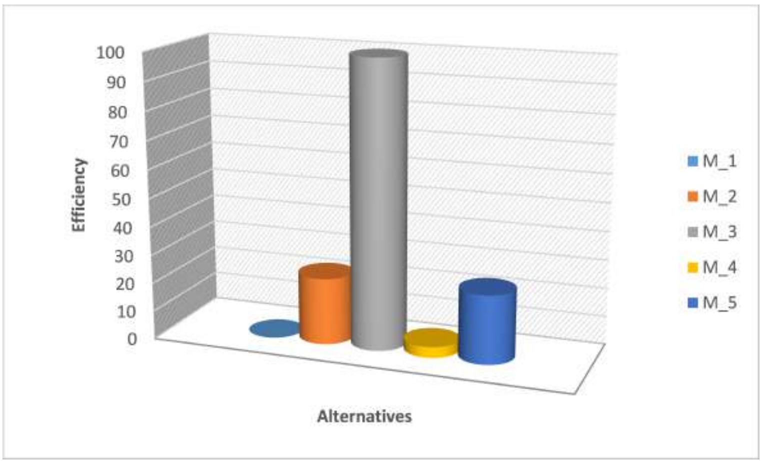

Figure 5. Utility degrees.

Now Eq. (4.19) can be converted into an LPP Problem as follows:

$$\begin{aligned} E_{max} =& 0.1217FK_1 + 0.2140FK_2 + 0.2120FK_3 + 0.1118FK_4 + 0.2742FK_5 \\ & + 0.0743FK_6 + 0.1995FK_7 + 0.1190FK_8 + 0.1456FK_9 + 0.1356FK_{10}, \end{aligned}$$

$$\text{subjected to conditions} \begin{cases} 0.07 \leq FK_1 \leq 0.1, \\ 0.08 \leq FK_2 \leq 0.09, \\ 0.06 \leq FK_3 \leq 0.2, \\ 0.05 \leq FK_4 \leq 0.09, \\ 0.04 \leq FK_5 \leq 0.8, \\ 0.1 \leq FK_6 \leq 0.3, \\ 0.04 \leq FK_7 \leq 0.11, \\ 0.09 \leq FK_8 \leq 0.2, \\ 0.08 \leq FK_9 \leq 0.25, \\ 0.08 \leq FK_{10} \leq 0.3. \\ \sum_{i=1}^{10} FK_i = 1. \end{cases} \tag{4.20}$$

Using MATLAB to solve Eq. (4.20), we have an optimal solution :

$$\begin{aligned} & FK_1 = 0.07, FK_2 = 0.08, FK_3 = 0.09, FK_4 = 0.2, FK_5 = 0.05, \\ & FK_6 = 0.08, FK_7 = 0.1, FK_8 = 0.11, FK_9 = 0.09, FK_{10} = 0.13. \end{aligned} \tag{4.21}$$

Using the same methodology as described in **(Case-A)**, again W_3 turns out to be the preferable one. Moreover, W_3 is found to be the best alternative for various values of weightage.

There are many real-world settings where this approach can be effectively applied to find solutions to MCDM problems associated with them. For instance,

(A). A person wants to purchase a piece of land among four types of properties available in the market. He can frame a list of criteria:

a. Developer goodwill
b. Construction quality
c. Connectivity
d. Location

(B). A college student aims to opt for one subject out of a list of six offered subjects. The student's objective is to select that subject which includes

Table 13. Computation of Fd_i, Fa_i

	W_1	W_2	W_3
Fd_i	**0.5**	**0.5**	**0.5**
Fa_i	2.5796	2.4658	2.5597

a. Future of the subject,
b. Student's interest
c. Seat availability,
d. Teacher availability

(C). Some Person wishes to host a party in a restaurant. To select a suitable restaurant his/her requirements are

a. Comfort
b. Prices
c. Services
d. Quality
e. The location

accuracy measure in the VIKOR approach

4.4 The Necessity For an Accuracy Measure in the VIKOR Method

Opricovic [63] introduced the VIKOR approach, which provides a compromised solution to a given MCDM problem. The key idea to this approach lies in the fact "near to perfect solution.". However, in some cases, the solution obtained by the distance measure is not feasible. Therefore replacement of distance measure by accuracy measure is preferable. We take several instances to show that the results in the VIKOR technique achieved by utilizing the distance measure are not consistent.

Example 4.1. Let W_1, W_2, W_3 be the alternatives and C_1, C_2 be two criteria weights. Take $W_1 = (0.7, 0.3)$, $W_2 = (0.5, 0.5)$, and $W_3 = (0.4, 0.6)$. Assuming that criteria weights are benefit criteria we now define mappings Fd_i and Fa_i $(\forall i, 1 \leq i \leq s)$ as follows

$$Fd_i = \frac{d(^*\Psi, W_i)}{d(^*\Psi,_* \Psi)}, \tag{4.22}$$

and

$$Fa_i = \frac{K_{Accy}(^*\Psi, W_i)}{K_{Accy}(^*\Psi,_* \Psi)}; \tag{4.23}$$

where $^*\Psi$ and $_*\Psi$ have usual meanings as defined earlier. The distance measure is given by Eq. (2.7). Mappings Fd_i and Fa_i are given by

From the above discussion, it is evident that distance measures can't discriminate between the alternatives. On the other hand, if an accuracy measure is used, then a clear-cut discrimination can be made. Let us take another example.

Example 4.2. Suppose we have a set of alternatives $\{W_1, W_2, W_3, W_4, W_5\}$ and two criteria weights C_1, C_2. Put $W_1 = \{0.7\ 0.3\}$, $W_2 = \{0.4\ 0.6\}$, $W_3 = \{0.5\ 0.5\}$, $W_4 = \{0.4\ 0.6\}$, and $W_5 = \{0.1\ 0.9\}$. Again considering that criteria weights as benefit criteria, we define mappings Fd_i and Fa_i $(\forall i, 1 \leq i \leq s)$ as follows From the above discussion, it is evident that distance measures can't discriminate between the alternatives. On the other hand, if an accuracy measure is used, then a clear-cut discrimination can be made.

The functions Fd_i and Fa_i are building components in the VIKOR technique. The examples shown above justify that distance measures in the VIKOR technique are not suitable to have an accurate result. Thus in this contest, the FAM-based VIKOR technique is more trustworthy.

Table 14. Computation of Fd_i, Fa_i

	W_1	W_2	W_3
Fd_i	**0.5**	**0.5**	**0.5**
Fa_i	2.9503	3.4218	2.4211

4.5 Comparison of Proposed FKM with Other Known Measures

We have included the following some well-known techniques and present a comparative study. Table 15 records the outcomes of various approaches along-with the output of the proposed approach whereas Table 16 produces comparison of these outcomes.

- TOPSIS [64]
- Ye [65]) .
- Singh et al. [27].
- Farhadinia [66].
- Singh et al. [67].
- Farhadinia[66], Nguyen[5].
- Singh and Ganie [51].

Table 15. Comparison Table

Decision Making Methods ↓	← Alternatives →				
	W_1	W_2	W_3	W_4	W_5
TOPSIS [64]	0.2512	0.5013	0.8556	0.2230	0.3661
Ye [65]	0.6379	0.8226	0.9174	0.6480	0.6505
Singh et al. [27]	0.5039	0.6276	0.6474	0.5881	0.6197
Singh et al. [67]	0.4729	0.5575	0.6538	0.4767	0.4856
Farhadinia[66]	-0.0648	0.2160	0.4872	-0.0407	-0.0381
Farhadinia[66], Nguyen measure [5]	-0.0641	0.2166	0.4867	-0.0406	-0.0379
Proposed one	0	0.3121	1	0.0627	0.1760

Here, we take α=1.5, β=2 for the measure given by [51].

Table 16. A Comparitive Analysis

DMM	Order of Preference	Best alternative
TOPSIS [64]	$W_3 \succ W_2 \succ W_5 \succ W_1 \succ W_4$	$W3$
Ye [65]	$W_3 \succ W_2 \succ W_5 \succ W_4 \succ W_1$	W_3
Singh et al. [27]	$W_3 \succ W_2 \succ W_5 \succ W_4 \succ W_1$	W_3
Singh et al. [67]	$W_3 \succ W_2 \succ W_5 \succ W_4 \succ W_1$	W_3
Farhadinia [66]	$W_3 \succ W_2 \succ W_5 \succ W_4 \succ W_1$	W_3
Farhadinia [66], Nguyen measure [5]	$W_3 \succ W_2 \succ W_5 \succ W_4 \succ W_1$	W_3
Proposed one	$W_3 \succ W_2 \succ W_5 \succ W_4 \succ W_1$	W_3

Discussion: Using Table 12, we deduce that W_3 is the most preffered alternative. To assert which approach is more reliable, a further investigation is required. According to TOPSIS [64], the most feasible alternative is one which is closest to the best solution. Simultaneously it is utmost

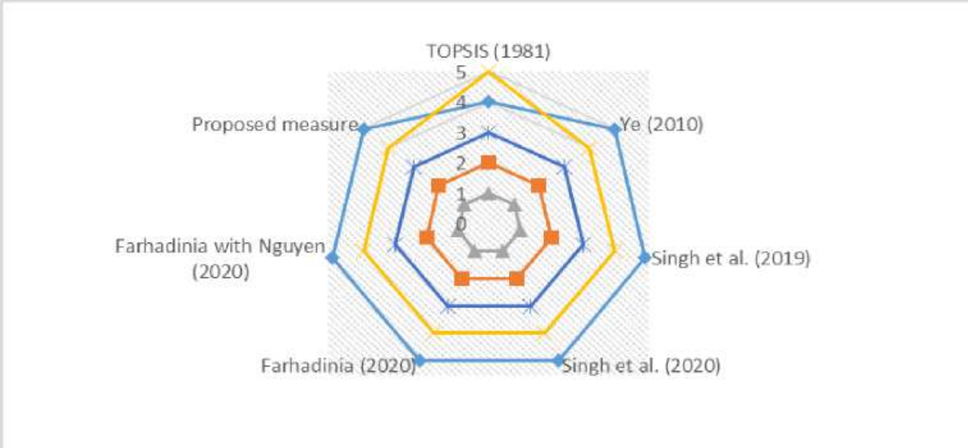

Figure 6. Comparison of existing methods and proposed measure.

from the worst solution. Contrary to TOPSIS, Opricovic [68] compared the VIKOR technique with the TOPSIS and asserted that it is not generally true that the solution nearest to the best solution is always the utmost to worst solution. Moreover, it is not always necessary for distance measures to obey criteria correlation. So we used the VIKOR approach in this work which is based on accuracy measure. This leads to a solution having maximum benefits. To determine the impact of criterion weights on the solution obtained, VIKOR ranking is done with several criterion weights. The VIKOR technique can also determine the interval of weight stability for any criteria. If weights (numerical value) remain outside this interval, then the obtained solution with basic weights has to be replaced. All criterion functions having identical fundamental weights are examined to determine the interval of weight stability for particular criteria. As an outcome, the VIKOR technique can be used to investigate the solution's preferred stability. The proposed VIKOR technique can be beneficial in MCDM situations where the decision creator is unable to define the preference sequence at the initial stage of system construction. The final answer is suited for each decision-maker. since it maximizes group utility while minimizing individual regret. Ye [65] only looked at the relationship between alternatives and the optimal answer. In certain cases, proximity to the optimal solution can be useful, but not every time, as it may lead to the loss of vital information in some cases. The work in [27, 67], addressed those MCDM problems which contain multiple conflicting criteria and obtained a solution that depends on parameters used for the given MCDM problem. Among these five alternatives used in our approach, one option outperforms the other counterparts in one or more areas whereas other approaches do not. In view of these limitations, the proposed technique's result is more feasible.

5 Conclusions

This work examines and verifies a novel FKM. Numerical examples are utilized to compute the performance of the proposed FKM to demonstrate its efficacy. We compare the new measure with a few previously established information measures. For measuring ambiguous content, measuring attribute weights for MCDM problems, and measuring structured language challenges, this knowledge measure turns out to be the most effective choice. When it comes to handling structured language variables, determining the degree of ambiguity in two different FSs, and determining objective weight, the suggested FKM proves to be an appropriate choice. Furthermore, the recommended FKM's utility is illustrated by a comparison with other well-known FIMs and FKMs. Furthermore, a new accuracy measure that employs a VIKOR-TOPSIS method is presented to address MCDM problems. We can summarizes advantages of the proposed method as follows:

5.1 Advantages of the suggested method

In the suggested method, we switched out the distance measure with the deduced accuracy measure. The VIKOR-TOPSIS technique uses two examples to demonstrate why it is necessary to use an accuracy measure over a distance measure. The derived accuracy measure has several fundamental features that are proven.

- It enables us to select the option that best suits all requirements.
- The suggested method explains why certain options are superior to others while making decisions.
- The suggested method eliminates the need for additional laborious computations.

The effectiveness of the recommended strategy is demonstrated through a comparative analysis with a few other well-known DMMs. An enhanced VIKOR technique is proposed (see Example 4.1 & 4.2) to solve an MCDM problem based on the suggested FKM and FAM. This strategy has produced noteworthy outcomes. The criteria weights in the suggested method are calculated in two different methods. When the weights of the criteria are unknown, it is better to apply the first strategy given by Eqs. (4.2 & 4.3); if they are known, just part of the way, the other approach is employed i.e., given by Eqs. (4.4 & 4.5). By resolving a case study that involves choosing the best e-commerce website, the efficacy of the recommended measure is determined. To illustrate the method's viability, a comparison research was carried out. This approach offers a great deal of potential for finding the best solution that satisfies every requirement. It illustrates why choosing a particular alternative is preferable when making decisions. The proposed approach is effective and low-processing, which makes it appropriate for a range of ambiguous situations.

6 Limitation of the suggested work

Every strategy yields outcomes dependent on the mindset that was established during the strategy's creation. In a certain circumstance, each of these is suitable. Under the same or different conditions, it is not always possible for one strategy to provide the same result as another. Therefore, an approach's foundational theory matters. There are some issues with the suggested work as well. Below are their details:

- The suggested method is exclusively applied to identify the optimal choice. It only provides information about the option that outperforms every other option. The preference order after the first position may differ between the proposed approach and any other approach, meaning that an alternative that is in the second position as determined by any other approach may not be in the second position as computed by the proposed approach.
- The suggested method produces absurd results, if the values of the VIKOR indices $_*\Psi$ (closest to weak ideal solutions) and VK_i (closest to strong ideal solutions) are zero for a given alternative.
- The suggested method's computations are straightforward yet laborious and time-consuming. The final response reflects any errors that we make at any stage. Without doing all of the suggested approach's computations, we are unable to fix this error.
- Experts are not always in a position to provide an accurate opinion. This is because they may not have enough time and may not have required knowledge about that specific profession. The outcomes of the other ways and the suggested approach could differ under those circumstances.
- There are limitations to this research when it comes to handling inconsistent and ambiguous data in a more precise setting.

Considering all of these drawbacks, we may conclude that the suggested strategy has certain issues. There are pros and downsides to everything in our world. Nobody in this world is flawless. However, considering the necessity of the suggested strategy, we can conclude that the suggested approach produces extremely reliable results.

6.1 Future extension of the present work

The present study can be expanded in the future in several different ways:

- IFS, PFS, Interval-valued IFS & PFS, Soft Sets, Q-rung Orthopair fuzzy sets, and T-spherical fuzzy sets can all be included in the proposed DMM.
- Image processing, cluster analysis, supplier selection, and medical diagnosis are among the additional challenges to which the suggested accuracy metric may be applied.
- From the suggested knowledge measure, we may derive a Novel Similarity and Dissimilarity measure, which can be applied to image processing, medical diagnosis, pattern recognition, cluster analysis, and other challenges.

In addition, the suggested work has been applied to numerous real-world problems. In the future, we'll also make an effort to eliminate all of the suggested work's constraints. This strategy offers a wide range of applications. Intuitionistic fuzzy sets (IFS), image fuzzy sets (PFS), neutrophilic fuzzy sets (NFS), hesitant fuzzy sets (HFS), interval-valued intuitionistic fuzzy sets (IvIFS), and so on could all be included in future research. The suggested method works with several systems, such as NFS, PFS, IvIFS, HFS, and IFS. Three domains can use the proposed FKM and FAM algorithms: voice recognition, feature detection, and thresholding.

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Author information

Meenu Goel, Department of Statistics, Mata Sundri College,University of Delhi (INDIA), India.
E-mail: meenugoel@ms.du.ac.in

Shiv Narain, Department of Mathematics, Arya P.G. College, Panipat, 132103 (INDIA), India.
E-mail: drnarainshiv@gmail.com

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