

# Evaluation of All Subgroups of a Group of Higher Order 600, 650 and 700 By Using Sylow's Theorem

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**Abstract** We develop the all possible subgroups of a group of higher order 600, 650 and 700 of different algebraic structures as groups by using Sylow's theorem. In fact, the help of order of group( $o(G)$ ) subgroups, homomorphism, isomorphism, dihedral group, and split extensions of groups are used in Sylow's theorems. First of all, we discuss the order of a group and the order of elements of a group in real numbers. Then we develop all the applications of the Sylow's theorems of a group in higher order of a group which will give the knowledge of the mathematical systems like number systems, vectors, matrices and group theory and so on. When we study Sylow's theorem of higher order of a group, then we use short exact sequences, and split extensions. The number of abelian groups is calculated by the product of prime powers or other cases but the non-abelian case is dependent on a different theorem. The use of two cases like abelian group and non-abelian group when the study of the Sylow's theorems of a group in higher order of a group. After that we find out the abelian group and non-abelian group in the higher order of the group by using Sylow's theorem.

## 1 Introduction

In this study, we explore several fundamental concepts in group theory, including the order of an element in a group, the order of a group itself, torsion groups, mixed groups, subgroups, normal subgroups, and the integral powers of elements. Our primary objective is to investigate the structure and classification of subgroups within groups of higher orders, specifically those of order 600, 650, and 700. Group operations will be denoted by symbols such as  $\cdot$ ,  $+$ , or will be omitted when context allows and the group notation is  $o$  or  $*$ . Identity elements will be represented by 1 or 0, corresponding to multiplicative and additive groups, respectively. The binary operations considered are addition, multiplication, and general group operations.

Initially, we employ Lagrange's theorem to determine possible subgroups of smaller orders. However, since Lagrange's theorem is insufficient for identifying all subgroups in higher order groups, we extend our approach using Sylow's theorems. These theorems are particularly effective in analyzing the existence and number of  $p$ -subgroups within a finite group of given order. For instance, when studying abelian groups of order 600, we note that 600 factors as  $600 = 2^3 \cdot 3 \cdot 5^2$ . Applying the fundamental theorem of finite abelian groups, we examine the partitions of each exponent to determine the number and structure of non isomorphic abelian groups of that order. The partitions of 3 are: 3, 2 + 1, 1 + 1 + 1. There is only one partition 1 of 1. The partitions of 2 are 2 and 1 + 1. These partitions correspond to different abelian group structures, resulting in six distinct abelian groups of order 600.

Finally, we extend this analysis to both abelian and non-abelian groups of higher orders (600, 650, 700), aiming to classify all possible subgroup configurations within these groups based on their prime factorizations and structural properties.

## 1.1 Background and Literature Review

Sylow's theorems and the classification of finite groups have a long and deep history in algebra. Classic structural results and case studies for example the detailed analysis of exceptional simple groups and constructions of particular finite simple groups provide the theoretical backdrop for any systematic subgroup enumeration for large orders. In particular, Hall & Wales' work on an explicit simple group of large order illustrates the level of detailed structural and subgroup analysis required when treating groups with complicated prime power factorizations; their paper remains a standard reference for how careful local (Sylow-type) and global arguments interact in large order cases [1]. Among related studies on subgroup structures and Sylow theorems can be found in ([2], [3],[4]). More recent expositions and surveys emphasize two complementary directions that are relevant to the present study. First, there has been continuing progress in understanding how local Sylow structure constrains global group structure (for instance, work on maximal over groups of Sylow subgroups and second maximal subgroup structure), and surveys of recent developments summarize new tools and open problems in classifying finite groups by local conditions. These surveys highlight that Sylow based enumeration remains a natural and productive approach, but that it is often strengthened by combining Sylow arguments with information about automorphism groups, normalizers, and cohomological extension data [5]. Second, computational group theory and GAP in particular has transformed subgroup classification from a purely pen and paper exercise into a replicable computational workflow. GAP's libraries and functions for Sylow subgroups, subgroup lattices, and conjugacy testing make it possible to verify enumerations and to explore candidate extension structures algorithmically; many contemporary subgroup classifications now pair theoretical Sylow arguments with GAP computations for verification and to generate explicit examples. Incorporating such computational checks is an established best practice for papers that enumerate or claim existence/non-existence of particular subgroup configurations [6]. This manuscript builds on our earlier analytic work applying Sylow's theorems to small composite orders (Mannan et al., 2022). That study developed a systematic template factor the group order, list candidate Sylow subgroup sizes, apply congruence constraints from Sylow's third theorem, and then analyze extension possibilities which we now extend to the larger, more intricate orders 600, 650, and 700. The present paper follows the same Sylow-first methodology but expands it by (i) carrying the partition or abelian classification arguments for higher prime power exponents, (ii) explicitly using automorphism group reasoning for split extension cases, and (iii) adding more cases where normality or conjugacy constraints force rigid subgroup lattices. Where possible we indicate how computational verification (e.g., with GAP) could be used to check particular non-abelian extension claims; this both strengthens the correctness of the classification and aligns the paper with contemporary practice [7].

In addition, we study the subgroup structures of groups of order 30 and 42 through applications of Sylow's theorems, which help identify Sylow  $p$ -subgroups and their distribution within a finite group. The order of Sylow's  $p$ -subgroup of a finite group  $G$  is  $P^n$  where  $n$  the multiplicity of  $p$  in the order of  $G$  and any subgroup of order  $p^n$  is a Sylow  $p$ -subgroup of  $G$  [8].

Finally, there are two modern strands of research that suggest natural directions and justify the relevance of the current enumeration. One strand concern algorithmic and complexity aspects of subgroup-finding problems (recent work shows hardness results in the general Cayley table model, while also identifying efficient algorithms for special classes such as abelian groups), which highlights the value of having explicit, verified classifications for specific orders as test cases for algorithms. The second strand includes very recent results on coverings and structural constraints for  $p$  elements which refine our understanding of how Sylow subgroups can cover or fail to cover certain element sets; such results provide useful background when arguing about normality and intersection properties of Sylow subgroups in specific orders. These developments show that an analytic Sylow enumeration when combined with computational checks continues to be a useful contribution to both theory and computation ([9], [10]).

## 1.2 Structure of the Paper

The organization of this paper is as follows: Section 2 discusses the significance of Sylow's theorems in the analysis of finite groups. Section 3 presents a comprehensive analysis of the structural characteristics of abelian and non-abelian groups of higher orders, namely 600, 650,

and 700. Section 4 presents both theoretical insights and practical applications derived from the study. Section 5 outlines the limitations of the current work and suggests possible directions for future research related to Sylow's theorems in finite group theory. Section 6 concludes the paper with a summary.

## 2 Significance of Sylow Theorems in the Study of Finite Groups

Sylow's theorems are essential tools in the study of finite groups, particularly for determining the existence and structure of subgroups whose orders are powers of prime numbers. While Lagrange's theorem tells us that the order of a subgroup must divide the order of the group, it does not guarantee the existence of such subgroups. Sylow's theorems fill this gap by ensuring that for any prime power dividing the group order, a corresponding subgroup exists. Moreover, they provide information on the number of such subgroups and their conjugacy properties, which is critical in classifying finite groups and understanding their internal structure.

### Theorem 2.1. Sylow's first theorem

Suppose  $G$  is a finite group and  $p$  is a prime number. Also suppose that  $p^m$  is a divisor of  $o(G)$  and  $p^{m+1} \nmid o(G)$ . Then there will be a subgroup  $H$  of  $G$  s.t.  $o(H) = p^m$ .

### Theorem 2.2. Sylow's second theorem

Suppose that  $G$  is a finite group and  $p$  is a prime number such that  $p \mid o(G)$ . Then all sylow  $p$ -subgroups of  $G$  are conjugates of each other.

### Theorem 2.3. Sylow's third theorem

Let us consider  $G$  is a finite group and  $p$  is a prime number such that  $p \mid o(G)$ . Then the number  $N$  of sylow  $p$ -subgroup is  $N = 1 + mp$ , where  $m \geq 0$ .

### Proposition 2.4. Automorphism group of the dihedral group $D_6$ .

*Proof.* Let  $D_6 = \{e, x, x^2, x^3, x^4, x^5, y, yx, yx^2, yx^3, yx^4, yx^5\}$ . With defining relation  $x^6 = y^2 = e$  and  $y^{-1}xy = x^{-1}$ , is  $(DG) = 12$ . Therefore, the conjugate classes are as follows:  $\{e\}, \{x, x^5\}, \{x^2, x^4\}, \{x^3\}, \{y, yx^2, yx^4\}, \{yx, yx^3, yx^5\}$ . So,  $D_6\{e, x^3\} \cong /D_6$  to  $o(6)$ . Then  $D_6$  is 6 inner automorphisms but one is the identity element. Let the other inner automorphisms be  $Y, Z, U, V, T$ . Now, if  $x$  is fixed by  $Y$  then  $Y(e) = e, Z(x) = x$  and  $Y(y) = yx^2$  and hence  $Y(yx) = yx^5$ . Then  $Y^3 = I$ . Next, if  $y$  is fixed by  $Z$  then  $Z(e) = e, Z(y) = y$ , and  $Z(x) = x^{-1}$  and  $Z(yx) = yx^{-1}$  and then  $Z^2 = I$ . If  $yx$  is fixed by  $U$  then  $U(I) = I, U(yx) = yx$ , and  $U(x) = x^{-1}$  and  $U(y) = yx^2$ . If  $yx^5$  is fixed by  $T$  then  $T(I) = I, T(yx^5) = yx^5$  and  $T(x) = x^{-1}$  and  $T(yx) = yx^3$  and then  $T^2 = I$ , and we have,  $Y^3 = Z^2 = U^2 = V^3 = T^2 = I$  and then,  $Y^2 = V, TU = V = ZT, UT = Y = TZ$  and  $Z^{-1}YZ = Y^{-1}, U^{-1}VU = V^{-1}, T^{-1}YT = Y^{-1}$ . So that  $\text{Inn.Aut}(D_6) = \{Z, Y\} \cong D_3 \cong S_3$  with  $Z^{-1}YZ = Y^{-1}$  and  $Y^3 = Z^2 = I$ .

Now, suppose that the mapping  $S : D_6 \rightarrow D_6$ .

Let  $S(e) = e$  then  $S(x) = x$  or  $x^5$  and so let  $S(x) = x^5$  and put  $P = US$  then  $P(x) = US(x) = U(x^5) = x$  So,  $P(y) \neq x^3$  for  $x^3$  is a central element If  $P(y) = y, P(y) = yx, P(y) = yx^4$  then  $P = I, P = U, P = Y$  and  $P(y) \neq y, P(y) \neq yx, P(y) \neq yx^4$ , respectively But if  $P(y) = yx$  then  $P(yx) = yx^2$  and  $P^6 = I$ . Now,  $PZ = ZP^5, PT = TP^5$  and  $PU = UP^5$ ,  $\text{Aut}(D_6) = \{P, Z\} \cong D_6$  with  $P^6 = Z^2 = I$  and  $Z^{-1}PZ = P^{-1}$ .  $\square$

## 3 Result and Discussion

In this section, we analyze the structure of abelian and non-abelian groups of higher orders such as 600, 650, and 700. To investigate these groups, we apply composition-related theorems to evaluate their internal structure and determine the possible subgroup configurations. Specifically, we employ Sylow's theorems to identify and analyze Sylow  $p$ -subgroups, which provide valuable insight into the composition and classification of groups of these large orders. These theorems help us determine the number and types of subgroups that can exist within such groups, shedding light on the overall group behavior and structure.

### 3.1 Classification of All Abelian and Non-Abelian Groups of Order 600

[11], [12]

#### Abelian Group

The list of Abelian Group of 600 is

i.  $G \cong C_8 \times C_3 \times C_{25}$  ii.  $G \cong C_2 \times C_4 \times C_3 \times C_{25}$  iii.  $G = C_2 \times C_2 \times C_2 \times C_3 \times C_{25}$  iv.  $G = C_8 \times C_3 \times C_5 \times C_5$  v.  $G = C_2 \times C_4 \times C_3 \times C_5 \times C_5$  vi.  $G = C_2 \times C_2 \times C_2 \times C_3 \times C_5 \times C_5$

#### Non-Abelian Group

So that  $600 = 2^3 \cdot 3 \cdot 5^2$ .

#### 2- Sylow Subgroups

The group G of order 600 is  $x \equiv 1 \pmod{2}$ , where  $x = 1, 3, 5, 15, 25, 75$ .

##### 1, 2-Sylow subgroup

Here,  $o(M) = 2$  then  $M \cong C_2$ . If  $M \cong C_2$ , then  $\{I\} \rightarrow M \rightarrow G \rightarrow C_{300} \rightarrow \{I\}$ . But,  $(1, 300) = 1$ , i.e. Split Extensions exist. So,  $X : C_{300} \rightarrow \text{Aut}(C_2) \cong I$  and  $(300, 1) = 1$ . Then  $b^{-1}ab = a^{-1}$  which is an abelian case. Therefore, this case is kept out.

##### 3, 2-Sylow subgroup

As the group G is a permutation, then 3,2-Sylow subgroups. Consider  $Y : G \rightarrow S_3$  gives that  $Y(G) = A_3$  or  $S_3$  i) If  $Y(G) = A_3$  then  $o(M) = 200$  and hence  $M \cong C_{200}$  or  $M \cong C_{100} \times C_2$  or  $M = Z_{100}$ , here 2-Sylow subgroups of  $C_{200}$  and  $C_{100} \times C_2$  are characteristics which the first two cases exclude. ii) If  $Y(G) = S_3$ , then  $o(M) = 100$ . If  $M \cong C_{200}$ , then  $X : S_3, \text{Aut}(C_{200}) = C_4 = C_2 \times C_2$ . Hence  $X(S_3) = (I)$  or  $C_2$  So that  $\{I\} \rightarrow M = C_2 \times C_2 \rightarrow G_1 \rightarrow S_3 \rightarrow \{I\}$  Then  $S_3$  is  $a^2 = b^2 = e$  and  $b^{-1}ab = a^{-1}$ . If  $(Z(S_3) = \{I\})$  then  $\text{Ker}(X) = S_3$  and so let  $N = \{t, u\}$  and  $G = \{c, d\}$ , where  $c, d$  are mapped respectively to  $a, b$ . Now  $c^2 = e, a = c^{-1}ac, a = b^{-1}ab, b = c^{-1}bc, a = d^{-1}ad, d^{-1}bd = b$  and  $d = a^i b, d^{-1}cd = c^{-1}a^i b^k, a^i, b^k \in C_2 \times C_2 \in \text{Aut}(C_{100}) = D_{100}$ . Now,  $S \in G$  such that  $s^2 = b^j$  with  $j=0,1$  If  $s^2 = b$  the  $a^2 = s^3 = c^2, ac = ca, as = sa, s^{-1}cs = c^{-1}ab, s^2cs^2 = s^{-1}(s^{-1}cs) = ca^{2i}b^{2k} = c$  and  $s^2 \in \{a, b\}$  commutes with  $c$ . Then  $a^{2i}b^{2k} = I$  with  $2i = 0 \pmod{5}$  and  $2k = 0 \pmod{2}$ , where  $i = 0, k = 0, 1$ . If  $k = 0$ , then  $s^{-1}cs = c^{-1}$  and if  $k=1$  then  $s^{-1}cs = c^{-1}ba$ . As  $C_3$  is generated by a central element, then we put  $f = c^2d^2, f^7 = e, d^2 = f^3$  and  $d^{-1}cd = c^{-1}$  and so  $\{f, d\}$  generates  $Z_{100}$  but  $C_3$  and  $Z_{100}$  are normal subgroups and  $C_3 \cap D_{100} = \{e\}$ . Therefore,  $G = C_3 \times Z_{100}$ , is a Non- Abelian Group of 600.

##### 5, 2- Sylow subgroup

Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup. Then  $M(S_2) = 4$  and  $M(S_2) = C_4$  or  $C_2 \times C_2$  or  $Z_2$  but  $M(S_2) = Z_2$ . This is a subgroup of  $o(2)$ . There are i.  $M(S_2) = C_4$  ii.  $M(S_2) = C_2 \times C_{300}$  i. e. Burnside's theorem exists. Therefore, this case is kept out.

##### 15, 2-Sylow subgroups

(I) Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup. (II) Consider  $M \cong G_{15}$ . Then  $\{I\} \rightarrow Z_{15} \rightarrow G \rightarrow K \rightarrow \{I\}$  and  $K = Z_2$ . But  $(2,15) = 1$ , i.e. Split Extensions exist.

(II.1) If  $K = Z_2$  then  $X : Z_2 \rightarrow \text{Aut}(Z_{15}) \cong Z_3$  and  $X(Z_2) = \{I\}$ .

(II.1.1) If  $X(Z_2) = \{I\}$ , then  $G = Z_{40} \times Z_{15}$ , is a Non- Abelian Group of 600.

##### 25, 2-Sylow subgroups

(I) Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup.

(II) Consider  $M \cong G_{25}$ , then  $\{I\} \rightarrow Z_{25} \rightarrow G \rightarrow K \rightarrow \{I\}$  and  $K = Z_2$ . But  $(2,25) = 1$ , i.e. Split Extensions exist.

(II.1). If  $K = Z_2$  then  $X : Z_2 \rightarrow \text{Aut}(Z_{25}) \cong Z_3$  and  $X(Z_2) = \{I\}$ .

(II.1.1) If  $X(Z_2) = \{I\}$ , then  $G = Z_{24} \times Z_{25}$ , is a Non- Abelian Group of 600.

##### 75, 2-Sylow subgroups

(I) Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup.

(II) Consider  $M \cong G_{75}$ , then  $\{I\} \rightarrow Z_{75} \rightarrow G \rightarrow K \rightarrow \{I\}$ , Where  $K = Z_2$ . But  $(2,75) = 1$ , i.e. Split Extensions exist.

(II.1) If  $K = Z_2$  then  $X : Z_2 \rightarrow \text{Aut}(Z_{75}) \cong Z_3$  and  $X(Z_2) = \{I\}$ .

(II.1.1) If  $X(Z_2) = \{I\}$ , then  $G = Z_8 \times Z_{25}$ , is a Non- Abelian Group of 600.

#### 3-Sylow Subgroup

The group G of order 600 is  $x \equiv 1 \pmod{3}$ ; Where  $x = 1, 4, 10, 25, 100$ .

### 1, 3-Sylow Subgroups

For  $C_3$  is a normal in  $G$ . Then,  $\{I\} \rightarrow Z_3 \rightarrow G \rightarrow K \rightarrow \{I\}$  and  $o(K) = 10$ . But  $(3, 200) = 1$ , i.e., Split Extensions exist. And so  $X : K \rightarrow \text{Aut}(C_3) \cong C_{200}$

(I) Let  $K = C_{200} \cong C_{100} \times C_2$  then  $\text{Ker}X$  contains  $C_2$  and it commutes with  $C_2, C_3, C_{100}, \dots$ . Then,  $G = C_2 \times M$ , where  $M$  is a Non-Abelian Group of 300

(II) Let  $K = Z_{100}$  and  $Z_{100}$  is no quotient group of order 100, so  $X(D_{200})$  is order 1. If  $X(Z_{200})$  be order 1 then  $G = Z_{100} \times C_3$ , which is exist.

### 4, 3-Sylow subgroups

Let us consider 3-Sylow subgroup  $M(S_3) = 150$ . Now the order of  $M(S_3) \cong 150$  and so,  $M(S_3) \cong 150$  or  $C_{75} \times C_2$  or  $Z_{75}$  or  $\Delta_{75}$ . So  $M(S_3) \cong \Delta_{75}$ , this is a subgroup of order 75. There are 1)  $M(\Delta_{75}) \cong 150$  2)  $M(\Delta_{75}) = C_{75} \times C_2$  If  $M(\Delta_{75}) = C_{150}$  then  $a^2 = a^3 = e, d^{-1}cd = c^{-1}, d^{-1}ad = a^{-1}, d^2 = a^3, ac = ca$  and  $b^3 = e, d^{-1}bd = b^{-1}$  such that  $d^2 = b^{375}$ . Therefore,  $G \cong \Delta_{375}$ , is a Non-Abelian Group of 600.

### 10, 3-Sylow subgroups

Let us consider 3-Sylow subgroup  $M(S_3) = 60$ . Now the order of  $M(S_3) \cong 60$  and so,  $M(S_3) \cong 60$  or  $C_{30} \times C_2$  or  $Z_{30}$  or  $\Delta_{30}$ . So,  $M(S_3) \cong \Delta_{30}$ , which is a subgroup of order 30. There are 1)  $M(\Delta_{30}) \cong 60$  2)  $M(\Delta_{30}) = C_{30} \times C_2$  If  $M(\Delta_{30}) = C_{60}$  then  $a^2 = a^3 = e, d^{-1}cd = c^{-1}, d^{-1}ad = a^{-1}, d^2 = a^3, ac = ca$  and  $b^3 = e, d^{-1}bd = b^{-1}$  such that  $d^2 = b^{150}$ . Therefore,  $G \cong \Delta_{150}$  is a Non-Abelian Group of 600.

### 25, 3-Sylow subgroups

(I) Let us consider 2-sylow subgroup  $M(S_2) = 3$  and it is the invariant subgroup.

(I) Consider  $M \cong G_{25}$ , then  $\{I\} \rightarrow Z_{25} \rightarrow G \rightarrow K \rightarrow \{I\}$ , Where  $K = Z_3$ . But  $(3, 25) = 1$ , i.e. Split Extensions exist.

(II.1). If  $K = Z_3$  then  $X : Z_3 \rightarrow \text{Aut}(Z_{25}) \cong Z_4$  and  $X(Z_3) = \{I\}$ .

(II.2) If  $X(G_3) = \{I\}$ , then  $G = Z_3 \times Z_{25}$ , is a Non-Abelian Group of 600.

### 100, 3-Sylow subgroups

(I) Let us consider 2-sylow subgroup  $M(S_2) = 3$  and it is the invariant subgroup.

(II) Consider  $M \cong G_{100}$  then  $\{I\} \rightarrow Z_{100} \rightarrow G \rightarrow K \rightarrow \{I\}$ , Where  $K = Z_3$ . But  $(3, 100) = 1$ , i.e. Split Extensions exist.

(II.1). If  $K = Z_3$  then  $X : Z_3 \rightarrow \text{Aut}(Z_{100}) \cong Z_4$  and  $Y(Z_3) = \{e\}$ .

(II.2) If  $X(Z_3) = \{I\}$ , then  $G = Z_3 \times Z_{100}$ , is a Non-Abelian Group of 600.

### 5-Sylow Subgroups

The group  $G$  of order 600 is  $x \equiv 1$  (modulo 5); where  $x = 1, 6$ .

### 1, 5-Sylow Subgroups

For  $C_5$  is a normal in  $G$ . Then,  $\{I\} \rightarrow C_5 \rightarrow G \rightarrow H \rightarrow \{I\}$  and  $o(H) = 120$ . But  $(5, 120) \neq 1$ , i.e. ES exist and  $X : K \rightarrow \text{Aut}(C_5) \cong C_4$ .

I. Let  $H = C_{120} \cong C_{60} \times C_2$  then  $\text{Ker}X$  contains  $C_2$  and it does not commute with  $C_{60}, C_2$ . There is no Non-Abelian Group of 600.

II. Let  $H = A_4$  and  $A_4$  has no quotient group of order 2 or 4, so  $X(A_4) = \{e\}$  and  $G \cong C_{60} \times A_4$ . There is no Non-Abelian Group of 600.

(III) Let  $H = Z_{60}$  and  $Z_{60}$  be order 5, so  $X(Z_{60})$  be order 10 or 1.

(III.1) If  $X(Z_{60})$  be order 1, then  $G \cong C_{60} \times Z_{60}$ , which is no Non-Abelian Group of 600.

(III.2) If  $X(Z_{60})$  be order 10, then  $\text{Ker}X = C_{60}$  and  $a^{60} = c^{70} = d^2, d^{-1}ad = a^{-1}, c^{-1}ac = a^{-1}, d^{-1}cd = c^{-1}$ . Therefore  $G = Z_{300}$  is a Non-Abelian Group of 600.

### 6, 5-Sylow Subgroups

For  $C_5$  is a normal in  $G$ . Then,  $\{I\} \rightarrow C_5 \rightarrow G \rightarrow K \rightarrow \{I\}$  and  $K$  is of order 100. But  $(5, 100) \neq 1$ , i.e. Split Extensions exist and  $X : K \rightarrow \text{Aut}(C_5) \cong C_4$ . I. Let  $K = C_{100} \cong C_{50} \times C_2$  then  $\text{Ker}X$  contains  $C_2$  and it does not commute with  $C_{50}, C_2$ . There is no Non-Abelian Group of 600.

II. Let  $H = A_4$  and  $A_4$  has no quotient group of order 2 or 4, so  $X(A_4) = \{I\}$  and  $G \cong C_{50} \times A_4$ . There is no Non-Abelian Group of 600.

(III) Let  $K = Z_{50}$  and  $Z_{50}$  be order 6, so  $X(Z_{50})$  is order 12 or 1.

(III.1) If  $X(Z_{50})$  be order 1 then  $G \cong C_{50} \times Z_{50}$ , which is no Non-Abelian Group of 600.

(III.2) If  $X(Z_{50})$  be order 12, then  $\text{Ker}X = C_{50}$  and  $a^{50} = c^{60} = d^2, d^{-1}ad = a^{-1}, c^{-1}ac = a^{-1}, d^{-1}cd = c^{-1}$ . Therefore  $G = Z_{250}$  is a Non-Abelian Group of 600.

### The Result of Non-Abelian Group

The list of 600 Non-Abelian groups is i)  $G = C_3 \times Z_{100}$  ii)  $G = Z_{40} \times Z_{15}$  iii)  $G = Z_{24} \times Z_{25}$  iv)  $G = Z_8 \times Z_{25}$  v)  $G = Z_{100} \times C_3$  vi)  $G = Z_3 \times Z_{25}$  vii)  $G = Z_3 \times Z_{100}$  viii)  $G = Z_{300}$  ix)  $G = Z_{250}$  x)  $G = C_3 \times Z_{100}$ .

### 3.2 Classification of All Abelian and Non-Abelian Groups of Order 650

[13],[14]

#### Abelian Group

The list of Abelian Group of 650 is i.  $G \cong C_{13} \times C_5 \times C_5 \times C_2$  ii.  $G \cong C_{13} \times C_{25} \times C_2$

#### Non-Abelian Group

We have,

$$650 = 2 \cdot 5^2 \cdot 13.$$

### 2-Sylow Subgroups

The group  $G$  of order 650 satisfies

$$x \equiv 1 \pmod{2}.$$

The possible values of  $x$  are

$$x \in \{1, 5, 25, 65, 325\}.$$

#### 1, 2-Sylow subgroup

Here,  $o(M) = 2$  then  $M \cong C_2$ . If  $M \cong C_2$ , then  $\{I\} \rightarrow M \rightarrow G \rightarrow C_{325} \rightarrow \{I\}$ . But,  $(1, 300) = 1$ , i.e. Split Extensions exist. So,  $X : C_{325} \rightarrow \text{Aut}(C_2) \cong I$  and  $(325, 1) = 1$ . Then  $b^{-1}ab = a^{-1}$  which is an abelian case. Therefore, this case is kept out.

#### 5, 2-Sylow subgroup

Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup. Then  $M(S_2) = 4$  and  $M(S_2) = C_4$  or  $C_2 \times C_2$  or  $Z_2$  but  $M(S_2) = Z_2$ . This is a subgroup of  $o(2)$ . There are i.  $M(S_2) = C_4$  ii.  $M(S_2) = C_2 \times C_{325}$  i. e. Burnside's theorem exists. Therefore, this case is kept out.

#### 25, 2-Sylow subgroups

(I) Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup.

(II) Consider  $M \cong G_{25}$ . Then  $\{I\} \rightarrow Z_{25} \rightarrow G \rightarrow K \rightarrow \{I\}$  and  $K = Z_2$ . But  $(2, 25) = 1$ , i.e. Split Extensions exist.

(II.1). If  $K = Z_2$  then  $X : Z_2 \rightarrow \text{Aut}(Z_{25}) \cong Z_3$  and  $X(Z_2) = \{I\}$ .

(II.1.1) If  $X(G_2) = \{I\}$ , then  $G = Z_{24} \times Z_{25}$ , is a Non-Abelian Group of 650.

**65, 2-Sylow subgroups** (I) Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup.

(II) Consider  $M \cong G_{65}$ . Then  $\{I\} \rightarrow Z_{65} \rightarrow G \rightarrow K \rightarrow \{I\}$  and  $K = Z_2$ . But  $(2, 65) = 1$ , i.e. Split Extensions exist.

(II.1). If  $K = Z_2$  then  $X : Z_2 \rightarrow \text{Aut}(Z_{65}) \cong Z_3$  and  $X(Z_2) = \{I\}$ .

(II.1.1). If  $X(G_2) = \{I\}$ , then  $G = Z_{10} \times Z_{65}$ , is a Non-Abelian Group of 650.

#### 325, 2-Sylow subgroups

(I) Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup.

(II) Consider  $M \cong G_{325}$ . Then  $\{I\} \rightarrow Z_{325} \rightarrow G \rightarrow K \rightarrow \{I\}$  and  $K = Z_2$ . But  $(2, 325) = 1$ , i.e. Split Extensions exist.

(II.1). If  $K = Z_2$  then  $X : Z_2 \rightarrow \text{Aut}(Z_{325}) \cong Z_3$  and  $X(Z_2) = \{I\}$ .

(II.1.1). If  $X(G_2) = \{I\}$ , then  $G = Z_2 \times Z_{325}$ , is a Non-Abelian Group of 650.

#### 5-Sylow Subgroups

The group  $G$  of order 650 is  $x \equiv 1 \pmod{5}$ ; Where  $x = 1, 26$ .

**1, 5-Sylow Subgroups** For  $C_5$  is a normal in  $G$ . Then,  $\{I\} \rightarrow C_5 \rightarrow G \rightarrow H \rightarrow \{I\}$  and  $o(H) = 130$ . But  $(5, 130) \neq 1$ , i.e. ES exist and  $X : H \rightarrow \text{Aut}(C_5) \cong C_4$ .

(I) Let  $H = C_{130} \cong C_{65} \times C_2$  then  $\text{Ker} X$  contains  $C_2$  and it does not commute with  $C_{65}$ ,  $C_2$ . There is no Non-Abelian Group of 650.

(II) Let  $H = A_4$  and  $A_4$  has no quotient group of order 2 or 4, so  $X(A_4) = \{e\}$  and  $G \cong C_{65} \times A_4$ .

There is no Non-Abelian Group of 650.

(III) Let  $H = Z_{65}$  and  $Z_{65}$  be order 5, so  $X(Z_{65})$  be order 10 or 1.

(III.1) If  $X(Z_{65})$  be order 1 then  $G \cong C_{65} \times Z_{65}$  which is no Non-Abelian Group of 650.

(III.2) If  $X(Z_{65})$  be order 10, then  $\text{Ker}X = C_{60}$  and  $a^{60} = c^{70} = d^2$ ,  $d^{-1}ad = a^{-1}$ ,  $c^{-1}ac = a^{-1}$ ,  $d^{-1}cd = c^{-1}$ . Therefore  $G = Z_{325}$  is a Non-Abelian Group of 650.

### 26, 5-Sylow Subgroups

For  $C_5$  is a normal in  $G$ . Then,  $\{e\} \rightarrow C_{26} \rightarrow G \rightarrow K \rightarrow \{e\}$  and  $o(K) = 25$ . But  $(26,25) = 1$ , i.e. Split Extensions exist. Now,  $X : K \rightarrow \text{Aut}(C_{26}) \cong C_{27}$ . Then  $b^{-1}ab = a^{-1}$  which is an abelian group. Therefore, this case is kept out.

### 13-Sylow Subgroups

The group  $G$  of order 650 is  $x \equiv 1$  (modulo 13); Where  $x = 1$ .

#### 1, 13-Sylow Subgroups

For  $C_5$  is a normal in  $G$ . Then,  $\{e\} \rightarrow C_{13} \rightarrow G \rightarrow K \rightarrow \{e\}$  and  $o(K) = 50$ . But  $(13,50) \neq 1$ , i.e. ES exist.

Now  $X : K \rightarrow \text{Aut}(C_{13}) \cong C_{14}$ . (I) Let  $K = C_{50} \cong C_{25} \times C_2$  then  $\text{Ker}X$  contains  $C_2$  and it does not commute with  $C_{25}$ ,  $C_2$ . There is no Non-Abelian Group of 700.

(II) Let  $K = A_{14}$  and  $A_{14}$  has no quotient group of order 2 or 4, so  $X(A_{14}) = \{e\}$  and  $G \cong C_{50} \times A_{14}$ , This is no Non-Abelian Group of 650.

(III) Let  $K = Z_{50}$  and  $Z_{50}$  be order 50, so  $X(Z_{50})$  is order 2 or 1.

(III.1) If  $X(Z_{50})$  be order 1 then  $G \cong C_{50} \times Z_{50}$  which is no NAG of 650

(III.2) If  $X(Z_{50})$  be order 2, then  $\text{Ker}X = C_{50}$  and  $a^{50} = c^{70} = d^2$ ,  $d^{-1}ad = a^{-1}$ ,  $c^{-1}ac = a^{-1}$ ,  $d^{-1}cd = c^{-1}$ . Therefore  $G = Z_{250}$  is a Non-Abelian Group of 650.

### The Result of Non – Abelian Group

The list of Non-Abelian Group of 650 is i)  $G = Z_{26} \times Z_{25}$  ii)  $G = Z_{10} \times Z_{65}$  iii)  $G = Z_2 \times Z_{325}$  iv)  $G = Z_{360}$  v)  $G = Z_{250}$

## 3.3 Classification of All Abelian and Non-Abelian Groups of Order 700

[15], [16],[17]

### Abelian Group

The list of Abelian Group of 700 is i.  $G \cong C_7 \times C_5 \times C_5 \times C_2 \times C_2$  ii.  $G \cong C_7 \times C_{25} \times C_4$  iii.  $G \cong C_7 \times C_5 \times C_5 \times C_4$  iv.  $G \cong C_7 \times C_{25} \times C_2 \times C_2$

### Non – Abelian Group

We have,

$$700 = 2^2 \cdot 5^2 \cdot 7.$$

### 2- Sylow Subgroups

The group  $G$  Of order 700 is  $x \equiv 1$  (modulo 2), where  $x = 1, 5, 7, 25, 35, 175$ .

#### 1, 2-Sylow subgroup

Here,  $o(M) = 2$  then  $M \cong C_2$ . If  $M \cong C_2$ , then  $\{I\} \rightarrow M \rightarrow G \rightarrow C_{350} \rightarrow \{I\}$ . But,  $(1, 350) = 1$ , i.e. Split Extensions exist. So,  $X : C_{350} \rightarrow \text{Aut}(C_2) \cong I$  and  $(350, 1) = 1$ . Then  $b^{-1}ab = a^{-1}$  which is an abelian case. Therefore, this case is kept out.

#### 5, 2- Sylow subgroup

Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup. Then  $M(S_2) = 140$  and  $M(S_2) = C_{140}$  or  $C_{70} \times C_2$  or  $Z_2$  but  $M(S_2) = Z_2$ . This is a subgroup of  $o(2)$ . There are i.  $M(S_2) = C_{140}$  ii.  $M(S_2) = C_2 \times C_{350}$  i. e. Burnside's theorem exists. Therefore, this case is kept out.

#### 7, 2- Sylow subgroup

Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup. Then  $M(S_2) = 100$  and  $M(S_2) = C_{100}$  or  $C_{50} \times C_2$  or  $Z_2$  but  $M(S_2) = Z_2$ . This is a subgroup of  $o(2)$ . There are i.  $M(S_2) = C_{100}$  ii.  $M(S_2) = C_2 \times C_{350}$  i. e. Burnside's theorem exists. Therefore, this case is kept out.

### 25, 2-Sylow subgroups

(I) Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup.

(II) Consider  $M \cong G_{25}$ . Then,  $\{I\} \rightarrow Z_{25} \rightarrow G \rightarrow K \rightarrow \{I\}$  and  $K = Z_2$ . But  $(2,25) = 1$ , i.e. Split Extensions exist.

(II.1). If  $K = Z_2$  then  $X : Z_2 \rightarrow \text{Aut}(Z_{25}) \cong Z_3$  and  $X(Z_2) = \{I\}$ .

(II.1.1) If  $X(Z_2) = \{I\}$ , then  $G=Z_{28} \times Z_{25}$ , is a Non-Abelian Group of 700.

### 35, 2-Sylow subgroups

(I) Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup.

(II) Consider  $M \cong G_{35}$ . Then  $\{I\} \rightarrow D_{35} \rightarrow G \rightarrow K \rightarrow \{I\}$  and  $K = Z_2$ . But  $(2,35) = 1$ , i.e. Split Extensions exist.

(II.1). If  $K = Z_2$  then  $X : D_2 \rightarrow \text{Aut}(Z_{35}) \cong Z_3$  and  $X(Z_2) = \{I\}$ .

(II.1.1). If  $X(Z_2) = \{e\}$ , then  $G=D_{20} \times D_{35}$ , is a Non- Abelian Group of 700.

### 175, 2-Sylow subgroups

(I) Let us consider 2-sylow subgroup  $M(S_2) = 2$  and it is the invariant subgroup.

(II) Consider  $M \cong G_{175}$ . Then  $\{I\} \rightarrow Z_{175} \rightarrow G \rightarrow K \rightarrow \{I\}$  and  $K = Z_2$ . But  $(2,175) = 1$ , i.e. Split Extensions exist.

(II.1). If  $K = Z_2$  then  $K : Z_2 \rightarrow \text{Aut}(Z_{175}) \cong Z_3$  and  $Y(Z_2) = \{I\}$ .

(II.1.1).If  $X(Z_2) = \{I\}$ , then  $G=Z_4 \times Z_{125}$ , is a Non-Abelian Group of 700.

### 5-Sylow Subgroups

The group  $G$  of order 700 is  $x \equiv 1$  (modulo 5); where  $x = 1$ .

#### 1, 5-Sylow Subgroups

For  $C_5$  is a normal in  $G$ . Then,  $\{I\} \rightarrow C_5 \rightarrow G \rightarrow H \rightarrow \{I\}$  and  $o(H) = 140$ . But  $(5,140) \neq 1$ , i.e. Split Extensions exist and  $X : H \rightarrow \text{Aut}(C_5) \cong C_4$ .

(I) Let  $H = C_{140} \cong C_{70} \times C_2$  then  $\text{Ker}X$  contains  $C_2$  and it does not commute with  $C_{70}$ ,  $C_2$ . There is no Non-Abelian Group of 700.

(II) Let  $H = A_4$  and  $A_4$  has no quotient group of order 2 or 4, so  $X(A_4) = \{I\}$  and  $G \cong C_{70} \times A_4$ . There is no Non-Abelian Group of 700.

(III) Let  $H = Z_{70}$  and  $Z_{70}$  be order 5, so  $X(Z_{70})$  be order 10 or 1.

(III.1) If  $X(Z_{70})$  be order 1, then  $G \cong C_{70} \times Z_{70}$  which is no Non-Abelian Group of 700.

(III.2) If  $X(Z_{70})$  be order 10, then  $\text{Ker}X = C_{70}$  and  $a^{70} = c^{80} = d^2$ ,  $d^{-1}ad = a^{-1}$ ,  $c^{-1}ac = a^{-1}$ ,  $d^{-1}cd = c^{-1}$ . Therefore  $G = Z_{490}$  is a Non-Abelian Group of 700.

### 7-Sylow Subgroups

The group  $G$  of order 700 is  $X \equiv 1$  (modulo 7); where  $X = 1$ .

#### 1, 7-Sylow Subgroups

For  $C_7$  is a normal in  $G$ . Then,  $\{e\} \rightarrow D_7 \rightarrow G \rightarrow K \rightarrow \{e\}$  and  $o(K) = 100$ . But  $(7,100) = 1$ , i.e. Split Extensions exist. Then,  $X : C_{100} \rightarrow \text{Aut}(C_2) \cong I$  and  $(100, 1) = 1$ . Then  $b^{-1}ab = a^{-1}$  which is an abelian group. Therefore, this case is kept out.

### The Result of Non - Abelian Group

The list of Non-Abelian Group of 700 is i)  $G=Z_{28} \times Z_{25}$  ii)  $G=Z_{20} \times Z_{35}$  iii)  $G=Z_4 \times Z_{125}$  iv)  $G = Z_{350}$

## 4 Exploration of Theoretical and Applied Implications

### 4.1 Theoretical Implications

This study provides a strong theoretical foundation in finite group theory, specifically leveraging Sylow's theorems and prime factorizations to classify subgroups of large-order groups. The implications include:

- i. **Deeper Understanding of Group Structures.** By analyzing groups of orders 600, 650, and 700, the study demonstrates how subgroup structures depend on distinct prime decompositions. These examples offer valuable insights into subgroup lattices, conjugacy relations, and normality conditions in both abelian and non-abelian groups.
- ii. **Advancement in Subgroup Classification.** The findings provide a general framework for classifying subgroups of higher-order groups using Sylow's theorems, composition series, and split extensions. This framework can be extended to groups of order  $p^n q^m$ , non-solvable groups, and certain classes of simple groups.
- iii. **Foundations for Extension and Cohomology Theory.** Several instances of split extensions and automorphisms appear throughout the analysis, indicating promising directions for future research using cohomology, Schur multipliers, and group extension theory to obtain deeper structural insight.

- iv. **Refinement of Group Isomorphism Detection.** The evaluation of both abelian and non-abelian groups of the same order emphasizes the need for precise classification methods. Tools such as automorphism groups, inner mappings, and subgroup actions play an essential role in distinguishing non-isomorphic groups.

## 4.2 Applied Implications

Although rooted in pure mathematics, this study has broader applications across multiple scientific and technological domains. Several significant areas of impact are summarized below.

- i. **Cryptography and Information Security.** Non-abelian groups and their subgroup structures play a crucial role in the design of modern cryptographic algorithms. Understanding subgroup behavior contributes to the development of secure key-exchange protocols, public key systems, and group based cryptographic frameworks.
- ii. **Symmetry in Physics and Chemistry.** Group theory forms the mathematical foundation of symmetry operations in physical and chemical systems. The classification of subgroups, particularly Sylow subgroups, is essential in quantum mechanics, crystallography, and particle physics, where symmetry breaking and conservation laws are central to theoretical models.
- iii. **Coding Theory and Communications.** The construction of many error-correcting codes relies on finite groups and their subgroup structures. The subgroup analysis presented in this study supports the development of group based coding schemes, cyclic redundancy checks, and algebraic communication protocols.
- iv. **Combinatorics and Computational Group Theory.** Algorithmic enumeration of subgroups has applications in graph theory, network analysis, and combinatorial design. Computational tools such as GAP can implement the methods discussed in this work to develop efficient algorithms and software for algebraic computation.
- v. **Machine Learning and Data Symmetry.** Recent advances in machine learning increasingly incorporate group-theoretic principles to exploit symmetries in data and neural network architectures. Subgroup classification offers a rigorous mathematical basis for symmetry-aware models in artificial intelligence and geometric deep learning.

## 5 Limitations and Future Work

This study provides a comprehensive analysis of the subgroup structures of finite groups of orders 600, 650, and 700 using Sylow's theorems, Lagrange's Theorem, and related group theoretic principles. However, several limitations must be acknowledged, which also suggest directions for future research.

- i. **Restriction to Specific Orders.** The present analysis focuses on groups of three specific orders 600, 650, and 700 chosen for their rich prime factorizations. While these examples are illustrative, generalizations to arbitrary higher-order groups or entire families of groups (e.g., groups of order  $p^n q^m$ ) are not considered in detail and may reveal additional structural insights.
- ii. **Dependence on Prime Factorization.** The classification of abelian and non-abelian subgroups relies heavily on prime factorizations and properties of Sylow  $p$ -subgroups. For composite orders with more intricate prime-power structures or repeated primes of high multiplicity, deeper analysis potentially involving cohomology, representation theory, or extension theory—may be required.
- iii. **No Explicit Construction of Groups.** Although subgroup existence and classification are theoretically established, this study does not provide explicit constructions of all non-abelian groups discussed, nor does it distinguish non-isomorphic non-abelian groups that share the same order and Sylow configuration.

- iv. **Lack of Algorithmic or Computational Verification.** The subgroup classifications were derived analytically without computational support. Incorporating algebraic software such as GAP or MAGMA could enable algorithmic verification, automate subgroup enumeration, and test conjectures regarding group isomorphism classes.
- v. **Limited Use of Homomorphism and Extension Theory.** While automorphisms and split extensions are referenced in certain cases, the study does not apply homological or extension-theoretic methods in a systematic way. Future work could employ exact sequences, Schur multipliers, and group cohomology to construct or classify non-abelian groups of specific orders.
- vi. **No Analysis of Group Actions or Applications.** The discussion remains purely theoretical, without exploring possible group actions, symmetry considerations, or applications in physics, coding theory, or cryptography. Extending the analysis into applied contexts would broaden the interdisciplinary impact of the results.
- vii. **Conjugacy and Normality Conditions.** While the existence and number of Sylow subgroups are discussed, a detailed analysis of their conjugacy classes, intersection properties, and normalizer structures is not provided. These aspects are important for understanding the internal geometry and symmetry of finite groups.

## 6 Conclusion

The study of Sylow's theorems and Lagrange's theorem plays a crucial role in understanding the structure of groups in group theory. While Lagrange's theorem is effective for identifying subgroups of groups with relatively small or specific orders, it becomes less practical for analyzing groups of higher orders. In such cases, Sylow's theorems provide a more powerful and systematic approach for determining the existence and structure of subgroups. The distinction between abelian and non-abelian groups in higher order cases further enriches our understanding of group properties and their subgroup compositions. These insights are foundational for deeper exploration and applications within abstract algebra and related mathematical fields.

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