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Finding different score values methods applied to complex spherical fuzzy soft set using aggregation operators

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Abstract The complex spherical fuzzy soft set (CSFSS) is a new extension of the complex FSS and the complex Pythagorean FSS (CPFSS). The CSFS decision matrix under aggregated operation is now available. TOPSIS methods provide a powerful approach for multi-criteria group decision making (MCGDM), which is a variety of extensions of FSSs. We present a scoring function that is based on the TOPSIS aggregation approach. They are given in order to identify the most advantageous choice in the area. Additionally, a numerical example is provided to illustrate the success of the suggested technique using flowchart based MCGDM. A variety of values for parameters were used to analyze the results. A wide range of parameter values have been utilized to assess the results. A comparison analysis has also been carried out to show how the suggested technique outperforms existing methods.

1 Introduction

Finding the best optional choices is indicated by the decision making (DM) problems. The discussion of multiple criterion decision making (MCDM) techniques was presented by Hwang et al. [1]. The matrix form of MCDM problem as:

$$D_{n\times m} = \begin{array}{c} \\ L_1 \\ L_2 \\ \vdots \\ L_n \end{array} \begin{array}{c} \begin{array}{cccc} M_1 & M_2 & \dots & M_m \end{array} \\ \begin{pmatrix} \ell_{11} & \ell_{12} & \dots & \ell_{1m} \\ \ell_{21} & \ell_{22} & \dots & \ell_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \ell_{n1} & \ell_{n2} & \dots & \ell_{nm} \end{pmatrix} \end{array}.$$

Here, $L_1, L_2, ..., L_n$ are possible choices, or options that decision makers, $M_1, M_2, ..., M_m$ are referred to as criteria, or factors that are used to calculate alternative effects; and ℓ_{ij} represents an estimate of L_i with respect to M_j. Numerous ideas have been put out to explain uncertainty, including fuzzy sets (FS) [2], which have membership grades (MG) ranging from 0 to 1. An intuitionistic FS (IFS) for $\mu, \nu \in [0, 1]$ was generated by Atanassov [3]. Each component has two MGs, positive μ and negative ν and $0 \leq \mu + \nu \leq 1$. The Pythagorean FS (PFS) logic deals by Yager [4] and is distinguished by its MG and non-membership grade (NMG) with $\mu + \nu \geq 1$ to $\mu^2 + \nu^2 \leq 1$. The use of IFSs and PFSs in many different kinds of areas is the subject of several research studies. They still have a limited capacity for data communication. As a result, the experts continued to struggle to interpret the data in these sets and the associated data. The idea of complex IFS with DOMBI prioritized AOs and its use for reliable green supplier selection have been covered by Wang et al. [5]. The MAIRCA methodology, which is based on interval-valued IFS distance measures by Mishra et al. [6] to investigate a method of assessing sustainable wastewater treatment technology. Cuong et al. [7] deals that the three basic concepts of the picture FS are positive MG (μ), neutral MG (α) and negative MG (ν). Also, it includes more benefits than PFS and IFS. Since $\mu, \alpha, \nu \in [0, 1]$, which is an expand of the IFS and $0 \leq \mu + \alpha + \nu \leq 1$. Following to the picture FS definition, expert opinions like "yes," "abstain," "no," and "refusal" will be communicated. Additionally, it will reduce results from being retained completely and promote consistency between the evaluation data and the real-world decision environment. Palanikumar et al. discussed the concept of complex Pythagorean normal interval valued fuzzy set, neutrosophic set (NSS) and its extension approach [8, 9, 10].

Shahzaib et al. [11] defined the spherical FS (SFS) basd on MADM. The SFS implies that $0 \leq \mu^2 + \alpha^2 + \nu^2 \leq 1$ instead of $0 \leq \mu + \alpha + \nu \geq 1$. Hussain et al. [12] were the concept of an intelligent decision support system for SFS logic. Biswas et al. [13] proposed a DM framework with Einstein aggregation to compare the growth of SMEs in quality 4.0. Linguistic SFS is defined by Jin et al. [14] using MADM . Rafiq et al. [15] gave a presentation on SFSs and their uses in DM consists of $\mu^2 + \nu^2 \geq 1$. Senapati et al. [16] presented Fermatean FS (FFS) in 2019 with the specification that $0 \leq \mu^3 + \nu^3 \leq 1$. Rahman et al. [17] are studying a geometric aggregation operator (AO) on interval-valued PFS (IVPFS). Using Einstein AO, Rahman et al. [18] proposed their notion of IVPFS as a successful MAGDM method. Peng et al. [19] handled interval valued PFSs and AOs. The notion of generalized orthopair FS was initially proposed by Yager [20]. Both the MG and the NMG have power q in the q-rung orthogonal pair FS (q-ROFS), but their sum can never be more than one. Asif et al. [21] were proposed Hamacher AOs for PFS and their application in MADM. The researchers [22, 23, 24, 25, 26, 27, 28] indicates that these sets may be applied to solve DM problems and that it has more benefits than IFS and PFS. The generalized PFS with AO were established by Liu et al. [29]. The concepts of square root vague sets [30], q-rung vague sets [31], q-rung interval-valued NSS [32], and q-rung neutrosophic soft set method [33] were recently discussed by Palanikumar et al. When the sum of the positive, neutral and negative MG values exceeds 1, the Pythagorean IVFS characteristics of the AOs [34]. The extension of SFSs, in which the square sum of three grade values positive, neutral and negative MGdoes not exceed one has been investigated by Ashraf et al. [11]. Fatmaa et al. [35] applied the TOPSIS methods. Liu et al. [36] analyzed particular forms of q-rung picture FS in 2020. Fermatean FS (FFS) and IVFFS are part of the Extended DEA method, as defined by Edalatpanah et al. [37, 38].Fuzzy Einstein averaging operations for complex q-rung picture set were defined by Edalatpanah et al. [39]. These notions cannot be used to explain neutral cases, which are neither favorable nor unfavorable. Using MCDM techniques, Sahoo et al. [40] concentrated on bibliometric analysis of material selection. Recently, new algebraic structures discussed by [41, 42]. Palanikumar et al. deals that the notion of possibility Pythagorean cubic fuzzy soft sets [43], neutrsophic Fermatean fuzzy soft set [44] and trigonometric DM and its application [45].

Adeel et al. [46], Akram et al. [47], Boran et al. [48], Eraslan et al. [49], Peng et al. [50], Xu et al. [51], and Zhang et al. [52] have all studied TOPSIS and VIKOR for DM problems. Zulqarnain et al. [53] talked TOPSIS methods extended to interval valued intuitionistic fuzzy soft sets (IVIFSS) in 2021. Under IVIFSS, he additionally talked about a novel kind of correlation coefficient. The TOPSIS technique, which includes distances to both positive ideal solutions (PIS) and negative ideal solutions (NIS), generates a preference order evaluated under relative closeness. Palanikumar et al. [54] discussed the concept of spherical Fermatean interval valued fuzzy soft set based on MCGDM. Recently, Many researchers new concepts such as meromorphic functions [55], fuzzy hypersoft set [56] and ring [57].

The six sections of this study are mentioned below. PFS and SFS were discussed in Section 2. The complex spherical fuzzy soft set (CSFSS) and some proposed methods are covered in Section 3. Section 4 contains the real-world applications. To solve problems based on these three approaches, we will utilize ranking. The comparison analysis, effectiveness, and benefits of the proposed approaches are shown in Section 5. We might arrive at the conclusion in Section 6.

2 Preliminaries

There are several important definitions in this part that we should go over again for further study.

Definition 2.1. Let $\mathbb{U}$ be a universe, SFS X in $\mathbb{U}$ is $X = \{u, \vartheta_X(u), \varpi_X(u), \tau_X(u) | u \in \mathbb{U}\}$, where $\vartheta_X(u)$, $\varpi_X(u)$ $\tau_X(u)$ represents the degree of PMG, neutral MG and NMG of X respectively. The mapping $\vartheta_X, \varpi_X, \tau_X : \mathbb{U} \to [0, 1]$ and

$$0 \le (\vartheta_X(u))^2 + (\varpi_X(u))^2 + (\tau_X(u))^2 \le 1. \tag{2.1}$$

The degree of refusal is defined as

$$\pi_X(u) = \left[\sqrt{1 - (\vartheta_X(u))^2 - (\varpi_X(u))^2 - (\tau_X(u))^2}\right] \tag{2.2}$$

Since $X = \langle \vartheta_X, \varpi_X, \tau_X \rangle$ is called a spherical fuzzy number(SFN).

Definition 2.2. Let E represent the collection of parameters. The term (Ψ, X) or Ψ_X is called a spherical FSS on $\mathbb{U}$ if $X \sqsubseteq E$, $\Psi : X \to SF^{\mathbb{U}}$, where $SF^{\mathbb{U}}$ is denote the of all spherical fuzzy subsets of $\mathbb{U}$. That is

$$\Psi_X = \left\{ \left(e, \left\{\frac{u}{(\vartheta_{\Psi_X}(u), \varpi_{\Psi_X}(u), \tau_{\Psi_X}(u))}\right\}\right) : e \in X \text{ and } u \in \mathbb{U} \right\}. \tag{2.3}$$

Definition 2.3. Let $p_{ij} = \vartheta_{\Psi_X}(e_j)(u_i)$, $q_{ij} = \varpi_{\Psi_X}(e_j)(u_i)$ and $r_{ij} = \tau_{\Psi_X}(e_j)(u_i)$, $1 \le i \le m$ and $1 \le j \le n$. Then the matrix form as

$$\Psi_X = [(\ell_{ij}, \wp_{ij}, \partial_{ij})]_{m\times n} = \begin{bmatrix} (\ell_{11}, \wp_{11}, \partial_{11}) & (\ell_{12}, \wp_{12}, \partial_{12}) & \dots & (\ell_{1n}, \wp_{1n}, \partial_{1n}) \\ (\ell_{21}, \wp_{21}, \partial_{21}) & (\ell_{22}, \wp_{22}, \partial_{22}) & \dots & (\ell_{2n}, \wp_{2n}, \partial_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\ell_{m1}, \wp_{m1}, \partial_{m1}) & (\ell_{m2}, \wp_{m2}, \partial_{m2}) & \dots & (\ell_{mn}, \wp_{mn}, \partial_{mn}) \end{bmatrix}_{m\times n} \tag{2.4}$$

Here Ψ_X is said to be spherical fuzzy soft matrix (SFSM).

Definition 2.4. The score function for any SFN $X = (\vartheta_X, \varpi_X, \tau_X)$ is defined as $S(X) = \vartheta_X^2 - \varpi_X^2 - \tau_X^2$, where $-1 \le S(X) \le 1$.

Definition 2.5. The cardinal set of the SFS set Ψ_X is denoted by $c\Psi_X$ and is defined as

$$c\Psi_X = \left\{\frac{e}{\left(\vartheta^c_{\alpha_1}(e), \varpi^c_{\beta_1}(e), \tau^c_{\gamma_1}(e)\right)} : e \in E\right\} \tag{2.5}$$

where $\vartheta^c_{\alpha_1}$, $\varpi^c_{\beta_1}$ and $\tau^c_{\gamma_1} : E \to [0,1]$ respectively, where $\vartheta^c_{\alpha_1}(e) = \frac{|\alpha_1(e)|}{|\mathbb{U}|}$, $\varpi^c_{\beta_1}(e) = \frac{|\beta_1(e)|}{|\mathbb{U}|}$ and $\tau^c_{\gamma_1}(e) = \frac{|\gamma_1(e)|}{|\mathbb{U}|}$ where $|\alpha_1(e)|$, $|\beta_1(e)|$ and $|\gamma_1(e)|$ denote the scalar cardinalities of the SFS sets $\alpha_1(e)$, $\beta_1(e)$ and $\gamma_1(e)$respectively, and $|\mathbb{U}|$ means cardinality of $\mathbb{U}$. The collection of all cardinal sets of SFS sets of $\mathbb{U}$ is denoted by $cSF^{\mathbb{U}}$. If $X \subseteq E = \{e_i\}$, then $c\Psi_X \in cSF^{\mathbb{U}}$ is defined in the matrix form by $[(\ell_{1j}, \wp_{1j}, \partial_{1j})]_{1\times n} = [(\ell_{11}, \wp_{11}, \partial_{11}), (\ell_{12}, \wp_{12}, \partial_{12}), ..., (\ell_{1n}, \wp_{1n}, \partial_{1n})]$, where $(\ell_{1j}, \wp_{1j}, \partial_{1j}) = \mu^c_{\Psi_X}(e_j)$, and $\forall i = 1, 2, ..., m, j = 1, 2, ..., n$.

Definition 2.6. Let $\Psi_X \in SF^{\mathbb{U}}$ and $c\Psi_X \in cSF^{\mathbb{U}}$. The SFS set $SFS_{agg} : cSF^{\mathbb{U}} \times SF^{\mathbb{U}} \to SFS(\mathbb{U}, E)$ is defined as

$$SFS_{agg}(c\Psi_X, \Psi_X) = \left\{\frac{u}{\mu_{\Psi^*_X}(u)} : u \in \mathbb{U}\right\} = \left\{\frac{u}{\left(\vartheta_{\alpha_1^*}(u), \varpi_{\beta_1^*}(u), \tau_{\gamma_1^*}(u)\right)} : u \in \mathbb{U}\right\}, \tag{2.6}$$

which is called aggregate SFS of Ψ_X, where $\vartheta_{\alpha_1^*}(u) : \mathbb{U} \to [0,1]$ by
$\vartheta_{\alpha_1^*}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\vartheta^c_{\alpha_1}(e), \vartheta_{\alpha_1}(e)\right)(u)$,
$\varpi_{\beta_1^*}(u) : \mathbb{U} \to [0,1]$ by $\varpi_{\beta_1^*}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\varpi^c_{\beta_1}(e), \varpi_{\beta_1}(e)\right)(u)$ and
$\tau_{\gamma_1^*}(u) : \mathbb{U} \to [0,1]$ by $\tau_{\gamma_1^*}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\tau^c_{\gamma_1}(e), \tau_{\gamma_1}(e)\right)(u)$. The set $SFS_{agg}(c\Psi_X, \Psi_X)$ is expressed in matrix form as

$$[(\ell_{i1}, \wp_{i1}, \partial_{i1})]_{m\times 1} = \begin{bmatrix} (\ell_{11}, \wp_{11}, \partial_{11}) \\ (\ell_{21}, \wp_{21}, \partial_{21}) \\ \vdots \\ (\ell_{m1}, \wp_{m1}, \partial_{m1}) \end{bmatrix} \tag{2.7}$$

where $[(\ell_{i1}, \wp_{i1}, \partial_{i1})] = \mu_{\Psi^*_X}(u_i)$ and i various from 1 to m. The above matrix said to be SFS aggregate matrix of $SFS_{agg}(c\Psi_X, \Psi_X)$ over $\mathbb{U}$.

Definition 2.7. Let $X = (\vartheta_{ij}, \varpi_{ij}, \tau_{ij}) \in SFSM_{m\times n}$. Then (i) (weights are equal) choice matrix of SFSM X is

$$\Xi(X) = \left[\left(\frac{\sum_{j=1}^{n}(\vartheta_{ij})^2}{n}, \frac{\sum_{j=1}^{n}(\varpi_{ij})^2}{n}, \frac{\sum_{j=1}^{n}(\tau_{ij})^2}{n}\right)\right]_{m\times 1}, for every i. \tag{2.8}$$

(ii) Weighted choice matrix of SFSM X (for every i, where $w_j > 0$ are weights) is

$$\Xi_w(X) = \left[\left(\frac{\sum_{j=1}^{n} w_j(\vartheta_{ij})^2}{\sum w_j}, \frac{\sum_{j=1}^{n} w_j(\varpi_{ij})^2}{\sum w_j}, \frac{\sum_{j=1}^{n} w_j(\tau_{ij})^2}{\sum w_j}\right)\right]_{m\times 1} \tag{2.9}$$

3 MCGDM based on CSFS sets

Definition 3.1. The cardinal set of the CSFS set Ψ_X is defined as

$$c\Psi_X = \left\{\frac{e}{\left(\vartheta^c_{\alpha_1}(e)exp^{i2\pi\vartheta^c_{\alpha_2}(e)}, \varpi^c_{\beta_1}(e)exp^{i2\pi\varpi^c_{\beta_2}(e)}, \tau^c_{\gamma_1}(e)exp^{i2\pi\tau^c_{\gamma_2}(e)}\right)} : e \in E\right\} \tag{3.1}$$

where $\vartheta^c_{\alpha_1}$, $\varpi^c_{\beta_1}$ and $\tau^c_{\gamma_1} : E \to [0,1]$ and $\vartheta^c_{\alpha_2}$, $\varpi^c_{\beta_2}$ and $\tau^c_{\gamma_2} : E \to [0,1]$ respectively, where $\vartheta^c_{\alpha_1}(e) = \frac{|\alpha_1(e)|}{|\mathbb{U}|}$, $\varpi^c_{\beta_1}(e) = \frac{|\beta_1(e)|}{|\mathbb{U}|}$ and $\tau^c_{\gamma_1}(e) = \frac{|\gamma_1(e)|}{|\mathbb{U}|}$, $\vartheta^c_{\alpha_2}(e) = \frac{|\alpha_2(e)|}{|\mathbb{U}|}$, $\varpi^c_{\beta_2}(e) = \frac{|\beta_2(e)|}{|\mathbb{U}|}$ and $\tau^c_{\gamma_2}(e) = \frac{|\gamma_2(e)|}{|\mathbb{U}|}$ where $|\alpha_1(e)|$, $|\beta_1(e)|$, $|\gamma_1(e)|$, $|\alpha_2(e)|$, $|\beta_2(e)|$ and $|\gamma_2(e)|$ denote the scalar cardinalities of the CSFS sets $\alpha_1(e)$, $\beta_1(e)$, $\gamma_1(e)$, $\alpha_2(e)$, $\beta_2(e)$ and $\gamma_2(e)$ respectively. The collection of all cardinal sets of CSFS sets of $\mathbb{U}$ is denoted by $cCSF^{\mathbb{U}}$. If $X \subseteq E = \{e_k : k = 1, 2, ..., n\}$, then $c\Psi_X \in cCSF^{\mathbb{U}}$ is defined in the matrix form by $[(\ell_{1j}, \wp_{1j}, \partial_{1j})]_{1\times n} = [(\ell_{11}, \wp_{11}, \partial_{11}), (\ell_{12}, \wp_{12}, \partial_{12}), ..., (\ell_{1n}, \wp_{1n}, \partial_{1n})]$, where $(\ell_{1j}, \wp_{1j}, \partial_{1j}) = \mu^c_{\Psi_X}(e_j), \forall j = 1, 2, ..., n$.

Definition 3.2. Let $\Psi_X \in CSF^{\mathbb{U}}$ and $c\Psi_X \in cCSF^{\mathbb{U}}$. The mapping $CSFS_{agg} : cCSF^{\mathbb{U}} \times CSF^{\mathbb{U}} \to CSFS(\mathbb{U}, E)$ is defined as $CSFS_{agg}(c\Psi_X, \Psi_X) = \left\{\frac{u}{\mu_{\Psi^*_X}(u)} : u \in \mathbb{U}\right\} =$

$$\left\{\frac{u}{\left(\vartheta_{\alpha_1^*}(u)exp^{i2\pi\vartheta_{\alpha_2^*}(u)}, \varpi_{\beta_1^*}(u)exp^{i2\pi\varpi_{\beta_2^*}(u)}, \tau_{\gamma_1^*}(u)exp^{i2\pi\tau_{\gamma_2^*}(u)}\right)} : u \in \mathbb{U}\right\} \tag{3.2}$$

which is called aggregate CSFS of Ψ_X and

$$\vartheta_{\alpha_1^*}(u) : \mathbb{U} \to [0,1]\, by\, \vartheta_{\alpha_1^*}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\vartheta^c_{\alpha_1}(e), \vartheta_{\alpha_1}(e)\right)(u)$$

$$\varpi_{\beta_1^*}(u) : \mathbb{U} \to [0,1] by \varpi_{\beta_1^*}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\varpi^c_{\beta_1}(e), \varpi_{\beta_1}(e)\right)(u)$$

and

$$\tau_{\gamma_1^*}(u) : \mathbb{U} \to [0,1] by \tau_{\gamma_1^*}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\tau^c_{\gamma_1}(e), \tau_{\gamma_1}(e)\right)(u)$$

where

$$\vartheta_{\alpha_2^*}(u) : \mathbb{U} \to [0,1], \vartheta_{\alpha_2^*}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\vartheta^c_{\alpha_2}(e), \vartheta_{\alpha_2}(e)\right)(u)$$

,

$$\varpi_{\beta_2^*}(u) : \mathbb{U} \to [0,1], \varpi_{\beta_2^*}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\varpi^c_{\beta_2}(e), \varpi_{\beta_2}(e)\right)(u)$$

$$\tau_{\gamma_2^*}(u) : \mathbb{U} \to [0,1], \tau_{\gamma_2^*}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\tau^c_{\gamma_2}(e), \tau_{\gamma_2}(e)\right)(u)$$

.

The set $CSFS_{agg}(c\Psi_X, \Psi_X)$ is expressed in matrix form as

$$[(\ell_{k1}, \wp_{k1}, \partial_{k1})]_{m\times 1} = \begin{bmatrix} (\ell_{11}, \wp_{11}, \partial_{11}) \\ (\ell_{21}, \wp_{21}, \partial_{21}) \\ \vdots \\ (\ell_{m1}, \wp_{m1}, \partial_{m1}) \end{bmatrix} \tag{3.3}$$

where $[(\ell_{k1}, \wp_{k1}, \partial_{k1})] = \mu_{\Psi_X^*}(u_k)$ and k various from 1 to m. The above matrix said to be CSFS aggregate matrix of $CSFS_{agg}(c\Psi_X, \Psi_X)$ over $\mathbb{U}$.

Definition 3.3. Let $X = (\vartheta_{kj}, \varpi_{kj}, \tau_{kj}) \in CSFSM_{m\times n}$. Then (i) Choice matrix of CSFSM X is

$$\Xi(X) = \left[\begin{pmatrix} \frac{\sum_{j=1}^{n}(\vartheta_{kj})^2}{n} exp^{i2\pi \frac{\sum_{j=1}^{n}(\vartheta_{kj})^2}{n}}, \frac{\sum_{j=1}^{n}(\varpi_{kj})^2}{n} exp^{i2\pi \frac{\sum_{j=1}^{n}(\varpi_{kj})^2}{n}}, \\ \frac{\sum_{j=1}^{n}(\tau_{kj})^2}{n} exp^{i2\pi \frac{\sum_{j=1}^{n}(\tau_{kj})^2}{n}} \end{pmatrix}\right]_{m\times 1} \tag{3.4}$$

for every k, when weights are equal.

(ii) Weighted choice matrix of CSFSM X (for every k, where $w_j > 0$ are weights) is

$$\Xi_w(X) = \left[\begin{pmatrix} \frac{\sum_{j=1}^{n} w_j(\vartheta_{kj})^2}{\sum w_j} exp^{i2\pi \frac{\sum_{j=1}^{n} w_j(\vartheta_{kj})^2}{\sum w_j}}, \frac{\sum_{j=1}^{n} w_j(\varpi_{kj})^2}{\sum w_j} exp^{i2\pi \frac{\sum_{j=1}^{n} w_j(\varpi_{kj})^2}{\sum w_j}}, \\ \frac{\sum_{j=1}^{n} w_j(\tau_{kj})^2}{\sum w_j} exp^{i2\pi \frac{\sum_{j=1}^{n} w_j(\tau_{kj})^2}{\sum w_j}} \end{pmatrix}\right]_{m\times 1} \tag{3.5}$$

Here, four methods are used to insert MCGDM based on CSFS sets:

Method-1

(i) Form CSFS set Ψ_X over $\mathbb{U}$.

(ii) Determine the cardinal set and cardinalities. $c\Psi_X$ of Ψ_X.

(iii) Compute aggregate CSFS set Ψ_X^* of Ψ_X.

(iv) Find the score function $S(u) = \left(\vartheta_u^2 - \varpi_u^2 - \tau_u^2\right) + \frac{1}{2\pi}\left(\vartheta_u^2 - \varpi_u^2 - \tau_u^2\right), \forall\, u \in \mathbb{U}$.

(v) Find the best alternative by $\max_i S(u_i)$.

4 Real life applications

Each robot has distinct characteristics, and robots vary greatly in size, design, and capabilities. However, many robots have some characteristics, making it easy to group them together.

(i) Medical robots (Ξ_1): A wide range of robotic devices known as medical robots are made to help patients in clinics, hospitals, rehabilitation facilities, and occasionally even at home. The da Vinci surgical system is an example of a medical robot that is intended to be teleoperated by a doctor during minimally invasive treatments.

(ii) Aerospace robots (Ξ_2): Aerospace robotics is an extensive field that includes both flying robots and robots for space applications. The Raven fixed-wing drone and the Smart Bird robotic seagull are examples of flying robots. They are used to capture pictures and get an aerial view of a particular location.

(iii) Aquatic robots (Ξ_3): Water-dwelling robots don't mind getting wet. They work on surveillance missions, check and maintain infrastructure, and gather environmental data on the world's oceans. Some of them float on the water's surface, while others sink to great depths.

(iv) Robotic automobile robots (Ξ_4): Robotic automobiles with cameras, lidar, GPS, computers, and other sensing and navigational tools that allow them to travel entirely on their own are known as autonomous vehicles. Early examples include Google's groundbreaking self-driving car, which eventually split off to become Waymo, and Boss and Stanley, which were constructed for DARPA's autonomous vehicle contests.

(v) Humanoid robots (Ξ_5): Since humanoid robots have a mechanical body with human-like arms, legs, and a head, they can walk and manipulate objects similarly to humans. Humanoids, like the friendly robot ambassador Asimo from Honda and the athletic, flexible Atlas from Boston Dynamics, typically have a machine-like appearance, but they can also look like humans.

(vi) Industrial robots (Ξ_6): Industrial robots carry out repetitive operations including choosing, moving, and assembling parts as well as duties that are commonly seen in production, such as cutting, welding, painting, polishing, and packing. An industrial robot with a fast-moving, accurate manipulator arm is a frequent kind.

(vii) Military and security robots (Ξ_7): A wide variety of strong, durable robotic devices that can conduct surveillance and other tasks that can be hazardous for humans are included in the category of military and security robots.

(viii) Research robots (Ξ_8): University and industry research laboratories are the birthplaces of experimental devices known as research robots. The main purpose of these robots is to assist researchers in doing research, even if some of them may be capable of useful tasks and fall under other robot categories.

(ix) Drones robots (Ξ_9): Drones are flying machines that enable you to take pictures and record data from a higher altitude. There are many different sizes and forms of drones. A quadrotor or quadcopter is a popular design that flies by using four rotors. Other drones, referred to as fixed-wing versions, fly similarly like little airplanes. Additionally, drones vary in their degree of autonomy.

(x) Telepresence robots (Ξ_{10}): You can be present at a location without physically visiting there thanks to telepresence robots. Through the Internet, you may access a robot avatar and move it around, seeing what it sees, hearing what it hears, and conversing with others you meet at the water cooler or in meetings. Telepresence robots can be used by doctors to check on patients and by employees to cooperate with coworkers at a remote location.

Five attributes of a robotics software stack enable you to construct robots quickly and consistently:

(i) Compatible with ROS (robot operating system) (e_1): In reality, a robot operating system (ROS) is a set of middle ware components and frameworks for creating robotics software rather than an operating system. The most often required tools geometry, mapping, goal-seeking, navigation, vision systems, and diagnosticsare pre-packaged in ROS.

(ii) Real-time capable (e_2): Robots, like humans, require real-time responses to external stimuli in order to identify targets or obstacles, position arms and appendages appropriately, and move toward or away from obstacles. However, the kind of application, the degree of dependability, and the hardware and operating system combination all have a significant impact on the lowest latency that satisfies real-time requirements.

(iii) Easily certified (e_3): The ability to quickly validate your stack is the next essential quality. It should be simple to certify a robotics software stack using a safety standard like ISO 10218, ISO 26262, IEC 61508, or IEC 62304. When a robot is doing a mission-critical duty, it should be safety-certified.

(iv) Manages cyber security (e_4): Nothing is more detrimental to a company's finances and reputation than having its products compromised on a global scale. Unfortunately, for hackers looking to disrupt or profit from their misdeeds, robots provide a new and lucrative platform. Given its mobility and physical item manipulation capabilities, a compromised robot might be more dangerous in the real world than current compromised servers. Consider an industrial robot that purposefully constructs vehicles with faulty welds to cause collisions, a medical robot that neglects to provide life-saving medication, or a robot that attacks specific employees in a warehouse.

(v) Consistent platform (e_5): Modern robots may have many CPUs, depending on their architecture. One job, such as limb management, executive functioning, communication, mapping and navigation systems, remote control, or user interface, may be the only focus of a CPU. As an alternative, a CPU may combine any or all of these tasks.

The top corporation company manufactures ten different types of robots $\mathbb{U} = \{\Xi_1, \Xi_2, ..., \Xi_{10}\}$ with five attributes namely $E = \{e_1, e_2, ..., e_5\}$ consists of compatible with ROS (Robot Operating System), real-time capable, easily certified, manages cyber-security, consistent platform respectively. The challenge we deal is the reality that the company must determine which robots to purchase ? Every robot is evaluated using a small number of attributes. That is $X = \{e_1, e_2, e_3, e_5\} \subseteq E$ and we applied to method-1.

Step-1: The definition of the CSFS set Ψ_X of $\mathbb{U}$ is as follows:

$$\Psi_X = \begin{bmatrix} e_1 = \left\{ \frac{\Xi_1}{(0.4e^{0.5} 0.15e^{0.25} 0.65e^{0.45})}, \frac{\Xi_4}{(0.45e^{0.25} 0.55e^{0.45} 0.2e^{0.35})}, \frac{\Xi_7}{(0.35e^{0.5} 0.45e^{0.35} 0.25e^{0.2})}, \right. \\ \left. \frac{\Xi_9}{(0.55e^{0.35} 0.5e^{0.55} 0.15e^{0.1})}, \frac{\Xi_{10}}{(0.2e^{0.5} 0.2e^{0.35} 0.25e^{0.2})} \right\} \\ e_2 = \left\{ \frac{\Xi_2}{(0.65e^{0.45} 0.25e^{0.35} 0.1e^{0.25})}, \frac{\Xi_3}{(0.45e^{0.35} 0.35e^{0.4} 0.25e^{0.2})}, \frac{\Xi_5}{(0.4e^{0.25} 0.45e^{0.25} 0.15e^{0.35})}, \right. \\ \left. \frac{\Xi_8}{(0.3e^{0.35} 0.5e^{0.45} 0.3e^{0.25})}, \frac{\Xi_{10}}{(0.6e^{0.45} 0.2e^{0.35} 0.2e^{0.15})} \right\} \\ e_3 = \left\{ \frac{\Xi_3}{(0.25e^{0.5} 0.2e^{0.3} 0.5e^{0.35})}, \frac{\Xi_4}{(0.2e^{0.2} 0.3e^{0.45} 0.45e^{0.25})}, \frac{\Xi_6}{(0.55e^{0.45} 0.2e^{0.25} 0.45e^{0.5})}, \right. \\ \left. \frac{\Xi_8}{(0.6e^{0.35} 0.2e^{0.35} 0.25e^{0.15})}, \frac{\Xi_9}{(0.45e^{0.25} 0.6e^{0.45} 0.1e^{0.25})} \right\} \\ e_5 = \left\{ \frac{\Xi_1}{(0.45e^{0.35} 0.45e^{0.35} 0.25e^{0.35})}, \frac{\Xi_2}{(0.6e^{0.45} 0.2e^{0.25} 0.25e^{0.2})}, \frac{\Xi_4}{(0.6e^{0.4} 0.25e^{0.35} 0.2e^{0.15})}, \right. \\ \left. \frac{\Xi_6}{(0.25e^{0.4} 0.4e^{0.55} 0.1e^{0.2})}, \frac{\Xi_7}{(0.55e^{0.25} 0.4e^{0.5} 0.2e^{0.25})}, \frac{\Xi_{10}}{(0.35e^{0.5} 0.3e^{0.35} 0.4e^{0.35})} \right\} \end{bmatrix}$$

Step-2: The cardinal set of Ψ_X as

$$c\Psi_X = \begin{bmatrix} \frac{e_1}{(0.2e^{0.21} 0.19e^{0.2} 0.15e^{0.13})}, \frac{e_2}{(0.24e^{0.19} 0.18e^{0.18} 0.1e^{0.12})}, \\ \frac{e_3}{(0.21e^{0.18} 0.15e^{0.18} 0.18e^{0.15})}, \frac{e_5}{(0.28e^{0.24} 0.2e^{0.24} 0.14e^{0.15})}. \end{bmatrix}$$

Step-3: Let Ψ_X be a CSFS set. Suppose that $M_{\Psi_X}, M_{c\Psi_X}, M^*_{\Psi_X}$ are matrices of $\Psi_X, c\Psi_X, \Psi^*_X$ respectively, then $M_{\Psi_X} \times M^{\mathbb{T}}_{c\Psi_X} = M^*_{\Psi_X} \times |E|$, where $M^{\mathbb{T}}_{c\Psi_X}$ is the transpose of $M_{c\Psi_X}$. The aggregate CSFS set Ψ^*_X of Ψ_X is $M_{\Psi^*_X} = \frac{M_{\Psi_X} \times M^{\mathbb{T}}_{c\Psi_X}}{|E|}$

$$= \frac{1}{5} \begin{bmatrix} 0.4e^{0.5} & 0e^0 & 0e^0 & 0e^0 & 0.45e^{0.35} \\ 0e^0 & 0.65e^{0.45} & 0e^0 & 0e^0 & 0.6e^{0.45} \\ 0e^0 & 0.45e^{0.35} & 0.25e^{0.5} & 0e^0 & 0e^0 \\ 0.45e^{0.25} & 0e^0 & 0.2e^{0.2} & 0e^0 & 0.6e^{0.4} \\ 0e^0 & 0.4e^{0.25} & 0e^0 & 0e^0 & 0e^0 \\ 0e^0 & 0e^0 & 0.55e^{0.45} & 0e^0 & 0.25e^{0.4} \\ 0.35e^{0.5} & 0e^0 & 0e^0 & 0e^0 & 0.55e^{0.25} \\ 0e^0 & 0.3e^{0.35} & 0.6e^{0.35} & 0e^0 & 0e^0 \\ 0.55e^{0.35} & 0e^0 & 0.45e^{0.25} & 0e^0 & 0e^0 \\ 0.2e^{0.5} & 0.6e^{0.45} & 0e^0 & 0e^0 & 0.35e^{0.5} \end{bmatrix} \begin{bmatrix} 0.2e^{0.21} \\ 0.24e^{0.19} \\ 0.21e^{0.18} \\ 0e^0 \\ 0.28e^{0.24} \end{bmatrix},$$

$$\frac{1}{5}\begin{bmatrix} 0.15e^{0.25} & 0e^{0} & 0e^{0} & 0e^{0} & 0.45e^{0.35} \\ 0e^{0} & 0.25e^{0.35} & 0e^{0} & 0e^{0} & 0.2e^{0.25} \\ 0e^{0} & 0.35e^{0.4} & 0.2e^{0.3} & 0e^{0} & 0e^{0} \\ 0.55e^{0.45} & 0e^{0} & 0.3e^{0.45} & 0e^{0} & 0.25e^{0.35} \\ 0e^{0} & 0.45e^{0.25} & 0e^{0} & 0e^{0} & 0e^{0} \\ 0e^{0} & 0e^{0} & 0.55e^{0.45} & 0e^{0} & 0.25e^{0.4} \\ 0.45e^{0.35} & 0e^{0} & 0e^{0} & 0e^{0} & 0.4e^{0.5} \\ 0e^{0} & 0.5e^{0.45} & 0.2e^{0.35} & 0e^{0} & 0e^{0} \\ 0.5e^{0.55} & 0e^{0} & 0.6e^{0.45} & 0e^{0} & 0e^{0} \\ 0.2e^{0.35} & 0.2e^{0.35} & 0e^{0} & 0e^{0} & 0.3e^{0.35} \end{bmatrix} \begin{bmatrix} 0.19e^{0.2} \\ 0.18e^{0.18} \\ 0.15e^{0.18} \\ 0e^{0} \\ 0.2e^{0.24} \end{bmatrix},$$

$$\frac{1}{5}\begin{bmatrix} 0.65e^{0.45} & 0e^{0} & 0e^{0} & 0e^{0} & 0.25e^{0.35} \\ 0e^{0} & 0.1e^{0.25} & 0e^{0} & 0e^{0} & 0.25e^{0.2} \\ 0e^{0} & 0.25e^{0.2} & 0.5e^{0.35} & 0e^{0} & 0e^{0} \\ 0.2e^{0.35} & 0e^{0} & 0.45e^{0.25} & 0e^{0} & 0.2e^{0.15} \\ 0e^{0} & 0.15e^{0.35} & 0e^{0} & 0e^{0} & 0e^{0} \\ 0e^{0} & 0e^{0} & 0.45e^{0.5} & 0e^{0} & 0.1e^{0.2} \\ 0.25e^{0.2} & 0e^{0} & 0e^{0} & 0e^{0} & 0.2e^{0.25} \\ 0e^{0} & 0.3e^{0.25} & 0.25e^{0.15} & 0e^{0} & 0e^{0} \\ 0.15e^{0.1} & 0e^{0} & 0.1e^{0.25} & 0e^{0} & 0e^{0} \\ 0.25e^{0.2} & 0.2e^{0.15} & 0e^{0} & 0e^{0} & 0.4e^{0.35} \end{bmatrix} \begin{bmatrix} 0.15e^{0.13} \\ 0.1e^{0.12} \\ 0.18e^{0.15} \\ 0e^{0} \\ 0.14e^{0.15} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} 0.0408e^{0.03745} \\ 0.0648e^{0.0378} \\ 0.03185e^{0.03045} \\ 0.05935e^{0.0363} \\ 0.0192e^{0.00925} \\ 0.03655e^{0.03455} \\ 0.04445e^{0.03275} \\ 0.039e^{0.0252} \\ 0.0399e^{0.02345} \\ 0.0562e^{0.06115} \end{bmatrix}, \begin{bmatrix} 0.02355e^{0.0262} \\ 0.01675e^{0.02435} \\ 0.01825e^{0.0252} \\ 0.03935e^{0.0502} \\ 0.01575e^{0.009} \\ 0.022e^{0.03485} \\ 0.03265e^{0.03715} \\ 0.0235e^{0.0288} \\ 0.0365e^{0.03765} \\ 0.0264e^{0.0427} \end{bmatrix}, \begin{bmatrix} 0.0265e^{0.0222} \\ 0.009e^{0.012} \\ 0.0225e^{0.0153} \\ 0.02735e^{0.0211} \\ 0.003e^{0.0084} \\ 0.01855e^{0.021} \\ 0.0131e^{0.0127} \\ 0.01475e^{0.0105} \\ 0.008e^{0.0101} \\ 0.0227e^{0.0193} \end{bmatrix} \end{bmatrix}$$

Hence,

$$\Psi_X^* = \begin{bmatrix} \frac{\Xi_1}{0.0408e^{0.03745}, 0.02355e^{0.0262}, 0.0265e^{0.0222}}, \frac{\Xi_2}{0.0648e^{0.0378}, 0.01675e^{0.02435}, 0.009e^{0.012}}, \\ \frac{\Xi_3}{0.03185e^{0.03045}, 0.01825e^{0.0252}, 0.0225e^{0.0153}}, \frac{\Xi_4}{0.05935e^{0.0363}, 0.03935e^{0.0502}, 0.02735e^{0.0211}}, \\ \frac{\Xi_5}{0.0192e^{0.00925}, 0.01575e^{0.009}, 0.003e^{0.0084}}, \frac{\Xi_6}{0.03655e^{0.03455}, 0.022e^{0.03485}, 0.01855e^{0.021}}, \\ \frac{\Xi_7}{0.04445e^{0.03275}, 0.03265e^{0.03715}, 0.0131e^{0.0127}}, \frac{\Xi_8}{0.039e^{0.0252}, 0.0235e^{0.0288}, 0.01475e^{0.0105}}, \\ \frac{\Xi_9}{0.0399e^{0.02345}, 0.0365e^{0.03765}, 0.008e^{0.0101}}, \frac{\Xi_{10}}{0.0562e^{0.06115}, 0.0264e^{0.0427}, 0.0227e^{0.0193}}. \end{bmatrix}$$

Step-4: The score function $S(\Xi_i)$ such as $\Xi_1 = 0.000443$, $\Xi_2 = 0.003948$, $\Xi_3 = 0.000184$, $\Xi_4 = 0.000964$, $\Xi_5 = 0.000101$, $\Xi_6 = 0.000434$, $\Xi_7 = 0.000664$, $\Xi_8 = 0.000703$, $\Xi_9 = 0.0000414$, $\Xi_{10} = 0.002192$.

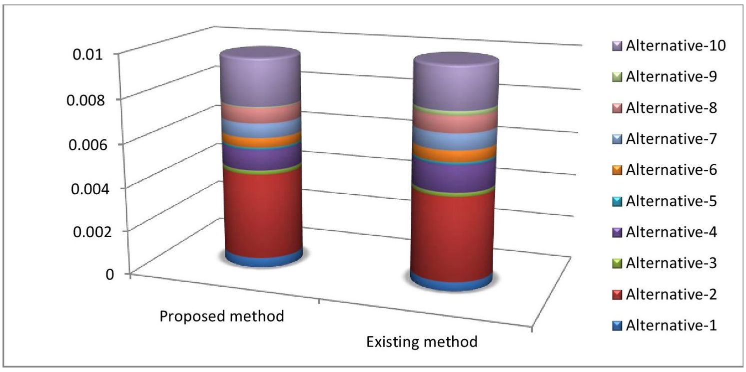

Figure 1. Graphical representation using MCGDM based on CSFS

Step 5: Here $\max_i S(\Xi_i) = 0.003948$.

Method-2

(i) Form CSFS matrix

(ii) Calculate the choice matrix for the PMG, neutral MG and NMG of CSFS matrix all weights are equal

(iii) Determine the score function $S(u) = \left(\vartheta_u^2 - \varpi_u^2 - \tau_u^2\right) + \frac{1}{2\pi}\left(\vartheta_u^2 - \varpi_u^2 - \tau_u^2\right), \forall\, u \in \mathbb{U}$

(iv) Find the best alternative by $\max_i S(u_i)$.

From the above data,

$$\Xi(X) = \begin{bmatrix} \begin{bmatrix} 0.0725e^{0.0745} \\ 0.1565e^{0.081} \\ 0.053e^{0.0745} \\ 0.1205e^{0.0525} \\ 0.032e^{0.0125} \\ 0.073e^{0.0725} \\ 0.085e^{0.0625} \\ 0.09e^{0.049} \\ 0.101e^{0.037} \\ 0.1045e^{0.1405} \end{bmatrix}, \begin{bmatrix} 0.045e^{0.037} \\ 0.0205e^{0.037} \\ 0.0325e^{0.05} \\ 0.091e^{0.1055} \\ 0.0405e^{0.0125} \\ 0.04e^{0.073} \\ 0.0725e^{0.0745} \\ 0.058e^{0.065} \\ 0.122e^{0.101} \\ 0.034e^{0.0735} \end{bmatrix}, \begin{bmatrix} 0.097e^{0.065} \\ 0.0145e^{0.0205} \\ 0.0625e^{0.0325} \\ 0.0565e^{0.0415} \\ 0.0045e^{0.0245} \\ 0.0425e^{0.058} \\ 0.0205e^{0.0205} \\ 0.0305e^{0.017} \\ 0.0065e^{0.0145} \\ 0.0525e^{0.037} \end{bmatrix} \end{bmatrix}$$

The score function $S(\Xi_i)$ such as $S(\Xi_1) = -0.00618$, $S(\Xi_2) = 0.024621$, $S(\Xi_3) = -0.00184$, $S(\Xi_4) = 0.00144$, $S(\Xi_5) = -0.00073$, $S(\Xi_6) = 0.001376$, $S(\Xi_7) = 0.00122$, $S(\Xi_8) = 0.003469$, $S(\Xi_9) = -0.00616$, $S(\Xi_{10}) = 0.009072$.

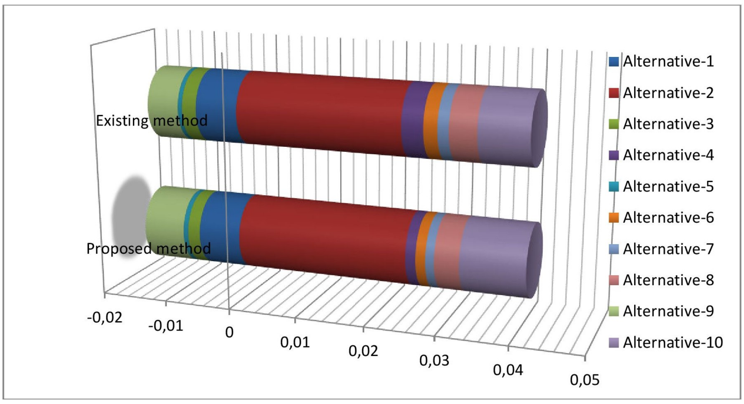

Figure 2. Graphical representation using MCGDM based on CSFS

Method-3

(i) Form CSFS matrix.

(ii) Find the choice matrix for the PMG, neutral MG and NMG of CSFS matrix all weights are unequal.

(iii) Find the score function $S(u) = \left(\vartheta_u^2 - \varpi_u^2 - \tau_u^2\right) + \frac{1}{2\pi}\left(\vartheta_u^2 - \varpi_u^2 - \tau_u^2\right), \forall\, u \in \mathbb{U}$.

(iv) Find the best alternative by $\max_i S(u_i)$.

Case-II: Weights $(w_j) = \{0.23, 0.14, 0.2, 0.2, 0.23\}$. From the above data,

$$\Xi_w(X) = \left[\begin{bmatrix} 0.083375e^{0.085675} \\ 0.14195e^{0.074925} \\ 0.04085e^{0.06715} \\ 0.137375e^{0.059175} \\ 0.0224e^{0.00875} \\ 0.074875e^{0.0773} \\ 0.09775e^{0.071875} \\ 0.0846e^{0.04165} \\ 0.110075e^{0.040675} \\ 0.087775e^{0.14335} \end{bmatrix}, \begin{bmatrix} 0.05175e^{0.04255} \\ 0.01795e^{0.031525} \\ 0.02515e^{0.0404} \\ 0.10195e^{0.11525} \\ 0.02835e^{0.00875} \\ 0.0448e^{0.082075} \\ 0.083375e^{0.085675} \\ 0.043e^{0.05285} \\ 0.1295e^{0.110075} \\ 0.0355e^{0.0735} \end{bmatrix}, \begin{bmatrix} 0.11155e^{0.07475} \\ 0.015775e^{0.01795} \\ 0.05875e^{0.0301} \\ 0.0589e^{0.04585} \\ 0.00315e^{0.01715} \\ 0.0428e^{0.0592} \\ 0.023575e^{0.023575} \\ 0.0251e^{0.01325} \\ 0.007175e^{0.0148} \\ 0.056775e^{0.040525} \end{bmatrix}\right]$$

The score function $S(\Xi_i)$ such as

$S(\Xi_1) = -0.00818$, $S(\Xi_2) = 0.020263$, $S(\Xi_3) = -0.0021$, $S(\Xi_4) = 0.003118$, $S(\Xi_5) = -0.00036$, $S(\Xi_6) = 0.001088$, $S(\Xi_7) = 0.001613$, $S(\Xi_8) = 0.004482$, $S(\Xi_9) = -0.00641$, $S(\Xi_{10}) = 0.00537$.

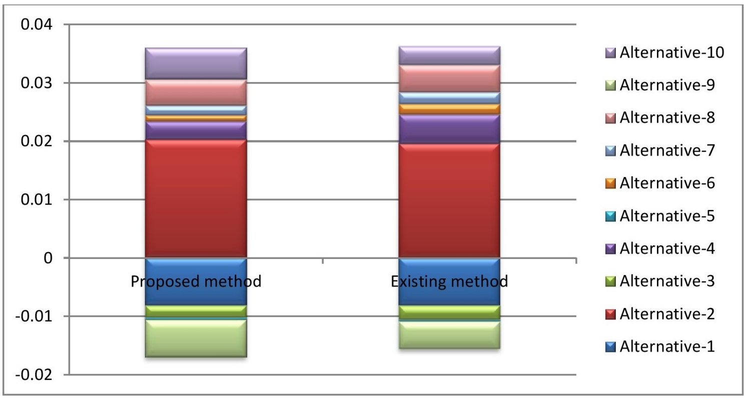

Figure 3. Graphical representation using MCGDM based on CSFS

Method-4

(i) Find the complex spherical fuzzy weighted averaging numbers (CSFWANs) $\Xi_i = \left(\sum_{j=1}^{n} w_j\vartheta_{ij}, \sum_{j=1}^{n} w_j\varpi_{ij}, \sum_{j=1}^{n} w_j\tau_{ij}\right)$.

(ii) Determine the score function $S(u) = \left(\vartheta_u^2 - \varpi_u^2 - \tau_u^2\right) + \frac{1}{2\pi}\left(\vartheta_u^2 - \varpi_u^2 - \tau_u^2\right), \forall\, u \in \mathbb{U}$.

(iii) Determine the best alternative by $\max_i S(u_i)$.

Weights $(w_j) = \{0.23, 0.14, 0.2, 0.2, 0.23\}$. From the above data,

$$\Xi_i = \left[\begin{bmatrix} 0.1955e^{0.1955} \\ 0.229e^{0.1665} \\ 0.113e^{0.149} \\ 0.2815e^{0.1895} \\ 0.056e^{0.035} \\ 0.1675e^{0.182} \\ 0.207e^{0.1725} \\ 0.162e^{0.119} \\ 0.2165e^{0.1305} \\ 0.2105e^{0.293} \end{bmatrix}, \begin{bmatrix} 0.138e^{0.138} \\ 0.081e^{0.1065} \\ 0.089e^{0.116} \\ 0.244e^{0.274} \\ 0.063e^{0.035} \\ 0.132e^{0.1765} \\ 0.1955e^{0.1955} \\ 0.11e^{0.133} \\ 0.235e^{0.2165} \\ 0.143e^{0.21} \end{bmatrix}, \begin{bmatrix} 0.207e^{0.184} \\ 0.0715e^{0.081} \\ 0.135e^{0.098} \\ 0.182e^{0.165} \\ 0.021e^{0.049} \\ 0.113e^{0.146} \\ 0.1035e^{0.1035} \\ 0.092e^{0.065} \\ 0.0545e^{0.073} \\ 0.1775e^{0.1475} \end{bmatrix}\right]$$

The score function $S(\Xi_i)$ such as
$S(\Xi_1) = -0.02601$, $S(\Xi_2) = 0.04233$, $S(\Xi_3) = -0.01351$, $S(\Xi_4) = -0.02398$, $S(\Xi_5) = -0.00166$, $S(\Xi_6) = -0.00522$, $S(\Xi_7) = -0.00914$, $S(\Xi_8) = 0.004446$, $S(\Xi_9) = -0.01692$, $S(\Xi_{10}) = -0.00446$.

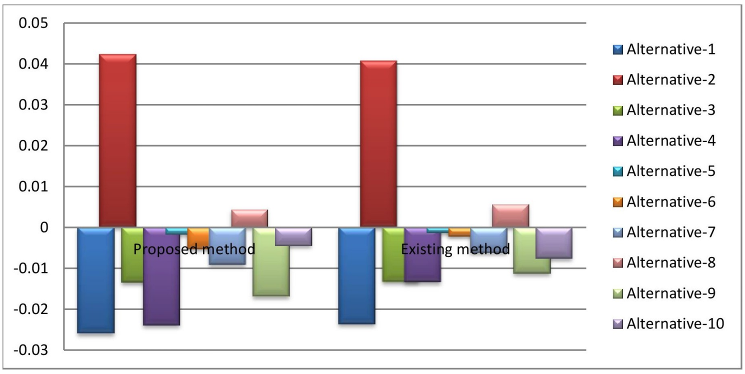

Figure 4. Graphical representation using MCGDM based on CSFS

5 Comparison of the CSFS-Methods:

Analysis of comparisons between the final rankings in Tables 1 and 2.

Table 1. Proposed methods

$Methods(proposed)$	$Ranking\ of\ alternatives$	$Optimal$
$Method - 1$	$\Xi_9 \le \Xi_5 \le \Xi_3 \le \Xi_6 \le \Xi_1 \le \Xi_7 \le \Xi_8 \le \Xi_4 \le \Xi_{10} \le \Xi_2$	Ξ_2
$Method - 2$	$\Xi_1 \le \Xi_9 \le \Xi_3 \le \Xi_5 \le \Xi_7 \le \Xi_6 \le \Xi_4 \le \Xi_8 \le \Xi_{10} \le \Xi_2$	Ξ_2
$Method - 3$	$\Xi_1 \le \Xi_9 \le \Xi_3 \le \Xi_5 \le \Xi_6 \le \Xi_7 \le \Xi_4 \le \Xi_8 \le \Xi_{10} \le \Xi_2$	Ξ_2
$Method - 4$	$\Xi_1 \le \Xi_4 \le \Xi_9 \le \Xi_3 \le \Xi_7 \le \Xi_6 \le \Xi_{10} \le \Xi_5 \le \Xi_8 \le \Xi_2$	Ξ_2

Table 2. Existing methods

$Methods(existing)$	$Ranking\ of\ alternatives$	$Optimal$
$Method-1$	$\Xi_5 \leq \Xi_3 \leq \Xi_9 \leq \Xi_1 \leq \Xi_6 \leq \Xi_7 \leq \Xi_8 \leq \Xi_4 \leq \Xi_{10} \leq \Xi_2$	Ξ_2
$Method-2$	$\Xi_1 \leq \Xi_9 \leq \Xi_3 \leq \Xi_5 \leq \Xi_7 \leq \Xi_6 \leq \Xi_4 \leq \Xi_8 \leq \Xi_{10} \leq \Xi_2$	Ξ_2
$Method-3$	$\Xi_1 \leq \Xi_9 \leq \Xi_3 \leq \Xi_5 \leq \Xi_6 \leq \Xi_7 \leq \Xi_{10} \leq \Xi_8 \leq \Xi_4 \leq \Xi_2$	Ξ_2
$Method-4$	$\Xi_1 \leq \Xi_4 \leq \Xi_3 \leq \Xi_9 \leq \Xi_{10} \leq \Xi_7 \leq \Xi_6 \leq \Xi_5 \leq \Xi_8 \leq \Xi_2$	Ξ_2

Hence the company Ξ_2 has to be big purchased.

The advantages of the novel approach are demonstrated by contrasting the proposed algorithm with a few existing methods. This uses the symbols ✓ and × to indicate whether or not the techniques are satisfied. Furthermore, it was claimed that the existing approaches only dealt with MGCDM problems. Using Table 3, we may evaluate and compare the proposed and existing models.

Table 3. Validity of the AO

$Existing$	$Method-1$	$Method-2$	$Method-3$	$Method-4$
Palanikumar et al. [54]	✓	✓	✓	✓

The findings demonstrate that compared to the existing methods, the proposed approach is noticeably more effective in overcoming DM problems.

5.1 Effectiveness test

The MCGDM methods reliability rankings differ for the various choices. Testing includes a lot of formulations. You can find the following ranks and closeness values. The various score values are shown in Table 3. The best outcomes with the most value were attained for all of them. To prove their superiority and validity, the suggested score values were put to the test using the available techniques. The recommended score values are more accurate and reliable than those of the current technique. We suggested a novel method for determining the best option for the MCGDM problem.

5.2 Advantages

The following advantages of the suggested method are made possible by the previous analysis. We use spherical fuzzy numbers and complex fuzzy numbers to introduce the CSFN. In addition to describing incomplete data, a CSFN may be used to understand both natural and human events. Although the fact that their squares were less than one, the sums of MG and NMG are larger than one. Because of its wider breadth, CSFN information is therefore more important to human cognition. When choosing a decision outcome, individual preferences should be taken into effect. Based on a set of criteria, an alternative can be graded using our four algorithms of methods. It is demonstrated that the outcomes obtained with novel CSFN are comparable to those obtained using traditional methods. To assess the suggested method's effectiveness and reliability. We compared it to some other algorithms that are currently in use. The domains or decision settings in which the suggested approach can be used may be restricted. A suggested technique should be assessed in view of the constraints and circumstances that may maximize its effectiveness. The proposed technique includes certain assumptions and simplifications to help with analysis. As a result, this application will be constrained because they could not always match actual events.

6 Conclusion:

This communication described the four algorithms that MCGDM uses under CSFS. At this point, we use a way to interface with CSFS's score values. In order to demonstrate the ranks of the options under consideration. Several kinds of data charts are also provided. The score values in data processing have a number of practical applications. Our goal was to make sure the suggested approach was both efficient and adaptable to various situations. Since concepts may be applied to solve problems in the real world, they must be difficult. Given the openness the subject has been upcoming researchers will benefit from the concepts presented here. To deal with future DM parameter inconsistency. We introducing a similarity measure to the analysis of (p, q, r)-CSFS and q-rung CSFS. We tackle MCGDM problems with known and unknown weights using these approaches.

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