Palestine Journal of Mathematics

Vol 14(4)(2025) , 727–740

FUZZY HYPERSOFT CONTINUOUS MAPS, IRRESOLUTE MAPS AND ITS APPLICATION

K. Chitirakala, D. Sobana, A. Vadivel and P. Surendra

Communicated by: Madeleine Al Tahan

MSC 2020 Classifications: Primary 03E72; Secondary 54A10, 54A40, 54C05, 54C10.

Keywords and phrases: fuzzy hypersoft continuous maps, fuzzy hypersoft semi continuous maps, fuzzy hypersoft irresolute maps, tangent similarity measure.

Corresponding Authors: K. Chitirakala and D. Sobana

Abstract. The purpose of this paper is to introduce and study fuzzy hypersoft continuous maps and fuzzy hypersoft semi continuous maps in fuzzy hypersoft topological spaces. Basic properties of fuzzy hypersoft continuous and semi continuous maps are analysed with examples. It is extended to fuzzy hypersoft irresolute maps and its related characteristics are also investigated. Moreover, an application in Covid-19 diagnosis is given using tangent similarity measure.

1 Introduction

The real world decision making problems in medical diagnosis, engineering, economics, management, computer science, artificial intelligence, social sciences, environmental science and sociology contains more uncertain and inadequate data. The traditional mathematical methods cannot deal with these kind of problems due to the imprecise data. To deal the problems with uncertainty, Zadeh [29] introduced the fuzzy set in 1965 which contains the membership value in [0,1]. A fuzzy set is a set where each element of the universe belongs to it but with some value or degree of belongingness which lies between 0 and 1 and such values are called membership value of an element in that set. The topological structure on fuzzy set was undertaken by Chang [8] as fuzzy topological space. Molodstov [10] introduced a new mathematical tool, soft set theory in 1999 to deal uncertainties in which a soft set is a collection of approximate descriptions of an object. Soft set is a parameterized family of subsets where parameters are the properties, attributes or characteristics of the objects. The soft set theory have several applications in different fields such as decision making, optimization, forecasting, data analysis etc. Shabir and Naz [19] presented soft topological spaces.

Smarandache [20] extended the notion of a soft set to a hypersoft set and then to plithogenic set by replacing function with a multi-argument function described in the cartesian product with a different set of attributes. This new concept of hypersoft set is more flexible than the soft set and more suitable in the decision-making issues involving different kind of attributes. Abbas et al. [1] defined the basic operations on hypersoft sets and hypersoft point in all the universe of discourses. Ajay and Charisma [3] introduced fuzzy hypersoft topology, intuitionistic hypersoft topology and neutrosophic hypersoft topology. Neutrosophic hypersoft topology is the generalized framework which generalizes intuitionistic hypersoft topology and fuzzy hypersoft topology. Ahsan et al. [2] studied a theoretical and analytical approach for fundamental framework of composite mappings on fuzzy hypersoft classes. Several researchers [5, 6, 11, 12, 13, 14, 15, 22, 23, 27, 28] studied different kind of open sets in neutrosophic soft, neutrosophic crisp, fuzzy hypersoft, neutrosophic hypersoft topological spaces and studied its maps.

Surya et al. [24, 25, 26] and Sri et al. [21] introduced q-Rung linear diophantine fuzzy hypersoft set and Bipolar linear diophantine fuzzy hypersoft set and studied its appliations in MADM. Ajay et al. [4] defined fuzzy hypersoft semi-open sets and developed an application in multiattribute group decision making. Kabbur et al. [9] and Arora et al. studied similarity measures in soft nano sets and pythagorean fuzzy sets. Said Broumi and Irfan Deli [16] studied application of medical disgnosis in neutrosophic refined sets. Saqlain et al. [17] studied single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft set.

In hypersoft environment, some kind of open sets are introduced and their applications are studied so far. No investigation on continuous maps is initiated. There is a need to study continuity in the hypersoft environment because it is a fundamental concept in topology and has many applications in contemporary mathematics. This leads us to develop continuity and semi continuity in fuzzy hypersoft topological spaces.

In this paper, we develop the concept of fuzzy hypersoft continuity and semi continuity in fuzzy hypersoft topological spaces and some of their basic properties are analyzed with examples. Added to that, we discuss some characterizations and properties of fuzzy hypersoft irresolute maps. Also, an application in Covid-19 diagnosis is explained with the example using tangent similarity measure.

2 Preliminaries

Definition 2.1. [29] Let $\mathfrak{M}$ be an initial universe. A function λ from $\mathfrak{M}$ into the unit interval I is called a fuzzy set in $\mathfrak{M}$. For every $\mathfrak{m} \in \mathfrak{M}$, $\lambda(\mathfrak{m}) \in I$ is called the grade of membership of $\mathfrak{m}$ in λ. Some authors say that λ is a fuzzy subset of $\mathfrak{M}$ instead of saying that λ is a fuzzy set in $\mathfrak{M}$. The class of all fuzzy sets from $\mathfrak{M}$ into the closed unit interval I will be denoted by $I^{\mathfrak{M}}$.

Definition 2.2. [10] Let $\mathfrak{M}$ be an initial universe, Q be a set of parameters and $\mathcal{P}(\mathfrak{M})$ be the power set of $\mathfrak{M}$. A pair $(\tilde{H}, Q)$ is called the a soft set over $\mathfrak{M}$ where $\tilde{H}$ is a mapping $\tilde{H} : Q \to \mathcal{P}(\mathfrak{M})$. In other words, the soft set is a parametrized family of subsets of the set $\mathfrak{M}$.

Definition 2.3. [20] Let $\mathfrak{M}$ be an initial universe and $\mathcal{P}(\mathfrak{M})$ be the power set of $\mathfrak{M}$. Consider $q_1, q_2, q_3, ..., q_n$ for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $Q_1,\ Q_2,\ ...,\ Q_n$ with $Q_i \cap Q_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$. Then the pair $(\tilde{H},\ Q_1 \times Q_2 \times ... \times Q_n)$ where $\tilde{H} : Q_1 \times Q_2 \times ... \times Q_n \to \mathcal{P}(\mathfrak{M})$ is called a hypersoft set over $\mathfrak{M}$.

Definition 2.4. [1] Let $\mathfrak{M}$ be an initial universal set and $Q_1, Q_2, ..., Q_n$ be pairwise disjoint sets of parameters. Let $\mathcal{P}(\mathfrak{M})$ be the set of all fuzzy sets of $\mathfrak{M}$. Let E_i be the nonempty subset of the pair Q_i for each $i = 1, 2, ..., n$. A fuzzy hypersoft set (briefly, $FHSs$) over $\mathfrak{M}$ is defined as the pair $(\tilde{H}, E_1 \times E_2 \times ... \times E_n)$ where $\tilde{H} : E_1 \times E_2 \times ... \times E_n \to \mathcal{P}(\mathfrak{M})$ and $\tilde{H}(E_1 \times E_2 \times ... \times E_n) = \{(q, \langle \mathfrak{m}, \mu_{\tilde{H}(q)}(\mathfrak{m})\rangle : \mathfrak{m} \in \mathfrak{M}) : q \in E_1 \times E_2 \times ... \times E_n \subseteq Q_1 \times Q_2 \times ... \times Q_n\}$ where $\mu_{\tilde{H}(q)}(\mathfrak{m})$ is the membership value such that $\mu_{\tilde{H}(q)}(\mathfrak{m}) \in [0, 1]$.

Definition 2.5. [1] Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be two $FHSs$'s over $\mathfrak{M}$. Then $(\tilde{H}, \wedge_1)$ is the fuzzy hypersoft subset of $(\tilde{G}, \wedge_2)$ if $\mu_{\tilde{H}(q)}(\mathfrak{m}) \leq \mu_{\tilde{G}(q)}(\mathfrak{m})$.

It is denoted by $(\tilde{H}, \wedge_1) \subseteq (\tilde{G}, \wedge_2)$.

Definition 2.6. [1] Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be $FHSs$'s over $\mathfrak{M}$. $(\tilde{H}, \wedge_1)$ is equal to $(\tilde{G}, \wedge_1)$ if $\mu_{\tilde{H}(q)}(\mathfrak{m}) = \mu_{\tilde{G}(q)}(\mathfrak{m})$.

Definition 2.7. [1] A $FHSs$ $(\tilde{H}, \wedge)$ over the universe set $\mathfrak{M}$ is said to be null fuzzy hypersoft set if $\mu_{\tilde{H}(q)}(\mathfrak{m}) = 0$, $\forall q \in \wedge$ and $\mathfrak{m} \in \mathfrak{M}$. It is denoted by $\tilde{0}_{(\mathfrak{M},Q)}$.

A $FHSs$ $(\tilde{G}, \wedge)$ over the universal set $\mathfrak{M}$ is said to be absolute fuzzy hypersoft set if $\mu_{\tilde{H}(q)}(\mathfrak{m}) = 1$ $\forall q \in \wedge$ and $\mathfrak{m} \in \mathfrak{M}$. It is denoted by $\tilde{1}_{(\mathfrak{M},Q)}$.

Clearly, $\tilde{0}^c_{(\mathfrak{M},Q)} = \tilde{1}_{(\mathfrak{M},Q)}$ and $\tilde{1}^c_{(\mathfrak{M},Q)} = \tilde{0}_{(\mathfrak{M},Q)}$.

Definition 2.8. [1] Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge)$ be $FHSs$ over $\mathfrak{M}$. $(\tilde{H}, \wedge)^c$ is the complement of $(\tilde{H}, \wedge)$ if $\mu^c_{\tilde{H}(q)}(\mathfrak{m}) = \tilde{1}_{(\mathfrak{M},Q)} - \mu_{\tilde{H}(q)}(\mathfrak{m})$ where $\forall q \in \wedge$ and $\forall \mathfrak{m} \in \mathfrak{M}$. It is clear that $((\tilde{H}, \wedge)^c)^c = (\tilde{H}, \wedge)$.

Definition 2.9. [1] Let $\mathfrak{M}$ be the universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be $FHSs$'s over $\mathfrak{M}$. Extended union $(\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)$ is defined as

$$\mu((\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)) = \begin{cases} \mu_{\tilde{H}(q)}(\mathfrak{m}) & \text{if } q \in \wedge_1 - \wedge_2 \\ \mu_{\tilde{G}(q)}(\mathfrak{m}) & \text{if } q \in \wedge_2 - \wedge_1 \\ max\{\mu_{\tilde{H}(q)}(\mathfrak{m}), \mu_{\tilde{G}(q)}(\mathfrak{m})\} & \text{if } q \in \wedge_1 \cap \wedge_2 \end{cases}$$

Definition 2.10. [1, 3] Let $\mathfrak{M}$ be the universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be $FHSs$'s over $\mathfrak{M}$. Extended intersection $(\tilde{H}, \wedge_1) \cap (\tilde{G}, \wedge_2)$ is defined as

$$\mu((\tilde{H}, \wedge_1) \cap (\tilde{G}, \wedge_2)) = \begin{cases} \mu_{\tilde{H}(q)}(\mathfrak{m}) & ifq \in \wedge_1 - \wedge_2 \\ \mu_{\tilde{G}(q)}(\mathfrak{m}) & ifq \in \wedge_2 - \wedge_1 \\ min\{\mu_{\tilde{H}(q)}(\mathfrak{m}), \mu_{\tilde{G}(q)}(\mathfrak{m})\} & ifq \in \wedge_1 \cap \wedge_2 \end{cases}$$

Definition 2.11. [3] Let $(\mathfrak{M}, Q)$ be the family of all $FHSs$'s over the universe set $\mathfrak{M}$ and $\tau \subseteq FHSs(\mathfrak{M}, Q)$. Then τ is said to be a fuzzy hypersoft topology (briefly, $FHSt$) on $\mathfrak{M}$ if

(i) $\tilde{0}_{(\mathfrak{M},Q)}$ and $\tilde{1}_{(\mathfrak{M},Q)}$ belongs to τ

(ii) the union of any number of $FHSs$'s in τ belongs to τ

(iii) the intersection of finite number of $FHSs$'s in τ belongs to τ.

Then $(\mathfrak{M}, Q, \tau)$ is called a fuzzy hypersoft topological space (briefly, $FHSts$) over $\mathfrak{M}$. Each member of τ is said to be fuzzy hypersoft open set (briefly, $FHSos$). A $FHSs$ $(\tilde{H}, \wedge)$ is called a fuzzy hypersoft closed set (briefly, $FHScs$) if its complement $(\tilde{H}, \wedge)^C$ is $FHSos$.

Definition 2.12. [3] Let $(\mathfrak{M}, Q, \tau)$ be a $FHSts$ over $\mathfrak{M}$ and $(\tilde{H}, \wedge)$ be a $FHSs$ in $\mathfrak{M}$. Then,

(i) the fuzzy hypersoft interior (briefly, $FHSint$) of $(\tilde{H}, \wedge)$ is defined as $FHSint(\tilde{H}, \wedge) = \cup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge)$ where $(\tilde{G}, \wedge)$ is $FHSos\}$.

(ii) the fuzzy hypersoft closure (briefly, $FHScl$) of $(\tilde{H}, \wedge)$ is defined as $FHScl(\tilde{H}, \wedge) = \cap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq (\tilde{H}, \wedge)$ where $(\tilde{G}, \wedge)$ is $FHScs\}$.

Definition 2.13. [4] Let $(\mathfrak{M}, Q, \tau)$ be a $FHSts$ over $\mathfrak{M}$ and $(\tilde{H}, \wedge)$ be a $FHSs$ in $\mathfrak{M}$. Then, $(\tilde{H}, \wedge)$ is called the fuzzy hypersoft semiopen set (briefly, $FHSSos$) if $(\tilde{H}, \wedge) \subseteq FHScl(int(\tilde{H}, \wedge))$.

A $FHSs$ $(\tilde{H}, \wedge)$ is called a fuzzy hypersoft semiclosed set (briefly, $FHSScs$) if its complement $(\tilde{H}, \wedge)^c$ is a $FHSSos$.

Definition 2.14. [2] Let $(\mathfrak{M}, L)$ and $(\mathfrak{N}, M)$ be classes of $FHSs$'s over $\mathfrak{M}$ and $\mathfrak{N}$ with attributes L and M respectively. Let $\omega : \mathfrak{M} \to \mathfrak{N}$ and $\nu : L \to M$ be mappings. Then a FHS mappings $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ is defined as follows, for a $FHSs$ $(\tilde{H}, \wedge)_A$ in $(\mathfrak{M}, L)$, $f(\tilde{H}, \wedge)_A$ is a $FHSs$ in $(\mathfrak{N}, M)$ obtained as follows, for $\beta \in \nu(L) \subseteq M$ and $\mathfrak{n} \in \mathfrak{N}$, $\mathfrak{h}(\tilde{H}, \wedge)_A(\beta)(\mathfrak{n}) = \bigcup_{\alpha \in \nu^{-1}(\beta) \bigcap A, s \in \omega^{-1}(\mathfrak{n})} (\alpha)\mu_s \mathfrak{h}(\tilde{H}, \wedge)_A$ is called a fuzzy hypersoft image of a $FHSs$ $(\tilde{H}, \wedge)$. Hence $((\tilde{H}, \wedge)_A, \mathfrak{h}(\tilde{H}, \wedge)_A) \in \mathfrak{h}$, where $(\tilde{H}, \wedge)_A \subseteq (\mathfrak{M}, L), \mathfrak{h}(\tilde{H}, \wedge)_A \subseteq (\mathfrak{N}, M)$.

Definition 2.15. [2] If $\mathfrak{h} : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping, then FHS class $(\mathfrak{M}, L)$ is called the domain of $\mathfrak{h}$ and the FHS class $(\tilde{G}, \wedge) \in (\mathfrak{N}, M) : (\tilde{G}, \wedge) = \mathfrak{h}(\tilde{H}, \wedge))$ for some $(\tilde{H}, \wedge) \in (\mathfrak{M}, L)$ is called the range of $\mathfrak{h}$. The FHS class $(\mathfrak{N}, M)$ is called co-domain of $\mathfrak{h}$.

Definition 2.16. [2] If $\mathfrak{h} : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping and $(\tilde{G}, \wedge)_B$, a $FHSs$ in $(\mathfrak{N}, M)$ where $\omega : \mathfrak{M} \to \mathfrak{N}$, $\nu : L \to M$ and $B \subseteq M$. Then $\mathfrak{h}^{-1}(\tilde{G}, \wedge)_B$ is a $FHSs$ in $(\mathfrak{M}, L)$ defined as follows, for $\alpha \in \nu^{-1}(B) \subseteq L$ and $\mathfrak{m} \in \mathfrak{M}$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)_B(\alpha)(\mathfrak{m}) = (\nu(\alpha))\mu_p(\mathfrak{m})\mathfrak{h}^{-1}(\tilde{G}, \wedge)_B$ is called a FHS inverse image of $(\tilde{G}, \wedge)_B$.

Definition 2.17. [2] Let $\mathfrak{h} = (\omega, \nu)$ be a FHS mapping of a FHS class $(\mathfrak{M}, L)$ into a FHS class $(\mathfrak{N}, M)$. Then

(i) $\mathfrak{h}$ is said to be a one-one (or injection) FHS mapping if for both $\omega : \mathfrak{M} \to \mathfrak{N}$ and $\nu : L \to M$ are one-one.

(ii) $\mathfrak{h}$ is said to be a onto (or surjection) FHS mapping if for both $\omega : \mathfrak{M} \to \mathfrak{N}$ and $\nu : L \to M$ are onto.

If $\mathfrak{h}$ is both one-one and onto, then $\mathfrak{h}$ is called a one-one onto (or bijective) correspondance of FHS mapping.

Definition 2.18. [2] If $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ and $g = (m, n) : (\mathfrak{N}, M) \to (P, N)$ are two FHS mappings, then their composite $\mathfrak{g} \circ \mathfrak{h}$ is a FHS mapping of $(\mathfrak{M}, L)$ into (P, N) such that for every $(\tilde{H}, \wedge)_A \in (\mathfrak{M}, L)$, $(\mathfrak{g} \circ \mathfrak{h})(\tilde{H}, \wedge)_A = \mathfrak{g}(\mathfrak{h}(\tilde{H}, \wedge)_A)$. For $\beta \in n(M) \subseteq N$ and $p \in P$, it is defined as $\mathfrak{g}(\mathfrak{h}(\tilde{H}, \wedge)_A(\beta)(p) = \bigcup_{\alpha \in n^{-1}(\beta) \bigcap \mathfrak{h}(A), s \in m^{-1}(p)} (\alpha)\mu_s$.

Definition 2.19. [2] Let $\mathfrak{h} = (\omega, \nu)$ be a FHS mapping where $\omega : \mathfrak{M} \to \mathfrak{M}$ and $\nu : L \to L$. Then $\mathfrak{h}$ is said to be a FHS identity mapping if for both ω and ν are identity mappings.

Definition 2.20. [2] A one-one onto FHS mapping $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ is called FHS invertable mapping. Its FHS inverse mapping is denoted by $\mathfrak{h}^{-1} = (\omega^{-1}, \nu^{-1}) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$.

Definition 2.21. [18] Let $\mathfrak{M}$ be an initial universal set and $Q_1, Q_2, ..., Q_n$ be pairwise disjoint sets of parameters. Let $\mathcal{P}(\mathfrak{M})$ be the set of all neutrosophic sets of $\mathfrak{M}$. Let E_i be the nonempty subset of the pair Q_i for each $i = 1, 2, ..., n$. A neutrosophic hypersoft set (briefly, N_sHSs) over $\mathfrak{M}$ is defined as the pair $(\tilde{H}, E_1 \times E_2 \times ... \times E_n)$ where $\tilde{H} : E_1 \times E_2 \times ... \times E_n \to \mathcal{P}(\mathfrak{M})$ and $\tilde{H}(E_1 \times E_2 \times ... \times E_n) = \{(q, \langle \mathfrak{m}, \mu_{\tilde{H}(q)}(\mathfrak{m}), \sigma_{\tilde{H}(q)}(\mathfrak{m}), \nu_{\tilde{H}(q)}(\mathfrak{m}) \rangle : \mathfrak{m} \in \mathfrak{M}) : q \in E_1 \times E_2 \times ... \times E_n \subseteq Q_1 \times Q_2 \times ... \times Q_n\}$ where $\mu_{\tilde{H}(q)}(\mathfrak{m})$ is the membership value of truthiness, $\sigma_{\tilde{H}(q)}(\mathfrak{m})$ is the membership value of indeterminacy and $\nu_{\tilde{H}(q)}(\mathfrak{m})$ is the membership value of falsity such that $\mu_{\tilde{H}(q)}(\mathfrak{m}), \sigma_{\tilde{H}(q)}(\mathfrak{m}), \nu_{\tilde{H}(q)}(\mathfrak{m}) \in [0, 1]$. Also, $0 \leq \mu_{\tilde{H}(q)}(\mathfrak{m}) + \sigma_{\tilde{H}(q)}(\mathfrak{m}) + \nu_{\tilde{H}(q)}(\mathfrak{m}) \leq 3$.

Definition 2.22. [17] Consider two neutrosophic hypersoft sets $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ over $\mathfrak{M}$. The tangent similarity measure for these two sets are given by $S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2)) = \{\frac{1}{n} \sum_{i=1}^{n} [1 - tan\frac{\pi}{12}(|\mu^i_H - \mu^i_G| + |\sigma^i_H - \sigma^i_G| + |\nu^i_H - \nu^i_G|)]\}$.

3 Fuzzy Hypersoft Continuous Maps

In this section, fuzzy hypersoft continuous maps are introduced and its related properties are discusssed.

Definition 3.1. Consider any two FHSts $(\mathfrak{M}, L, \tau)$ and $(\mathfrak{N}, M, \sigma)$. A map $\mathfrak{h} : (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is called as FHS

(i) continuous (in short,$FHSCts$) if the inverse image of each $FHSos$ in $(\mathfrak{N}, M, \sigma)$ is a $FHSos$ in $(\mathfrak{M}, L, \tau)$.

(ii) semi-continuous (in short, $FHSSCts$) if the inverse image of each $FHSos$ in $(\mathfrak{N}, M, \sigma)$ is a $FHSSos$ in $(\mathfrak{M}, L, \tau)$.

Example 3.2. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the FHS initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$$
$$Q'_1 = \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_3)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$
$$(\tilde{H}_2, \wedge_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$
$$(\tilde{H}_3, \wedge_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$
$$(\tilde{H}_4, \wedge_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_3), (\tilde{H}_4, \wedge_3)\}$ is $FHSts$.

Let the $FHSs's$ $(\tilde{G}_1, \wedge_1), (\tilde{G}_2, \wedge_2), (\tilde{G}_3, \wedge_3), (\tilde{G}_4, \wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1, \wedge_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.6}, \frac{\mathfrak{n}_2}{0.8}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.7}\}\rangle \end{Bmatrix}$$
$$(\tilde{G}_2, \wedge_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.3}, \frac{\mathfrak{n}_2}{0.2}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{Bmatrix}$$
$$(\tilde{G}_3, \wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.3}, \frac{\mathfrak{n}_2}{0.2}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.7}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{Bmatrix}$$
$$(\tilde{G}_4, \wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.6}, \frac{\mathfrak{n}_2}{0.8}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.7}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1, \wedge_1), (\tilde{G}_2, \wedge_2), (\tilde{G}_3, \wedge_3), (\tilde{G}_4, \wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:

$$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$$
$$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$$
$$\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_1, \wedge_1), \mathfrak{h}^{-1}(\tilde{G}_2, \wedge_2) = (\tilde{H}_2, \wedge_2),$$
$$\mathfrak{h}^{-1}(\tilde{G}_3, \wedge_3) = (\tilde{H}_3, \wedge_3), \mathfrak{h}^{-1}(\tilde{G}_4, \wedge_3) = (\tilde{H}_4, \wedge_3)$$

The inverse image of each $FHSos$ in $(\mathfrak{N}, M, \sigma)$ is a $FHSos$ in $(\mathfrak{M}, L, \tau)$.
$\therefore \mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ is $FHSCts$.

Example 3.3. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the FHS initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$$
$$Q'_1 = \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_3)$, $(\tilde{H}_5, \wedge_1)$, $(\tilde{H}_6, \wedge_2)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$
$$(\tilde{H}_2, \wedge_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{H}_3, \wedge_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{Bmatrix}$$
$$(\tilde{H}_4, \wedge_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{Bmatrix}$$
$$(\tilde{H}_5, \wedge_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{Bmatrix}$$
$$(\tilde{H}_6, \wedge_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.3}, \frac{\mathfrak{m}_2}{0.4}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_3), (\tilde{H}_4, \wedge_3)\}$ is $FHSts$.

Let the $FHSs$ $(\tilde{G}_1, \wedge_1)$ be defined as

$$(\tilde{G}_1, \wedge_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.5}\} \rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1, \wedge_1)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:

$$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$$
$$\nu(a_1, b_1) = (c_2, d_1),\ \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$$
$$\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_5, \wedge_1)$$

$(\tilde{G}_1, \wedge_1)$ is $FHSos$ in $\mathfrak{N}$ and $\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_5, \wedge_1)$ is $FHSSos$ in $\mathfrak{M}$.
The inverse image of each $FHSos$ in $(\mathfrak{N}, M, \sigma)$ is a $FHSSos$ in $(\mathfrak{M}, L, \tau)$.
$\therefore \mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ $FHSSCts$.

Proposition 3.4. *Each $FHSCts$ is $FHSSCts$. But the converse need not be true.*

Proof. Let $(\tilde{G}, \wedge)$ be a $FHSos$ in $\mathfrak{N}$. Since $\mathfrak{h}$ is $FHSCts$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHSos$ in $\mathfrak{M}$. Since all $FHSos$ are $FHSSos$ [4], $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHSSos$ in $\mathfrak{M}$. Hence $\mathfrak{h}$ is a $FHSSCts$. □

Example 3.5. In Example 3.3, $\mathfrak{h}$ is $FHSSCts$ but not $FHSCts$ because $(\tilde{G}_1, \wedge_1)$ is $FHSos$ in $\mathfrak{M}$ but $\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_5, \wedge_1)$ is not $FHSos$ in $\mathfrak{N}$.

Theorem 3.6. *A map $\mathfrak{h} : (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is $FHSCts$ iff the inverse image of each $FHScs$ in $\mathfrak{N}$ is $FHScs$ in $\mathfrak{M}$.*

Proof. Let $(\tilde{G}, \wedge)$ be a $FHScs$ in $\mathfrak{N}$. This implies that $(\tilde{G}, \wedge)^c$ is $FHSos$ in $\mathfrak{N}$. Since $\mathfrak{h}$ is $FHSCts$, $\mathfrak{h}^{-1}((\tilde{G}, \wedge)^c)$ is $FHSos$ in $\mathfrak{M}$. Since $\mathfrak{h}^{-1}((\tilde{G}, \wedge)^c) = (\mathfrak{h}^{-1}(\tilde{G}, \wedge))^c, \mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is a $FHScs$ in $\mathfrak{M}$.

Conversely, let $(\tilde{G}, \wedge)$ be a $FHSos$ in $\mathfrak{N}$. Then $(\tilde{G}, \wedge)^c$ is a $FHScs$ in $\mathfrak{N}$. By hypothesis, $\mathfrak{h}^{-1}((\tilde{G}, \wedge)^c)$ is $FHScs$ in $\mathfrak{M}$. Since, $\mathfrak{h}^{-1}((\tilde{G}, \wedge)^c) = (\mathfrak{h}^{-1}(\tilde{G}, \wedge))^c, (\mathfrak{h}^{-1}(\tilde{G}, \wedge))^c$ is $FHScs$ in $\mathfrak{M}$. Therefore, $(\mathfrak{h}^{-1}(\tilde{G}, \wedge))$ is a $FHSos$ in $\mathfrak{M}$. Hence, $\mathfrak{h}$ is $FHSCts$. □

Theorem 3.7. *Let $\mathfrak{h} : (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ be a $FHSCts$ map and $\mathfrak{g} : (\mathfrak{N}, M, \sigma) \to (P, N, \rho)$ be a $FHSCts$, then $\mathfrak{g} \circ \mathfrak{h} : (\mathfrak{M}, L, \tau) \to (P, N, \rho)$ is a $FHSCts$.*

Proof. Let $(\tilde{K}, \wedge)$ be a $FHSos$ in P. Then $\mathfrak{g}^{-1}(\tilde{K}, \wedge)$ is a $FHSos$ in $\mathfrak{N}$, by hypothesis. Since $\mathfrak{h}$ is a $FHSCts$ map $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\tilde{K}, \wedge))$ is a $FHSos$ in $\mathfrak{M}$. Hence $\mathfrak{g} \circ \mathfrak{h}$ is a $FHSCts$ map. □

Theorem 3.8. *Let $\mathfrak{h} : (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ be a $FHSCts$ map. Then the following conditions are hold:*

(i) $\mathfrak{h}(FHScl(\tilde{H}, \wedge)) \leq FHScl(\mathfrak{h}(\tilde{H}, \wedge))$, *for all* $FHScs(\tilde{H}, \wedge)$ *in* $\mathfrak{M}$.

(ii) $FHScl(\mathfrak{h}^{-1}(\tilde{G}, \wedge)) \leq \mathfrak{h}^{-1}(FHScl(\tilde{G}, \wedge))$, *for all* $FHScs(\tilde{G}, \wedge)$ *in* $\mathfrak{N}$.

Proof. (i) As $FHScl(\mathfrak{h}(\tilde{H},\wedge))$ is a $FHScs$ in $\mathfrak{N}$ and $\mathfrak{h}$ is $FHSCts$, $\mathfrak{h}^{-1}(FHScl(\mathfrak{h}(\tilde{H},\wedge)))$ is a $FHScs$ in $\mathfrak{M}$. Now, as $(\tilde{H},\wedge) \leq \mathfrak{h}^{-1}(FHScl(\mathfrak{h}(\tilde{H},\wedge)))$, we have $FHScl(\tilde{H},\wedge) \leq \mathfrak{h}^{-1}(FHScl(\mathfrak{h}(\tilde{H},\wedge)))$. Therefore, $\mathfrak{h}(FHScl(\tilde{H},\wedge)) \leq FHScl(\mathfrak{h}(\tilde{H},\wedge))$.

(ii) By replacing $(\tilde{H},\wedge)$ with $(\tilde{G},\wedge)$ in (i), it is followed that $\mathfrak{h}(FHScl(\mathfrak{h}^{-1}(\tilde{G},\wedge))) \leq FHScl(\mathfrak{h}(\mathfrak{h}^{-1}(\tilde{G},\wedge))) \leq FHScl(\tilde{G},\wedge)$. Hence, $FHScl(\mathfrak{h}^{-1}(\tilde{G},\wedge)) \leq \mathfrak{h}^{-1}(FHScl(\tilde{G},\wedge))$. □

Remark 3.9. If $\mathfrak{h}$ is $FHSCts$ then,

(i) $\mathfrak{h}(FHScl(\tilde{H},\wedge))$ need not be equal to $FHScl(\mathfrak{h}(\tilde{H},\wedge))$ where $(\tilde{H},\wedge) \in \mathfrak{M}$.

(ii) $FHScl(\mathfrak{h}^{-1}(\tilde{G},\wedge))$ need not be equal to $\mathfrak{h}^{-1}(FHScl(\tilde{G},\wedge))$ where $(\tilde{G},\wedge) \in \mathfrak{N}$.

Example 3.10. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the FHS initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q_1' \times Q_2'$ respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$$
$$Q_1' = \{c_1, c_2, c_3\}, Q_2' = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{H}_1, \wedge_1)$ and $(\tilde{H}_2, \wedge_2)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \begin{Bmatrix} \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.4}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.4}, \frac{\mathfrak{m}_2}{0.2}\}\rangle \end{Bmatrix}$$
$$(\tilde{H}_2, \wedge_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (a_2, b_2), \{\frac{\mathfrak{m}_1}{0.6}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1)\}$ is $FHSts$.

Let the $FHSs's$ $(\tilde{G}_1, \wedge_1)$ and $(\tilde{G}_2, \wedge_2)$ over the universe $\mathfrak{N}$ be defined as

$$(\tilde{G}_1, \wedge_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.3}, \frac{\mathfrak{n}_2}{0.4}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.2}, \frac{\mathfrak{n}_2}{0.4}\}\rangle \end{Bmatrix}$$
$$(\tilde{G}_2, \wedge_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.3}, \frac{\mathfrak{n}_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.6}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1, \wedge_1)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:

$$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$$
$$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_2, d_1),\ \nu(a_1, b_2) = (c_1, d_2),\ \nu(a_2, b_2) = (c_2, d_2)$$
$$\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_1, \wedge_1), \mathfrak{h}^{-1}(\tilde{G}_2, \wedge_2) = (\tilde{H}_2, \wedge_2)$$

Then $\mathfrak{h}$ is $FHSCts$.

(i) $\mathfrak{h}(FHScl(\tilde{H}_2, \wedge_2)) = \tilde{0}_{(\mathfrak{N},Q)}$, but $FHScl(\mathfrak{h}(\tilde{H}_2, \wedge_2)) = (\tilde{G}_1, \wedge_1)^c$. Hence $\mathfrak{h}(FHScl(\tilde{H}_2, \wedge_2)) \neq FHScl(\mathfrak{h}(\tilde{H}_2, \wedge_2))$.

(ii) $FHScl(\mathfrak{h}^{-1}(\tilde{G}_2, \wedge_2)) = \tilde{0}_{(\mathfrak{M},Q)}$, but $\mathfrak{h}^{-1}(FHScl(\tilde{G}_2, \wedge_2)) = (\tilde{H}_1, \wedge_1)^c$. Hence $FHScl(\mathfrak{h}^{-1}(\tilde{G}_2, \wedge_2)) \neq \mathfrak{h}^{-1}(FHScl(\tilde{G}_2, \wedge_2))$.

Theorem 3.11. $\mathfrak{h}$ *is* $FHSCts$ *iff* $\mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge)) \leq FHSint(\tilde{G},\wedge)$ *for all* $FHScs$ $(\tilde{G},\wedge)$ *in* $\mathfrak{N}$.

Proof. Let $\mathfrak{h}$ be a $FHSCts$ and $(\tilde{G},\wedge) \in \mathfrak{N}$. $FHSint(\tilde{G},\wedge)$ is $FHSos$ in $\mathfrak{N}$ and hence, $\mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge))$ is a $FHSos$ in $\mathfrak{M}$. Therefore, $FHSint(\mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge))) = \mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge))$. Also, $FHSint(\tilde{G},\wedge) \leq (\tilde{G},\wedge)$ implies that $\mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge)) \leq \mathfrak{h}^{-1}(\tilde{G},\wedge)$. Therefore, $FHSint(\mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge))) \leq FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$. That is, $\mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge)) \leq FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$.

Conversely, let $\mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge)) \leq FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$ for all subset $(\tilde{G},\wedge)$ of $\mathfrak{N}$. If $(\tilde{G},\wedge)$ is $FHSos$ in $\mathfrak{N}$, then $FHSint(\tilde{G},\wedge) = (\tilde{G},\wedge)$. By assumption, we have

$\mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge)) \leq FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$. Thus $(\mathfrak{h}^{-1}(\tilde{G},\wedge) \leq FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$. But $FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge)) = \mathfrak{h}^{-1}(\tilde{G},\wedge)$. Therefore, $FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge)) = \mathfrak{h}^{-1}(\tilde{G},\wedge)$. That is, $\mathfrak{h}^{-1}(\tilde{G},\wedge)$ is $FHSos$ in $\mathfrak{M}$, for all $FHSos(\tilde{G},\wedge)$ in $\mathfrak{N}$. Therefore, $\mathfrak{h}$ is $FHSCts$ on $\mathfrak{M}$. □

Remark 3.12. If $\mathfrak{h}$ is $FHSCts$, then $FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$ need not be equal to $\mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge))$ where $(\tilde{G},\wedge) \in \mathfrak{N}$.

Example 3.13. Consider the Example 3.10.
$\mathfrak{h}^{-1}(\tilde{G}_2,\wedge_2) = (\tilde{H}_2,\wedge_2)$
$FHSint(\tilde{H}_2,\wedge_2) = \tilde{0}_{(\mathfrak{M},Q)}$
$\therefore FHSint(\mathfrak{h}^{-1}(\tilde{G}_2,\wedge_2)) = \tilde{0}_{(\mathfrak{M},Q)}$.
But $FHSint(\tilde{G}_2,\wedge_2) = (\tilde{G}_1,\wedge_1)$
$\mathfrak{h}^{-1}(\tilde{G}_1,\wedge_1) = (\tilde{H}_1,\wedge_1)$
$\therefore \mathfrak{h}^{-1}(FHSint(\tilde{G}_2,\wedge_2)) = (\tilde{H}_1,\wedge_1)$.
Hence, $FHSint(\mathfrak{h}^{-1}(\tilde{G}_2,\wedge_2)) \neq \mathfrak{h}^{-1}(FHSint(\tilde{G}_2,\wedge_2))$.
Therefore $FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$ need not be equal to $\mathfrak{h}^{-1}(FHSint(\tilde{G},\wedge))$.

4 Fuzzy Hypersoft Irresolute Maps

Fuzzy hypersoft irresolute maps are introduced and studied in this section.

Definition 4.1. A map $\mathfrak{h} : (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is called a FHS irresolute (in short, $FHSIrr$) map if $\mathfrak{h}^{-1}(\tilde{G},\wedge)$ is a $FHSSos$ in $(\mathfrak{M}, L, \tau)$ for every $FHSSos$ $(\tilde{G},\wedge)$ of $(\mathfrak{N}, M, \sigma)$.

Example 4.2. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the FHS initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$$
$$Q'_1 = \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{H}_1,\wedge_1)$, $(\tilde{H}_2,\wedge_2)$, $(\tilde{H}_3,\wedge_3)$, $(\tilde{H}_4,\wedge_3)$, $(\tilde{H}_5,\wedge_1)$, $(\tilde{H}_6,\wedge_2)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1,\wedge_1) = \left\{ \begin{array}{c} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{array} \right\}$$
$$(\tilde{H}_2,\wedge_2) = \left\{ \begin{array}{c} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{array} \right\}$$
$$(\tilde{H}_3,\wedge_3) = \left\{ \begin{array}{c} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{array} \right\}$$
$$(\tilde{H}_4,\wedge_3) = \left\{ \begin{array}{c} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{array} \right\}$$
$$(\tilde{H}_5,\wedge_1) = \left\{ \begin{array}{c} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{array} \right\}$$
$$(\tilde{H}_6,\wedge_2) = \left\{ \begin{array}{c} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.3}, \frac{\mathfrak{m}_2}{0.4}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{array} \right\}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{H}_1,\wedge_1), (\tilde{H}_2,\wedge_2), (\tilde{H}_3,\wedge_3), (\tilde{H}_4,\wedge_3)\}$ is $FHSts$.

Let the $FHSs(\tilde{G}_1,\wedge_1)$ be defined as

$$(\tilde{G}_1,\wedge_1) = \left\{ \begin{array}{c} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.2}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{array} \right\}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1, \wedge_1)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:

$$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$$
$$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$$
$$\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_5, \wedge_1)$$

$(\tilde{G}_1, \wedge_1)$ is $FHS\mathcal{S}os$ in $\mathfrak{N}$ and $\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_5, \wedge_1)$ is $FHS\mathcal{S}os$ in $\mathfrak{M}$.
The inverse image of each $FHS\mathcal{S}os$ in $(\mathfrak{N}, M, \sigma)$ is a $FHS\mathcal{S}os$ in $(\mathfrak{M}, L, \tau)$.
$\therefore \mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ $FHSIrr$.

Theorem 4.3. *Let $\mathfrak{h} : (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ be a $FHSIrr$ map. Then $\mathfrak{h}$ is a $FHS\mathcal{S}Cts$ map. But not conversely.*

Proof. Let $\mathfrak{h}$ be a $FHSIrr$ map. Let $(\tilde{G}, \wedge)$ be any $FHSos$ on $\mathfrak{N}$. Since every $FHSos$ is a $FHS\mathcal{S}os$, $(\tilde{G}, \wedge)$ is a $FHS\mathcal{S}os$ in $\mathfrak{N}$. By hypothesis, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is a $FHS\mathcal{S}os$ in $\mathfrak{M}$. Hence, $\mathfrak{h}$ is a $FHS\mathcal{S}Cts$ map. □

Example 4.4. Consider the Example 3.10.
Here $\mathfrak{h}$ is $FHS\mathcal{S}Cts$ because, $(\tilde{G}_1, \wedge_1)$ is $FHSos$ in $\mathfrak{N}$ and $\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_1, \wedge_1)$ is $FHS\mathcal{S}os$ in $\mathfrak{M}$ but $\mathfrak{h}$ is not $FHSIrr$ because $(\tilde{G}_2, \wedge_2)$ is $FHS\mathcal{S}os$ in $\mathfrak{N}$ and $\mathfrak{h}^{-1}(\tilde{G}_2, \wedge_2) = (\tilde{H}_2, \wedge_2)$ is not $FHS\mathcal{S}os$ in $\mathfrak{M}$. Then $\mathfrak{h}$ is a $FHS\mathcal{S}Cts$ map but not $FHSIrr$ map. Hence the converse part is proved.

Theorem 4.5. *Let $\mathfrak{h} : (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ and $\mathfrak{g} : (\mathfrak{N}, M, \sigma) \to (P, N, \rho)$ be $FHSIrr$ maps, then $\mathfrak{g} \circ \mathfrak{h} : (\mathfrak{M}, L, \tau) \to (P, N, \rho)$ is a $FHSIrr$ map.*

Proof. Let $(\tilde{K}, \wedge)$ be a $FHS\mathcal{S}os$ in P. Then $\mathfrak{g}^{-1}(\tilde{K}, \wedge)$ is a $FHS\mathcal{S}os$ in $\mathfrak{N}$. Since $\mathfrak{h}$ is a $FHSIrr$ map, $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\tilde{K}, \wedge))$ is a $FHS\mathcal{S}os$ in $\mathfrak{M}$. Hence $\mathfrak{g} \circ \mathfrak{h}$ $FHSIrr$ map. □

Theorem 4.6. *Let $\mathfrak{h} : (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ be a $FHSIrr$ map and $\mathfrak{g} : (\mathfrak{N}, M, \sigma) \to (P, N, \rho)$ is a $FHS\mathcal{S}Cts$ map, then $\mathfrak{g} \circ \mathfrak{h} : (\mathfrak{M}, L, \tau) \to (P, N, \rho)$ is a $FHS\mathcal{S}Cts$ map.*

Proof. Let $(\tilde{K}, \wedge)$ be a $FHS\mathcal{S}os$ in P. Then $\mathfrak{g}^{-1}(\tilde{K}, \wedge)$ is a $FHS\mathcal{S}os$ in $\mathfrak{N}$. Since, $\mathfrak{h}$ is a $FHSIrr$ map, $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\tilde{K}, \wedge))$ is a $FHS\mathcal{S}os$ in $\mathfrak{M}$. Hence, $\mathfrak{g} \circ \mathfrak{h}$ is a $FHS\mathcal{S}Cts$ map. □

5 Application in Covid-19 Diagnosis using Tangent Similarity Measure

Proposition 5.1. *The tangent similarity measure $S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2))$, satisfies the following properties:*

(i) $0 \leq S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2)) \leq 1$.

(ii) $S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2)) = S_T((\tilde{G}, \wedge_2), (\tilde{H}, \wedge_1))$.

(iii) $(\tilde{H}, \wedge_1) = (\tilde{G}, \wedge_2)$ *iff* $S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2)) = 1$.

(iv) If $(\tilde{P}, \wedge_3)$ is a $FHSs$ in $\mathfrak{M}$ and $(\tilde{H}, \wedge_1) \subseteq (\tilde{G}, \wedge_2) \subseteq (\tilde{P}, \wedge_3)$, then $S_T((\tilde{H}, \wedge_1), (\tilde{A}, \wedge_3)) \leq S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2))$ and $S_T((\tilde{H}, \wedge_1), (\tilde{A}, \wedge_3)) \leq S_T((\tilde{G}, \wedge_2), (\tilde{A}, \wedge_3))$.

Proof. (i) Since the value of tangent function and the membership value of $FHSs$'s are in the interval $[0, 1]$, the similarity measure based on the tangent functions which is arithmetic mean of these tangent functions, are also in $[0, 1]$.
Therefore, $0 \leq S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2)) \leq 1$.

(ii) Proof is obvious.

(iii) For any two $FHSs$'s $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ in $\mathfrak{M}$, if $(\tilde{H}, \wedge_1) = (\tilde{G}, \wedge_2)$, then $\mu^i_{(\tilde{H}, \wedge_1)} = \mu^i_{(\tilde{G}, \wedge_2)}$, for $i = 1, 2, ..., n$. Thus, we obtain $|\mu^i_{(\tilde{H}, \wedge_1)} - \mu^i_{(\tilde{G}, \wedge_2)}| = 0$.

And so the tangent similarity measure $S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2)) = 1$. Conversely, let $S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2)) = 1$. Since $tan\frac{\pi}{4} = 1$, this implies that $|\mu^i_{(\tilde{H}, \wedge_1)} - \mu^i_{(\tilde{G}, \wedge_2)}| = 0$.

Therefore, we obtain $\mu^i_{(\tilde{H}, \wedge_1)} = \mu^i_{(\tilde{G}, \wedge_2)}$, for $i = 1, 2, 3, ..., n$. Hence, $(\tilde{H}, \wedge_1) = (\tilde{G}, \wedge_2)$.

(iv) If $(\tilde{H}, \wedge_1) \subseteq (\tilde{G}, \wedge_2) \subseteq (\tilde{P}, \wedge_3)$, then $\mu^i_{(\tilde{H},\wedge_1)} \leq \mu^i_{(\tilde{G},\wedge_2)} \leq \mu^i_{(\tilde{P},\wedge_3)}$, for $i = 1, 2, 3, ..., n$.

Thus, we have

$|\mu^i_{(\tilde{H},\wedge_1)} - \mu^i_{(\tilde{G},\wedge_2)}| \leq |\mu^i_{(\tilde{H},\wedge_1)} - \mu^i_{(\tilde{P},\wedge_3)}|$, $|\mu^i_{(\tilde{G},\wedge_2)} - \mu^i_{(\tilde{P},\wedge_3)}| \leq |\mu^i_{(\tilde{H},\wedge_1)} - \mu^i_{(\tilde{P},\wedge_3)}|$

Hence, $(\tilde{H}, \wedge_1) \subseteq (\tilde{G}, \wedge_2) \subseteq (\tilde{P}, \wedge_3)$. Then, $S_T((\tilde{H}, \wedge_1), (\tilde{P}, \wedge_3)) \leq S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2))$ and $S_T((\tilde{H}, \wedge_1), (\tilde{P}, \wedge_3)) \leq S_T((\tilde{G}, \wedge_2), (\tilde{P}, \wedge_3))$.

As the tangent function is increasing with the interval $[0, \frac{\pi}{4}]$, the proof is completed.

Hence, the tangent similarity measure for these two sets are given by

$S_T((\tilde{H}, \wedge_1), (\tilde{G}, \wedge_2)) = \{\frac{1}{n} \sum_{i=1}^{n} [1 - tan\frac{\pi}{12}(|\mu^i_H - \mu^i_G|)]\}$. □

5.1 Algorithm

Now, the algorithm based on the proposed similarity measure is given.

As per the medical history, the various symptoms of Covid-19 are Fever, Headache, Dry Cough, Body pain, Chest pain and Difficulty in breathing. We categorize these symptoms as the distinct set of severe symptoms, most common symptom and less common symptoms.

Severe symptoms = Difficulty in breathing, Chest pain

Most common symptoms = Fever, Dry cough

Less common symptoms = Headache, Body pain

We can formulate the symptoms of the Covid-19 patients collected from the hospital records as FH_ySs's by considering the membership values as 'Covid-19' and 'No Covid-19'. Now, consider the patients visiting hospital with Covid-19 symptoms. Let us formulate those patients' symptoms as the FH_ySs's using the defined category of the symptoms. Using the proposed tangent similarity measure, the examination can be done by comparing the symptoms of the Covid-19 patients and the patients visiting hospital with the symptoms related to Covid-19. Thus, a decision can be made whether the patients have the possibility of suffering from Covid-19 or not.

We next give the implementation steps of the proposed algorithm based on cotangent similarity measure for FH_ySs's in which the flow chart of the proposed alogorithm is shown in the figure.

Step 1: Formulate the symptoms of Covid-19 patients as a FH_ySs by considering the degree of relation between the Covid-19 patients and the Covid-19 symptoms.

Step 2: Formulate the symptoms of the two patients visited the hospital as FH_ySs's by considering the relation between the patients and the Covid-19 symptoms.

Step 3: Find the similarity between the symptoms of the Covid-19 patients and the 1st patient visited hospital using the proposed tangent similarity measure.

Step 4: Find the similarity between the symptoms of the Covid-19 patients and the 2nd patient visited hospital using the proposed cotangent similarity measure.

Step 5: Compare both the similarity measures. The more the similarity, there is a higher chance for the patient to be suffering from Covid-19.

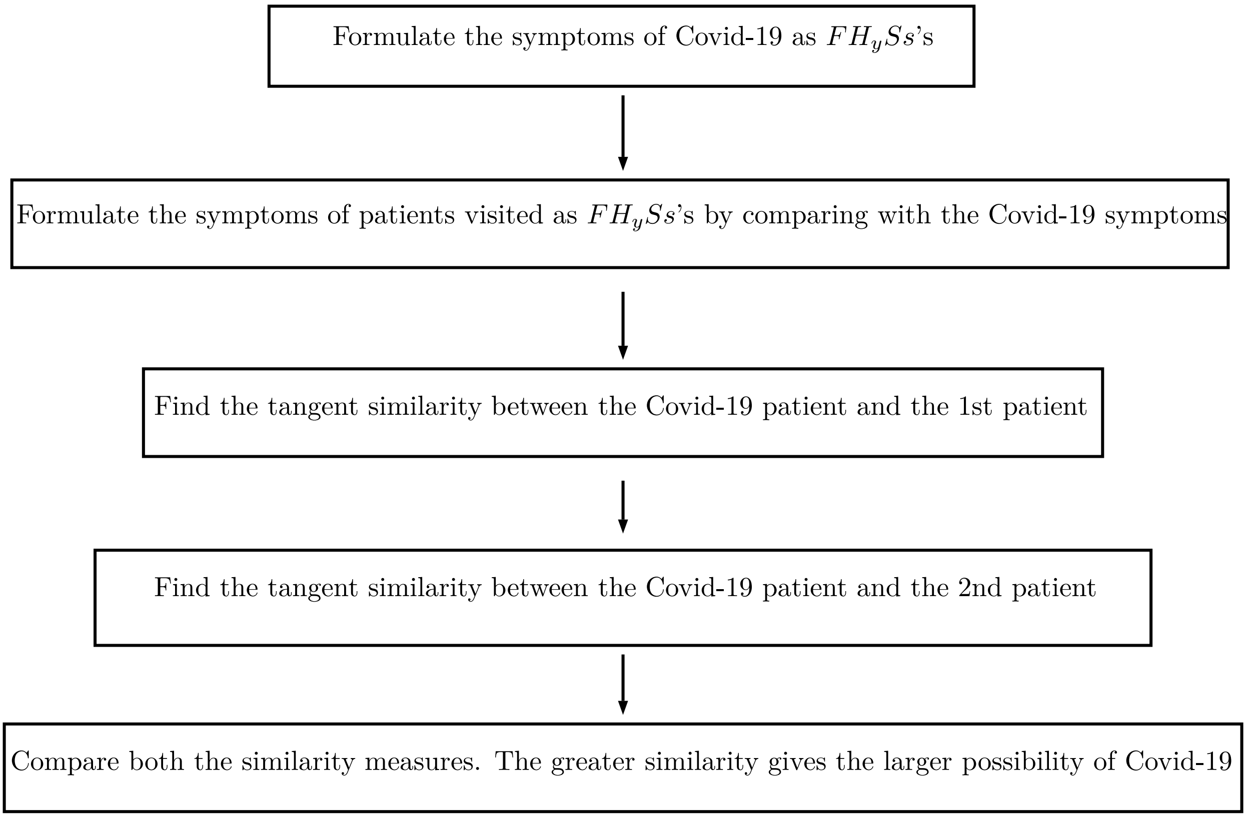

Figure 1. Flowchart of the proposed algorithm

Example 5.2. Consider 2 patients visiting hospital with the following symptoms: Fever, Dry cough, Head ache, Body pain, Difficulty in breathing and Chest pain. The symptoms of Covid-19 patients can be categorized as
Severe symptoms = Difficulty in breathing, Chest pain
Most common symptoms = Fever, Dry cough
Less common symptoms = Headache, Body pain
Using the fuzzy hypersoft model problem, the examination can be done whether the patients have the possibility of suffering from Covid-19 or not. Let $\mathfrak{M}$ be the universal set $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\} = \{\text{Covid-19, No Covid-19}\}$. The attributes are given as:

$$Q_1 = \{a_1 = \text{Difficulty in breathing}, a_2 = \text{Chest pain}\}$$
$$Q_2 = \{b_1 = \text{Fever}, b_2 = \text{Dry cough}\}$$
$$Q_3 = \{c_1 = \text{Headache}, c_2 = \text{Body pain}\}$$

We define the fuzzy hypersoft sets which give the degree of association between the Covid-19 patients and the Covid-19 symptoms and between the 2 patients visited and their symptoms.
The $FHSs$ $(\tilde{H}, \wedge)$ describes the evaluation of the Covid-19 patients and their symptoms as per the hospital records.

$$(\tilde{H}, \wedge) = \left\{ \begin{array}{c} \langle (a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{1.0}, \frac{\mathfrak{m}_2}{0.2}\} \rangle, \\ \langle (a_1, b_1, c_2), \{\frac{\mathfrak{m}_1}{0.9}, \frac{\mathfrak{m}_2}{0.1}\} \rangle, \\ \langle (a_1, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.9}, \frac{\mathfrak{m}_2}{0.2}\} \rangle, \\ \langle (a_1, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.2}\} \rangle, \\ \langle (a_2, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.9}, \frac{\mathfrak{m}_2}{0.1}\} \rangle, \\ \langle (a_2, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.1}\} \rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.1}\} \rangle, \\ \langle (a_2, b_1, c_2), \{\frac{\mathfrak{m}_1}{0.9}, \frac{\mathfrak{m}_2}{0.1}\} \rangle \end{array} \right\}$$

The $FHSs$'s $(\tilde{G}, \wedge)$ and $(\tilde{P}, \wedge)$ describe the evaluation of the 2 patients visited and their symptoms respectively.

$$(\tilde{G}, \wedge) = \left\{ \begin{matrix} \langle (a_1, b_1, c_1), \{\frac{m_1}{0.1}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_1, b_1, c_2), \{\frac{m_1}{0.1}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_1, b_2, c_1), \{\frac{m_1}{0.0}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_1, b_2, c_2), \{\frac{m_1}{0.1}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_2, b_1, c_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_2, b_2, c_1), \{\frac{m_1}{0.1}, \frac{m_2}{0.8}\} \rangle, \\ \langle (a_2, b_2, c_2), \{\frac{m_1}{0.1}, \frac{m_2}{0.9}\} \rangle, \\ \langle (a_2, b_1, c_2), \{\frac{m_1}{0.1}, \frac{m_2}{0.9}\} \rangle \end{matrix} \right\}$$

$$(\tilde{P}, \wedge) = \left\{ \begin{matrix} \langle (a_1, b_1, c_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.3}\} \rangle, \\ \langle (a_1, b_1, c_2), \{\frac{m_1}{0.7}, \frac{m_2}{0.2}\} \rangle, \\ \langle (a_1, b_2, c_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.4}\} \rangle, \\ \langle (a_1, b_2, c_2), \{\frac{m_1}{0.6}, \frac{m_2}{0.4}\} \rangle, \\ \langle (a_2, b_1, c_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.2}\} \rangle, \\ \langle (a_2, b_2, c_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.3}\} \rangle, \\ \langle (a_2, b_2, c_2), \{\frac{m_1}{0.7}, \frac{m_2}{0.3}\} \rangle, \\ \langle (a_2, b_1, c_2), \{\frac{m_1}{0.7}, \frac{m_2}{0.2}\} \rangle \end{matrix} \right\}$$

Using tangent similarity measure, we get
$S_T((\tilde{H}, \wedge), (\tilde{G}, \wedge)) = 0.7976$
$S_T((\tilde{H}, \wedge), (\tilde{P}, \wedge)) = 0.9623.$
As the similarity between the Covid-19 patient and the 2nd patient is lesser than 1st patient, there is larger possibility for the 2nd patient suffering from Covid-19.

There are several similarity measures in fuzzy environment such as tangent similarity measure, cotangent similarity measure, cosine similarity measure etc. If the similarity between the two sets is more close to 1, there is a possiblity of more similarity between the given two sets. Using this concept, we have arrived for a decision in the above example. The other kind of similarities also give the same results. All the similarities can be applied in both fuzzy and neutrosophic environments depends on the membership functions.

6 Conclusion

In this paper, $FHSCts$ and $FHSSCts$ maps are introduced and its properties are analyzed with the examples. Then $FHSCts$ maps are compared with $FHSSCts$ maps by means of theorems and examples. In addition, these maps are extended to $FHSIrr$ maps which is compared with $FHSCts$ and $FHSSCts$ maps. Further, an application in diagnosing Covid-19 using tangent similarity measure is discussed with an example. In coding, facial recognition and patterns, similarity measures have extensive scope of applications. In future, these findings can be extended to fuzzy hypersoft open mapping, fuzzy hypersoft closed mapping and fuzzy hypersoft homeomorphic functions. The application work can be extended with the other similarity measures, entropy, distance, value and score functions.

References

[1] M. Abbas, G. Murtaza and F. Smarandache, *Basic operations on hypersoft sets and hypersoft point*, Neutrosophic Sets and Systems, **35**, (2020), 407-421.

[2] M. Ahsan, M. Saeed and A.U. Rahman, *A Theoretical and Analytical Approach for Fundamental Framework of Composite Mappings on Fuzzy Hypersoft Classes*, Neutrosophic Sets and Systems, **45**, (2021), 268-285.

[3] D. Ajay and J. Joseline Charisma, *Neutrosophic hypersoft topological spaces*, Neutrosophic Sets and Systems, **40**, (2021), 178-194.

[4] D. Ajay, J. Joseline Charisma, N. Boonsatit, P. Hammachukiattikul and G. Rajchakit *Neutrosophic semiopen hypersoft sets with an application to MAGDM under the COVID-19 scenario*, Hindawi Journal of Mathematics, **2021**, (2021), 1-16.

[5] S. Aranganayagi, M. Saraswathi and K. Chitirakala, *More on open maps and closed maps in fuzzy hypersoft topological spaces and application in Covid-19 diagnosis using cotangent similarity measure*, International Journal of Neutrosophic Science, **21**(2), (2023), 32-58.

[6] S. Aranganayagi, M. Saraswathi, K. Chitirakala and A. Vadivel, *The e-open sets in neutrosophic hypersoft topologial spaces and application in Covid-19 diagnosis using normalized hamming distance*, Journal of the Indonesian Mathematical Society, **29**(2), (2023), 177-196.

[7] H. D. Arora and Anjali Naithani, *An analysis of customer preferences of airlines by means of dynamic approach to logarithmic similarity measures for pythagorean fuzzy sets*, Palestine Journal of Mathematics, **12** (1), (2023), 306-317.

[8] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., **24** (1968), 182-190.

[9] N. S. Kabbur and U. I. Neeli, *On soft nano semi generalized closed sets in soft nano topological spaces and application in deision making problems*, Palestine Journal of Mathematics, **10** (Special issue II), (2021), 143-150.

[10] D. Molodtsov, *Soft set theory-first results*, Comput. Math. Appl., **37**, (1999), 19-31.

[11] P. Revathi, K. Chitirakala and A. Vadivel, *Soft e-separation axioms in neutrosophic soft topological spaces*, Journal of Physics:Conference Series, **2070** (1), (2021), 012028.

[12] P. Revathi, K. Chitirakala and A. Vadivel, *Neutrosophic soft e-compact spaces and application using Entropy measure*, Applications and Applied Mathematics: An International Journal (AAM) **17** (1), (2022), 243-256.

[13] P. Revathi, K. Chitirakala and A. Vadivel, *Neutrosophic soft e-open maps, neutrosophic soft e-closed maps and neutrosophi soft e-homeomorphisms in neutrosophic soft topological spaces*, Springer Proceedings in Mathematics and Statistics, **384** (2022), 47-57.

[14] P. Revathi, K. Chitirakala and A. Vadivel, *Neutrosophic soft contra e-continuous maps, contra e-irresolute maps and application using distance measure*, Applications and Applied Mathematics: An International Journal (AAM) **18** (1), (2023), Article 13, 15 pages.

[15] P. Revathi, K. Chitirakala and A. Vadivel, *e-continuous maps and e-irresolute maps in neutrosophic soft topological spaces*, AIP Conf. Proc., **2850**, (2024), 050006.

[16] Said Broumi and Irfan Deli, *Correlation measure for neutrosophic refined sets and its application in medical diagnosis*, Palestine Journal of Mathematics, **5** (1), (2016), 135-143.

[17] M. Saqlain, N. Jafar, S. Moin, M. Saeed and S. Broumi, *Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Set*, Neutrosophic Sets and Systems, **32**, (2020), 317-329.

[18] M. Saqlain, S. Moin, M.N. Jafar, M. Saeed and F. Smarandache, *Aggregate operators of neutrosophic hypersoft set*, Neutrosophic Sets and Systems, **32** (1), (2020), 294-306.

[19] M. Shabir and M. Naz, *On soft topological spaces*, Comput. Math. Appl., **61**, (2011), 1786-1799.

[20] F. Smarandache, *Extension of soft set to hypersoft set, and then to plithogenic hypersoft set*, Neutrosophic Sets and Systems, **22**, (2018), 168-170.

[21] S. N. Sri, J. Vimala, N. Kausar, E. Ozbilge, E. Özbilge and D. Pamucar, *An MCDM approach on Einstein aggregation operators under Bipolar Linear Diophantine Fuzzy Hypersoft Set*, Heliyon, (2024), 10(9).

[22] P. Surendra, K. Chitirakala and A. Vadivel, *δ-open sets in neutrosophic hypersoft topological spaces*, International Journal of Neutrosophic Science, **20** (4), (2023), 93-105.

[23] P. Surendra, A. Vadivel and K. Chitirakala, *δ-separation axioms on fuzzy hypersoft topological spaces*, International Journal of Neutrosophic Science, **23** (1), (2024), 17-26.

[24] A. N. Surya, J. Vimala and K. Ashma Banu, *A Hybrid Fuzzy Extension and Its Application in Multi-Attribute Decision Making*, Contemporary Mathematics, (2024), 2456-2481.

[25] A. N. Surya, J. Vimala, N. Kausar, Z. Stevic and M. A. Shah, *Entropy for q-rung linear diophantine fuzzy hypersoft set with its application in MADM*, Scientific Reports, **14** (1), (2024), 5770.

[26] A. N. Surya, J. Vimala, and M. Vizhi, *Bonferroni mean aggregation operators under q-rung linear diophantine fuzzy hypersoft set environment and its application in multi-attribute decision making*, International Journal of Information Technology, (2024), 1-24.

[27] A. Vadivel and C. John Sundar, *On almost γ-continuous functions in N-neutrosophic crisp topological spaces*, Palestine Journal of Mathematics, **11** (3), (2022), 424-432.

[28] A. Vadivel, M. Seenivasan and C. John Sundar, *An introduction to δ-open sets in a neutrosophic topological spaces*, Journal of Physics: Conference series, **1724** (2021), 012011.

[29] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8** (3), (1965), 338-353.

Author information

K. Chitirakala, Department of Mathematics, M.Kumarasamy College of Engineering, Karur - 639 113, India.
E-mail: `chitrakalalaksana@gmail.com`

D. Sobana, Department of Mathematics, Kandaswamy Kandar's College, P.Velur, Namakkal - 638 182, India.
E-mail: `slmsobana@gmail.com`

A. Vadivel, PG and Research Department of Mathematics, Arignar Anna Government Arts College, Namakkal - 637 002
Department of Mathematics, Annamalai University, Annamalai Nagar - 608 002, India.
E-mail: `avmaths@gmail.com`

P. Surendra, Department of Mathematics, Annamalai University, Annamalai Nagar - 608 002, India.
E-mail: `surendrasarathi47@gmail.com`

Received: 2024-10-05
Accepted: 2025-10-27