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Impact of Fractional Order Parameter on Non-local Thermoelastic Behaviors in Elastic Medium due to Laser Heating Pulse

Rajneesh Kumar, Sachin Kaushal, Vikram and Nidhi Sharma

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Corresponding Author: S.Kaushal

Abstract This article examines the impact of the fractional order parameter on thermoelastic materials subjected to laser heating pulses, with and without the consideration of non-local effects. The model is formulated and solved using normal mode analysis, applied under impedance boundary conditions at the surface. Key physical field variables, including displacement, stress, and temperature distribution, are derived analytically and illustrated graphically for various values of the fractional order parameter, both with and without the non-local influence. A specific case has also been derived from the general model.

1 Introduction

In non-local elasticity theory, the stress at any given point depends on the strain at all other points in the material, unlike classical elasticity, which links the stress at a point solely to the strain at that same point. Fractional order derivatives play a key role in modeling certain physical phenomena that classical elasticity cannot adequately address. For example, materials like colloids, amorphous media, glassy substances, and porous materials often behave in ways that classical thermoelasticity, which relies on Fourier's law of heat conduction, fails to describe accurately. In such cases, a generalized thermoelasticity theory, incorporating heat conduction models with time-fractional derivatives, is required. This approach offers a more accurate representation of heat conduction in these complex materials. Research by Povstenko [1] explored non-local generalizations of Fourier's law using time and space fractional derivatives, while Youssef [2] proposed a thermoelasticity model that incorporates fractional-order heat conduction and proved the uniqueness of this theory. Additionally, Ezzat [3] studied thermoelectric fluid systems with fractional-order heat transfer, applying state-space and Laplace transform methods to solve one-dimensional problems. These studies highlight the limitations of classical methods and the growing importance of fractional calculus in describing complex materials and heat transfer processes.

Various researchers have developed non-local elasticity theory through different assumptions and approaches. Notable contributors include Eringen and Edelen [4], Edelen and Law [5], Eringen [6-11], McCay [12], McCay and Narsimhan [13], with a comprehensive overview provided in Eringen's [14] book. Mustafa et al. [15] investigated exponential decay in thermoelastic systems, while Somaiah and Lasiecka [16] explored the effects of rotation on radial vibrations in a microelastic solid with a spherical cavity. Kumar et al. [17] conducted a transient analysis of non-local microstretch thermoelastic thick circular plates with phase-lags. Hobiny and Abbas [18] examined the influence of non-local effects on thermoelastic materials, and Usman et al. [19] derived solutions to fractional kinetic equations involving generalized hypergeometric functions. Lu et al. [20] studied the thermoelastic response of a rod subjected to a moving heat source, applying fractional-order thermoelasticity theory.

This paper explores the impact of the fractional order parameter on thermoelastic materials, examining scenarios both with and without the inclusion of a non-local parameter, with laser heating as the heat source. The problem is solved using normal mode analysis, applying impedance boundary conditions at the surface. Analytical solutions are derived for key physical field variables such as displacement, stresses, and temperature distribution, and these results are illustrated graphically for various values of the fractional order parameter. Furthermore, a special case is extracted from the general solution to highlight specific behavior.

The significant feature of this kind of study is the analytical approach to the problem. If the analytical solution of a special problem can be available, then it can be used as a benchmark to prove the accuracy of any numerical approach. The use of integral transforms is one of the powerful semi-analytical tools for solving linear partial differential equations arising in heat transfer, earth-quake engineering, soil dynamics, and other areas of applied mechanics. However, the applicability of integral transform method is quite limited and is confined to linear problems.

2 Basic Equations

Following Tzou and Guo [21] and Sherief et al. [22], we have
(i) Constitutive Relations

$$t_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \gamma_1 T), \tag{2.1}$$

(ii) Equation of motion

$$(\lambda + 2\mu)\nabla(\nabla \cdot \overrightarrow{u}) - \gamma_1 \nabla T + \mu \triangle \overrightarrow{u} = \rho(1 - \xi_1^2 \triangle)\frac{\partial^2 \overrightarrow{u}}{\partial t^2}, \tag{2.2}$$

(iii) Heat conduction equation

$$K^* \triangle T = (1 + \tau_0 \frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}})(\rho C_e \frac{\partial T}{\partial t} - Q + \gamma_1 T_0 \frac{\partial e}{\partial t}). \tag{2.3}$$

In the equations (2.1)-(2.3), ξ_1 - non-local parameter, τ_0 - thermal relaxation times with $\tau_0 \geq 0$, α_0 -fractional parameter, K^* - thermal conductivity, $\triangle$- Laplacian operator, ∇- nabla(gradient) operator, Other symbols have usual meanings.

3 Statement of the Problem

For the assumed model, we have

$$\overrightarrow{u} = (u_1(x_1, x_3, t), 0, u_3(x_1, x_3, t)), \quad T(x_1, x_3, t) \tag{3.1}$$

Using (3.1) in (2.2) and (2.3), recast the following equations

$$(\mu + \lambda)\frac{\partial e}{\partial x_1} - \gamma_1 \frac{\partial T}{\partial x_1} + \mu \triangle u_1 = \rho(1 - \xi_1^2 \triangle)\frac{\partial^2 u_1}{\partial t^2}, \tag{3.2}$$

$$(\mu + \lambda)\frac{\partial e}{\partial x_3} - \gamma_1 \frac{\partial T}{\partial x_3} + \mu \triangle u_3 = \rho(1 - \xi_1^2 \triangle)\frac{\partial^2 u_3}{\partial t^2}, \tag{3.3}$$

$$K^* \triangle T = (1 + \tau_0 \frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}})(\rho C_e \frac{\partial T}{\partial t} - Q + \gamma_1 T_0 \frac{\partial e}{\partial t}). \tag{3.4}$$

Following dimensionless quantiites are used

$$\xi_1' = \frac{\omega^*}{c_1}\xi_1, \quad (u_i', x_i') = \frac{\omega^*}{c_1}(u_i, x_i), \quad (t', \tau_0') = \omega_1(t, \tau_0), \quad t_{ij}' = \frac{t_{ij}}{\gamma_1 T_0}, \quad \omega' = \frac{\omega}{\omega^*}$$

$$T' = \frac{\gamma_1}{\rho c_1^2}T, \quad , Q' = \frac{C_e}{K^* \omega^{*2}}Q, \quad (Z_1', Z_2') = \frac{c_1}{\gamma_1 T_0}(Z_1, Z_2), \quad Z_3' = \frac{c_1}{K^*}Z_3, \quad (i = 1, 3) \tag{3.5}$$

where

$$\omega^* = \frac{\rho C_e c_1^2}{K^*}, \qquad c_1^2 = (\frac{\lambda + 2\mu}{\rho})$$

ω^*-characteristic frequency and c_1 - longitudinal wave velocity.
Equations (3.2)-(3.4) with the aid of (3.5), reduce to the following equations after suppressing the primes

$$\frac{(\mu+\lambda)}{\rho c_1^2}\frac{\partial e}{\partial x_1} - \frac{\partial T}{\partial x_1} + \frac{\mu}{\rho c_1^2}\triangle u_1 = (1 - \xi_1^2\triangle)\frac{\partial^2 u_1}{\partial t^2}, \tag{3.6}$$

$$\frac{(\mu+\lambda)}{\rho c_1^2}\frac{\partial e}{\partial x_3} - \frac{\partial T}{\partial x_3} + \frac{\mu}{\rho c_1^2}\triangle u_3 = (1 - \xi_1^2\triangle)\frac{\partial^2 u_3}{\partial t^2}, \tag{3.7}$$

$$\triangle T = (1 + \tau_0(\omega^*)^{\alpha_0 - 1}\frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}})(\frac{\rho C_e c_1^2}{K^*\omega^*}\frac{\partial T}{\partial t} - \frac{\gamma_1}{\rho C_e}Q + \frac{\gamma_1^2 T_0}{\rho K^*\omega^*}\frac{\partial e}{\partial t}). \tag{3.8}$$

where $\triangle = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$ and $e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}$.
Equations (3.6)-(3.8) can be expressed as

$$n_{11}\frac{\partial e}{\partial x_1} + n_{12}\triangle u_1 - \frac{\partial T}{\partial x_1} = (1 - \xi_1^2\triangle)\frac{\partial^2 u_1}{\partial t^2}, \tag{3.9}$$

$$n_{11}\frac{\partial e}{\partial x_3} + n_{12}\triangle u_3 - \frac{\partial T}{\partial x_3} = (1 - \xi_1^2\triangle)\frac{\partial^2 u_3}{\partial t^2}, \tag{3.10}$$

$$\triangle T = (1 + \tau_0(\omega^*)^{\alpha_0 - 1}\frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}})(n_{13}\frac{\partial T}{\partial t} + n_{14}\frac{\partial e}{\partial t} - n_{15}Q). \tag{3.11}$$

where

$$n_{11} = \frac{(\lambda+\mu)}{\rho c_1^2}, \qquad n_{12} = \frac{\mu}{\rho c_1^2}, \qquad n_{13} = \frac{\rho C_e c_1^2}{K^*\omega^*}, \qquad n_{14} = \frac{\gamma_1^2 T_0}{\rho K^*\omega^*}, \qquad n_{15} = \frac{\gamma_1}{\rho C_e}.$$

4 Solution Procedure

Displacement components $u_1(x_1, x_3, t)$ and $u_3(x_1, x_3, t)$ to the scalar potential functions $\phi(x_1, x_3, t)$ and $\psi(x_1, x_3, t)$ in dimensionless form are given by

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, \quad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}. \tag{4.1}$$

With the aid of (4.1), equations (3.9)-(3.11) yield

$$\left((n_{11} + n_{12})\triangle - (1 - \xi_1^2\triangle)\frac{\partial^2}{\partial t^2}\right)\phi - T = 0, \tag{4.2}$$

$$\left(n_{12}\triangle - (1 - \xi_1^2\triangle)\frac{\partial^2}{\partial t^2}\right)\psi = 0, \tag{4.3}$$

$$\begin{aligned}&\left(1 + \tau_0(\omega^*)^{\alpha_0 - 1}\frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}}\right)n_{14}\triangle\frac{\partial\phi}{\partial t} \\ &+ \left(1 + \tau_0(\omega^*)^{\alpha_0 - 1}\frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}}\right)n_{13}\frac{\partial T}{\partial t} - \triangle T = \left(1 + \tau_0(\omega^*)^{\alpha_0 - 1}\frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}}\right)n_{15}Q.\end{aligned} \tag{4.4}$$

Using (4.2) in (4.4) yields

$$\left\{\left[-\left(n_{11}+n_{12}+\xi_1^2\frac{\partial^2}{\partial t^2}\right)\triangle^2-\left(1+\tau_0(\omega^*)^{\alpha_0-1}\frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}}\right)n_{13}\frac{\partial^3}{\partial t^3}\right]\right.$$
$$\left.+\left[\left(1+\tau_0(\omega^*)^{\alpha_0-1}\frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}}\right)\left(n_{14}\frac{\partial}{\partial t}+n_{13}(n_{11}+n_{12})\frac{\partial}{\partial t}+(\xi_1^2-1)n_{13}\frac{\partial^3}{\partial t^3}\right)+\frac{\partial^2}{\partial t^2}\right]\triangle\right\}\phi=$$
$$\left(1+\tau_0(\omega^*)^{\alpha_0-1}\frac{\partial^{\alpha_0}}{\partial t^{\alpha_0}}\right)n_{15}Q. \tag{4.5}$$

5 Normal Mode Analysis

Solution of considered physical variables in terms of normal modes is as follows:

$$(\phi,\psi,T,Q)=(\overline{\phi},\overline{\psi},\overline{T},\overline{Q})e^{\iota(kx_1-\omega t)} \tag{5.1}$$

The plate is illuminated by heat source

$$Q=\frac{I_0\gamma^*}{2\pi r^2t_0^2}te^{-t/t_0}e^{-x_1^2/r^2}e^{-\gamma^*x_3}.$$

where ω - complex time constant, k-wave number in x_1-direction, I_0 - energy absorbed and γ^* - absorption depth of heating energy.
Using (5.1), equations (4.3) and (4.5) take the form

$$(D^4+AD^2+B)(\overline{\phi},\overline{T})=n_{24}Q_0e^{\gamma^*x_3}, \tag{5.2}$$

$$(D^2-m_3^2)\overline{\psi}=0. \tag{5.3}$$

where

$$D=\frac{d}{dx_3},\quad A=\frac{n_{22}-2n_{21}k^2}{n_{21}},\quad B=\frac{n_{21}k^4-n_{22}k^2-n_{23}}{n_{21}},\quad m_3^2=\frac{\xi_1^2k^2+1-\frac{n_{12}k^2}{\omega^2}}{\xi_1^2-\frac{n_{12}}{\omega^2}},$$

$$n_{21}=-(n_{11}+n_{12}-\omega^2\xi_1^2),\quad n_{22}=(1+(-\iota\omega)^{\alpha_0})(-\iota\omega(n_{14}+n_{13}(n_{11}+n_{12}))+(\xi_1^2-1)n_{14}\iota\omega^3-\omega^2)$$

$$n_{23}=(1+\tau_0(\omega^*)^{\alpha_0-1})n_{13}\iota\omega^3,\quad n_{24}=(1+\tau_0(\omega^*)^{\alpha_0-1})(-\iota\omega)^{\alpha_0}n_{15},\quad Q_0=\frac{I_0\gamma^*}{2\pi r^2t_0^2}te^{-t/t_0}e^{-x_1^2/r^2}.$$

Bounded solution of equations (5.2) and (5.3) are

$$\{\overline{\phi},\overline{T}\}=\sum_{i=1}^{2}(1,a_i)[A_ie^{-m_ix_3}+f_1e^{-\gamma^*x_3}], \tag{5.4}$$

$$\overline{\psi}=A_3e^{-m_3x_3} \tag{5.5}$$

a_1 and a_2 are coupling constants given by

$$a_i=\frac{[-(m_i^2-k^2)n_{14}\iota\omega(1+\tau_0(\omega^*)^{\alpha_0-1})(-\iota\omega)^{\alpha_0}]}{[-(m_i^2-k^2)-n_{13}\iota\omega(1+\tau_0(\omega^*)^{\alpha_0-1})(-\iota\omega)^{\alpha_0}]},\qquad i=1,2.$$

$$f_1=\frac{n_{24}Q_0}{\gamma^{*4}+A\gamma^{*2}+B}.$$

By substituting the values of $\overline{\psi}$, $\overline{\phi}$ and $\overline{T}$ from equations (5.4) and (5.5) into equation (2.1), along with equations (3.1),(3.5),(4.1) and (5.1), and solving the resulting system, we obtain

$$t_{33}=\frac{1}{\triangle}\sum_{i=1}^{3}(a_{1i}\triangle_ie^{-m_ix_3}+a_{14}f_1e^{-\gamma^*x_3}), \tag{5.6}$$

$$t_{31} = \frac{1}{\triangle}\sum_{i=1}^{3}(a_{2i}\triangle_i e^{-m_i x_3} + a_{24} f_1 e^{-\gamma^* x_3}), \tag{5.7}$$

$$T = \frac{1}{\triangle}\sum_{i=1}^{2}(a_i\triangle_i e^{-m_i x_3} + f_1 e^{-\gamma^* x_3}), \tag{5.8}$$

where

$$a_{1i} = (2n_{31}+n_{32})m_i^2 - n_{32}k^2 - n_{33}a_i, \quad a_{13} = -2n_{31}\iota k m_3, \quad a_{14} = [(2n_{31}+n_{32})\gamma^{*2} - n_{32}k^2 - n_{33}],$$

$$a_{2i} = -2n_{31}\iota k m_i, \quad a_{23} = -(m_3^2 + k^2), \quad a_{24} = -2n_{31}\iota k\gamma^*, \qquad i = 1, 2.$$

$$n_{31} = \frac{\mu}{\gamma_1 T_0}, \quad n_{32} = \frac{\lambda}{\gamma_1 T_0}, \quad n_{33} = \frac{\rho c_1^2}{\gamma_1 T_0}$$

6 Boundary Conditions

Impedence boundary conditions at $x_3 = 0$ are

$$t_{33} + \omega_1 Z_1 u_3 = 0, \quad t_{31} + \omega_1 Z_2 u_1 = 0, \quad K^*\frac{\partial T}{\partial x_3} + \omega_1 Z_3 T = 0 \tag{6.1}$$

Z_i $(i = 1, 2, 3)$ are impedance real valued parameters. Z_1 and Z_2 have dimension of stress/velocity and Z_3 has dimension $\mathrm{Nm}^{-1}\mathrm{K}^{-1}$. ω_1 is the wave circular frequency.
Using (3.5) in (6.1) and after suppressing the primes, we get

$$t_{33} + \omega_1 Z_1 u_3 = 0, \quad t_{31} + \omega_1 Z_2 u_1 = 0, \quad \frac{\partial T}{\partial x_3} + \omega_1 Z_3 T = 0. \tag{6.2}$$

Substituting the expression of variables considered into (6.2), we obtain

$$\sum_{i=1}^{3} n_{i4} A_i = F_1, \tag{6.3}$$

$$\sum_{i=1}^{3} n_{i5} A_i = F_2, \tag{6.4}$$

$$\sum_{i=1}^{3} n_{i6} A_i = F_3. \tag{6.5}$$

where

$$A_i = \frac{\Delta_i}{\Delta}, \quad i = 1, 2, 3, \qquad \triangle = n_{61}(n_{42}n_{53} - n_{43}n_{52}) - n_{62}(n_{41}n_{53} - n_{51}n_{43}),$$

$$n_{41} = \omega_1 Z_1 m_1 + n_{32}k^2 + n_{33}a_1 - (2n_{31} + n_{32})m_1^2,$$

$$n_{42} = \omega_1 Z_1 m_2 + n_{32}k^2 + n_{33}a_1 - (2n_{31} + n_{32})m_2^2,$$

$$n_{43} = 2\iota k(m_3 - \omega_1 Z_1), \quad n_{51} = 2\iota k(n_{31}m_1 - \omega_1 Z_1),$$

$$n_{52} = \iota k(2n_{31}m_2 - \omega_1 Z_2), \quad n_{53} = m_3^2 + k^2 - \iota k\omega_1 Z_2,$$

$$n_{61} = m_1 a_1 - \omega_1 Z_3 a_1, \quad n_{62} = m_2 a_2 - \omega_1 Z_3 a_2,$$

$$F_1 = [(2n_{31} + n_{32})\gamma^* 2 + n_{32}k^2 + n_{33} + \omega_1 Z_1 \gamma^*] f e^{-\gamma x_3},$$

$$F_2 = [-2n_{31}\iota k\gamma^* + \iota k\omega_1 Z_2] f_1 e^{-\gamma^* x_3}, \quad F_3 = [\omega_1 Z_3 - \gamma^*] f_1 e^{-\gamma^* x_3}.$$

Putting $[F_1, F_2, F_3]^T$ in i^{th} column of Δ respectively determine Δ_i $(i = 1, 2, 3)$.

7 Particular Case

Taking $\xi_1 = 0$ in equations (5.6)-(5.8) gives the resulting expressions for thermoelastic medium due to laser pulse heating.

8 Validation

The effect of the fractional parameter on thermoelastic materials with non-local considerations has not yet been explored. In the absence of the non-local parameter, the results of this study align with those previously discussed by Wang et al. [23].

9 Numerical Implementation

For numerical computations, following [24], we take the copper material.

$$\lambda = 7.76 \times 10^{10} Kgm^{-1}s^{-2}, \quad \mu = 3.86 \times 10^{10} Kgm^{-1}s^{-2}, \quad T_0 = 0.293 \times 10^3 K,$$

$$C_e = 0.3891 \times 10^3 JKg^{-1}K^{-1}, \alpha_t = 1.78 \times 10^{-5} K^{-1}, \quad \alpha_c = 1.98 \times 10^{-4} m^3 Kg^{-1},$$

$$\rho = 8.954 \times 10^3 Kgm^{-3}, \quad K = 0.386 \times 10^3 Wm^{-1}K^{-1}, \quad r = 100\mu m, \quad t = 0.01s,$$

$$t_0 = 2nans, \quad \gamma^* = 1m^{-1}, \quad \tau_0 = 0.2s, \quad \xi = 0.395 \times 10^{-9} m, \quad \omega = 2 - 0.1t, \quad \omega_1 = 1,$$

$$Z_1 = 1Nm^{-1}s^{-1}, \quad Z_2 = 2Nm^{-1}s^{-1}, \quad Z_3 = 4Nm^{-1}K^{-1}$$

Graphs are computed using software Matlab(R2016a).
In figures 1-6:

- Solid line represents $\alpha_0 = 0.1$.
- Line with small dashes denotes $\alpha_0 = 0.5$.
- Line with small dashes denotes $\alpha_0 = 1.0$.
- Impact of different values of fractional parameter on stress components and temperature distribution on non-local thermoelastic material is shown in figures 1-3.
- Figures 4-6 show the impact of fractional order parameter on thermoelastic material without non-local effect.

Figure 1 demonstrates trend of t_{33} vs. x_3. t_{33} shows low damp and jump for high values of fractional parameter and with increasing distance converge to zero for all values of α_0.
Figure 2 displays trend of t_{31} vs.x_3. t_{31} shows high damp and jump for moderate values of fractional parameter and converge to zero with increasing distance for all values of fractional parameter.
Figure 3 depicts trend of T vs.x_3. T shows high variations for high values of α_0 and low variations for low value of α_0.
Figure 4 shows trend of t_{33} vs.x_3. t_{33} shows high oscillations for low values of fractional order parameter, while these oscillations decrease for increasing values of α_0.
Figure 5 displays trend of t_{31} vs.x_3. t_{31} shows high damp and jump for low values of α_0 and less variations for high values of α_0.
Figure 6 depicts trend of T vs.x_3. For increasing values of α_0, T shows low variations.

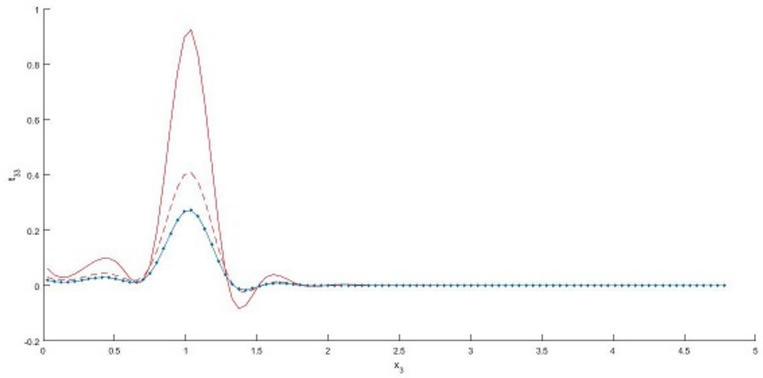

Figure 1. Profile of t_{33} vs. x_3

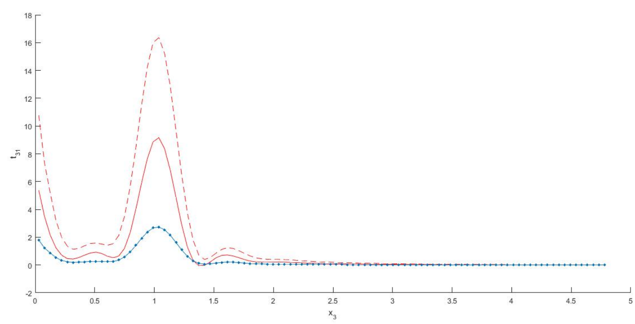

Figure 2. Profile of t_{31} vs. x_3

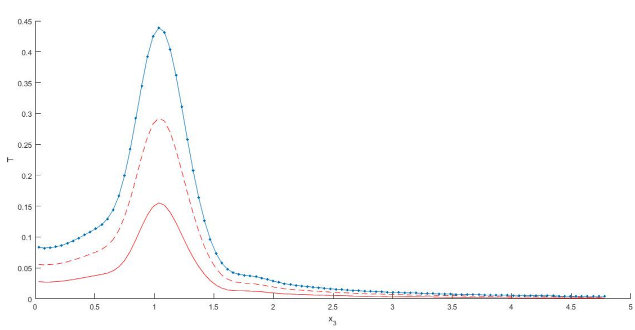

Figure 3. Profile of T vs. x_3

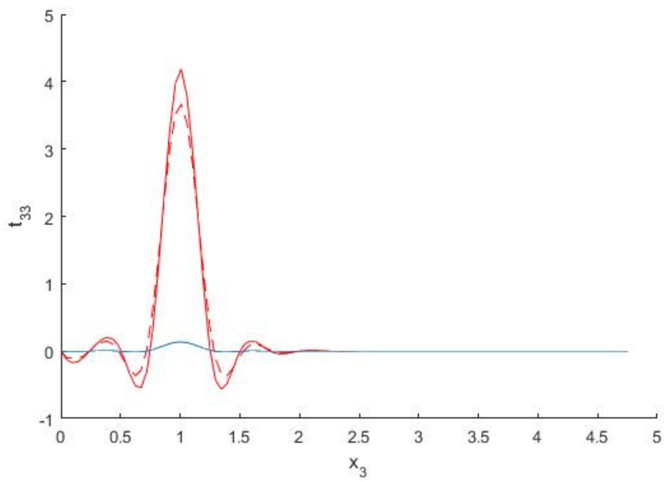

Figure 4. Profile of t_{33} vs. x_3

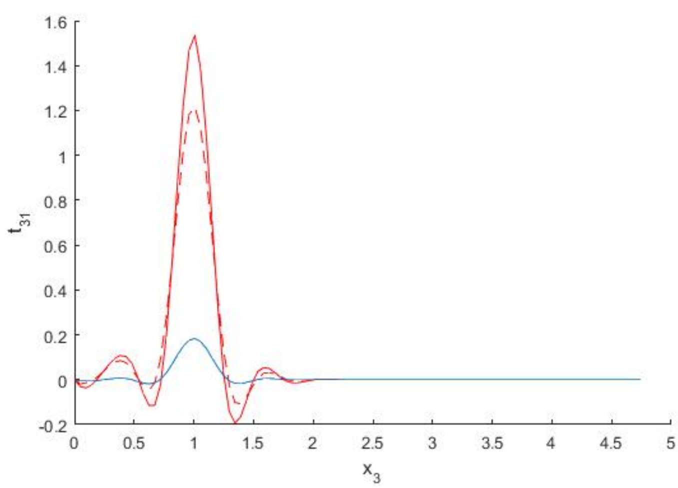

Figure 5. Profile of t_{31} vs. x_3

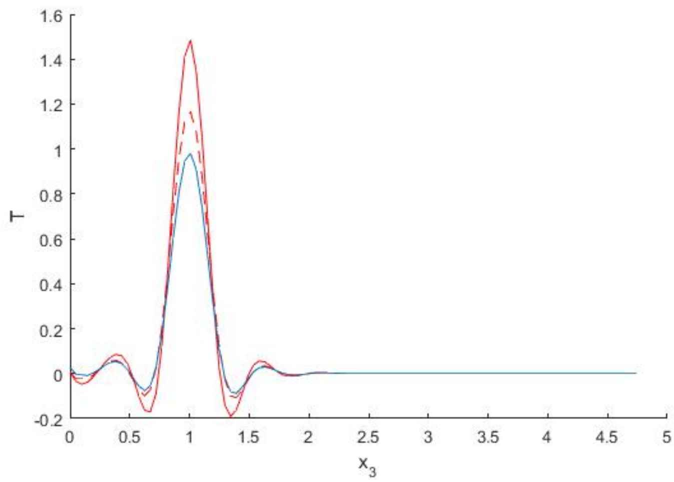

Figure 6. Profile of T vs. x_3

10 Conclusion remarks

The fractional parameter has a significant influence on both the stress components and temperature distribution. Analytical solutions for thermoelastic problems in solids, based on normal mode analysis, have been developed and examined. For non-local thermoelastic materials subjected to a laser heating pulse, the magnitudes of all physical quantities diminish to zero with increasing distance, and all functions remain continuous. It has been observed that in thermoelastic materials, stress components and temperature distribution exhibit oscillatory behavior, with these oscillations decreasing as the fractional order parameter increases. Furthermore, as the distance from the heat source increases, the fractional order parameter has a negligible effect on the physical quantities. This research provides valuable insights for those working in seismology and related fields.

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Author information

Rajneesh Kumar, Department of Mathematics, Kurukshetra University, Kurukshetra, Haryana, India.
E-mail: `rajneesh_kuk@rediffmail.com`

Sachin Kaushal, Department of Mathematics, School of Chemical Engineering and Physical sciences, Lovely Professional Universiy-Phagwara, India, India.
E-mail: `sachin_kuk@yahoo.co.in`

Vikram, Department of Mathematics, Lovely Professional Universiy-Phagwara, India, India.
E-mail: `dhaiyavikram91@gmail.com`

Nidhi Sharma, Department of Mathematics,Maharishi Markandeshwar (DU)- Mullana-Ambala, India.
E-mail: `nidhi_kuk26@rediff.com`