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Profit Analysis of a System of Non-Identical Units with Varying Demand

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This study is based on ghee manufacturing plant. I am very thankful to all the members of the industry who helped me throughout my studies and helped me for the collection of estimate data based on repairs from the industry as well.

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Abstract This innovation uses a stochastic approach to study a system of two dissimilar units with parallel working and varying demands. When identical units are not monetarily possible, industries often use non-identical units that range in dependability and desirability but accomplish the same tasks. Enhancing the system's reliability, accessibility, and financial performance requires optimizing the priority of unit repairs. An example would be a ghee manufacturing factory with an original central unit and an ordinary (replica) unit. The two units must be operational to meet the increase in demand for ghee during winter; otherwise, the system operates at a lower capacity. Every unit failure is assigned to a single technician, the only one constantly present in the system. Setting a repair priority for the various units is crucial. The study found that it would be more beneficial to concentrate on repairing the original unit when the repair rate of the ordinary (replica) unit is high. The system performance is most affected by the maintenance rate. The units' failure/repair time distribution follows a negative exponential distribution. The units' failure/repair time distribution follows a negative exponential distribution. The authors compute several system efficacy parameters using the semi-Markov and regeneration point techniques. Additionally, they plotted graphs to draw attention to the significant findings. They also assess the system's profit for a few fixed repairs and other cost values.

1 Introduction

When discussing day-to-day activities and modern society's needs, we should also mention that machines are part of our lives. However, these machines will benefit the user if their work is trouble-free. In such a situation, reliability plays a vital role due to its characteristics, such as overcoming sudden failures and breakdowns and providing a safe environment. The literature on reliability is becoming increasingly affluent daily, as many researchers are making many contributions in the field by incorporating new ideas/concepts/studies. In reliability research, it is common practice to assume that two or more identical systems, without considering the parallel working of both units. For example, Taneja et al. worked in a sugar mill with three identical units with varied production [1]. The user shut down the mill during the shortage of raw materials. Otherwise, the system will work at total capacity. Ram et al. investigated the reliability of two-unit standby systems. Initially, one unit was operational while the other was in standby mode [2]. Their system may also fail due to improper starting of the system; the reason behind it was untrained and inexperienced system analysts. Additionally, Malhotra and Taneja analyzed two identical standby systems, assuming that both units may be operational simultaneously due to increased demand [3, 4]. Furthermore, the authors worked on a comparative study of a cable manufacturing plant, considering that demand is not constant. They used regenerative and semi-Markov processes to find numerous system effectiveness measures. According to Levitin et al., redundancy is awidely applied technique for high reliability [7]. They also discussed how its failure made reliability non-monotonic and affected other parameters. Also, the elements of the system are assumed to be non-repairable. Afterward, Yang et al. took a system with M primary and S spare units [8]. All units were repairable and did not require waiting space; therefore, the repairman immediately repaired the failed unit. The researchers used the matrix analysis method to compute steady-state availability. Malhotra also developed a two-unit redundant system where one unit was in working mode while the other was cold redundant by enchanting the activation time [17]. After that, they worked on a hot standby system with varying demands. In this model, identical standby units remain operational from the initial state. Afterward, Kumar et al. worked on two similar unit systems. The authors assume that a single unit can run the system smoothly [18]. The other unit can fail due to unused or any other environmental issues. The repairman will inspect the failed unit and decide whether it will be repaired or replaced. In contrast to this, some authors took systems with non-identical units. For instance, Wang et al. studied non-identical redundant allocation problems with degrading components [10]. The main concern is cold standby units that suffer performance degradation when exposed to extreme standby environments for long-term storage. Moreover, Levitin et al. discussed the factors that affect industrial systems, such as deterioration, corrosion, etc [11]. They performed fixed planning to renew the worn element by using redundant aspects to improve its functioning successfully. Gao et al. studied warm and cold standby systems assuming unreliable repair [9]. They performed preventive maintenance and repair according to idle time and unit failure. The authors used the Markov process approach to solve the equations. And they assumed that the repair unit was as good as the new one. Kamal et al. analyzed the cost-benefit of a system having two dissimilar units by considering one with high quality and the other with low quality [14]. They used the concept of repairing or replacing according to the type of failure. The central unit kept its high quality. According to Juybari et al., mixed redundancy is a powerful technique to improve the reliability of a system [13]. Also, assume that all components are under environmental shocks and may deteriorate by internal or external shocks. Also, Shekhar et al. evaluated the reliability of multi-unit systems having several failures, degradation, random delays, and probabilistic imperfections [19]. In addition, they discussed their impact on production and the system performance. According to Kundu et al., comparing extremes (maximum or minimum) in fixed or random sample sizes has been a significant topic in various fields [22]. Li et al. worked on non-identical units by assuming either repair is a priority or it may not [23]. They did not consider the concept of varied demand. In the present study, the authors observed repair going on a prior basis, and the demand varies. Thus, many studies have taken the concepts of either identical units working or assuming equal repair is required by both units. But practically, there is a lack of consideration. Due to the high unit cost, every industry must buy different units. In such a situation, duplicate units play a vital role. Now the question arises: which unit will be on a prior basis for repair after failure? Some researchers assumed either the original unit or a duplicate unit. Non-identical units have the same working with different reliability and availability. So, it is essential to give priority to one of the units at a time. A practical, real-life example exists, like a system with a transformer and generator. In this case, our repair priority is the transformer rather than the generator. The reason behind this is the high cost of operation of later. In conclusion, prioritizing original unit repair is not beneficial when duplicate unit repair rates are high. To the best of our knowledge, none of the extensive literature on reliability considered non-identical units with different degrees of reliability, prior repair, and varied production due to varied seasons simultaneously [24, 25, 26]. Hence, there is a considerable gap. The authors will try to fill it in. The proposed study investigates the stochastic analysis of two separate bleacher-earth machines operating simultaneously. There is an original central unit and an ordinary (replica) unit. On visiting the manufacturing plant in Punjab, the authors observed that although both units work in tandem, their availability and dependability may vary. Initially, our system worked at total capacity and could meet the increased demand in winter. Failure of any unit causes the system to reduce capacity and cannot meet market demand on time, especially in winter. This variation affects the reliability, availability, and profit of the plant. The user applied repair techniques to both units to prevent such a situation. A single technician is available to do the desired job with total efficiency. The reliability characteristics of the system model have been studied numerically and graphically for various parameter values [27, 28, 29]. The authors use the Markov process to analyze the system's profit for some fixed repair and maintenance cost values [30, 31].

2 Model Description and Measures of System Effectiveness

The description of the proposed model is as follows. The Nomenclature (Shown in Table 1)

Table 1. Nomenclature

Symbol	Description
λ/μ	The failure rate of the main bleacher machine / ordinary machine (replica).
B_0, B'_0	The main bleacher machine and the ordinary both work simultaneously.
B_{ur}, B'_0	The main machine goes under repair after failing, and the ordinary machine still works.
B_0, B'_{ur}	The main machine works, and the ordinary machine is in repair.
B_{UR}, B'_{wr}	The main machine is under repair from the previous state, and the ordinary machine is waiting for repair.
B_{wr}, B'_{UR}	The main machine is waiting for repair, and the ordinary machine is in repair from its previous state.
$g(t)/h(t)$	The probability density function of the repair times of the central / standby units.
$\star/\sim$	Symbol of Laplace / Stieltjes transform.
© /Ⓢ	Laplace / Stieltjes convolution.
p_{ij}, p^k_{ij}	Steady-state probabilities.
$\mu_i(t)$	Mean sojourn time in regenerative state before transiting to any other state.
$Q_{ij}, Q^k_{ij}/q_{ij}, q^k_{ij}$	CDF / PDF of first passage time.
$M_i(t)/m_{ij}$	The probability that the system is up initially / Contribution to the mean sojourn time in regenerative state i.
q_{ij}, p_{ij}, p^k_{ij}	Transition / steady-state probabilities.

3 Model Description and Assumptions

The authors observed two different bleacher earth units in the visiting plant, the original central and ordinary (replica) as shown in Figure 1. Initially, both units work simultaneously (state S_0). The failure rates of the original and ordinary units are λ and μ respectively. The state will move to state(S_1) or (S_2) based on their failure. Also, during the failure of any one of them, other units get corrupted. Then, states will move to either (state S_3) or (state S_4) accordingly. After repair, all units return to their previous state due to satisfactory repair service. Both units work instantaneously in parallel mode. Both units have regular and complete failures. Various other assumptions are as follows:

- A single unit can run the system smoothly but cannot meet the increased demand in winter.
- The system will function if any of the units are working. Otherwise, it is assumed to fail.
- Single technicians are available to do the desired job efficiently and never leave the system during repair.
- Repair time distributions are general, while the failure time distribution of units follows a negative exponential.

4 Transition Probabilities

$$dQ_{01}e^{(-\lambda+\mu)t}dt$$

$$dQ_{02}(t) = \mu e^{(-\lambda+\mu)t}dt$$

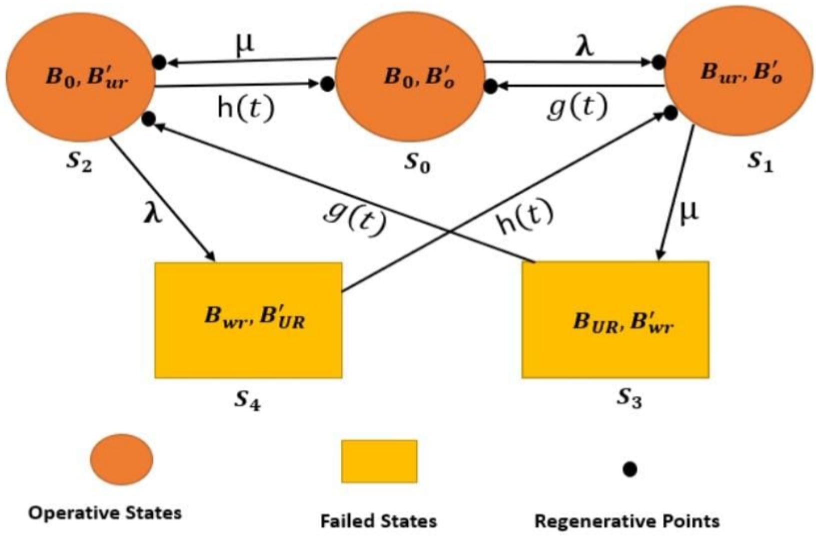

Figure 1. State transition diagram of the model

$$dQ_{10}(t) = g(t)e^{(-\mu t)}dt$$
$$dQ_{13}(t) = \mu e^{(-\mu)t}G(t)dt$$
$$dQ_{12}^{3}(t) = d[Q_{13}(t)[s]Q_{32}(t)]$$
$$dQ_{20}(t) = h(t)e^{(-\lambda t)}dt$$
$$dQ_{24}(t) = \lambda e^{(-\lambda t)}H(t)dt$$
$$dQ_{20}^{4}(t) = d[Q_{24}(t)[s]Q_{41}(t)]$$
$$dQ_{32}(t) = g(t)dt$$
$$dQ_{41}(t) = h(t)dt$$

Changes in variables, such as wear and tear in a mechanical system, fluctuations in demand, or modifications in environmental elements that affect the process, can cause these probabilities to change over time. Time-independent probabilities, p_ij, on the other hand, are steady-state or invariant probabilities that characterize a system's long-term behavior. The hypothesis explains this transition, in which methods frequently attain a steady state where the chances of changing conditions over time become constant. The process is known as the "steady-state" or "stationary" assumption. Markov chains analyze time-dependent and time-independent systems as they describe systems that switch between distinct states. Simple probabilistic considerations yield the following expressions for the non-zero elements:

$$p_{ij} = Q_{(i,j)}(\infty) = \int_0^\infty dQ_{(i,j)}(t)\,dt = \tilde{Q}_{(i,j)}(0) = \int_0^\infty q_{(i,j)}(t)\,dt \tag{4.1}$$

Thus,

$$p_{ij} = \lim_{s\to 0} q_{ij}^{*}(s)$$

After taking the Laplace Transformation of equation (1), we get

$$p_{01} = \frac{\lambda}{\lambda+\mu},$$
$$p_{02} = \frac{\mu}{\lambda+\mu},$$
$$p_{10} = g^{*}(\mu),$$

$$p_{13} = 1 - g^*(\mu),$$
$$p_{12}^3 = 1 - g^*(\mu),$$
$$p_{20} = h^*(\lambda),$$
$$p_{24} = 1 - h^*(\lambda),$$
$$p_{21}^4 = 1 - h^*(\lambda),$$
$$p_{32} = p_{41} = 1.$$

Based on these probabilities, it is clear that

$$p_{01} + p_{02} = 1,$$
$$p_{10} + p_{13} = 1,$$
$$p_{10} + p_{12}^3 = 1,$$
$$p_{20} + p_{24} = 1,$$
$$p_{20} + p_{21}^4 = 1,$$
$$p_{32} = p_{41} = 1.$$

Mean sojourn time (μ_i) in the state S_i are

$$\mu_0 = \frac{1}{\lambda + \mu},$$
$$\mu_1 = \frac{1}{\alpha + \mu},$$
$$\mu_1' = \frac{1}{\alpha},$$
$$\mu_2 = \frac{1}{\lambda + \beta},$$
$$\mu_2' = \frac{1}{\beta}.$$

Also,

$$m_{ij} = \int_0^\infty t\, d\{Q_{ij}(t)\} = -q_{ij}^{*'}(0)$$
$$m_{01} + m_{02} = \mu_0,$$
$$m_{10} + m_{13} = \mu_1,$$
$$m_{20} + m_{24} = \mu_2,$$
$$m_{10} + m_{12}^3 = \mu_1',$$
$$m_{20} + m_{21}^4 = \mu_2'.$$

5 Mean Time to System Failure (MTSF)

In determining MTSF, assume the failed states to be absorbing states. Recursive relations for $\phi_i(t)$ are:

$$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1(t) + Q_{02}(t)Ⓢ\phi_2(t), \quad (5.1)$$
$$\phi_1(t) = Q_{10}(t)Ⓢ\phi_0(t) + Q_{13}(t), \quad (5.2)$$
$$\phi_2(t) = Q_{20}(t)Ⓢ\phi_0(t) + Q_{24}(t) \quad (5.3)$$

Taking the Laplace–Stieltjes Transform (L.S.T.) on (5.1,5.2,5.3) and solving for $\tilde{Q}_0(s)$, then the Mean Time to System Failure (MTSF) is given by

$$\lim_{s\to 0} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{N}{D}$$

where

$$N = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2$$

and

$$D = 1 - p_{01}p_{10} - p_{02}p_{20}.$$

6 Availability

The chance that a system does not currently suffer a failure at a time t is known as the availability $A(t)$. It is true even if the system might have previously failed to resume normal operating conditions. The availability is given by

$$A_i(t) = M_i(t) + \sum_{j\neq i} q_{i,j}^k(t)Ⓒ A_j(t), \tag{6.1}$$

The recursive relations for the system availability are as follows:

$$A_0(t) = M_0(t) + q_{01}(t)Ⓒ A_1(t) + q_{02}(t)Ⓒ A_2(t), \tag{6.2}$$

$$A_1(t) = M_1(t) + q_{10}(t)Ⓒ A_0(t) + q_{12}^3(t)Ⓒ A_2(t), \tag{6.3}$$

$$A_2(t) = M_2(t) + q_{20}(t)Ⓒ A_0(t) + q_{21}^4(t)Ⓒ A_1(t) \tag{6.4}$$

Here,

$$M_0(t) = \int_0^t e^{-(\lambda+\mu)\tau}\, d\tau,$$

$$M_1(t) = \int_0^t e^{-\mu\tau}\bar{G}(\tau)\, d\tau,$$

$$M_2(t) = \int_0^t e^{-\lambda\tau}\bar{H}(\tau)\, d\tau$$

Taking the Laplace Transform on (6) and solving for $\tilde{A}_0(s)$, we have the steady-state availability given by

$$A(\infty) = \lim_{s\to 0} s\tilde{A}_0(s) = \frac{N}{D} \tag{6.5}$$

where

$$N = p_{10}\mu_0 + p_{20}p_{12}^3\mu_0 + \mu_1 - \mu_1 p_{02}p_{20} + \mu_2 - \mu_2 p_{01}p_{10},$$

and

$$D = p_{10}\mu_0 + p_{20}p_{12}^3\mu_0 + \mu_1' - p_{02}p_{20}\mu_1' + \mu_2' - \mu_2' p_{01}p_{10}.$$

7 Busy Period Due to Repair

Recursive relations for $B_i^R(t)$ in general are

$$B_i^R(t) = W_f(t) + \sum_{j\neq i} q_{i,j}^k(t)Ⓒ B_j^R(t) \tag{7.1}$$

Recursive relations for $B_i^R(t)$ are as follows:

$$B_0^R(t) = q_{01}(t)Ⓒ B_1^R(t) + q_{02}(t)Ⓒ B_2^R(t), \tag{7.2}$$

$$B_1^R(t) = W_1^R(t) + q_{10}(t)Ⓒ B_0^R(t) + q_{12}^3(t)Ⓒ B_2^R(t), \tag{7.3}$$

$$B_2^R(t) = W_2^R(t) + q_{20}(t)Ⓒ B_0^R(t) + q_{21}^4(t)Ⓒ B_1^R(t) \tag{7.4}$$

Here, we have

$$W_1^R(t) = e^{-\mu t}\bar{G}(t) + (\mu e^{-\mu t}Ⓒ 1)\bar{G}(t),$$

$$W_2^R(t) = e^{-\lambda t}\bar{H}(t) + (\lambda e^{-\lambda t}Ⓒ 1)\bar{H}(t)$$

The busy time of the server due to repair is:

$$B^R(\infty) = \lim_{s\to 0} s\tilde{B}_0^R(s) = \frac{N^R}{D} \tag{7.5}$$

where

$$N^R = W_1^R - W_1^R p_{02}p_{20} + W_2^R - W_2^R p_{01}p_{10},$$

and D is already mentioned.

8 Expected Number of Repairs

The expected number of repairs for the unit is given by

$$E_0^N(t) = Q_{01}(t)Ⓢ[1 + E_1^N(t)] + Q_{02}(t)Ⓢ[1 + E_2^N(t)], \tag{8.1}$$

$$E_1^N(t) = Q_{10}(t)ⓈE_0^N(t) + Q_{12}^3(t)ⓈE_2^N(t), \tag{8.2}$$

$$E_2^N(t) = Q_{20}(t)ⓈE_0^N(t) + Q_{21}^4(t)ⓈE_1^N(t) \tag{8.3}$$

Taking the Laplace–Stieltjes Transform (L.S.T) of the above equations and solving for $\tilde{E}_i^N(s)$, we have

$$\tilde{E}_i^N(s) = \frac{E^N(s)}{D(s)} \tag{8.4}$$

where

$$E^N(s) = 1 - p_{12}^3 Q_{21}^4,$$

and $D(s)$ is already defined.

9 The Profit Analysis

Profit is the income obtained after paying all costs:

$$P = (C_0 A) - (C_1 B^R + C_2 E^N) \tag{9.1}$$

All values mentioned below:

$$C_0 = \text{revenue per active unit of the system,}$$

$$C_1 = \text{cost per unit period for which the server is busy repairing the unit,}$$

$$C_2 = \text{cost per unit for the time that the server is visited.}$$

Also, A, B^R, and E^N are taken earlier.

10 Particular Case

The authors considered dissimilar random variables in the model to follow an exponential distribution with different parameters. Let the PDF of all the random variables be given as:

$$g(t) = \alpha e^{-\alpha t},$$

$$h(t) = \beta e^{-\beta t}.$$

There are two non-identical bleacher earth machines with different failure rates. One is the original unit, while the other is an ordinary one. Hence, repair rates are also different for both units. The results are:

$$p_{01} = \frac{\lambda}{\lambda + \mu},$$

$$p_{02} = \frac{\mu}{\lambda + \mu},$$

$$p_{10} = \frac{\alpha}{\alpha + \mu},$$

$$p_{13} = \frac{\mu}{\alpha + \mu},$$

$$p_{12}^3 = \frac{\mu}{\alpha + \mu},$$

$$p_{20} = \frac{\beta}{\beta + \lambda},$$

$$p_{24} = \frac{\lambda}{\beta + \lambda},$$

$$p_{21}^{4} = \frac{\lambda}{\beta + \lambda},$$

$$\mu_0 = \frac{1}{\lambda + \mu},$$

$$\mu_1 = \frac{1}{\alpha + \mu},$$

$$\mu_1' = \frac{1}{\alpha},$$

$$\mu_2 = \frac{1}{\lambda + \beta},$$

$$\mu_2' = \frac{1}{\beta}.$$

All the formulae given above: **Mean Time to System Failure (MTSF)**
Mean Time to System Failure (MTSF)

$$\begin{aligned}\text{MTSF}\, T &= \frac{N}{D}\\ &= \frac{\lambda^2 + \lambda\mu + \lambda\alpha + \lambda\beta + \mu^2 + \mu\beta + \mu\alpha + \alpha\beta}{\lambda^2\mu + \lambda\mu^2 + \lambda\mu\alpha + \lambda\beta\mu + \beta^2\mu^2 - \mu^2\beta}\end{aligned}$$

Availability

$$\begin{aligned} A &= \frac{N}{D^*}\\ \text{where}\quad N &= \alpha^2\beta^3 + \alpha^2\mu\beta^2 + \mu\alpha\beta^3 + \lambda\alpha^2\beta^2 + \lambda\alpha^2\mu\beta + \lambda\mu\alpha\beta^2\\ &\quad + \lambda\alpha\beta^3 + \alpha\beta^2\lambda^2 + 2\alpha\beta^2\lambda\mu + \alpha\lambda^2\mu\beta + \alpha\lambda\mu^2\beta + \lambda\mu\alpha\beta^2\\ &\quad + \beta^2\alpha^2\mu + \alpha\mu^2\beta^2 + \alpha\lambda\beta\mu^2 + \mu^2\alpha^2\beta + \alpha\beta\mu^3\\ D^* &= \alpha^2\beta^3 + 2\alpha^2\mu\beta^2 + \mu\alpha\beta^3 + \lambda\alpha^2\beta^2 + 2\lambda\alpha^2\mu\beta + 4\lambda\mu\alpha\beta^2\\ &\quad + \lambda\mu\beta^3 + \beta^3\lambda\alpha + 2\beta^2\lambda\mu^2 + \beta^2\lambda^2\mu + \beta^2\lambda^2\alpha\\ &\quad + \lambda^2\mu^2\beta + 2\alpha\lambda^2\mu\beta + \lambda\beta\mu^3 + 3\alpha\lambda\beta\mu^2\\ &\quad + \alpha\lambda^2\mu^2 + \mu^2\alpha^2\beta + \mu^2\alpha^2\lambda + \alpha\mu^2\beta^2 + \alpha\beta\mu^3 + \alpha(\lambda\mu)^3\end{aligned}$$

Busy Period Due to Inspection

$$\begin{aligned} B^R &= \frac{N^R}{D^*}\\ \text{where}\quad N^R &= \alpha(\lambda\beta)^3 + \lambda\mu\beta^3 + 3\lambda\mu\alpha\beta^2 + \mu^2\beta^2\lambda\\ &\quad + \lambda^2\beta^2\alpha + \beta^2\lambda^2\mu + 2\alpha\lambda^2\mu\beta + \mu^2\lambda^2\beta + \lambda\mu^2\beta^2\\ &\quad + 3\alpha\lambda\beta\mu^2 + \alpha\lambda^2\mu^2 + \beta^2\alpha^2\mu + \lambda\alpha^2\mu\beta\\ &\quad + \mu^2\alpha^2\beta + \mu^2\alpha^2\lambda + \alpha\mu^2\beta^2 + \alpha\beta\mu^3 + \alpha(\lambda\mu)^3\end{aligned}$$

Expected Number of Repairs

$$E^N = \frac{N^E}{D^*}$$

$$\text{where} \quad N^E = \lambda\mu\alpha\beta^3 + \alpha\lambda\mu^2\beta^2 + \alpha\mu^2\beta^3 + \alpha\mu^3\beta^2 + \beta^2\alpha^2\lambda^2 + \lambda^2\alpha^2\mu\beta + \beta^2\alpha^2\mu\lambda + \mu^2\alpha^2\lambda\beta + \beta^3\alpha^2\lambda + \beta^2\alpha^2\lambda\mu + \beta^3\alpha^2\mu + \beta^2\alpha^2\mu^2$$

11 Results and Discussions

The study examines certain significant dependability aspects of a two-unit repairable system.The MTSF, availability, and profit functions are shown graphically in Figures 2,3,5,6,7,8,9 respectively, so we can see how they react to the values of the parameters corresponding to the failure and repair rates.

Table 2. MTSF for the variable repair rate of the main unit (α) vs failure rate of the ordinary unit (μ)

μ	$\alpha = 0.03$ hr	$\alpha = 0.04$ hr	$\alpha = 0.05$ hr
0.01	55449.2796	72561.98673	89065.65748
0.02	27225.3327	35638.92778	43758.30011
0.03	17829.1754	23346.17472	28673.54744
0.04	13139.502	17210.42296	21143.77574
0.05	10332.0768	13537.04256	16635.49576
0.06	8465.50731	11094.51388	13637.57158
0.07	7136.35484	9355.061449	11502.39495

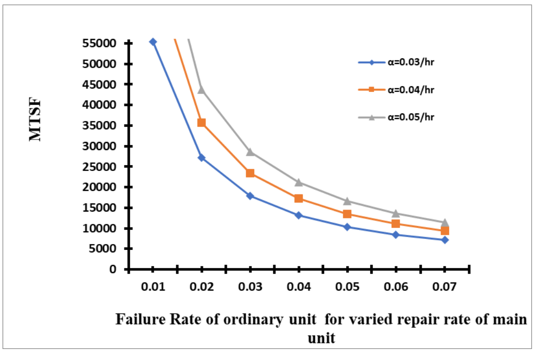

Figure 2. MTSF for the varied repair rate of the central unit (α) v/s failure rate of the ordinary unit (μ)

Figure 2 and Table 2 makes it abundantly evident that when the failure rate of the ordinary unit increases from 0.01 to 0.07, it results in a decrease in the MTSF of the system. Furthermore, with higher repair rates α = 0.03/h, 0.04/h, and 0.05/hr. Respectively, the MTSF shows an upward trend in the basic parameters. Supplementary parameters are β =0.3/hr, λ= 0.7/hr., μ= (0.02, 0.03, etc.) / hour.Here, α and β are the repair rates of the central and ordinary units, respectively. Also, λ and μ are the central and ordinary unit failure rates.

In Figure 3 and Table 3, a highly reliable system can run continuously, but its availability will drop if not maintained correctly. On the other hand, a low-reliability machine may experience numerous failures, but its availability increases with appropriate care and prompt repairs. Figure 3 and Table 3 provides clear and convincing evidence that the system's availability rapidly drops for a rising failure rate of the ordinary unit μ from 0.02 to 0.04; additionally, when the repair rate

Table 3. Availability for varied failure rates of ordinary unit (μ) vs repair rate of the main unit (α)

α	$\mu = 0.02$ hr	$\mu = 0.03$ hr	$\mu = 0.04$ hr
0.01	0.227364262	0.173867141	0.144132653
0.02	0.277696227	0.215533199	0.180131658
0.03	0.299238855	0.233820733	0.196172738
0.04	0.310930075	0.243893794	0.205097087
0.05	0.318127889	0.250164047	0.2106974

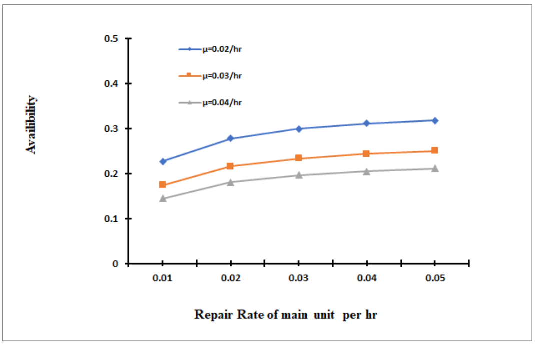

Figure 3. Availability for various failure rates of ordinary unit (μ) v/s repair rate of a central unit (α)

of the prominent unit α increases from 0.01 to 0.05, so does the system's availability. Therefore, repairs should occur on ordinary units to improve the system's availability. Other parameters are assumed to be $\beta = 0.008$ hr and $\lambda = 0.1$ hr.

Table 4. Availability for variable repair rate (α) of main unit vs failure rates of ordinary unit (μ)

μ	$\alpha = 0.00105$ hr	$\alpha = 0.00305$ hr	$\alpha = 0.00505$ hr
0.01	0.13885628	0.196166191	0.214094832
0.03	0.107714	0.152500875	0.166886916
0.05	0.10166327	0.143737172	0.15725774
0.07	0.09911666	0.140019131	0.153155721
0.09	0.0977169	0.137968067	0.150888791
0.11	0.09683234	0.13666926	0.149451894

Figure 4 and Table 4 shows that the system experiences a significant decrease in availability due to an increasing failure rate of the ordinary unit μ from 0.01 to 0.11. Furthermore, availability shows an upward trend with an increase in the repair rate of the main unit α from 0.00105 to 0.00505 per hour. There is a hike in the system's availability with the growth of the repair rate of the central unit. The authors plotted the graph by considering the rest of the parameters as $\lambda = 0.005$ and $\beta = 0.001$ per hour. More repairs are carried out on ordinary units to make the system more available.

Figure 5 and Table 5 illustrates how profit increases significantly when the repair rate of ordinary units (β) increases from 0.002 to 0.004 per hour and decreases with rising repair costs (C_1). The cut-off points shown in the figure help determine the repair costs the company needs to pay to achieve a profit. Consider $C_2 = 50$ (costs are in Indian rupees). Also, $\mu = 0.1$ hr^{-1}, $\alpha = 0.01$ hr^{-1}, $\beta = 0.002$ hr^{-1}, 0.003 hr^{-1}, 0.004 hr^{-1}, $\lambda = 0.1$ hr^{-1}. $C_0 =$ INR 5000, with C_1 values of (140, 160, 180, 200, 220, 240).

Although revenue is the income generated before costs, profit is the income obtained after paying all expenses. Figure 6 and Table 6 illustrate how fluctuations in revenue and costs affect the system's profitability. The system improves profitability when the central unit's repair rate

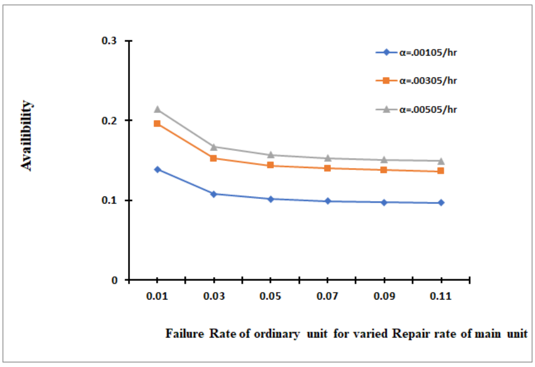

Figure 4. Availability for varied repair rate (α) of the primary unit vs failure rates of the ordinary unit (μ)

Table 5. Profit from the varied repair rate (β) of the ordinary unit vs repair costs (C_1)

C_1	$\beta = 0.002$/hr	$\beta = 0.003$/hr	$\beta = 0.004$/hr
140	23.9796975	85.58812643	137.761194
160	3.99854761	65.61625583	117.7985075
180	-15.982602	45.64438522	97.8358209
200	-35.963752	25.67251462	77.87313433
220	-55.944902	5.700644015	57.91044776
240	-75.926052	-14.27122659	37.94776119

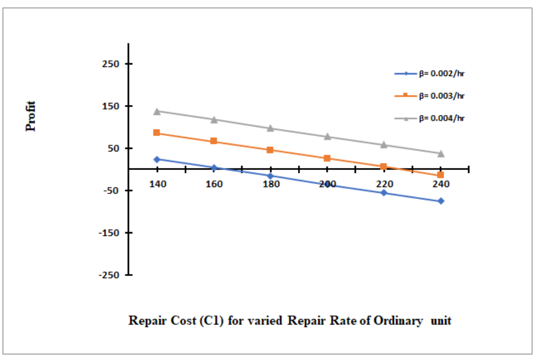

Figure 5. Profit for the variable repair rate (β) of the ordinary unit vs. repair costs (C1)

Table 6. Revenue cost (C_0) for varied repair rates of a main unit (α)

C_0	$\alpha = 0.05$ hr^{-1}	$\alpha = 0.07$ hr^{-1}	$\alpha = 0.09$ hr^{-1}
2300	-42.133527	-24.8864592	-14.3555713
2400	-26.894049	-8.99074852	1.917698406
2500	-11.654572	6.904962153	18.1909682
2600	3.58490566	22.80067283	34.464238
2700	18.8243832	38.69638352	50.73750779
2800	34.0638607	54.5920942	67.01077759

rises rapidly. Cut-off points help decide the repair rate's upper limit; the profit becomes harmful beyond this limit, and the company faces financial loss. Also, $\mu = 0.1$ hr^{-1}, $\alpha = 0.09$ hr^{-1}, $\beta = 0.01$ hr^{-1}, $\lambda = 0.1$ hr^{-1}. $C_0 = \{2300, 2400, 2500, 2600, 2700, 2800\}$, $C_1 = 400$, $C_2 = 50$ (Costs are in Indian rupees).

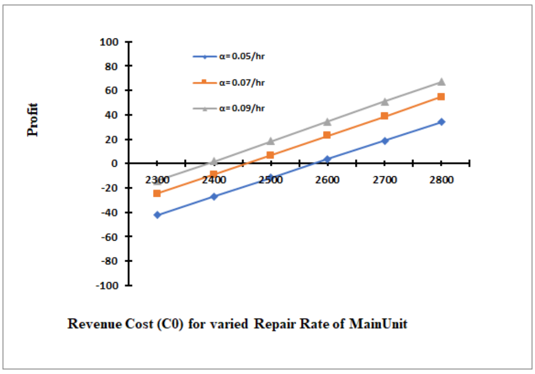

Figure 6. Revenue cost (C0) for different repair rates of the central unit (α)

Table 7. Profit from varied repair rates of ordinary unit (β) vs revenue cost (C_0)

C_0	$\beta = 0.001$ hr^{-1}	$\beta = 0.011$ hr^{-1}	$\beta = 0.012$ hr^{-1}
2200	-57.3730	-31.79661	-7.379280
2300	-42.1335	-15.425947	10.070237
2400	-26.8940	0.9447163	27.519754
2500	-11.6545	17.315380	44.9692
2600	3.584905	33.686044	62.418788
2700	18.82438	50.056708	79.868305

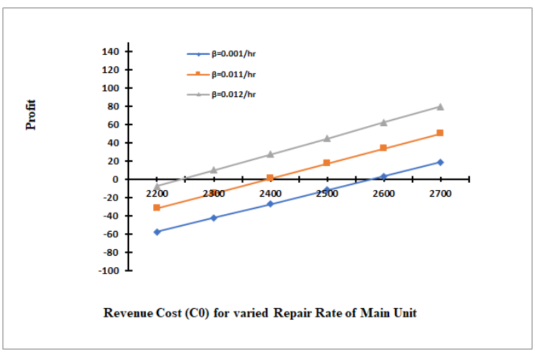

Figure 7. Profit for varied repair rates of ordinary unit (β) v/s revenue cost (C0)

The assertion is clearly illustrated in Figure 7 and Table 7 that the company experiences more profitability when its ordinary unit's repair rate (β) is higher and its failure rate is lower. The authors plotted graphs by taking $C_0 = \{2200, 2300, \ldots\}$, $C_1 = 400$, and $C_2 = 50$ (costs are in Indian rupees). Also, $\mu = 0.1$ hr^{-1}, $\alpha = 0.09$ hr^{-1}, $\beta = \{0.001$ hr^{-1}, 0.011 hr^{-1}, 0.012 hr$^{-1}\}$, and $\lambda = 0.1$ hr^{-1}. The company would be more profitable if the user performed more repairs on ordinary units. Profit is income earned after deducting expenses, not revenue. It demonstrates that reducing costs or increasing revenue can boost the plant's or company's net profit. The Figure 8 and Table 8 illustrates how costs, revenue, and repair impact system profitability. Profit shows an upward trend with lower values of repair costs.Cut-off points help estimate the minimum cost of repair to get maximum profit.

Figure 9 and Table 9 makes it abundantly evident how the profit increases with a remarkable value when the repair rate (β) increases from 0.0058 to 0.007 per hr. and declines with rising repair cost (C2). Cut-off points shown in the figure help decide how much repair costs are to be paid by the company to make a profit.

Table 8. Profit V/S revenue (C_0) for varied repair cost (C_1)

C0	C1 = INR 200	C1 = INR 400	C1 = INR 600
20000	-271.5195371	-323.446345	-375.373153
25000	-175.836302	-227.7631099	-279.6899179
30000	-80.15306694	-132.0798749	-184.0066828
35000	15.53016815	-36.39663977	-88.32344769
40000	111.2134032	59.28659532	7.359787403
45000	206.8966383	154.9698304	103.0430225

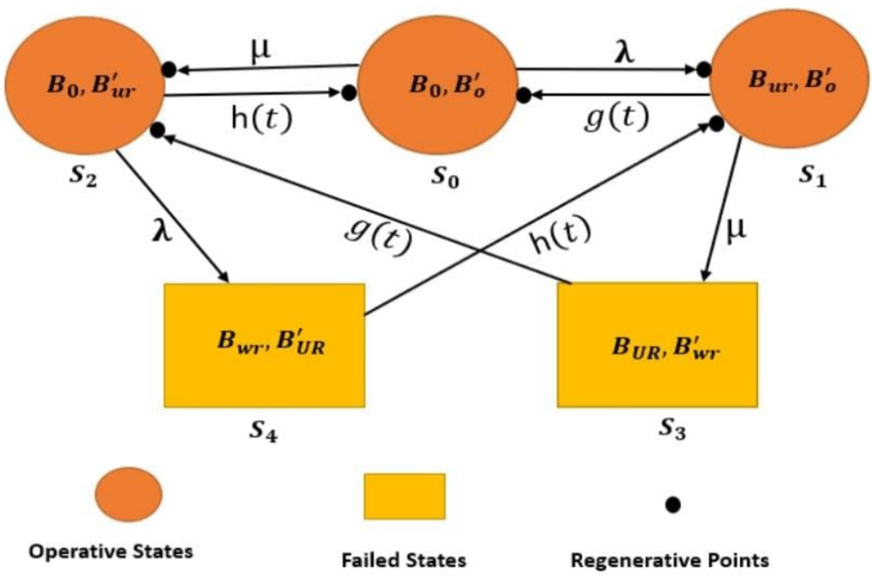

Figure 8. .Profit V/S revenue (C0) for varied repair cost (C1)

Table 9. Profit V/S repair cost (C_2) for varied repair rates (β) of ordinary unit

C2	$\beta = 0.0058$/hr	$\beta = 0.0064$/hr	$\beta = 0.007$/hr
5000	-202.7190166	770.3246726	1521.664158
7000	-496.7770843	503.3252479	1277.225653
9000	-790.835152	236.3258233	1032.787148
11000	-1084.89322	-30.67360142	788.3486426
13000	-1378.951287	-297.6730261	543.9101375
15000	-1673.009355	-564.6724508	299.4716323
17000	-1967.067423	-831.6718755	55.03312712

12 Conclusion

Due to the high unit cost, every industry must buy different units. In such a situation, duplicate units play a vital role. Also, prioritizing original unit repair is not beneficial when duplicate unit repair rates are high.The proposed study is on Bleacher Earth Machines, non-identical units at the visited ghee manufacturing plant. Here, both units work in parallel mode, but the degree of reliability and availability may differ. A single server that promptly calls the system whenever needed handled the repair activities of both units. Repair actions on the part of the server are considered perfect, and thus, the repaired unit works as a new one. The authors analyzed the system's effectiveness measures using the Markov process and the system's profit for fixed repair

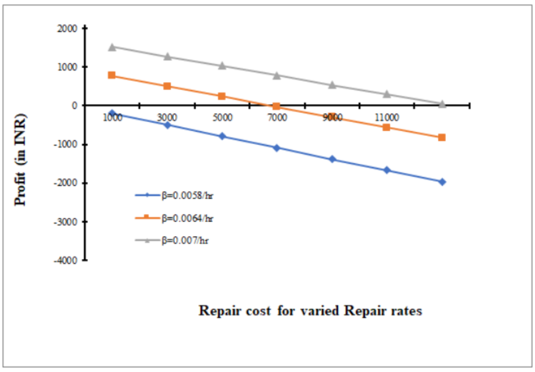

Figure 9. Profit V/S repair cost (C2) for varied repair rates (β) of ordinary unit

values and other maintenance costs. They plotted graphs by taking various parameters with the help of computer software such as MS Excel, etc. MTSF increases as the repair rates of the central unit increase; it declines with higher repair rates of the ordinary unit. However, the profit decreases if ordinary units have higher repair rates. Consequently, when the repair rate of the ordinary (replica) unit is high, it could be more profitable to provide priority to repair the original unit. Thus, the repair rate is the most significant parameter influencing performance measures. Since the model is general, any plant or business with a similar circumstance can utilize it. All random times considered in this paper are assumed to follow an exponential distribution.

Future Scope The authors wish to take random times with a more general distribution (such as Weibull distribution) in the future. In the proposed model, the authors prioritize repairing the duplicate unit to keep it operative all the time. In the future, the authors are thinking of working by giving priority to repair to the original unit with varying demand. Afterward, the authors will compare two models to optimize the profit.

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