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Effects of Memory-Dependent Heat Transfer and Impedance Boundary Conditions on Wave Propagation in a Micropolar Thermoelastic Material

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Abstract *The current study investigates Rayleigh wave propagation in micropolar thermoelastic materials under the influence of memory-dependent heat transfer and impedance boundary conditions. Rayleigh waves, also known as ground roll, are very useful for diagnosing and detecting surface defects in materials, and micropolar thermoelastic materials are used in the development of smart materials and sensors. The frequency equation of Rayleigh waves is obtained analytically, and numerical computation has been used to analyze the impact of various parameters. Some particular cases are mentioned to authenticate the results. The effects of key factors like the time delay parameter, various kernels, and the impedance parameter on the wave speed have been shown graphically. The velocity of Rayleigh waves in micropolar thermoelastic material is notably impacted by the impedance parameter and memory-dependent heat transfer.*

1 Introduction

Fractional order derivatives have recently been used in a number of studies to understand the behavior and modeling of complex systems. Fractional order derivatives are especially helpful in understanding the memory and hereditary properties of a system. Based on fractional derivatives, Wang and Li [1] proposed a novel concept of derivative termed as memory-dependent (MD) derivative, expressed in integral form as

$$D_\chi(f(t)) = \frac{1}{\chi}\int_{t-\chi}^{t} G(t-s)f'(s)ds. \tag{1.1}$$

Where $G(t-s)$ represents a kernel function and χ is the delay time. The kernel function can be conceptualized as the intensity of influence from past events on the current state. As available in literature, the constant, linear, and quadratic forms of the kernel are taken as 1, $[1-(t-s)/\chi]^1$, and $[1-(t-s)/\chi]^2$, respectively, which are useful to study the impact of the previous state on the current state of the function. These functions should also satisfy the inequality $0 \leq G(t-s) \leq 1$ for $s \in [t-\chi, t]$, and in the limiting case when $\chi \to 0$ and $G(t-s) = 1$ in equation (1.1), the MD derivative approaches the common derivative of the function. This type of derivative has recently been applied in a number of fields such as materials science, control theory, solid mechanics, and signal processing, where systems with complex dynamics may be better modeled using MD derivatives rather than traditional derivatives. Recently, there are a number of research articles [2, 3] and [7] utilize MD derivatives to address the problem of generalized thermoelasticity. The studies [4]–[6] investigate wave propagation and damping by applying MD derivatives. Mondal et al. [8] applied MD derivatives to study thermoelastic interaction in a thermoelastic rod. Sarkar and Mondal [9, 10] solved a two-temperature problem using memory-dependent deriva-

tives. Mondal and Othman [11] examined the effects of MD derivatives in piezo-thermoelastic materials under three theories. Purkait et al. [13] studied an elasto-thermodiffusive problem under the effects of memory-dependent heat transfer. The research articles [12], [14]–[16] can be referred to for applications of MD derivatives in magneto-thermoelastic problems. .

Rayleigh waves represent a category of surface waves that propagate in close proximity to the surface of solid materials, and these waves have many applications in seismology, non-destructive testing techniques, and structural health monitoring. Rayleigh waves were first mathematically predicted by Lord Rayleigh [17]. Many researchers have explored the propagation of these surface waves in micropolar thermoelastic solids. Micropolar thermoelastic materials are a class of materials that exhibit both thermal and mechanical responses while also incorporating microstructural effects. Eringen [18] proposed a mathematical formulation to study the deformation in these types of materials, known as micropolar thermoelasticity theory. This theory has garnered significant interest recently as it provides insights into the deformation characteristics of solids that cannot be accurately modeled using conventional approaches. There are number of investigations which studies wave propagation in micropolar materials such as, Rao and Reddy [19] discussed the propagation of Rayleigh-type waves in a micropolar cylindrical system, providing insights into the wave characteristics in such a material. Kumar and Singh [20] explored the behavior of wave propagation in a micropolar generalized thermoelastic body, incorporating the effects of stretch in the material system. The articles [21, 22] investigated Rayleigh wave propagation in a micropolar elastic materials highlighting the unique aspects of wave propagation in such materials. The papers [23]–[25] explored the micropolar characteristics of materials under heating effects.

In most of the Rayleigh wave problems, the boundary conditions are assumed to be traction-free surfaces where stresses diminish on the surface. There are few studies that used impedance boundary (IB) conditions that are prescribed on the boundary. The areas of acoustics, electromagnetism, and seismology all heavily rely on these kinds of boundary conditions. When Rayleigh waves propagate in a medium under IB conditions, the behavior of the waves can be significantly affected. Tiersten [26] used impedance type conditions to study the problem of a thin layer deposited on an elastic substrate. Malischewsky [27] modified "Tiersten's conditions" and expressed them in stress and displacement components to derive the frequency equation of Rayleigh waves. Godoy et al. [28] demonstrated that surface waves exist in an elastic half-space and derived the secular equation with IB conditions. There are some studies such [29]–[32] which employed IB conditions to explore the Rayleigh waves in different types of materials. Recently, the papers [33, 34] investigated Rayleigh waves under impedance boundary conditions under non-local theory of elasticity.

In the current study, using memory-dependent heat transfer and IB conditions, Rayleigh waves are analyzed in a micropolar thermoelastic half-space. An analytical method is employed to derive the frequency equation of Rayleigh waves. Numerical computations have been carried out by taking a particular material, and results are presented graphically. Analysis reveals that both the delay time and kernel play crucial roles in determining the Rayleigh wave speed.

2 Governing Equations

The basic governing equations of motion of particles in an isotropic and homogeneous micropolar thermoelastic solid are given as [18]

$$\sigma_{ij} = \lambda\mu_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \epsilon_{ijr}\phi_r) - \nu T\delta_{ij}, \tag{2.1}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \tag{2.2}$$

$$(\mu + K)\nabla^2\mathbf{u} + (\lambda + \mu)\nabla(\nabla\cdot\mathbf{u}) + K\nabla\times\phi - \nu\nabla T = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{2.3}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla\cdot\phi) - \gamma\nabla\times(\nabla\times\phi) + K\nabla\times\mathbf{u} - 2K\phi = \rho j\frac{\partial^2\phi}{\partial t^2}. \tag{2.4}$$

A new heat conduction equation carrying the memory effects with time delay parameter χ is *given as [?]*

$$K^*\nabla^2 T = \left(\rho C^* \frac{\partial}{\partial t} T + \nu T_0 \frac{\partial}{\partial t} \nabla \cdot \mathbf{u}\right) + \frac{\tau_0}{\chi}\left[\int_{t-\chi}^{t} G(t-s)\left(\rho C^* \frac{\partial^2 T}{\partial s^2} + \nu T_0 \frac{\partial \nabla \cdot \mathbf{u}}{\partial s^2}\right) ds\right]. \tag{2.5}$$

The meaning of symbols used in the above equations has been given in Table 1.

Symbol	Meaning
σ_{ij}	Stress tensor
m_{ij}	Couple stress tensor
δ_{ij}	Kronecker delta
$\mathbf{u}$	Displacement vector
ρ	Density
j	Microinertia
ϕ	Microrotation vector
λ, μ	Lame's constants
α, β, γ, K	Micropolar constants
α_t	Thermal expansion coefficient
T_0, T	Reference temperature of body, temperature change
τ_0	Thermal relaxation time
$\nu = (3\lambda + 2\mu + K)\alpha_t$	Coupling constant

Table 1. Meanings of symbols used in governing equations.

To determine the general results representing all kernels, the function $G(t-s)$ is considered as:

$$G(t-s) = 1 - \frac{2g}{\chi}(t-s) + \frac{h^2}{\chi^2}(t-s)^2$$

$$G(t-s) = \begin{cases} G_1 = 1 & g = 0, h = 0 \\ G_2 = 1 - \frac{(t-s)}{\chi} & g = 0, h = \frac{1}{2} \\ G_3 = \left(1 - \frac{(t-s)}{\chi}\right)^2 & g = 1, h = 1 \end{cases}$$

3 Problem Formulation and Solution

Our analysis focuses on a micropolar thermoelastic half space that is both homogeneous and isotropic in its undeformed state, at an initial constant temperature of T_0. The origin is situated at the surface of the plane, with the y-axis oriented towards the interior of the half-space. In order to ensure that particles vibrating along a line parallel to the z-axis are equally displaced, we assume that the wave propagates along the x-axis. As a result, all field values will not depend on the z-coordinate. For a two-dimensional problem, we consider

$$\mathbf{u} = (u, v, 0) \quad \textit{and} \quad \phi = (0, 0, \phi). \tag{3.1}$$

From equations (2.3), (2.4), and (3.1), we obtain:

$$(\lambda + 2\mu + K)\frac{\partial^2 u}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} + (\mu + K)\frac{\partial^2 u}{\partial y^2} + K\frac{\partial \phi}{\partial y} - \nu\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{3.2}$$

$$(\lambda + 2\mu + K)\frac{\partial^2 v}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} + (\mu + K)\frac{\partial^2 v}{\partial x^2} + K\frac{\partial \phi}{\partial x} - \nu\frac{\partial T}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{3.3}$$

$$\gamma\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) + K\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - 2K\phi = \rho j\frac{\partial^2 \phi}{\partial t^2}. \tag{3.4}$$

Using Helmholtz's theorem, the displacement components u and v can be expressed as

$$u = \frac{\partial \phi_1}{\partial x} + \frac{\partial \psi_1}{\partial y}, \quad v = \frac{\partial \phi_1}{\partial y} - \frac{\partial \psi_1}{\partial x}. \tag{3.5}$$

Where ϕ_1 and ψ_1 are potential functions.

Substituting equation (3.5) into equations (2.5) and (3.2)-(3.4), we obtain:

$$(\lambda + 2\mu + K)\nabla^2 \phi_1 - \nu T = \rho \frac{\partial^2 \phi_1}{\partial t^2}, \tag{3.6}$$

$$(\mu + K)\nabla^2 \psi_1 + K\phi = \rho \frac{\partial^2 \psi_1}{\partial t^2}, \tag{3.7}$$

$$\gamma \nabla^2 \phi - 2K\phi - K\nabla^2 \psi_1 = \rho j \frac{\partial^2 \phi}{\partial t^2}. \tag{3.8}$$

Considering the surface wave solution for the equations (3.6)-(3.9) as

$$\{\phi_1, \psi_1, T, \phi\} = \{\bar{\phi}_1(y), \bar{\psi}_1(y), \bar{T}(y), \bar{\phi}(y)\} e^{ik(x-ct)}, \tag{3.10}$$

where c denotes the phase velocity and k represents the wave number. It is assumed that c is complex with Re$(c) > 0$.

Using equation (3.10) in (3.6)-(3.9), we get:

$$\left[\frac{d^4}{dy^4} - P_q \frac{d^2}{dy^2} + Q_q\right] \left(\bar{\phi}_1(y), \bar{T}(y)\right) = 0, \tag{3.11}$$

$$\left[\frac{d^4}{dy^4} - P \frac{d^2}{dy^2} + Q\right] \left(\bar{\psi}_1(y), \bar{\phi}(y)\right) = 0. \tag{3.12}$$

where,

$$\begin{aligned}
P_q &= k^2 - \frac{(1 + A_2)}{A_1} \left(\frac{ikc}{\tau^*}\right) \eta_q + k^2 \left(1 - \frac{c^2}{c_1^2}\right), \\
Q_q &= k^2 \left(k^2 - \frac{A_3^q}{A_1}\right) \left(1 - \frac{c^2}{c_1^2}\right) - \frac{A_2}{A_1} \left(\frac{ik^3 c}{\tau^*}\right) \eta_q, \quad q = 1, 2, 3, 4 \\
P &= k^2 - \frac{\omega^2 \rho j}{\gamma} + \frac{2K}{\gamma} - \frac{K^2}{\gamma}(\mu + K) + k^2 \left(1 - \frac{c^2}{c_2^2}\right), \\
Q &= k^2 \left[\left(k^2 - \frac{\omega^2 \rho j}{\gamma} + \frac{2K}{\gamma}\right) \left(1 - \frac{c^2}{c_2^2}\right) - \frac{K^2}{\gamma}(\mu + K)\right], \\
c_1^2 &= \frac{\lambda + 2\mu + K}{\rho}, \quad c_2^2 = \frac{\mu + K}{\rho}, \quad \tau^* = \tau_0 + \frac{i}{\omega}, \\
A_1 &= \frac{K^*}{\rho C^* \tau^*}, \quad A_2 = \frac{\nu^2 T_0}{\rho^2 c_1^2 C^*}.
\end{aligned}$$

$$\eta_q = \begin{cases} \eta_1 = 1 + \frac{\tau_0}{\chi}\left(1 - e^{ikc\chi}\right), & \textit{for } G_1 \\ \eta_3 = 1 + \frac{\tau_0}{\chi} + \frac{\tau_0}{\chi^2(ikc)}\left(1 - e^{ikc\chi}\right), & \textit{for } G_2 \\ \eta_4 = 1 + \frac{\tau_0}{\chi} + \frac{2\tau_0}{\chi^2(ikc)} - \frac{2\tau_0\left(1 - e^{ikc\chi}\right)}{\chi^3(k^2c^2)}, & \textit{for } G_3. \end{cases}$$

Using the conditions $\bar{\phi}_1(y), \bar{\psi}_1(y), \bar{T}(y), \bar{\phi}(y) \to 0$ as $y \to \infty$, the general solution of equations (3.11) and (3.12) can be expressed as

$$\phi_1 = \left(R_1 e^{-b_1 y} + R_2 e^{-b_2 y}\right) e^{ik(x-ct)}, \tag{3.13}$$

$$T = \left(r_1 R_1 e^{-b_1 y} + r_2 R_2 e^{-b_2 y}\right) e^{ik(x-ct)}, \tag{3.14}$$

$$\psi_1 = \left(R_3 e^{-b_3 y} + R_4 e^{-b_4 y}\right) e^{ik(x-ct)}, \tag{3.15}$$

$$\phi = \left(r_3 R_3 e^{-b_3 y} + r_4 R_4 e^{-b_4 y}\right) e^{ik(x-ct)}. \tag{3.16}$$

Where b_1, b_2 and b_3, b_4 are the roots of equations (3.11) and (3.12), respectively, and satisfy the following conditions:

$$b_1^2 + b_2^2 = P_q, \quad b_1^2 b_2^2 = Q_q, \tag{3.17a}$$

$$b_3^2 + b_4^2 = P, \quad b_3^2 b_4^2 = Q. \tag{3.17b}$$

The coefficients r_1, r_2, r_3, and r_4 are given by:

$$r_1 = \frac{k^2}{\nu}\left[(\lambda + 2\mu + K)\left(\frac{b_1^2}{k^2} - 1\right) + \rho c^2\right], \tag{3.18a}$$

$$r_2 = \frac{k^2}{\nu}\left[(\lambda + 2\mu + K)\left(\frac{b_2^2}{k^2} - 1\right) + \rho c^2\right], \tag{3.18b}$$

$$r_3 = \frac{k^2(\mu + K)}{K}\left[1 - \frac{c^2}{c_2^2} - \frac{b_3^2}{k^2}\right], \tag{3.18c}$$

$$r_4 = \frac{k^2(\mu + K)}{K}\left[1 - \frac{c^2}{c_2^2} - \frac{b_4^2}{k^2}\right]. \tag{3.18d}$$

Here, R_1, R_2, R_3, and R_4 are arbitrary constants.

4 Derivation of Secular Equations

The boundary conditions at the surface $y = 0$ in terms of impedance parameters Z_i ($i = 1, 2, 3$) are taken as [26]:

$$\begin{aligned} \sigma_{21} + \omega Z_1 u &= 0, \\ \sigma_{22} + \omega Z_2 \nu &= 0, \\ m_{23} + \omega Z_3 \phi &= 0, \\ \frac{\partial T}{\partial y} + hT &= 0. \end{aligned} \tag{4.1}$$

The parameters Z_i are real-valued with dimensions of stress/length, and ω represents the circular frequency. In thermal conditions, $h \to 0$ corresponds to an insulated surface, and $h \to \infty$ represents an isothermal surface.

Using equations (2.1), (2.2), (3.5), (3.13)–(3.16), and boundary conditions (4.1), we obtain a system of four homogeneous equations in the unknowns R_1, R_2, R_3, and R_4. For a non-trivial solution, the following frequency equation of Rayleigh waves is derived:

$$m_3\left[T_1(l_2 n_4 - n_2 l_4) - T_2(l_1 n_4 - n_1 l_4)\right] = m_4\left[T_1(l_2 n_3 - n_2 l_3) - T_2(l_1 n_3 - n_1 l_3)\right]. \tag{4.2}$$

The parameters are defined as follows:

$$l_i = k\left[kV_1 Z_1^* - b_i\left(2 + \frac{K}{\mu}\right)\right], \quad (i = 1, 2),$$

$$l_j = kV_1 Z_1^* - k^2 - \left(1 + \frac{K}{\mu}\right)\left(2b_j^2 - k^2\left(1 - \frac{c^2}{c_2^2}\right)\right), \quad (j = 3, 4),$$

$$n_j = k\left[\left(2 + \frac{K}{\mu}\right)b_j - V_1 Z_2^*\right], \quad (j = 3, 4),$$

$$n_i = k^2\left(2 + \frac{K}{\mu} - V_1^2\right) - kb_i V_1 Z_2^*, \quad (i = 1, 2),$$

$$m_i = (\mu k V_1 Z_3^* - \gamma b_i)\left[1 - \frac{c^2}{c_2^2} - \frac{b_i^2}{k^2}\right], \quad (i = 3, 4),$$

$$T_i = b_i \left[\left(2 + \frac{\lambda + K}{\mu} \right) \left(\frac{b_i^2}{k^2} - 1 \right) + V_1^2 \right], \quad (i = 1, 2) \textit{ (for insulated boundary)},$$

$$T_i = \left[\left(2 + \frac{\lambda + K}{\mu} \right) \left(\frac{b_i^2}{k^2} - 1 \right) + V_1^2 \right], \quad (i = 1, 2) \textit{ (for isothermal boundary)}.$$

Here,

$$V_1 = \sqrt{\frac{\rho c^2}{\mu}}, \quad Z_i^* = \frac{Z_i}{\sqrt{\rho\mu}}, \quad (i = 1, 2, 3).$$

5 Particular Cases and Validation of Results

Case (i)

In the particular case when the kernel $G(t-s) = 1$ and $\chi \to 0$, the operator $D_\chi(f(t))$ becomes the ordinary derivative as follows:

$$D_\chi(f(t)) = \frac{1}{\chi} \int_{t-\chi}^{t} f'(s)\, ds = \frac{f(t) - f(t-\chi)}{\chi} \to f'(t) \textit{ as } \chi \to 0.$$

In this case, P_q and Q_q are modified as:

$$P_q = k^2 \left[2 - \frac{c^2}{A_1} \left(1 + A_2 + \frac{A_1}{c_1^2} \right) \right],$$
$$Q_q = \frac{k^4}{A_1} \left[\frac{c^4}{c_1^2} + A_1 - c^2 \left(1 + A_2 + \frac{A_1}{c_1^2} \right) \right].$$

The equation (4.2) with these modified values of P_q and Q_q becomes the dispersive relation governing Rayleigh waves in micropolar thermoelastic material, excluding MD derivatives. These equations are in agreement with the solution derived by Kumar et al. [32].

Case (ii)

In the case where micropolar effects vanish ($K = j = 0$), we get:

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho},$$
$$P = k^2 \left(1 - \frac{c^2}{c_2^2} \right) + k^2, \quad Q = k^4 \left(1 - \frac{c^2}{c_2^2} \right),$$

$$m_3 = (\mu k V_1 Z_3^* - \gamma b_3) \left[1 - \frac{c^2}{c_2^2} - \frac{b_3^2}{k^2} \right],$$
$$m_4 = (\mu k V_1 Z_3^* - \gamma b_4) \left[1 - \frac{c^2}{c_2^2} - \frac{b_4^2}{k^2} \right].$$

Using equations (3.17), in this particular case, we have:

$$b_3^2 = k^2 \left(1 - \frac{c^2}{c_2^2} \right), \quad b_4^2 = k^2,$$

which implies that $m_3 = 0$ and m_4 will be non-zero. Substituting $m_3 = 0$ and m_4 into equation (4.2), we obtain:

$$l_3 (n_1 T_2 - n_2 T_1) - n_3 (T_2 l_1 - T_1 l_2) = 0. \tag{5.1}$$

The above equation is the same dispersion relation for Rayleigh waves derived by Singh [31] for thermoelastic materials. The parameters are defined as:

$$n_1 = k^2(2 - V_1^2) - kb_1V_1Z_2^*, \quad n_2 = k^2(2 - V_1^2) - kb_2V_1Z_2^*,$$
$$n_3 = k(2b_3 - V_1Z_2^*),$$

$$l_1 = k\left(kV_1Z_1^* - 2b_1\right), \quad l_2 = k\left(kV_1Z_1^* - 2b_2\right),$$
$$l_3 = kV_1Z_1^* - \frac{\omega^2}{c_2^2}.$$

For an insulated boundary:

$$T_1 = b_1\left[\left(2 + \frac{\lambda}{\mu}\right)\left(\frac{b_1^2}{k^2} - 1\right) + V_1^2\right],$$
$$T_2 = b_2\left[\left(2 + \frac{\lambda}{\mu}\right)\left(\frac{b_2^2}{k^2} - 1\right) + V_1^2\right].$$

For an isothermal surface:

$$T_1 = \left[\left(2 + \frac{\lambda}{\mu}\right)\left(\frac{b_1^2}{k^2} - 1\right) + V_1^2\right],$$
$$T_2 = \left[\left(2 + \frac{\lambda}{\mu}\right)\left(\frac{b_2^2}{k^2} - 1\right) + V_1^2\right].$$

Case (iii)

On taking $Z_1^* = Z_2^* = Z_3^* = 0$ in equation (5.1), we derive the dispersion relation of Rayleigh waves with ordinary stress-free boundary conditions.

Case (iv)

Further, by removing the thermal effects in equation (5.1), we obtain:

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 = 4\sqrt{1 - \frac{c^2}{c_1^2}}\sqrt{1 - \frac{c^2}{c_2^2}}. \tag{5.2}$$

Equation (5.2) is the well-established secular equation of Rayleigh waves in the elastic half-space.

6 Numerical Analysis and Discussions

Numerical calculations are performed to calculate the non-dimensional Rayleigh wave speed in a micropolar thermoelastic material to demonstrate the theoretical findings. The pertinent parameters of the aluminum epoxy material for numerical computation are outlined in Table 2 [35].

Considering c as a complex value with $\Re(c) = V \geq 0$ and $V_1 = \sqrt{\frac{\rho V^2}{\mu}}$, the non-dimensional Rayleigh wave speed V_1 has been calculated. The impact of impedance parameters Z_1^*, Z_2^*, Z_3^*, time delay χ, and kernels G_1, G_2, G_3 on V_1 with respect to non-dimensional wave number has been scrutinized and presented graphically in Figures 1-3.

It has been observed that as the delay time increases, the wave speed decreases. Figures 2 and 3 show the effects of kernels G_2 and G_3. The decreasing effect of the memory-dependent parameter is clearly visible in all cases.

Figures 4-6 describe the changes in Rayleigh wave speed with respect to impedance parameters Z_1^*, Z_2^*, Z_3^* under thermally insulated and isothermal heat transfer conditions for kernel

Table 2. Aluminum epoxy material parameters for numerical computation

Parameter	Value
ρ	2.19×10^3 kg/m^3
j	0.196×10^4 m^2
λ	7.59×10^{10} N/m^2
K^*	0.492×10^2 W/m
μ	1.89×10^{10} N/m^2
C^*	1.89×10^{10} J/kg
α	0.01×10^6 N
τ_0	0.5×10^{-10} s
β	0.015×10^6 N
T_0	298 K
γ	0.268×10^6 N
α_t	2.36×10^{-6} K^{-1}
K	0.0149×10^{10} N/m^2

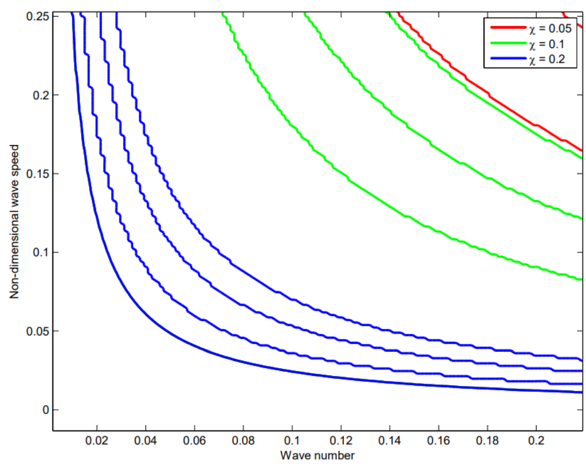

Figure 1. Variation of V_1 with wave number and time delay parameter for kernel G_1.

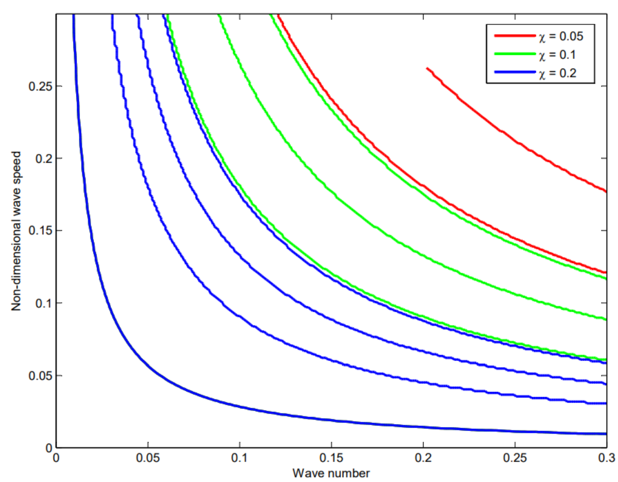

Figure 2. Effect of kernel G_2 on Rayleigh wave speed.

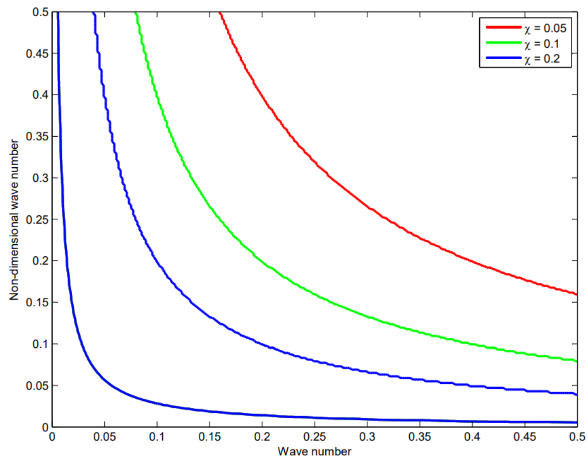

Figure 3. Effect of kernel G_3 on Rayleigh wave speed.

G_3 (at a constant time delay parameter $\chi = 0.05\,s$). The variations in Rayleigh wave speed are similar for thermally insulated and isothermal surfaces for impedance parameters Z_1^* and Z_2^*. It is noticed that the wave speed decreases gradually. However, for impedance parameter Z_3^*, the Rayleigh wave speed remains constant as Z_3^* increases. The wave speed is higher in the case of isothermal conditions compared to insulated conditions, as shown in Figures 4-6.

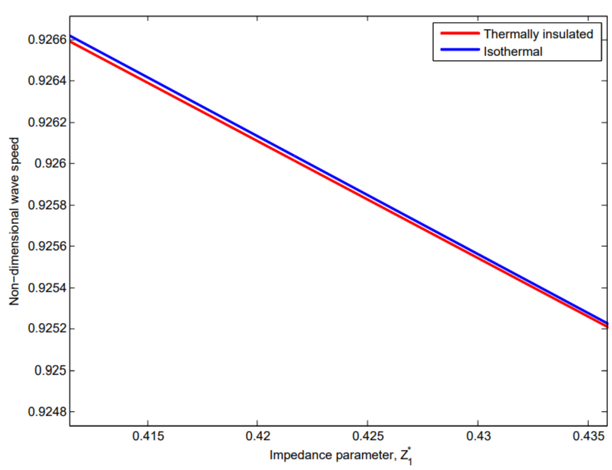

Figure 4. Effect of Z_1^* on Rayleigh wave speed for kernel G_3.

Figure 7 illustrates the variation in Rayleigh wave speed with different wave numbers, depending on impedance parameters Z_1^*, Z_2^*, Z_3^*. The kernel used to generate the graph is $G(t-s) = \left[1 - \frac{t-s}{\chi}\right]^2$, with the time delay parameter χ kept constant at $0.05\,s$.

7 Conclusion remarks

This study obtained the secular equation within the framework of memory-dependent heat transfer for the propagation of Rayleigh waves in micropolar thermoelastic material under impedance boundary (IB) conditions. The following conclusions can be drawn from this study:

- *The wave speed is significantly influenced by different kernels.*
- *The time delay parameter has a decreasing effect on the wave speed.*

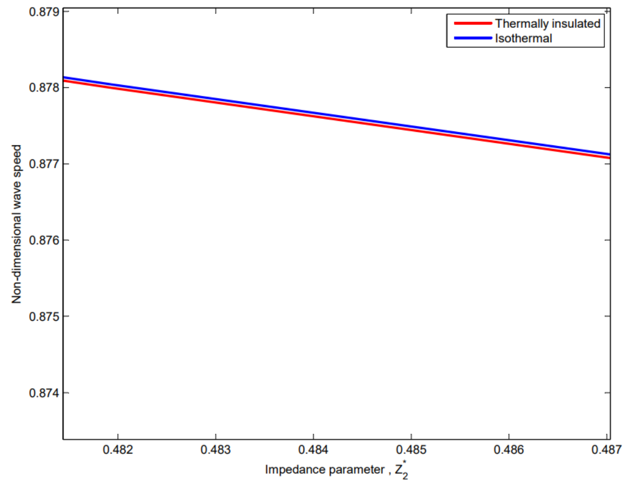

Figure 5. Effect of Z_2^* on Rayleigh wave speed for kernel G_3.

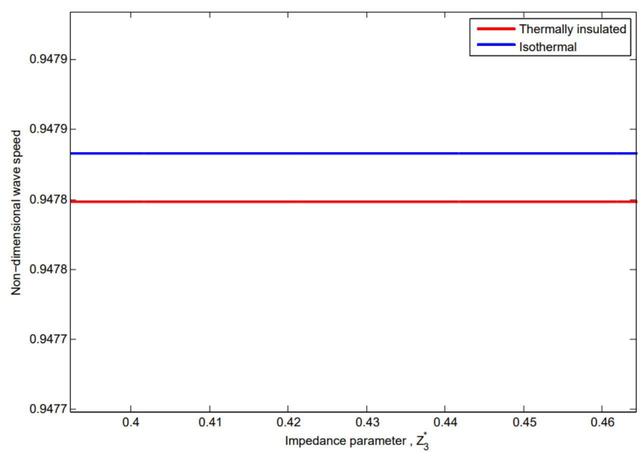

Figure 6. Effect of Z_3^* on Rayleigh wave speed for kernel G_3.

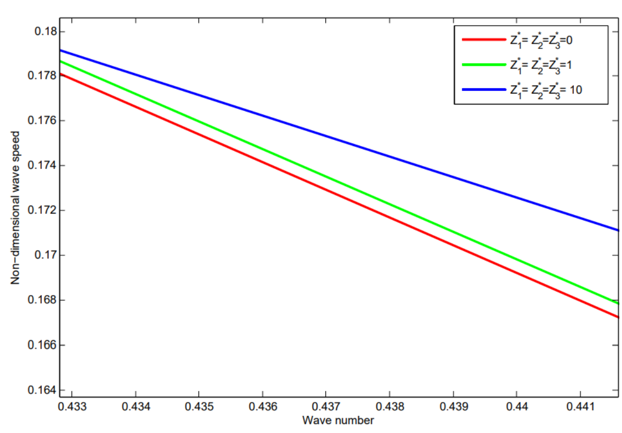

Figure 7. Variation of wave speed with different wave numbers and impedance parameters.

- *The phase velocity is dispersive in nature, and its velocity decreases with an increase in wave number.*

- *The Rayleigh wave speed depends on the frequency and other parameters of the micropolar thermoelastic material.*

The study concludes that understanding Rayleigh waves in micropolar thermoelastic material with time-delay memory-dependent derivatives can provide valuable insights for researchers working on sensors, structural health monitoring, and seismology, particularly for experimental verification.

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