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SMART PARKING SYSTEM PERFORMANCE EVALUATION: A COMPREHENSIVE RELIABILITY AND MARKOV MODELLING STUDY

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Abstract

The research investigates the performance of a smart parking system (SPS) using a dependability technique. It talks about urbanization-related problems like parking space scarcity and traffic congestion. The work integrates the ideas of reliability engineering and Markov modeling to assess several SPS reliability measurements. The authors have considered various hardware and software components, such as sensors, user interfaces, central management software, and communication networks. The proposed system is modeled with the help of Markov model, and the Kolmogorov Chapman differential equations are generated from the transition state diagram. Laplace transforms are used to solve these equations, yielding the mean time to failure (MTTF), availability, and dependability of the system. Moreover, sensitivity analysis identifies critical components and profit analysis directs decisions on repair facilities.

1 Introduction

In the fast-paced world we inhabit, time has become the most precious commodity. In the middle of this never-ending pursuit, timeliness has become a highly valued yet elusive virtue. It's sometimes an enormous undertaking to arrive on time for anything be it for a meeting, an appointment, or even a friendly get-together. There are several causes for this persistent delay, but insufficient parking infrastructure is a major one. Parking spots are uncommon and precious in many cities. Insufficient parking facilities frequently result in cars being parked carelessly, which exacerbates traffic congestion. Moreover, the time spent searching for a parking spot is time wasted, leading to further delays. The domino effect of this can be seen in missed deadlines, hurried work, and increased stress levels. In the search of convenient parking spots much of the time is wasted leading to waste of time, fuel, and energy. It also leads to air and sound pollution for that matter. Inadequate parking systems disrupt our schedules but also add to our daily stress levels. As we move forward, it is imperative to find effective solutions to these issues to ensure smoother journeys, better time management, and ultimately, a better quality of life. The Internet of Things (IoT) has become a very integral part of our lives which makes life easier, and it accounts for many day-to-day activities. The Internet of Things (IoT) has a wide range of applications in our daily lives, transforming the way we live and work. Some applications of IoT can be seen in Smart security systems, Smart home hubs, connected cars, smart retails, and smart cities. The use of IoT and cloud computing systems provides a solution to the perennial problem of parking in urban cities by efficiently managing parking spaces, reducing costs, and improving the overall parking experience. In our paper, we have data formulated and created a smart parking model using IOT and other components such as sensors, cameras, processors to collect data, software, and user interface. Here, the Internet of Things (IoT) uses sensors to connect physical parking infrastructures with information and communication technologies, where cloud-based smart management services are provided. This paper has mainly highlighted how a fault in any component of the system leads to inefficient functioning of the system and failure of which component will lead to complete shutdown or dysfunction of the complete system. Different applications have been proposed in the literature to predict parking spaces, these methods vary in the types of collected data and in the used methods to analyse these data. The remainder of the paper is organised as follows: In Section 2, the literature review is given. In Section 3, system description, system assumption, and notation are provided. In Section 4, mathematical formulation and solution of the problem are given. In Section 5, the smart parking system's performance measure, availability, reliability, mean time to failure, sensitivity of the system's reliability, and expected profit are discussed. In Section 6, results and discussions are presented. In Section 7, conclusions and future scope of the study are provided.

1.1 Problem Statement

It is challenging for traditional parking management strategies to keep up with the growing number of cars in urban areas. To address this issue, we need a system that provides real-time information on parking spaces that are available. This research proposes a reliability-based approach that examines the interactions between the system components to evaluate the efficacy of smart parking systems. It is crucial to ensure that every component functions at its best since users find system outages caused by malfunctions to be extremely annoying. Thus, to evaluate the overall dependability of Smart Parking Systems, we have created a mathematical model.

1.2 Objectives

(i) To develop a mathematical model using reliability approach.

(ii) To determine the mean time between failures, dependability, and time-dependent availability of the system.

(iii) To determine which part of the system is most crucial.

(iv) To determine expected profit of the proposed system.

2 Literature Review

Finding parking places has become increasingly difficult because of growing urbanization and an increase in the number of cars. To solve this problem and raise the effectiveness of parking operations, researchers have realized that creative solutions, such smart parking systems, are required. [1] demonstrated a smart parking system, which was an integrated system that let customers reserve parking spots ahead of time through a special app. [2] that offered visual input on parking space availability was shown by them in their paper. Parking space availability was detected and indicated by IR sensors and LED lights. [1, 3] utilized cloud storage and IoT sensors to make it easier for cars to locate open parking spaces. The parking spots can be accessed and exited via QR codes created by registered users. The microcontroller NodeMCU ESP8266 was used to test the system's performance. In one article [4] a multi-story office parking lot demonstrated a smart parking and energy management system in which the occupied parking spaces were digitally saved to the cloud so that a central system could access them and route oncoming automobiles to vacant spaces. To manage parking spaces, the system made use of Honeywell sensors and controllers. In [5] The goal of their similarly titled project was to develop a system that made it simple for drivers of private automobiles to locate parking in designated locations and enabled them reserve spaces using an Android application. The other system used computer vision techniques and made use of Internet of Things technologies. [6] used convolution neural network and Mask R-CNN algorithms for vehicle and parking slot detection with high accuracy. [7] Smart parking system that used M2M communication in IoT to monitor and manage available parking spaces in real-time, reducing human effort and cost. Mobile Application, Wi-Fi, LED Indicator, IR Sensors, RFID, Cloud Database, A* Algorithms, and RaspberryPI3. Some authors such as in [8] used Bluetooth Low Energy beacons and particle filtering for improved accuracy. The proposed system correctly predicts the user's parking spot and estimates their distance from the beacon. [9] designed and implemented a prototype of Reservation-based Smart Parking System (RSPS) permits drivers to effectively locate and withhold the vacant parking spaces in mentioned has smoothen the operations of parking systems, as well as reduced traffic congestion caused by searching for parking. The system uses four important methods: POST, GET, PUT, DELETE. In the other article, the authors of [10] proposed a smart parking system based on improved Optical Character Recognition (OCR) model, which consists of three stages: 1) OCR based parking slot detection, 2) User notification based on IoT approach, 3) Smart parking meter based on Simple Mail Transfer Protocol (SMTP). It aimed to reduce the time and fuel wasted in searching for parking slots in urban areas. Smart parking management system that depended on Arduino components, android applications, and IoT technology to allow users to check available parking spaces and reserve a spot [11, 13] . This system [12] used the Message Queue Telemetry Transport Protocol (MQTT) for communication. [13] allows the user quick access which helps in reduction of time in searching the parking spot, reduction in traffic congestion and can increase the quality of services in cities and can improve productivity and reliability. In this paper, the author [14] proposes an Edge-Cloud-Dew architecture for smart parking systems, aiming to make parking systems smarter through the integration of cloud and dew architecture technology. It has Innovative design, including LAN-level deployment, platform-as-a-dew Service (PaaDS), dew version of license plate recognition, and the dew type of machine learning model training. [15] has come up with the model of a complex outdoor smart parking IoT system based on the mini-PC platform with the pilot implementation, which would provide a solution for the problem and has the potential in its expansion and integration with other IoT services. With the view of the pandemic [16] article of a smart parking system with Node MCU and IoT-based RFID was proposed to filter incoming vehicles, increase security and efficiency in parking areas, and reduce the impact of the spread of the Covid-19 virus. [19] proposed similar smart parking system using IoT to solve parking problems. The infrared sensors detect parking spaces, connected to the ESP12-E module. This study designed a prototype of a parking monitoring system using NodeMCU. Users access parking space information through a smartphone application. Registered users have a login code for security and convenience. The system works effectively for organizing parking and helping drivers find spots. In one of the articles, the authors [17] explored different SPS communication models by varying the number of occupancy data collators, their positions, hybrid power cycles and data aggregation strategies, and proposed a concise data format for effective data dissemination. A similar article has been proposed by [18] to analyse the smart car parking system. The paper examines different approaches used to investigate the smart car parking system. The Car-Park Occupational Information System (COINS) is mentioned as a framework for acquiring automobile residency data. [20] examines issues with conventional parking lots and their inefficiency. The authors proposed a smart parking system using IoT technology, ESP8266, servo motor, and IR sensors. The system allows users to locate open parking spaces, reserve them, and make payments through a mobile application, and administrators can offer free parking spots as well. Servo motor and IR sensors are used for gate control and exit procedures. The system uses embedded cameras and machine learning to track parking spaces. IoT-AIPS enhances performance in terms of cost-effectiveness and accuracy. Many systems have been proposed where machine learning is implemented. In this article, [23] has proposed a framework that can analyse the performance of smart parking systems at an enormous scale using an ML algorithm, and the experimental results show that predicting network usage of large-scale smart parking system using the ML framework is 1500x faster than the simulation time of the CloudSim simulator. Further, [21] proposed a digitized parking system with a proof-of-concept implementation that combines multiple technological concepts into one solution with the advantages of using IoT for real-time tracking of parking availability. Machine learning models, including XGboost, were used to estimate occupancy rate and similarly in another paper [22] a fuzzy logic integrated machine learning algorithm (FL-MLA) is proposed for smart parking and traffic management in a metropolis, which uses fuzzy induction to distinguish between parked and moving vehicles while calculating traffic flow. FL-MLA reduces energy consumption and improves traffic estimation accuracy. There are also some surveys which are conducted on smart parking system. [24] provides a comprehensive classification of smart parking solutions based on their functionalities and problematic focuses. It identifies three macro-themes: informa-

tion collection, system deployment, and service dissemination. The survey highlights the main methodologies used in existing works and summarizes their common goals and visions to solve current parking difficulties. The paper offers engineering insights and discusses challenges and open issues in the field of smart parking. For more mathematical modelling related concepts one can refer to [25],[26],[27].
Although many studies have been conducted on smart parking systems, our research offers a distinct perspective by conducting an extensive reliability and Markov Modelling analysis, providing a strong basis for evaluating system performance and identifying potential flaws of the system.

3 System Description and Assumptions

3.1 The following is the description of the smart parking system

(i) **Sensors:** Sensors in a smart parking system are like watchful eyes and ears, continuously collecting information on available parking spaces and vehicles. They are the foundation of smart parking systems, facilitating intelligent decision-making and smooth communication to improve the parking experience. They play a critical role in accurate and effective parking management that benefits operators and drivers alike. These sensors are used to detect the presence or absence of vehicles in real-time and are strategically placed within parking lots or on-street parking slots. The system's ability to reliably detect parking space occupancy is jeopardized in the event of sensor failure, which will affect user experience and efficiency of operation.

(ii) **User Interface:** It acts as the main interface between the system and its users, giving them access to key features like real-time parking availability updates, reservation or payment processing, and the ability to locate and navigate available parking spaces. A well-thought-out user interface increases usability, improves user experience, and helps the smart parking system become more successful and widely used. The total efficacy and efficiency of the smart parking system may be impacted by a poorly designed user interface, which can also make the system less adaptable and difficult to operate.

(iii) **Communication System:** The communication system acts as the nervous system, seamlessly connecting all components and facilitating smooth information flow. They make it possible for cloud servers, user apps, gateway devices, and parking spot sensors to communicate in real-time. They effectively communicate critical information, such as reservations and space availability, over robust communication protocols like Wi-Fi, Bluetooth, or cellular networks. This smooth communication structure minimizes traffic, maximizes the use of parking resources, and enhances user experience. If the communication system fails, the smart parking system may become unable to provide real-time updates on parking availability and guidance to drivers.

(iv) **Central Management System:** It acts as the primary control point, enabling effective parking space management, real-time occupancy monitoring, and data analytics to maximize parking operations. It is crucial for optimizing space utilization, cutting traffic, and simplifying parking management—all of which enhance traffic flow and urban mobility. Overall, this critical component plays a vital role in orchestrating data, decisions, and operations, ultimately determining the success of the entire system. If the Central Management System (CMS) fails in a smart parking system, the system may experience partial shutdown or limited functionality.

(v) **Software:** To ensure smooth data gathering and analysis for efficient parking management, the software synchronizes communication between IoT devices, sensors, and the central system. Software acts as the brain of the system, ingesting data from sensors, cameras, and user interactions. It uses algorithms to process this data and assess parking availability in real-time, forecast occupancy in the future, and spot trends. In addition to making, it easier for users to locate parking, this data analysis allows for dynamic pricing models, which maximize resource allocation and reduce traffic. Information about parking availability and automatic features would be disrupted if the software in a smart parking system is disrupted.

3.2 Assumptions of the system

The system analysis is based on the following assumptions:

(i) All required components are correctly initialized and operating when the system starts from a predetermined initial state.

(ii) Throughout the system's operation, the Internet of Things devices, including sensors and user interfaces have dependable and steady network connectivity.

(iii) The functionality of IoT devices and the system's overall performance are not significantly affected by external factors like the weather or physical obstacles.

(iv) Power supply to all components is stable and uninterrupted.

(v) A sufficient repair facility is available.

(vi) The system does not stop working in a degraded state and works only for a short period.

3.3 Notation

This paper will use the following notion given in **Table 1** to build the mathematical model.

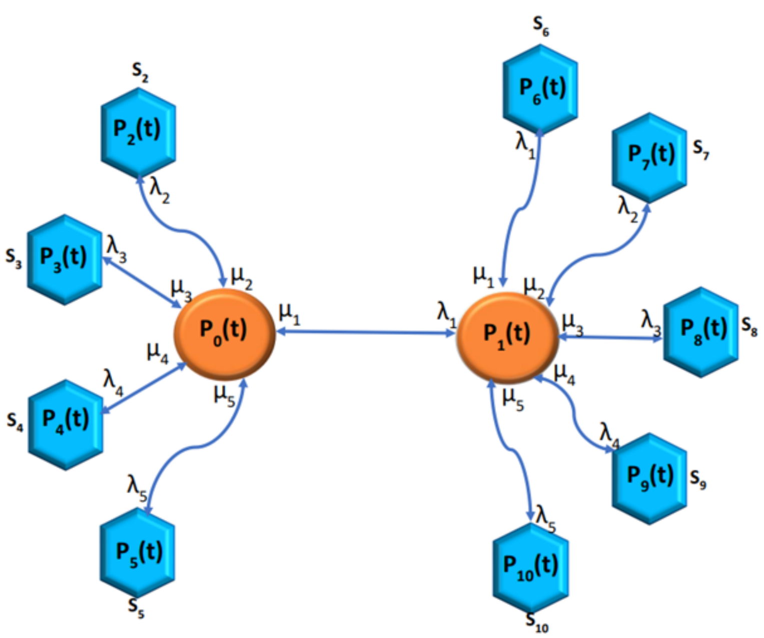

Figure 1. State Transition Diagram of the System.

Table 1. Notations and state description

Notation	Description
t	Time variable
s	The transformation variable of Laplace equation
λ_i	Failure rate of component i
μ_i	The repair rate of i^{th} component.
$P_i(t)$	The probability of the system being in the i^{th} state.
$\bar{P}_i(s)$	The probability that the system is in the i^{th} state.
S_0	Every component of the system is functioning properly
S_1	First subcomponent (camera/sensor) of the first component of the system fails and the system is in a degraded state
S_2	Second component (user interface) of the system fails, and the system fails completely
S_3	Third component (communication system) of the system fails, and the system fails completely
S_4	Fourth component (central management system) of the system fails, and the system fails completely
S_5	Fifth component (Software) of the system fails, and the system fails completely
S_6	First and second subcomponents (camera and sensor) of the first component fail, and the system completely fails
S_7	First subcomponent (camera/sensor) of the first component fails along with the second component (user interface) and the system fails completely
S_8	First subcomponent (camera/sensor) of the first component fails along with the third component (communication system) and the system fails completely
S_9	First subcomponent (camera/sensor) of the first component fails along with the fourth component (central management system) and the system fails completely
S_{10}	First subcomponent (camera/sensor) of the first component fails along with the fifth component (software) and the system fails completely

4 Smart Parking System's Mathematical Modelling

4.1 Mathematical Formulation and Solution of the Problem

Based on Figure 1 of the state transition diagram of the system, the authors formulate Chapman-Kolmogorov differential equations in this section. The following set of differential equations can be created by allowing the system to transition at a certain time.

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5\right]P_0(t)=\mu_1P_1(t)+\mu_2P_2(t)+\mu_3P_3(t)+\mu_4P_4(t)+\mu_5P_5(t) \quad (1)$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\mu_1\right]P_1(t)=\mu_1P_6(t)+\mu_2P_7(t)+\mu_3P_8(t)+\mu_4P_9(t)+\mu_5P_{10}(t)+\lambda_1P_0(t) \quad (2)$$

$$\left[\frac{d}{dt}+\mu_2\right]P_2(t)=\lambda_2P_0(t) \quad (3)$$

$$\left[\frac{d}{dt}+\mu_3\right]P_3(t)=\lambda_3P_0(t) \quad (4)$$

$$\left[\frac{d}{dt}+\mu_4\right]P_4(t)=\lambda_4P_0(t) \quad (5)$$

$$\left[\frac{d}{dt}+\mu_5\right]P_5(t)=\lambda_5P_0(t) \quad (6)$$

$$\left[\frac{d}{dt}+\mu_1\right]P_6(t)=\lambda_1P_1(t) \quad (7)$$

$$\left[\frac{d}{dt}+\mu_2\right]P_7(t)=\lambda_2P_1(t) \quad (8)$$

$$\left[\frac{d}{dt}+\mu_3\right]P_8(t)=\lambda_3P_1(t) \quad (9)$$

$$\left[\frac{d}{dt}+\mu_4\right]P_9(t)=\lambda_4P_1(t) \quad (10)$$

$$\left[\frac{d}{dt}+\mu_5\right]P_{10}(t)=\lambda_5P_1(t) \quad (11)$$

$$P_i(0)=\begin{cases}1 & i=0\\ 0 & i\neq 0\end{cases} \quad (12)$$

The results from applying the Laplace inverse transformation to the equations from (1) to (11) are given as:

$$[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5]\,\bar{P}_0(s)=1+\mu_1\bar{P}_1(s)+\mu_2\bar{P}_2(s)+\mu_3\bar{P}_3(s)+\mu_4\bar{P}_4(s)+\mu_5\bar{P}_5(s) \quad (13)$$

$$[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\mu_1]\,\bar{P}_1(s)=\mu_1\bar{P}_6(s)+\mu_2\bar{P}_7(s)+\mu_3\bar{P}_8(s)+\mu_4\bar{P}_9(s)+\mu_5\bar{P}_{10}(s)+\lambda_1\bar{P}_0(s) \quad (14)$$

$$[s+\mu_2]\,\bar{P}_2(s)=\lambda_2\bar{P}_0(s) \quad (15)$$

$$[s+\mu_3]\,\bar{P}_3(s)=\lambda_3\bar{P}_0(s) \quad (16)$$

$$[s+\mu_4]\,\bar{P}_4(s)=\lambda_4\bar{P}_0(s) \quad (17)$$

$$[s+\mu_5]\,\bar{P}_5(s)=\lambda_5\bar{P}_0(s) \quad (18)$$

$$[s+\mu_1]\,\bar{P}_6(s)=\lambda_1\bar{P}_1(s) \quad (19)$$

$$[s+\mu_2]\,\bar{P}_7(s)=\lambda_2\bar{P}_1(s) \quad (20)$$

$$[s+\mu_3]\,\bar{P}_8(s)=\lambda_3\bar{P}_1(s) \quad (21)$$

$$[s+\mu_4]\,\bar{P}_9(s)=\lambda_4\bar{P}_1(s) \quad (22)$$

$$[s+\mu_5]\,\bar{P}_{10}(s)=\lambda_5\bar{P}_1(s) \quad (23)$$

On solving equations (13)-(23), one can easily obtain:

$$T_1 = \left[s + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \mu_1 - \frac{\mu_1\lambda_1}{s+\mu_1} - \frac{\mu_2\lambda_2}{s+\mu_2} - \frac{\mu_3\lambda_3}{s+\mu_3} - \frac{\mu_4\lambda_4}{s+\mu_4} - \frac{\mu_5\lambda_5}{s+\mu_5}\right] \tag{24}$$

$$T_2 = \left[s + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 - \frac{\mu_1\lambda_1}{T_1} - \frac{\mu_2\lambda_2}{s+\mu_2} - \frac{\mu_3\lambda_3}{s+\mu_3} - \frac{\mu_4\lambda_4}{s+\mu_4} - \frac{\mu_5\lambda_5}{s+\mu_5}\right] \tag{25}$$

$$\bar{P}_1(s) = \frac{\lambda_1 \bar{P}_0(s)}{T_1} \tag{26}$$

$$\bar{P}_0(s) = \frac{1}{T_2} \tag{27}$$

After solving the above equations, one can easily determine the up state and the down state of the system. In reliability theory, the system's "up state" means that the system is performing its intended task. The system's up state equation is given below:

$$\bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) \tag{28}$$

Similarly, the system's down state means that the system is not performing its intended task. The equation for the system down state is given below:

$$\bar{P}_{down}(s) = \bar{P}_1(s)+\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_4(s)+\bar{P}_5(s)+\bar{P}_6(s)+\bar{P}_7(s)+\bar{P}_8(s)+\bar{P}_9(s)+\bar{P}_{10}(s) \tag{29}$$

5 The Smart Parking System's Performance Measures

5.1 Performance measures of the smart parking system

Reliability-based performance metrics for smart parking systems are essential for assessing the dependability and efficacy of the system. In this sense, "reliability" refers to the system's capacity to deliver parking-related services and information in a consistent and correct manner. The system's uptime, or the proportion of time it runs without errors, is one example of a key performance indicator. Reliability is also influenced by the precision with which parking spaces are detected and instantaneous updates on available spaces. To provide customers confidence that the system would accurately direct them to available parking spots, a trustworthy parking system should reduce the number of false informational incidents.

The dependability of a parking system can be increased by regular maintenance, strong sensor technologies, and effective communication protocols, which will ultimately increase customer happiness and optimize overall system performance.

5.2 Availability of the Proposed System

The percentage of time the parking system is operational and usable is referred to as availability. It is computed by dividing the total time the system is available by the sum of the available time and downtime:

$$\text{Availability} = \frac{\text{uptime}}{\text{uptime} + \text{downtime}} \tag{30}$$

Now to calculate the availability of the parking system, take the Laplace inverse of equation (28) and set the repair and failure rates per hour as given below:

$$\lambda_1 = 0.14 \times 10^{-4},\ \lambda_2 = 0.10 \times 10^{-4},\ \lambda_3 = 0.7 \times 10^{-4},\ \lambda_4 = 0.40 \times 10^{-4},\ \lambda_5 = 0.2 \times 10^{-4}$$

$$\mu_1 = 0.5,\ \mu_2 = 0.25,\ \mu_3 = 0.20,\ \mu_4 = 0.125,\ \mu_5 = 0.25$$

One can obtain the availability expression as given below:

$$\begin{aligned} A(t) = 0.9992106228 - 7.345575 \times 10^{-8} \times e^{-0.5027774086t} + 7.283691 \times 10^{-8} \times e^{-0.49774805189t} \\ + 0.0001203992435 \times e^{-0.2500300517t} - 9.4580481 \times 10^{-11} \times e^{-0.2499700587t} \\ + 0.0003498295445 \times e^{-0.2000699952t} + 3.4021829 \times 10^{-11} \times e^{-0.1999533502t} \\ + 0.0003191478289 \times e^{-0.1250399531t} + 3.1888540 \times 10^{-12} \times e^{-0.1249866637t} \end{aligned} \quad (31)$$

By changing the time unit between 0 and 4000 hours, one can obtain Table 2 and Figure 2, which represent the availability of the system.

Table 2. Availability of the smart parking system

Time (hours)	System Availability
0	0.9999
400	0.9992
800	0.9992
1200	0.9992
1600	0.9992
2000	0.9992
2400	0.9992
2800	0.9992
3200	0.9992
3600	0.9992
4000	0.9992

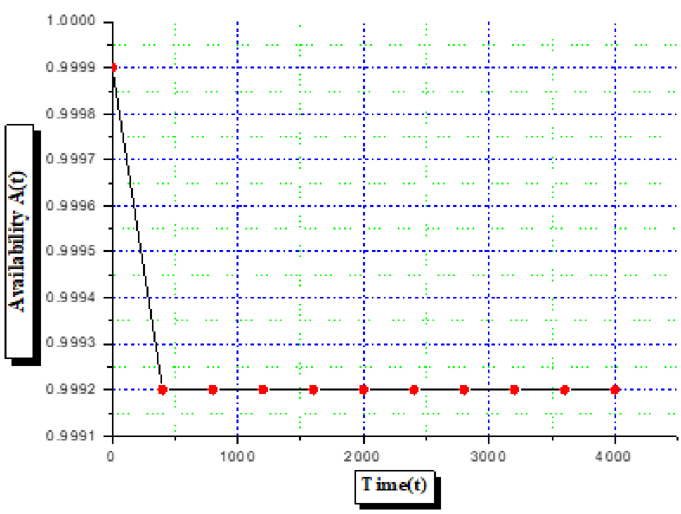

Figure 2. Availability of the smart parking system.

5.3 Reliability of the system

The likelihood that a system will function as intended for the designated amount of time under specified operating conditions is known as the system's reliability. Set all repair rates to zero to calculate the system's time-dependent reliability in equation (28) and take the inverse Laplace transformation of the equation. Then, set the failure rates as:

$\lambda_1 = 0.14 \times 10^{-4}$, $\lambda_2 = 0.10 \times 10^{-4}$, $\lambda_3 = 0.7 \times 10^{-4}$, $\lambda_4 = 0.40 \times 10^{-4}$, $\lambda_5 = 0.2 \times 10^{-4}$

The following equation represents the smart parking system's reliability:

$$R(t) = 2 \times 10^{-6} \times e^{-0.000154t}(500000 + 7t) \tag{32}$$

Now, by varying the time unit t from 0 to 4000, one can obtain Table 3 and Figure 3, which represent the system's reliability.

Table 3. Reliability of the smart parking system

Time (hours)	System Reliability
0	1.0000
400	0.9455
800	0.8940
1200	0.8452
1600	0.7991
2000	0.7555
2400	0.7142
2800	0.6752
3200	0.6383
3600	0.6034
4000	0.5703

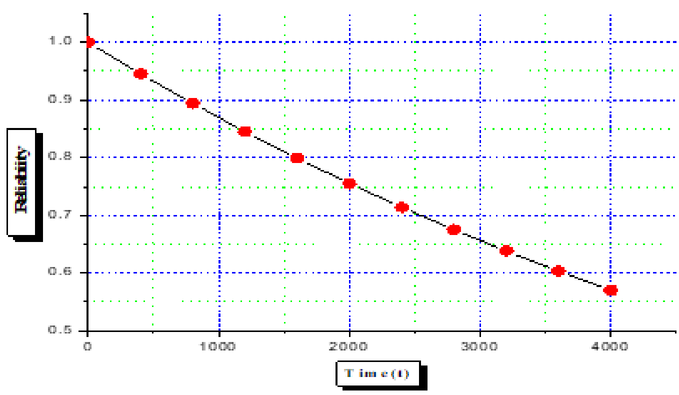

Figure 3. Reliability of the smart parking system.

5.4 Meantime to failure of the system

Mean time to failure (MTTF) is the average duration a system or component is expected to operate before experiencing a failure. To calculate the mean time to failure of the smart parking system, set all repair rates to zero in equation (28) and take the limit $s \to 0$. One can obtain the MTTF of the system as given in the equation below:

$$\text{MTTF} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} + \frac{\lambda_1}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5)^2} \tag{33}$$

Set the failure rates in equation (33) as:

$\lambda_1 = 0.14 \times 10^{-4}$, $\lambda_2 = 0.10 \times 10^{-4}$, $\lambda_3 = 0.7 \times 10^{-4}$, $\lambda_4 = 0.40 \times 10^{-4}$, $\lambda_5 = 0.2 \times 10^{-4}$

One may generate Table 4 and Figure 4 of the MTTF of the smart parking system by varying each failure rate one at a time.

Table 4. Mean Time to Failure (MTTF) of the Smart Parking System

Variation in the failure rate	**MTTF w.r.t.** λ_1	**MTTF w.r.t.** λ_2	**MTTF w.r.t.** λ_3	**MTTF w.r.t.** λ_4	**MTTF w.r.t.** λ_5
0.1×10^{-4}	7111.1111	7083.8253	12222.7252	8975.0260	7619.5987
0.2×10^{-4}	7031.2500	6618.0845	10909.7633	8242.3702	7083.8252
0.3×10^{-4}	6920.4152	6209.5389	9849.1843	7619.5987	6618.0844
0.4×10^{-4}	6790.1235	5848.2987	8975.0260	7083.8253	6209.5389
0.5×10^{-4}	6648.1994	5526.6234	8242.3702	6618.0845	5848.2987
0.6×10^{-4}	6500.0000	5238.3698	7619.5988	6209.5389	5526.6234
0.7×10^{-4}	6349.2063	4978.6008	7083.8253	5848.2987	5238.3698
0.8×10^{-4}	6198.3471	4743.3035	6618.0845	5526.6234	4978.6007
0.9×10^{-4}	6049.1493	4529.1840	6209.5389	5238.3699	4743.3036

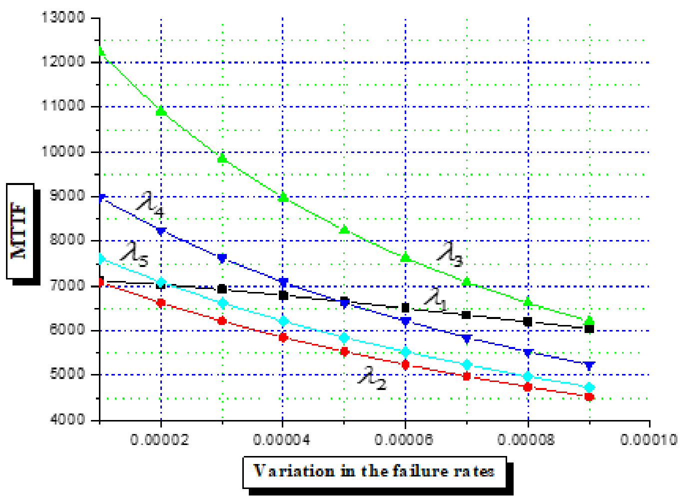

Figure 4. Mean time to failure (MTTF) of the smart parking system

5.5 Sensitivity of the reliability of the system

Set all the failure rates in equation (28) to zero, and we take the inverse of Laplace transform of the equation to carry out the sensitivity analysis of the system's reliability. Now that this equation has been differentiated regarding the failure rate and varied between 0 and 4000, it is simple to derive Table 5 and Figure 5 of the system's reliability sensitivity.

5.6 Expected profit

It is always necessary to calculate the expected profit that a system will generate in the times to come. Therefore, the author calculated the expected profit of the smart parking system. The following function can be used to estimate the profit of the system in the time interval $[0, t)$:

Table 5. Sensitivity of Reliability of Smart Parking System

Time (hours)	$\frac{\partial R(t)}{\partial \lambda_1}$	$\frac{\partial R(t)}{\partial \lambda_2}$	$\frac{\partial R(t)}{\partial \lambda_3}$	$\frac{\partial R(t)}{\partial \lambda_4}$	$\frac{\partial R(t)}{\partial \lambda_5}$
0	0	0	0	0	0
400	-2.0480	-367.7667	-367.7667	-367.7667	-367.7667
800	-7.4900	-676.2409	-676.2409	-676.2409	-676.2409
1200	-15.4082	-932.5635	-932.5635	-932.5635	-932.5635
1600	-25.0448	-1143.1143	-1143.1143	-1143.1143	-1143.1143
2000	-35.7786	-1313.5880	-1313.5880	-1313.5880	-1313.5880
2400	-47.1057	-1449.0622	-1449.0622	-1449.0622	-1449.0622
2800	-58.6212	-1554.0596	-1554.0596	-1554.0596	-1554.0596
3200	-70.0045	-1632.6040	-1632.6040	-1632.6040	-1632.6040
3600	-81.0061	-1688.2709	-1688.2709	-1688.2709	-1688.2709
4000	-91.4366	-1724.2334	-1724.2334	-1724.2334	-1724.2334

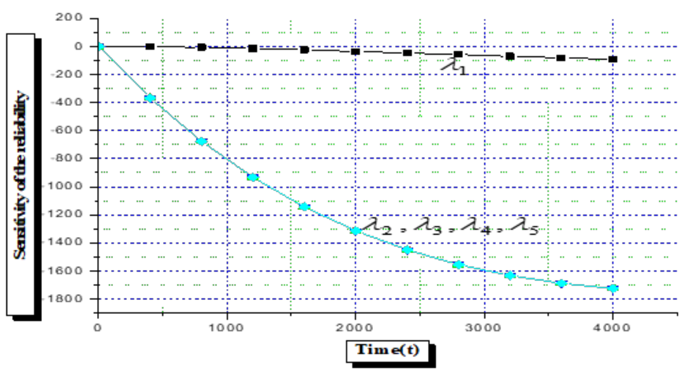

Figure 5. Sensitivity of reliability of smart parking system

$$E_P(t) = K_1 \int_0^t A(t)\,dt - K_2 t \tag{34}$$

Here, the expression $\int_0^t A(t)\,dt$ gives the total uptime of the system, and t represents the total time. The parameter K_1 represents the revenue generated by the system per unit time, and K_2 represents the expenditure of the system per unit time. Now, set $K_1 = 20$, and vary the value of K_2 and t. We can obtain the following Table 6 of the expected profit of the smart parking system.

Table 6. Expected Profit of the Smart Parking System

Time (hours)	$E_p(t) \mid K_2 = 2$	$E_p(t) \mid K_2 = 4$	$E_p(t) \mid K_2 = 6$	$E_p(t) \mid K_2 = 8$
0	0	0	0	0
400	7193.7806	6393.7806	5593.7806	4793.7806
800	14387.4656	12787.4656	11187.4656	9587.4656
1200	21581.1505	19181.1506	16781.1506	14381.1505
1600	28774.8355	25574.8355	22374.8356	19174.8355
2000	35968.5205	31968.5206	27968.5206	23968.5205
2400	43162.2055	38362.2055	33562.2055	28762.2055
2800	50355.8905	44755.8905	39155.8905	33555.8905
3200	57549.5755	51149.5755	44749.5755	38349.5755
3600	64743.2604	57543.2604	50343.2605	43143.2604
4000	71936.9455	63936.9455	55936.9455	47936.9454

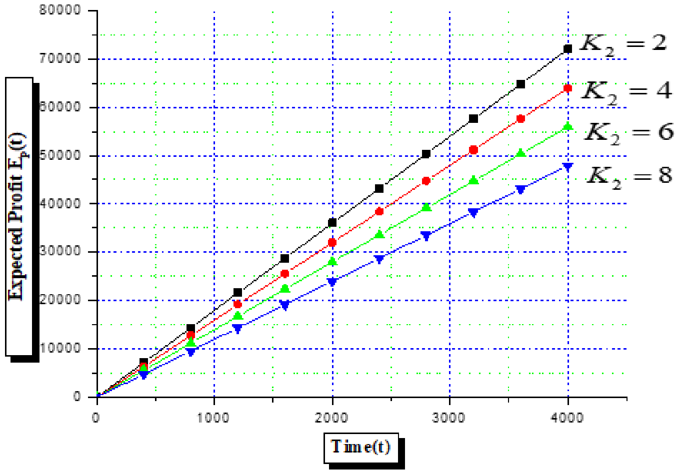

Figure 6. Expected Profit of smart parking system.

6 Results and Discussions

The authors of this research measured the smart parking system's reliability indices using a Markov model. Both hardware and software components have been considered to evaluate the system's reliability. The suggested system has produced the following outcomes:

(i) **Table 2** and **Figure 2** show the availability of the system as a function of time. Initially, at time $t = 0$, the availability of the system is 0.9999, which is very high. From 400 hours onward, it maintains a consistently high availability of 0.9992, despite a minor drop. Hence, in the whole duration, the system is highly available for its usage.

(ii) **Table 3** and **Figure 3** show the system's reliability as a function of time. With a value of 1.0000 at 0 hours, the system shows excellent reliability at first. But with time, there is a discernible drop in reliability, suggesting system degradation. The reliability falls to 0.9455 by 400 hours and then gradually declines over the next intervals, reaching 0.5703 by 4000 hours. This pattern indicates that, in order to maintain system performance over extended periods of time, periodic maintenance or reliability improvement actions are required.

(iii) **Table 4** and **Figure 4** provide the system's Mean Time Between Failures (MTTF) based on changes in the component failure rates. When it comes to changes in the user interface failure rate, the system's Mean Time Between Failures (MTTF) is low, whereas it is high when it comes to changes in the communication system failure rate. The software failure rate fluctuation is reflected in the low mean time between failures (MTTF) of the system. Software and the user interface should therefore be given top attention when it comes to maintenance and repairs.

(iv) **Table 5** and **Figure 5** indicate the system's sensitivity to reliability. Software, central management systems, communication systems, and user interfaces all depend on the dependability of the system.

(v) **Table 6** and **Figure 6** give the expected profit of the proposed system with respect to variation in time. It is quite clear from the diagram that as the system's expenditure goes up, the profit goes down. Therefore, it is mandatory to keep the expenditure of the organization in control to get the maximum profit.

7 Conclusion and Future Scope of the Study

Upon careful analysis, the system begins with commendable availability but undergoes a gradual decline in reliability over time, hinting at potential deterioration. This underscores the urgent need for maintenance or enhancement measures to uphold performance over prolonged periods. Notably, the user interface and software components emerge as critical areas due to their lower Mean time to failure and heightened sensitivity to failure rates, necessitating prioritized attention. Furthermore, the sensitivity analysis emphasizes the substantial impact of user interface, communication system, central management system and software on system reliability. Considering that system profitability dwindles as expenditure rises. Adopting cost-effective maintenance strategies becomes indispensable to optimize profitability. Hence, it is suggested that user interface and software are the main critical components hence their proper maintenance is the top priority so that system's uptime can be increased. The future scope of smart parking systems looks promising with advancements in technology like AI, IoT, and data analytics. With real-time monitoring, predictive analytics, and handy mobile apps, these systems maximize parking space use, lessen traffic, and enhance user experience. As urbanization increases, the demand for efficient parking solutions will grow, making smart parking a crucial component of smart cities worldwide.

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