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A Semi-analytical approach via Yang Transform on Fractional-order Sawada-Kotera Equation

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Abstract The fractional-order Sawada-Kotera equation has been addressed in this current by applications of the Yang transformation approach. The proposed approach leads to approximate-analytical solutions in the form of a series, which are suitably reliant on fractional-order derivative values and have simple, understandable mechanics. The stability of the numerical technique are investigated when the Caputo fractional derivative is used. The analytical solution of the approach is shown by numerical examples, and it is investigated if the offered procedures are reliable, effective, and minimise the amount of numerical calculations.

1 Introduction

Fractional partial differential equations are encountered in various domains of science and engineering, including physics, electrochemistry, biology, rheology, control theory, system identification, viscoelasticity, and signal processing [1]. These equations play a pivotal role in modelling complex phenomena and processes within these diverse fields. As is generally known, since they are crucial in describing non-linear events, accurate solutions to nonlinear partial differential equations are being studied by more and more mathematicians and physicists. Many researchers in this subject are currently focused on finding precise solutions to non-linear models with fractional order that have been produced [2].

When used to certain physical models that are represented by non-linear partial differential equations, these solutions may exhibit certain insight qualities. However, employing an analytical approach makes it more difficult to find some precise solutions. Numerous approaches have recently been put out to address the new precise solutions, including the generalised Darboux transformation, Residual Power technique, the inverse scattering technique, Adomian decomposition method, the multi-wave approach and many more.

In order to cope with a variety of complex produced partial diiferential equations, a unique approach called the Yang transformation is used in addition to the Caputo derivative in this study. Non-linear fractional order partial differential equations can be solved quickly and effectively using the Yang transformation. In a number of application disciplines, the current approach may be utilised to compute the solutions to additional high non-linear problems.

The Sawada-Kotera equation is known to belong to the totally integrable hierarchy of higher-order KdV equations and to have several sets of conservation laws. The classical Sawada-Kotera equation holds a fundamental position as a mathematical model with broad applicability across various physical scenarios. It characterizes the propagation of long waves in shallow water under the influence of gravity and in a one-dimensional non-linear lattice [3]. This equation finds extensive applications in fields like quantum mechanics and non-linear optics, where it plays a vital role in describing wave behaviour and complex phenomena.

Here, the fifth-order Sawada-Kotera equation is given as [4]-

$$w_t + 45w^2 w_y + 15\left(w_y w_{yyy} + w_{yyy} w\right) + w_{yyyyy} = 0$$

There have been several research on the classical Sawada-Kotera equation, with some significant results. However, according to the author's best understanding, the full investigation of the nonlinear fractional order Sawada-Kotera equation is only getting started.

1.1 Preliminaries

This part will go over some fundamental notions and characteristics of the fractional calculus theory that will be used in our study..

Definition 1. Yang transform is defined as follows [7]-

$$\mathbb{Y}\left[w(t)\right] = Y(w) = \int_0^\infty e^{-\frac{t}{w}} w(t)dt;\ t > 0,\ w \,\epsilon\, (-t_1, t_2)$$

Definition 2. The Inverse of the Yang transform is defined as [6]-

$$\mathbb{Y}^{-1}\left[Y(w)\right] = w(t)$$

Definition 3. For n^{th} derivatives, the Yang transform is defined as [6]-

$$\mathbb{Y}\left(w^n(t)\right) = \frac{Y(w)}{w^n} - \sum_{k=0}^{n-1} \frac{w^k(0)}{w^{n-k-1}};\ \forall n = 1, 2, 3\ldots$$

Definition 4. Yang transform of Caputo fractional derivative is defined as [6]-

$$\mathbb{Y}\left(w^\mu(t)\right) = \frac{Y(w)}{w^\lambda} - \sum_{k=0}^{n-1} \frac{w^k(0)}{w^{\lambda-(k+1)}};\ 0 < \lambda \leq n$$

Definition 5. Linearity property of Yang transform is defined as [6]-
If

$$\mathbb{Y}\left[w_a(t)\right] = Y_a(w)$$

and

$$\mathbb{Y}\left[w_b(t)\right] = Y_b(w),$$

Then,

$$\mathbb{Y}\left[\alpha_a w_a(t) + \alpha_b w_b(t)\right] = \alpha_a \mathbb{Y}\left[w_a(t)\right] + \alpha_b \mathbb{Y}\left[w_b(t)\right],$$

$$\mathbb{Y}\left[\alpha_a w_a(t) + \alpha_b w_b(t)\right] = \alpha_a Y_a(w) + \alpha_b Y_b(w)$$

Where, α_a and α_b are the arbitrary constants.

Definition 6. Linearity property of inverse of the Yang transform is defined as [6]-
If

$$\mathbb{Y}^{-1}\left[w_a(t)\right] = Y_a(w)$$

and

$$\mathbb{Y}^{-1}\left[w_b(t)\right] = Y_b(w),$$

Then,

$$\mathbb{Y}^{-1}\left[\alpha_a w_a(t) + \alpha_b w_b(t)\right] = \alpha_a \mathbb{Y}^{-1}\left[w_a(t)\right] + \alpha_b \mathbb{Y}^{-1}\left[w_b(t)\right],$$

$$\mathbb{Y}^{-1}\left[\alpha_a w_a(t) + \alpha_b w_b(t)\right] = \alpha_a Y_a(w) + \alpha_b Y_b(w).$$

Table 1. Chart regarding Yang transform [5]

	$w(t)$	$\mathbb{Y}[w(t)] = Y(w)$
1.	$\mathbb{Y}[1]$	w
2.	$\mathbb{Y}[w]$	w^2
3.	$\mathbb{Y}[w^i]$	$\Gamma(i+1)w^{i+1}$
4.	$\mathbb{Y}[e^{\text{ct}}]$	$\frac{w}{1-cw}$
5.	$\mathbb{Y}[w(\text{ct})]$	$\frac{1}{c}Y\left(\frac{w}{c}\right)$

Table 2. Chart regarding inverse Yang transform [5]

	$Y(w)$	$w(t) = \mathbb{Y}^{-1}\left[Y(w)\right]$
1.	w	1
2.	w^2	t
3.	$\Gamma(i+1)w^{i+1}$	t^i
4.	$\frac{w}{1-cw}$	e^{ct}
5.	$\frac{1}{c}Y\left(\frac{w}{c}\right)$	ct

This study is arranged in the following way: In Section 2 we will revise the Yang transformation method in addition with Caputo derivative. In Section 3 we obtain explicit exact solutions. Finally, some conclusions will be given.

2 Methodology

In order to demonstrate the Yang transformation method's solution procedure, we use the following non-linear fractional differential equation.

$$D_t^{\lambda}\left[w(y,t)\right] = \mathfrak{l}\left[w(y,t)\right] + \mathfrak{n}\left[w(y,t)\right] + \mathfrak{h}(y,t) \tag{2.1}$$

Where, D_t^{λ} represents the Caputo operator, $\mathfrak{l}$ is the linear term, $\mathfrak{n}$ is the non-linear term and $\mathfrak{h}(a,t)$ is the source term.

Applying Yang transform in Eq.(2.1):

$$\Rightarrow \mathbb{Y}[D_t^{\lambda}\left[w(y,t)\right]] = \mathbb{Y}[\mathfrak{l}\left[w(y,t)\right]] + \mathbb{Y}[\mathfrak{n}\left[w(y,t)\right]] + \mathbb{Y}[\mathfrak{h}(y,t)]], \tag{2.2}$$

From Eq.(2.2):

$$\Rightarrow \frac{\mathbb{Y}\left(w(t)\right)}{w^{\lambda}} - \sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}} = \mathbb{Y}[\mathfrak{l}\left[w(y,t)\right]] + \mathbb{Y}[\mathfrak{n}\left[w(y,t)\right]] + \mathbb{Y}[\mathfrak{h}(y,t)]],$$

$$\Rightarrow \frac{\mathbb{Y}\left(w(t)\right)}{w^{\lambda}} = \mathbb{Y}[\mathfrak{l}\left[w(y,t)\right]] + \mathbb{Y}[\mathfrak{n}\left[w(y,t)\right]] + \mathbb{Y}[\mathfrak{h}(y,t)]] + \sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}},$$

$$\Rightarrow \mathbb{Y}\left(w(t)\right) = w^{\lambda}\left[\mathbb{Y}\left[\mathfrak{l}\left[w(y,t)\right]\right] + \mathbb{Y}\left[\mathfrak{n}\left[w(y,t)\right]\right] + \mathbb{Y}\left[\mathfrak{h}(y,t)\right]\right] + w^{\lambda}[\sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}}],$$

$$\Rightarrow w(t) = \mathbb{Y}^{-1}[w^{\lambda}\left[\mathbb{Y}\left[\mathfrak{l}\left[w(y,t)\right]\right] + \mathbb{Y}\left[\mathfrak{n}\left[w(y,t)\right]\right] + \mathbb{Y}\left[\mathfrak{h}(y,t)\right]\right]] + \mathbb{Y}^{-1}[w^{\lambda}[\sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}}]],$$

$$\Rightarrow w(t) = \mathbb{Y}^{-1}\left[w^{\lambda}\left\{\sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}} + \mathbb{Y}\left[\mathfrak{l}\left[w(y,t)\right] + \mathfrak{n}\left[w(y,t)\right] + \mathfrak{h}(y,t)\right]\right\}\right].$$

Where,

$\mathfrak{l}\left[w(y,t)\right] = \mathfrak{n}\left[w_0(y,t)\right] + \sum_{k=1}^{\infty}\left[\mathfrak{n}\left(\sum_{j=0}^{k} w_j(y,t)\right) - \mathfrak{n}\sum_{j=0}^{k-1} w_j(y,t)\right],$

$$\mathfrak{l}\left[w(y,t)\right]=\mathfrak{l}\left[w_0(y,t)\right]+\sum_{k=1}^{\infty}\left[\mathfrak{l}\left(\sum_{j=0}^{k}w_j(y,t)\right)-\mathfrak{l}\left(\sum_{j=0}^{k-1}w_j(y,t)\right)\right]$$

$$\Rightarrow\sum_{\infty}^{r=0}w_r(y,t)=\mathbb{Y}^{-1}\left[w^{\lambda}\sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}}+\mathfrak{h}(y,t)\right],$$

$$\Rightarrow\sum_{r=0}^{\infty}w_r(y,t)=\mathbb{Y}^{-1}\left[w^{\lambda}\left\{\sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}}+\mathbb{Y}\left[\mathfrak{h}(y,t)\right]\right\}\right]+\mathbb{Y}^{-1}\left[w^{\lambda}\mathbb{Y}\left\{\mathfrak{l}\left[w(y,t)\right]+\mathfrak{n}\left[w(y,t)\right]\right\}\right]$$

$$\sum_{r=0}^{\infty}w_r(y,t)=\mathbb{Y}^{-1}\left[w^{\lambda}\left\{\sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}}+\mathbb{Y}\left[\mathfrak{h}(y,t)\right]\right\}\right]+\mathbb{Y}^{-1}\left[w^{\lambda}\mathbb{Y}\left\{\sum_{k=0}^{\infty}\mathfrak{l}\left[\left[w_k(y,t)\right]\right]+\sum_{k=0}^{\infty}\mathfrak{n}\left[\left[w_k(y,t)\right]\right]\right\}\right]$$

$$\begin{aligned}\Rightarrow\sum_{r=0}^{\infty}w_r(y,t)=&\mathbb{Y}^{-1}\left[w^{\lambda}\left\{\sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}}+\mathbb{Y}\left[\mathfrak{h}(y,t)\right]\right\}\right]\\&+\mathbb{Y}^{-1}\left[w^{\lambda}\mathbb{Y}\left\{\mathfrak{l}\left[w_0(y,t)\right]+\mathfrak{n}\left[w_0(y,t)\right]+\sum_{k=1}^{\infty}\left[w_k(y,t)\right]+\sum_{k=1}^{\infty}\mathfrak{n}\left[w_k(y,t)\right]\right\}\right]\end{aligned}$$

$$\begin{aligned}\sum_{r=0}^{\infty}w_r(y,t)=&\mathbb{Y}^{-1}\left[w^{\lambda}\left\{\sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}}+\mathbb{Y}\left[\mathfrak{h}(y,t)\right]\right\}\right]\\&+\mathbb{Y}^{-1}\left[w^{\lambda}\mathbb{Y}\left\{\mathfrak{l}\left[w_0(y,t)\right]+\mathfrak{n}\left[w_0(y,t)\right]\right\}\right]\\&+\mathbb{Y}^{-1}\left[w^{\lambda}\mathbb{Y}\left\{\sum_{k=1}^{\infty}\mathfrak{l}\left[w_k(y,t)\right]+\sum_{k=1}^{\infty}\mathfrak{n}\left(\sum_{j=0}^{k}w_k(y,t)-\sum_{k=1}^{\infty}\mathfrak{n}\left(\sum_{j=0}^{k-1}w_k(y,t)\right)\right)\right\}\right]\end{aligned}$$

$$\begin{aligned}\sum_{r=0}^{\infty}w_r(y,t)=&\mathbb{Y}^{-1}\left[w^{\lambda}\left\{\sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}}+\mathbb{Y}\left[\mathfrak{h}(y,t)\right]\right\}\right]\\&+\mathbb{Y}^{-1}\left[w^{\lambda}\mathbb{Y}\left\{\mathfrak{l}\left[w_0(y,t)\right]+\mathfrak{n}\left[w_0(y,t)\right]\right\}\right]\\&+\mathbb{Y}^{-1}\left[w^{\lambda}\mathbb{Y}\left\{\sum_{k=1}^{\infty}\left(\mathfrak{l}\left[w_k(y,t)\right]+\mathfrak{n}\left(\sum_{j=0}^{k}w_k(y,t)-\sum_{k}^{j=0}w_k(y,t)-\mathfrak{n}\left(\sum_{k}^{j=0}w_k(y,t)\right)\right)\right)\right\}\right]\end{aligned}$$

Extracted formulae from above equation:

$$w_0(y,t)=\mathbb{Y}^{-1}\left[w^{\lambda}\left\{\sum_{k=0}^{n-1}\frac{w^k(0)}{w^{\lambda-(k+1)}}+\mathbb{Y}\left[\mathfrak{h}(y,t)\right]\right\}\right],\tag{2.3}$$

$$w_1(y,t)=\mathbb{Y}^{-1}\left[w^{\lambda}\mathbb{Y}\left\{\mathfrak{l}\left[w_0(y,t)\right]+\mathfrak{n}\left[w_0(y,t)\right]\right\}\right],\tag{2.4}$$

And,

$$\Rightarrow w_{k+1}(y,t)=\mathbb{Y}^{-1}\left[w^{\lambda}\mathbb{Y}\left\{\sum_{k=1}^{\infty}\left(\mathfrak{l}\left[w_k(y,t)\right]+n\left(\sum_{k}^{j=0}w_k(y,t)-\mathfrak{n}\left(\sum_{k}^{j=0}w_k(y,t)\right)\right)\right)\right\}\right].\tag{2.5}$$

For $r=1,2,3\ldots$

3 Application of Yang transformation method for solving non-linear time-fractional Sawada-Kotera equation

We analyze the time-fractional Sawada-Kotera equation to demonstrate the usefulness and accuracy of the suggested technique [9].

$$D_t^{\lambda} + 45w^2 w_y + 15\left(w_y w_{yyy} + w_{yyy} w\right) + w_{yyyyy} = 0; t > 0,\ 0 < \lambda \leq 1 \tag{3.1}$$

With initial condition-

$$w(y,0) = 2A^2 sech^2[A(y-B)]$$

Where, A and B are arbitrary constants and $A \neq 0$.
By applying Yang transformation on eq.(3.1)-

$$\Rightarrow Y\left[\frac{\partial^{\lambda} w}{\partial t^{\lambda}}\right] = Y\left[-(45w^2 w_y + 15(w_y w_{yyy} + w_{yyy} w) + w_{yyyyy})\right]$$

$$\Rightarrow \frac{Y[w]}{w^{\lambda}} - \sum_{k=0}^{n-1} \frac{w^k(0)}{w^{\lambda-(k+1)}} = Y\left[-(45w^2 w_y + 15(w_y w_{yyy} + w_{yyy} w) + w_{yyyyy})\right]$$

$$\Rightarrow \frac{Y[w]}{w^{\lambda}} = \sum_{k=0}^{n-1} \frac{w^k(0)}{w^{\lambda-(k+1)}} + Y\left[-(45w^2 w_y + 15(w_y w_{yyy} + w_{yyy} w) + w_{yyyyy})\right]$$

$$\Rightarrow Y[w] = w^{\lambda}\left\{\sum_{k=0}^{n-1} \frac{w^k(0)}{w^{\lambda-(k+1)}} + Y\left[-(45w^2 w_y + 15(w_y w_{yyy} + w_{yyy} w) + w_{yyyyy})\right]\right\}$$

Applying inverse Yang transformation to equation:

$$\Rightarrow [w] = Y^{-1}\left[w^{\lambda}\left\{\sum_{k=0}^{n-1} \frac{w^k(0)}{w^{\lambda-(k+1)}} + Yw^{\lambda}\left[-(45w^2 w_y + 15(w_y w_{yyy} + w_{yyy} w) + w_{yyyyy})\right]\right\}\right]$$

$$\Rightarrow \sum_{n=0}^{\infty} w_n = Y^{-1}\left[w^{\mu} \sum_{k=0}^{n-1} \frac{w^k(0)}{w^{\lambda-(k+1)}}\right] + Y^{-1} w^{\lambda} Y\left[-\sum_{n=0}^{\infty} 45w^2 w_y - \sum_{n=0}^{\infty} 15(w_y w_{yyy} + w_{yyy} w) - \sum_{n=0}^{\infty} w_{yyyyy}\right]$$

Now, by using the proposed analytical technique on Equation, we have:

$$\Rightarrow w_0 = Y^{-1}\left[w^{\lambda} \sum_{k=0}^{n-1} \frac{w^k(0)}{w^{\lambda-(k+1)}}\right],$$

Consider $n = 1$:

$$\Rightarrow w_0 = Y^{-1}\left[w^{\lambda}.\frac{w(0)}{w^{\lambda+1}}\right] \Rightarrow w_0 = Y^{-1}\left[\frac{w(0)}{w}\right],$$

$$\Rightarrow w_0 = w(0) Y^{-1}\left[\frac{1}{w}\right] \Rightarrow w_0 = w(0)$$

$$\Rightarrow w_0 = 2A^2 sech^2[A(y-B)]$$

$$\Rightarrow w_1 = Y^{-1} w^{\lambda} Y[-(45w_0^2 w_y + 15\left((w_0)_y (w_0)_{yyy} + (w_0)_{yyy} w_0\right) + (w_0)_{yyyyy})]$$

$$\Rightarrow w_1 = 64A^7 \tanh\left[A(y-B)\right] sech^2[A(y-B)]\, \frac{t^{\lambda}}{\Gamma(\lambda+1)}$$

$$\Rightarrow w_2 = Y^{-1} w^{\lambda} Y[-(45w_1^2 w_y + 15\left((w_1)_y (w_1)_{yyy} + (w_1)_{yyy} w_1\right) + (w_1)_{yyyyy})]$$

$$\Rightarrow w_2 = 512A^{12} sech^4[A(y-B)]\left[2\cosh^2\left[A(y-B)\right] - 3\right]\frac{t^{2\lambda}}{\Gamma(2\lambda+1)}$$

$$\Rightarrow w_n = Y^{-1} w^{\lambda} Y[-(45w_{n-1}^2 w_y + 15\left((w_{n-1})_y (w_{n-1})_{yyy} + (w_{n-1})_{yyy} w_{n-1}\right) + (w_{n-1})_{yyyyy})]$$

The result of the series is-

$$\Rightarrow w(y,t) = w_0(y,t) + w_1(y,t) + w_2(y,t) + \ldots$$

$$\begin{aligned} w(y,t) =& 2A^2 \text{sech}^2[A(y-B)] \\ &+ 64A^7 \tanh[A(y-B)]\text{sech}^2[A(y-B)]\frac{t^{\lambda}}{\Gamma(\lambda+1)} \\ &+ 512A^{12}\text{sech}^4[A(y-B)][2\cosh^2[A(y-B)] - 3]\frac{t^{2\lambda}}{\Gamma(2\lambda+1)} + \ldots \end{aligned}$$

For $\lambda = 1$, the exact solution to the preceding equation is given by:

$$\Rightarrow w(y,t) = 2A^2 sech^2[A(y - 16A^4 t - B)]$$

Remark: The previous result is equivalent to the exact solution of the fractional Sawada-Kotera equation[8].

4 Conclusion

In this study, the Yang transformation approach has been used to solve the time fractional fifth-order Sawada-Kotera equation. The resulting findings are then contrasted with precise solutions. The numerical solutions to the aforementioned issues show quick convergence with few calculations. The current method for getting numerical solutions to the fifth-order Sawada-Kotera equation is relatively straightforward, efficient, and practical. The suggested method's successful use in solving the time-fractional Sawada-Kotera equation validates its ease of use, effectiveness, and adaptability. Therefore, this suggested method can be applied in a wider range.

Conflict of Interest - Not applicable.
Data Availability Statement - All the data is provided in the manuscript.

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