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Multi-objective Fractional Programming Problem Analysis in the Manufacturing Sector Using Situation-Based S-Shaped Nonlinear Fuzzy Numbers

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Abstract In this paper, we have proposed a method for solving linear fractional programming problems by incorporating a non-linear S-shaped fuzzy membership function into a fuzzy goal programming approach. The comparison of the proposed methodology based on the S-shaped membership function is also done with the existing technique, which is based on the linear membership function. The main motivation behind the enhancement of the membership function for fuzzy goal programming from linear to non-linear is that if the result obtained by using the linear membership function is not satisfactory for decision-makers, there should be a scope to switch to a non-linear version of it. This is because many situations do not provide the optimal solution through linear membership functions due to different uncertain scenarios in real-life problems. One numerical example illustrated using the proposed method as well as a linear function-based approach, and it is also being implemented on a case study of the garment industry of Bangladesh for the sake of result analysis of the real-world data set where we considered the optimization of profit to investment of the garment industry to demonstrate the effectiveness of this approach for its utility in decision-making.

1 Introduction

Linear-fractional programming problem (LFPP) is an extension of linear programming problem (LPP). In a mathematical optimization method, the objective function in linear-fractional problems is a proportion of two linear functions. Linear programming is a particular instance of linear-fractional programming in which the denominator is a one-unit constant function. Fractional programming is the optimization problem related to a single or several ratios of functions. Many researchers have been interested in fractional programming in the past. Many real-world problems and challenges rely on the fraction of physical or economic value that can be expressed as a linear function, such as cost on volume, gain over the cost, or any other metric that assesses the efficiency of a system.

In past various methods for dealing with LFP have been proposed by researchers. The LFP problem was first recognized by [1], who then used the LPP sequence to solve it. In order to convert the LFP problem to the LPP, Charnes and Cooper [2] used a variable transformation approach. The concept of a fuzzy set was first proposed by Zadeh [3] as a means of mathematically handling uncertainty [4]. Fuzzy set theory was gradually integrated into optimization problems following the seminal research of Bellman and Zadeh [5],they presented fuzzy decision making to reflect daily uncertainties as crisp parameters fail. Zimmerman [6] pioneered the use of fuzzy ideas in mathematical programming by proposing a fuzzy approach to linear programming with multiple target functions. The extensive study on fuzzy numbers discussed by [7, 8].

A multi-objective programming approach is used to describe challenges in many real-world contexts [9].To address various optimization problems with multiple objectives, an extensive number of strategies have been proposed in the literature Gulia et.[10].Mohamad [11] introduced the relationship between fuzzy programming and goal programming and their similarities and also discussed how one can leads to the other .The extensions and modification is being discussed by researches discussed in [12] and [13] proposed fuzzy logic approaches to solve linear fractional programing problem with numerical point of view.Multifractional programming refers to the resolution of multi-objective problems that involve ratios.Chang [14] applied absolute function on fractional programming with goal programming approach and B. Mishra et al [15] proposed multi objective goal programming in optimization of land use in agriculture.

A fundamental principle of goal programming (GP) is to identify a solution that is both feasible and satisfies the given constraints. GP approaches quantify the degree of variation between a solution and each objective. The goal is to maximize the ratio of profit to expenditure in the manufacture of different items with restrictions or constraints. Bal and Pal [16] discussed on dynamic programming. In a fuzzy decision-making context, achieving the necessary levels of objective goals is primarily determined by attaining the highest possible degree of their related membership values and also the result of goal programming is being use by [17]. Membership functions and numerous fuzzy techniques have also been implemented in [18, 19, 20]. Companies and organizations from a variety of industries, such as technology and finance, need to make sure that logistics-related operations have a positive ratio. To create the desired goods in the industrial sector, a manufacturing system integrates a number of resources, some of which may have competing needs. This research demonstrates how to resolve a multi-objective optimization problem using the concept of fractional fuzzy programming. Industrial production systems use diverse resources with conflicting needs that must be integrated to produce products. A non-linear functions used in multi objective linear function in [21] that has also grabbed the interest of researchers from throughout the world .

So far most of research work used Zimmermann's approach of linear membership function in fuzzy goal programming. In this paper we enhanced Zimmermann's approach by applying non- linear S-shaped membership function instead of linear function in goal programming. The multi objective linear fractional programing is discussed in fuzzy environment using non-linear [22] S-shaped [23] membership function and the solutions and results compared with the usual Zimmerman approach on linear membership function .The proposed methodology is applied to a case study of a knit garment manufacturing unit, utilizing secondary data [24] and LINGO software is used to solve fractional programming problems directly without reducing them into Linear form.

In fractional programming, to address uncertainty, the majority of studies focus on linear fuzzy numbers, such as triangular and trapezoidal fuzzy numbers. In numerous intricate real-world scenarios, non-linear fuzzy numbers may yield more advantageous solutions compared to linear fuzzy numbers. This study addresses this issue by employing a non-linear fuzzy number specifically, an S-shaped fuzzy number. We modified our methodology to encompass the S-shaped fuzzy number, and our proposed approach, which employs S-shaped fuzzy numbers, produces better outcomes than linear fuzzy numbers.

The layout of paper is as follows : Section 2 covers definitions from literature [5], in section 3 dealt with methodology where we discussed both the Zimmerman as well as non-linear function based proposed methodology. In section 4 one numerical example is discussed with both methods and in section 5 these methodologies applied on a case study based on garment industry. In section 6 Result analysis is done in tabular and graphical form followed by conclusion in section 7.

2 Preliminaries

We begin by going through some basic definitions and conclusions with regard to fuzzy numbers.

2.1 Fuzzy set

$\widetilde{\mathcal{F}}$:$\{(x, \mu_{\widetilde{\mathcal{F}}}(x))/x \in \mathcal{X}\}$ defines fuzzy set over real numbers set $\mathcal{X}$,where $\mu_{\widetilde{\mathcal{F}}}(x)$ represents a membership function of $x \in \widetilde{\mathcal{F}}$ that associates elements of $\mathcal{X}$ to real numbers 0 to 1.

2.2 Normal Fuzzy set

A fuzzy set $\widetilde{\mathcal{F}}$ known as normal if $\mu_{\widetilde{\mathcal{F}}}$ (x) attains value 1 at the minimum one element x $\in \mathcal{X}$.

2.3 Fuzzy number

A fuzzy set $\widetilde{\mathcal{F}}$ with membership functions $\mu_{\widetilde{\mathcal{F}}}(x) : \mathbb{R} \to [0,1]$represents a fuzzy numbers if below mentioned characteristics are met:
i. $\mu_{\widetilde{\mathcal{F}}}(\lambda x + (1-\lambda)\,y) \geq min\ \{\mu_{\widetilde{\mathcal{F}}}(x),\ \mu_{\widetilde{\mathcal{F}}}(y)\}$, for every$x, y\ \in \mathcal{X}\ \&\ \lambda \in [0,1]$. i.e. $\widetilde{\mathcal{F}}$ is convex
ii. $\widetilde{\mathcal{F}}$ is normal i.e.$\mu_{\widetilde{\mathcal{F}}}(x_0) = 1$.
iii. $\mu_{\widetilde{\mathcal{F}}}(x)$ is piecewise continuous.

3 Methodology

The basic problem of linear fractional programming with assumption is presented as follows.

$$\begin{gathered} \max\ Z_k(x) = \tfrac{p_k X + \alpha_k}{q_k X + \beta_k};\ k = 1, 2, \ldots, K \\ \text{subjected to: } x \in S = \{x \in R^n : Ax \leqq B,\ x \geq 0\} \\ A \in R^{m \times n}\ ; B \in R^m;\ p_k\ , q_k \in R^n;\ \alpha_k,\ \beta_k \in R. \end{gathered} \tag{3.1}$$

Let us assume the u_k assigned as the aspiration level to the objective function $Z_k(x)$ and let ℓ_k be the lower bound limit to the fuzzy goal then eq 3.1 can be stated as follow:

$$Z_k(x) = \frac{p_k \mathcal{X} + \alpha_k}{q_k \mathcal{X} + \beta_k} \geq u_k. \tag{3.2}$$

Now we find the solution for eq 3.1 by the existing approach and then we will apply the proposed nonlinear membership and will compare the results obtained from these two methodologies.

(i) By Zimmermann's Approach, the fuzzy objective eq 3.2 describes the linear membership function as follows:

$$m(Z_k(x)) = \begin{cases} 1\ if\ Z_k(X) \geq u_k, \\ \frac{Z_k(X) - l_k}{u_k - l_k}\ if\ \ell_k < Z_k(X) < u_k, \\ 0\ if\ Z_k(X) \leq \ell_k. \end{cases} \tag{3.3}$$

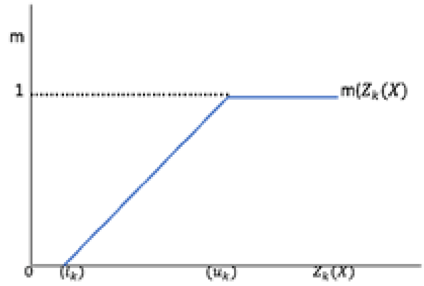

Figure 1. Linear membership function

Figure 1 describes the Zimmerman's linear membership function.Since the largest value is 1 for the membership function mentioned in eq 3.3 can be expressed as given in figure 1. Following model by converting into single objective function problem

$$\begin{gathered} \min\ \psi \\ \text{subjected to: } \tfrac{Z_k(X) - l_k}{u_k - l_k} + D_k^- - D_k^+ = 1, \\ A \in R^{m \times n}\ ; B \in R^m;\ p_k\ , q_k \in R^n;\ \alpha_k,\ \beta_k \in R, \\ \text{A}x \leq \text{B}, \\ \text{where } D_k^- \geq 0,\ D_k^+ \geq 0, \\ D_k^-.D_k^+ \geq 0, \\ 0 \leq \psi \leq 1, \\ \psi \geq D_k^-,\ D_k^+, \\ x \geq 0. \end{gathered} \tag{3.4}$$

(i) By proposed methodology, the fuzzy objective function eq 3.2 using non-linear (S-shaped) membership function is defined as follows:

$$m\left(Z_k\left(x\right)\right)=\begin{cases} 1\ if\ Z_k\left(X\right)\geq u_k, \\ 1-\left(\frac{1}{1+Ae^{\alpha\left(\frac{Z_k(X)-l_k}{u_k-l_k}\right)}}\right)\ if\ \ell_k<Z_k\left(X\right)<u_k, \\ 0\ if\ Z_k\left(X\right)\leq \ell_k. \end{cases} \tag{3.5}$$

Above figure 2 is of non-linear membership function. Since the largest value is 1 for the mem-

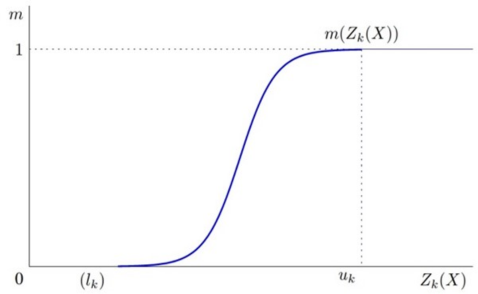

Figure 2. Non-linear (S-shaped) membership function

bership functions mentioned in eq 3.5 then the single-objective problem can be formulated as follows:

$$\begin{gathered} \min\ \psi \\ \text{subjected to: } 1-\left(\frac{1}{1+Ae^{\alpha\left(\frac{Z_k(X)-l_k}{u_k-l_k}\right)}}\right)+D_k^- - D_k^+ = 1, \\ \mathrm{A}x\leq \mathrm{B}, \\ \text{where } D_k^-\geq 0\ , D_k^+\geq 0, \\ 0\leq\psi\leq 1, \\ \psi\geq D_k^-,\ D_k^+, \\ D_k^-.D_k^+\geq 0, \\ x\geq 0. \end{gathered} \tag{3.6}$$

Figure 3 explains the procedure to follow using proposed methodology based on non-linear S-shaped membership function.

4 Numerical Example

Let us investigate a MOLFPP having the following objectives [25]:

$$\begin{gathered} \max\ Z_1\left(y\right)=\frac{-3y_1+2y_2}{y_1+y_2+3} \\ \max\ Z_2\left(y\right)=\frac{7y_1+y_2}{5y_1+2y_2+1} \\ \text{subjected to: } y_1-y_2\geq 1, \\ 2y_1+3y_2\leq 15, \\ y_1\geq 3, \\ y_1,y_2\geq 0. \end{gathered} \tag{4.1}$$

For setting aspiration levels for Z_1 and Z_2 selected as maxZ_1 =1/2 and maxZ_1 =7/5 by setting lower value for Z_1 and Z_2 as 0.

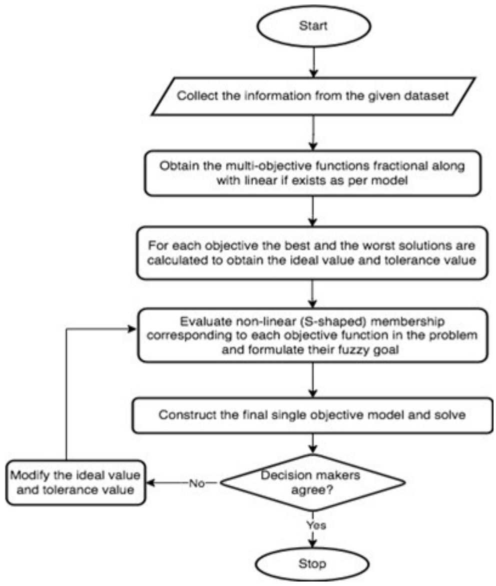

Figure 3. Flow chart to explain proposed methodology using S-shaped membership function

4.1 By Zimmerman's approach

Linear Membership function formulation:

$$m\left(Z_1\left(y\right)\right)= \begin{cases} 1 & \text{if } Z_1 \geq 1/2, \\ \frac{\frac{-3y_1+2y_2}{y_1+y_2+3}-0}{1/2} & \text{if } 0<Z_1<1/2, \\ 0 & \text{if } Z_1 \leq 0. \end{cases} \tag{4.2}$$

$$m\left(Z_2\left(y\right)\right)= \begin{cases} 1 & \text{if } Z_2 \geq 7/5, \\ \frac{\frac{7y_1+y_2}{5y_1+2y_2+1}-0}{7/5} & \text{if } 0<Z_2<7/5, \\ 0 & \text{if } Z_2 \leq 0. \end{cases} \tag{4.3}$$

Converting into single objective function problem:

$$\begin{gathered} \min\ \psi \\ \frac{-6y_1+y_2}{y_1+y_2+3}+\mathrm{D}_1^- - \mathrm{D}_1^+ = 1, \\ \frac{35y_1+5y_2}{35y_1+14y_2+7}+\mathrm{D}_2^- - \mathrm{D}_2^+ = 1, \\ 0 \ \leq \ \phi \leq \ 1, \\ \phi \geq \mathrm{D}_1^-, \mathrm{D}_1^+,\ \mathrm{D}_2^-, \mathrm{D}_2^+ \geq 0, \\ \mathrm{D}_1^-.\mathrm{D}_1^+ = 0, \\ \mathrm{D}_2^-.\mathrm{D}_2^+ = 0, \\ y_1 - y_2 \geq 1, \\ 2y_1 + 3y_2 \leq 15, \\ y_1 \geq 3, \\ y_1, y_2 \geq 0. \end{gathered} \tag{4.4}$$

By solving through LINGO software, we got $y_1 = 3$, $y_2 = 0.4268153$, $\mathrm{D}_1^- = 0.4268153$, $\mathrm{D}_2^- = 0.42$ and $Z_1 = -0.80076$and $Z_2 = 1.27134$.

4.2 By proposed methodology

Non-linear (S-shaped) membership function:

$$m\left(Z_1(y)\right)=\begin{cases} 1 & \text{if } Z_1 \geq 1/2, \\ 1-\left(\dfrac{1}{1+0.001001e^{13.813\left(\frac{\frac{-3y_1+2y_2}{y_1+y_2+3}-0}{0.5}\right)}}\right) & \text{if } 0<Z_1<\frac{1}{2}, \\ 0 & \text{if } Z_1 \leq 0. \end{cases} \tag{4.5}$$

$$m\left(Z_2\left(y\right)\right)=\begin{cases} 1 & \text{if } Z_2 \geq \frac{7}{5}, \\ \left(1-\left(\dfrac{1}{1+0.001001e^{13.813\left(\frac{\frac{7y_1+y_2}{5y_1+2y_2+1}-0}{\frac{7}{5}}\right)}}\right)\right) & \text{if } 0<Z_2<\frac{7}{5}, \\ 0 & \text{if } Z_2 \leq 0. \end{cases} \tag{4.6}$$

Converting into single objective function problem:

$$\begin{gathered} \min\ \phi \\ 1-\left(\dfrac{1}{1+0.001001e^{13.813\left(\frac{\frac{-3y_1+2y_2}{y_1+y_2+3}-0}{0.5}\right)}}\right)+\mathrm{D}_1^- - \mathrm{D}_1^+ = 1, \\ 1-\left(\dfrac{1}{1+0.001001e^{13.813\left(\frac{\frac{7y_1+y_2}{5y_1+2y_2+1}-0}{\frac{7}{5}}\right)}}\right)+\mathrm{D}_2^- - \mathrm{D}_2^+ = 1, \\ 0 \ \leq\ \phi \leq\ 1, \\ \phi \geq \mathrm{D}_1^-, \mathrm{D}_1^+,\ \mathrm{D}_2^-, \mathrm{D}_2^+ \geq 0, \\ \mathrm{D}_1^-.\mathrm{D}_1^+ = 0, \\ \mathrm{D}_2^-.\mathrm{D}_2^+ = 0, \\ y_1 - y_2 \geq 1, \\ 2y_1 + 3y_2 \leq 15, \\ y_1 \geq 3, \\ y_1, y_2 \geq 0. \end{gathered} \tag{4.7}$$

By solving through LINGO software, we got $y_1 = 3$, $y_2 = 2$, $\mathrm{D}_1^- = 1$, $\mathrm{D}_2^- = 0.11$ and $Z_1 = -0.62$ and $Z_2 = 1.15$.

5 Case Study

We worked with secondary data used in [21]. In the present study, a knit garment manufacturing unit from Bangladesh has been taken into consideration. This plant is located in the Gazipur district of Bangladesh. From the case industry, information has been gathered that includes the monthly resource usage amount, product volume, and profit per unit for a variety of product categories. The case industry manufactures a wide variety of knitted garments. The collated data served as the input for the linear programming model that was proposed. There are currently eight different kinds of clothing that are being manufactured by the company that we are examining. The purpose of this study is to determine the current level of resource consumption, production cost, time utilized, and monthly profit, and then compare these values to the ideal answer that was reached by solving the FLPP model that we built. We used LINGO Solver to solve the.model. The following tables 1,2,3 of the paper provide a summary of the pertinent information that was acquired from the case company. This information includes the amount of

Table 1. Details of industry production, profit and time utilization required for different products

Product	Industry Production	Profit (Per piece)	Profit by industry	Time				
				Cutting	Sewing	Trimming	Finishing	Packing
GT	8000	42	336000	0.4	6.4	0.4	0.4	0.4
KBLJ	14000	36	504000	0.3	5.3	0.3	0.3	0.3
BCH	8000	40	320000	0.5	5.5	0.5	0.5	0.5
BUW	7000	30	210000	0.2	5.2	0.2	0.2	0.2
GC	12000	35	420000	0.6	7.6	0.3	0.6	0.6
GL	10000	40	400000	0.5	6.5	0.3	0.5	0.5
GCS	12000	30	360000	0.4	5.4	0.4	0.4	0.4
GTL/S	11000	25	275000	0.5	7.5	0.4	0.5	0.5
Total			2825000					

time required to make various items, the monthly production, the profit, the cost, and the material utilization per unit. According to the information: Given in table 1.
Restrictions:Given in table 2.
Decision variable:Given in table 3.

Table 2. Material and cost requirement for fabric thread and labour

Over	Material	Over	Cost
Fabric	12156000	Labour	2853000
Tread	11295000	Material	13194000

Table 3. Symbols and notations for different garment products

Symbol (number of product)	Product
y_1	T-shirt for girls(GT)
y_2	Long-johns made by Keiki Boy(KBLJ)
y_3	Hoodie for Boys in College(BCH)
y_4	Underwear for Boys(BUW)
y_5	Cardigan for girls (GC)
y_6	Leggings for girls(GL)
y_7	College shirt for girls(GCS)
y_8	T-shirt for girls, L/S (GTL/S)

Objective function: Profit/Investment

$$\text{Max P} = \frac{42y_1 + 36y_2 + 40y_3 + 30y_4 + 35y_5 + 40y_6 + 30y_7 + 25y_8}{159y_1 + 169y_2 + 285y_3 + 142y_4 + 300y_5 + 150y_6 + 185y_7 + 165y_8} \tag{5.1}$$

Time utilization:
Cutting:

$$\text{Max C} = 0.4y_1 + 0.3y_2 + 0.5y_3 + 0.2y_4 + 0.6y_5 + 0.5y_6 + 0.4y_7 + 0.5y_8 \tag{5.2}$$

Sewing:

$$\text{Max S} = 6.4y_1 + 5.3y_2 + 5.5y_3 + 5.2y_4 + 7.6y_5 + 6.5y_6 + 5.4y_7 + 7.5y_8 \tag{5.3}$$

Trimming:

$$\text{Max T} = 0.4y_1 + 0.3y_2 + 0.5y_3 + 0.2y_4 + 0.3y_5 + 0.3y_6 + 0.4y_7 + 0.4y_8 \tag{5.4}$$

Finishing:

$$\text{Max F} = 0.4y_1 + 0.3y_2 + 0.5y_3 + 0.2y_4 + 0.6y_5 + 0.5y_6 + 0.4y_7 + 0.5y_8 \tag{5.5}$$

Packing:

$$\text{Max Pc} = 0.4y_1 + 0.3y_2 + 0.5y_3 + 0.2y_4 + 0.6y_5 + 0.5y_6 + 0.4y_7 + 0.5y_8 \tag{5.6}$$

Restrictions:
Budget (labour cost):

$$28y_1 + 25y_2 + 65y_3 + 22y_4 + 60y_5 + 25y_6 + 30y_7 + 25y_8 \leq 2853000 \tag{5.7}$$

Budget (material cost):

$$131y_1 + 144y_2 + 220y_3 + 120y_4 + 240y_5 + 125y_6 + 155y_7 + 140y_8 \leq 13194000 \tag{5.8}$$

Fabric:

$$128y_1 + 121y_2 + 246y_3 + 100y_4 + 180y_5 + 131y_6 + 155y_7 + 120y_8 \leq 12156000 \tag{5.9}$$

Thread:

$$120y_1 + 110y_2 + 220y_3 + 70y_4 + 220y_5 + 120y_6 + 120y_7 + 115y_8 \leq 11295000 \tag{5.10}$$

Cutting Time:

$$0.4y_1 + 0.3y_2 + 0.5y_3 + 0.2y_4 + 0.6y_5 + 0.5y_6 + 0.4y_7 + 0.5y_8 \leq 35300 \tag{5.11}$$

Sewing time:

$$6.4y_1 + 5.3y_2 + 5.5y_3 + 5.2y_4 + 7.6y_5 + 6.5y_6 + 5.4y_7 + 7.5y_8 \leq 509300 \tag{5.12}$$

Trimming time:

$$0.4y_1 + 0.3y_2 + 0.5y_3 + 0.2y_4 + 0.3y_5 + 0.3y_6 + 0.4y_7 + 0.4y_8 \leq 28600 \tag{5.13}$$

Finishing time:

$$0.4y_1 + 0.3y_2 + 0.5y_3 + 0.2y_4 + 0.6y_5 + 0.5y_6 + 0.4y_7 + 0.5y_8 \leq 35300 \tag{5.14}$$

Packing time:

$$0.4y_1 + 0.3y_2 + 0.5y_3 + 0.2y_4 + 0.6y_5 + 0.5y_6 + 0.4y_7 + 0.5y_8 \leq 35300 \tag{5.15}$$

Non-negativity restrictions:

$$y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8 \geq 0. \tag{5.16}$$

Max P=$\overline{P}$=0.2666667 at point (0,0,0,0,70600,0,0)
Max C= $\overline{C}$=35300 at point (0,0,25281,0,0,45319,0,0)
Max S=$\overline{S}$= 509299.5 at point (0,0,1,0,0,1,0,67905)
Max T=$\overline{T}$=28600 at point (0,0,31570,1,0,0,0,32037)
Worst point of P^w =0.11667
Worst point of C^w=0
Worst point of S^w =0
Worst point of T^w =0

5.1 By Zimmerman's approach

Linear Membership function formulation:

$$m\left(P\left(y\right)\right)=\begin{cases} 1 & \text{if P}\left(y\right)\geq 0.2666667, \\ \dfrac{\frac{42y_1+36y_2+40y_3+30y_4+35y_5+40y_6+30y_7+25y_8}{159y_1+169y_2+285y_3+142y_4+300y_5+150y_6+185y_7+165y_8}-0.11687}{0.149997} & \text{if } 0.11687<\text{ P}\left(y\right)<0.2666667, \\ 0 & \text{if P}\left(y\right)\leq 0.11687. \end{cases} \tag{5.17}$$

$$m\left(C\left(y\right)\right)=\begin{cases} 1 & \text{if } C\left(y\right)\geq 35300, \\ \dfrac{0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.6y_5+0.5y_6+0.4y_7+0.5y_8}{35300} & \text{if } 0<\text{C}\left(y\right)<35300, \\ 0 & \text{if C}\left(y\right)\leq 0. \end{cases} \tag{5.18}$$

$$m\left(S\left(y\right)\right)=\begin{cases} 1 & \text{if } S\left(y\right)\geq 509299.5, \\ \dfrac{6.4y_1+5.3y_2+5.5y_3+5.2y_4+7.6y_5+6.5y_6+5.4y_7+7.5y_8}{509299.5} & \text{if } 0<\text{S}\left(y\right)<509299.5, \\ 0 & \text{if S}\left(y\right)\leq 0. \end{cases} \tag{5.19}$$

$$m\left(\text{T}\left(y\right)\right)=\begin{cases} 1 & \text{if } T\left(y\right)\geq 28600, \\ \dfrac{0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.3y_5+0.3y_6+0.4y_7+0.4y_8}{28600} & \text{if } 21180<T\left(y\right)<28600, \\ 0 & \text{if } T\left(y\right)\leq 21180. \end{cases} \tag{5.20}$$

Converting into single objective function problem:

$$\begin{gathered} \min\ \psi \\ \text{subjected to; } \dfrac{\frac{42y_1+36y_2+40y_3+30y_4+35y_5+40y_6+30y_7+25y_8}{159y_1+169y_2+285y_3+142y_4+300y_5+150y_6+185y_7+165y_8}-0.11687}{0.149997}+\text{D}_1^- - \text{D}_1^+ = 1, \\ \frac{0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.6y_5+0.5y_6+0.4y_7+0.5y_8}{35300}+\text{D}_2^- - \text{D}_2^+ = 1, \\ \frac{6.4y_1+5.3y_2+5.5y_3+5.2y_4+7.6y_5+6.5y_6+5.4y_7+7.5y_8}{509299.5}+\text{D}_3^- - \text{D}_3^+ = 1, \\ \frac{0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.3y_5+0.3y_6+0.4y_7+0.4y_8}{28600}+\text{D}_4^- - \text{D}_4^+ = 1, \\ 0\leq\phi\leq\ 1, \\ \phi\geq\text{D}_1^-,\text{D}_1^+,\ \text{D}_2^-,\text{D}_2^+,\ \text{D}_3^-,\text{D}_3^+,\text{D}_4^-,\text{D}_4^+, \\ \text{D}_1^-,\text{D}_1^+,\ \text{D}_2^-,\text{D}_2^+,\ \text{D}_3^-,\text{D}_3^+,\text{D}_4^-,\text{D}_4^+\geq 0, \\ \text{D}_1^-,\text{D}_1^+,\ \text{D}_2^-,\text{D}_2^+,\ \text{D}_3^-,\text{D}_3^+,\text{D}_4^-,\text{D}_4^+=0, \\ 128y_1+121y_2+246y_3+100y_4+180y_5+131y_6+155y_7+120y_8\leq 12156000, \\ 120y_1+110y_2+220y_3+70y_4+220y_5+120y_6+120y_7+115y_8\leq\ 11295000, \\ 28y_1+25y_2+65y_3+22y_4+60y_5+25y_6+30y_7+25y_8\leq 2853000, \\ 131y_1+144y_2+220y_3+120y_4+240y_5+125y_6+155y_7+140y_8\leq 13194000, \\ 0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.6y_5+0.5y_6+0.4y_7+0.5y_8\leq 35300, \\ 6.4y_1+5.3y_2+5.5y_3+5.2y_4+7.6y_5+6.5y_6+5.4y_7+7.5y_8\leq 509300, \\ 0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.3y_5+0.3y_6+0.4y_7+0.4y_8\leq 28600, \\ y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8\geq 0,\ \in Z. \end{gathered} \tag{5.21}$$

By solving through LINGO software P= 0.2651431, C= 34941.3, S= 509299.3, T= 28309.4 at point (45892,0,9,0,0,33159,0,1). Value of deviation parameter is 0.01016084. According to which profit is 3254209 and investment is 12273408.

5.2 By proposed methodology

Non-linear (S-shaped) membership function:

$$m\left(P\left(y\right)\right)=\begin{cases} 1 & \text{if P}\left(y\right)\geq 0.267,\\ 1-\left(\dfrac{1}{1+0.001001e^{13.813\left(\frac{\frac{42y_1+36y_2+40y_3+30y_4+35y_5+40y_6+30y_7+25y_8}{159y_1+169y_2+285y_3+142y_4+300y_5+150y_6+185y_7+165y_8}-0.1169}{0.149997}\right)}}\right) & \text{if } 0.117<\text{ P}\left(y\right)<0.26\\ 0 & \text{if P}\left(y\right)\leq 0.1169. \end{cases} \tag{5.22}$$

$$m\left(C\left(y\right)\right)=\begin{cases} 1 & \text{if } C\left(y\right)\geq 35300,\\ 1-\left(\dfrac{1}{1+0.001001e^{13.813\left(\frac{0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.6y_5+0.5y_6+0.4y_7+0.5y_8}{35300}\right)}}\right) & \text{if } 0<\text{C}\left(y\right)<35300,\\ 0 & \text{if C}\left(y\right)\leq 0. \end{cases} \tag{5.23}$$

$$m\left(S\left(y\right)\right)=\begin{cases} 1 & \text{if } S\left(y\right)\geq 509299.5,\\ 1-\left(\dfrac{1}{1+0.001001e^{13.813\left(\frac{6.4y_1+5.3y_2+5.5y_3+5.2y_4+7.6y_5+6.5y_6+5.4y_7+7.5y_8}{509299.5}\right)}}\right) & \text{if } 0<\text{S}\left(y\right)<509299.5,\\ 0 & \text{if S}\left(y\right)\leq 0. \end{cases} \tag{5.24}$$

$$m\left(\text{T}\left(y\right)\right)=\begin{cases} 1 & \text{if } T\left(y\right)\geq 28600,\\ 1-\left(\dfrac{1}{1+0.001001e^{13.813\left(\frac{0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.3y_5+0.3y_6+0.4y_7+0.4y_8}{28600}\right)}}\right) & \text{if } 21180<T\left(y\right)<28600,\\ 0 & \text{if } T\left(y\right)\leq 21180. \end{cases} \tag{5.25}$$

Converting into single objective function problem:

$$\min \phi$$

$$\text{subjected to;}1-\left(\frac{1}{1+0.001001e^{13.813\left(\frac{\frac{42y_1+36y_2+40y_3+30y_4+35y_5+40y_6+30y_7+25y_8}{159y_1+169y_2+285y_3+142y_4+300y_5+150y_6+185y_7+165y_8}-0.11687}{0.149997}\right)}}\right)+\mathrm{D}_1^- -\mathrm{D}_1^+=1,$$

$$1-\left(\frac{1}{1+0.001001e^{13.813\left(\frac{0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.6y_5+0.5y_6+0.4y_7+0.5y_8)}{35300}\right)}}\right)+\mathrm{D}_2^- -\mathrm{D}_2^+=1,$$

$$1-\left(\frac{1}{1+0.001001e^{13.813\left(\frac{6.4y_1+5.3y_2+5.5y_3+5.2y_4+7.6y_5+6.5y_6+5.4y_7+7.5y_8}{509299.5}\right)}}\right)+\mathrm{D}_3^- -\mathrm{D}_3^+=1,$$

$$1-\left(\frac{1}{1+0.001001e^{13.813\left(\frac{0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.3y_5+0.3y_6+0.4y_7+0.4y_8}{28600}\right)}}\right)+\mathrm{D}_4^- -\mathrm{D}_4^+=1.$$

$$0\leq\ \phi\leq 1,$$
$$\phi\geq \mathrm{D}_1^-,\mathrm{D}_1^+,\ \mathrm{D}_2^-,\mathrm{D}_2^+,\ \mathrm{D}_3^-,\mathrm{D}_3^+,\mathrm{D}_4^-,\mathrm{D}_4^+,$$
$$\mathrm{D}_1^-,\mathrm{D}_1^+,\ \mathrm{D}_2^-,\mathrm{D}_2^+,\ \mathrm{D}_3^-,\mathrm{D}_3^+,\mathrm{D}_4^-,\mathrm{D}_4^+\geq 0,$$
$$\mathrm{D}_1^-,\mathrm{D}_1^+,\ \mathrm{D}_2^-,\mathrm{D}_2^+,\ \mathrm{D}_3^-,\mathrm{D}_3^+,\mathrm{D}_4^-,\mathrm{D}_4^+=0,$$
$$128y_1+121y_2+246y_3+100y_4+180y_5+131y_6+155y_7+120y_8\leq 12156000,$$
$$120y_1+110y_2+220y_3+70y_4+220y_5+120y_6+120y_7+115y_8\leq\ 11295000,$$
$$28y_1+25y_2+65y_3+22y_4+60y_5+25y_6+30y_7+25y_8\leq 2853000,$$
$$131y_1+144y_2+220y_3+120y_4+240y_5+125y_6+155y_7+140y_8\leq 13194000,$$
$$0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.6y_5+0.5y_6+0.4y_7+0.5y_8\leq 35300,$$
$$6.4y_1+5.3y_2+5.5y_3+5.2y_4+7.6y_5+6.5y_6+5.4y_7+7.5y_8<=509300,$$
$$0.4y_1+0.3y_2+0.5y_3+0.2y_4+0.3y_5+0.3y_6+0.4y_7+0.4y_8\leq 28600,$$
$$y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8\geq 0,\ \in Z. \tag{5.26}$$

By solving through LINGO software C= 34939.5, S= 509278.5, T= 28307.9, P = 0.2651461 at point (45886, 0,1,0,0,33158,14,0). Value of deviation parameter is 0.001151189. According to which profit is 3253992 and investment is 12272449.

6 Result Analysis

Table 4. Deviation of objective functions from ideal points

Objective Function	Deviation From Linear membership	Ideal Points Non-linear membership
Profit/Investment(P)	0.571349%	0.15206%
Cutting time(C)	1.0161473%	1.021246%
Sewing time (S)$\times 10^4$	0.000039269%	0.000041233105%
Trimming time (T)	1.0160839%	1.0213286%
Total	2.603619469%	2.194675833%

In table 4 we analyzed that deviation of objective function from its ideal point is reduced for non-linear membership function as compared to linear membership function. We can see that the deviation of objective function profit over investment using linear membership function for fuzzy goal programming is 0.571349% from it ideal point but it reduces to 0.15206% when we considered the same problem for non-linear membership function in fuzzy goal programming. The comparison between the linear membership and non-linear membership function over the deviation from its ideal solution can be seen in figure 4, where we can see that there is huge gap between two for the objective function profit over investment. Lesser the deviation value from it ideal point better is the result.

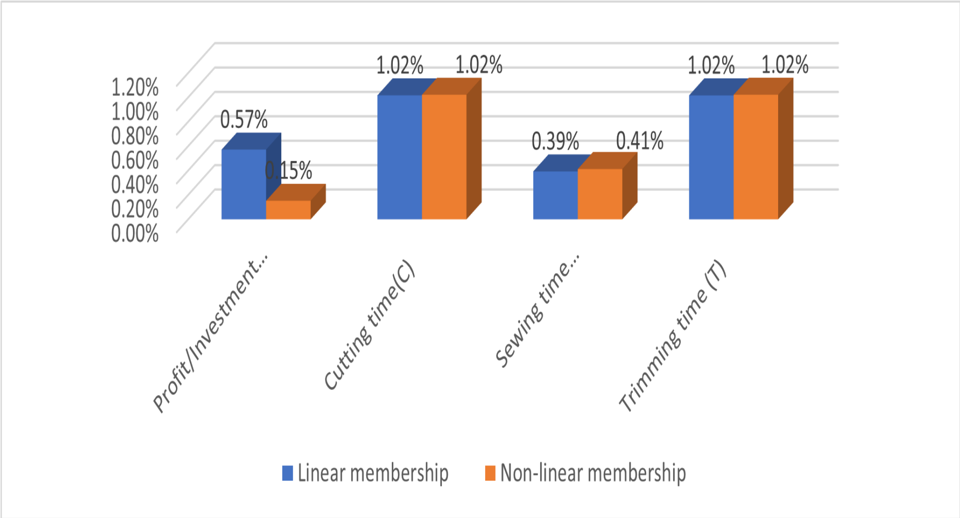

Figure 4. Comparison between linear and nonlinear membership functions over deviation from ideal solution

Table 5. Percentage change in single objective and multi-objective LPP

	In Single objective LPP	In Multi-objective LPP	Percentage of Change
Profit	3420618	3253992	-4.871%
Cost	15542248	12272449	21.038%
Fabric	11386410	10219522	10.248%
Thread	10381550	9487180	8.6149%
Cutting time utilization	30002.2	34939.5	14.1309%
Sewing time utilization	509299.2	509278.5	-0.004644%
Trimming time utilization	28600	28307.9	-1.0213%
Total			49.1459%

In table 5 we compared the results obtained in single objective LPP with multi-objective LPP and analyzed that overall, the percentage change between them is 49.11459% that describes that optimal utility of all the resources is better in case of multi-objective programming problem as compare to the single objective problem. Even though the profit in single objective is little higher as compared to multi-objective but as we can see from table that cost and other important parameters reduced with higher amount in case of multi-objective LPP than the single objective LPP which is always required for optimal solution. The results are also presented graphically in figure 5.

Table 6. Percentage change in profit and cost components via linear and non-linear functions

	Linear	Non-linear	Percentage of change
Profit	3254209	3253992	-0.000066683%
Cost	12273408	12272449	0.000078136%
Total			0.000011453%

In table 6 described the profit and cost values obtained by linear function and by non-linear

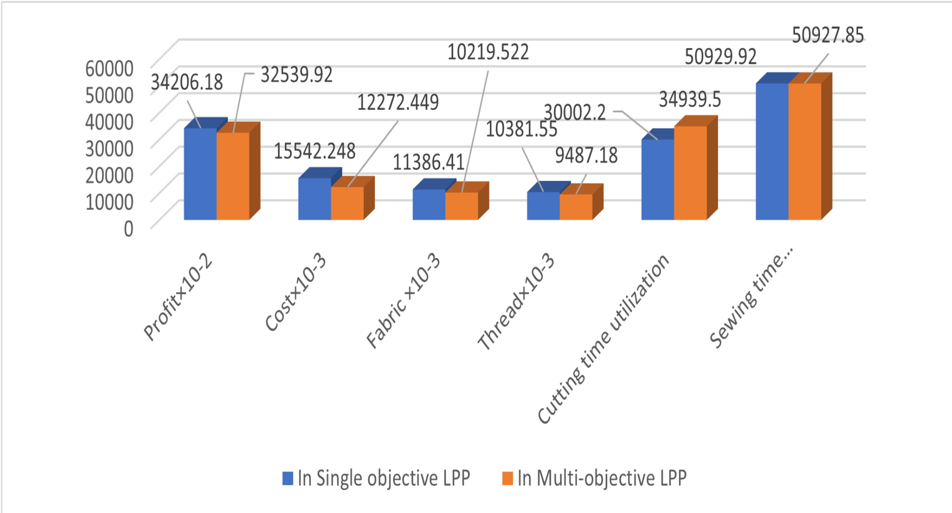

Figure 5. Graphical representation of percentage change between single objective to multi-objective

function and the net percentage change of these factors by two methodologies. We observed that overall percentage of change is positive. In table 7 we have discussed about the results obtained

Table 7. Percentage change in Fabric and Thread via linear and non-linear functions

	Linear	Non-linear	Percentage of change
Fabric	10220339	10219522	0.000079938%
Thread	9488215	9487180	0.010908%
Total			0.0109879%

in two important component of garment industry fabric and thread and we observed that the overall as well as individual percentage changes between linear and non-linear is positive that describes that non-linear methodology has enhanced the results as compare to the linear function-based methodology.

7 Conclusion and future work

This study used the non-linear S-shaped membership function to find an optimal solution to the fuzzy multiple-purpose linear fractional programming Problems. We compared the results and solutions with the Zimmerman's approach using the linear membership function. We found that when an S-shaped membership function solves the objective function profit over the investment, the percentage deviation from the ideal point is lower than the deviation achieved by the linear membership function technique. In the future, we will extend our proposed approach to more advanced fuzzy sets that provide the ideal solution to many real-world issues.

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