

FACE RECOGNITION USING GENERALIZED ALMOST DISTRIBUTIVE FUZZY LATTICES IN IMAGE PROCESSING FOR FUZZY MORPHOLOGICAL EROSION

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Abstract This paper explores face identification and recognition using MATLAB software, implementing fuzzy morphological erosion based on Generalized Almost Distributive Fuzzy Lattices (GADFL). Face recognition involves identifying a face biometric process used to identify or verify a person by analyzing their facial features. The focus here is on recognizing faces from binary images. Fuzzy morphological erosion is one of the most widely used fuzzy techniques in image processing due to its foundation in distributive fuzzy lattices. This operation is primarily applied to process various face reactions (binary images), where a partial ordering and a fuzzy lattice structure are evident. Face recognition is achieved by analyzing intensity value measurements of input images. The training dataset was compiled from various sources, incorporating metrics such as Peak Signal-to-Noise Ratio (PSNR), Signal-to-Noise Ratio (SNR), and the Structural Similarity Index (SSIM). The system demonstrated effective performance, achieving an 80% recognition accuracy. MATLAB software was utilized for implementing the proposed methodology.

1 Introduction

The study of shape and structure is referred to as “mathematical morphology.” Morphological operations have a wide range of applications, including morphology that deals with transformations. The analysis of binary images was the first use of mathematical morphology [1]. The binary pictures are maps $A : U \rightarrow [0, 1]$, where U denotes the Euclidean plane R^2 or the Cartesian grid Z^2 respectively, and the image value at each point $x \in U$ can only be 0 or 1, representing black and white [2, 3]. Serra proposed grayscale morphological expansion of the binary morphological operator. Fuzzy mathematical morphology’s purpose is to apply several approaches have been attempted [4]. For example, the notion of fuzzy subset inclusion grade is used by Xu, J., and Giardina, C.R. to define the core operations of fuzzy mathematical morphology [5]. Zadeh’s $A \leq B$ iff $A(x) \leq B(x) \forall x \in U$, which has been used by numerous writers to build weaker conceptions, is one example of several approaches to the fuzzy subsets of A, B of a universe U.

Since the 1990s, face recognition algorithms have been created. One of the most difficult elements of the technology is face recognition systems, which require of action with better discriminatory strength [6]. Many dimension reduction methods linear discriminant analysis (LDA) [7], the principal component of analysis (PCA) [8], and (ICA) [9], have been proposed in previous research. Many computer vision applications, like as activity recognition, car safety, and surveillance [10], rely on object detection and tracking. A face detection system utilising MATLAB is presented here, which can recognise not only a human face but also eyes and the upper body. Humans find face detection to be a simple operation, while computers do not. Due to huge intra-class variances induced by changes in facial look, lighting, and expression, it has been

recognised as the most complex and challenging subject in the field of computer vision [11]. In any space that is linear to the original picture space, such differences cause the face distribution to be very nonlinear and complex.

The significance of lattice theory (order theory) as a fundamental mathematical topic that gives a new perspective on a collection of elements and an ordering relation known as a partial order [12] has long been recognised. The phrase 'y includes x' denotes this partial order, which is symbolised by $x \geq y$. In this work, we will focus on generalised virtually distributive fuzzy lattices in addition to lattice algebra [13]. Using a set R and a nearly distributive fuzzy lattice L , we define a fuzzy subset A of R as a function $A : R \times R \rightarrow [0, 1]$, where $A((r))$ is the degree of membership of element x accordingly (R, A) [14, 15, 16]. $L(R, A)$ can then be enlarged to the point where (R, A) is effectively a distributive fuzzy lattice by inserting the fuzzy morphological dilation. As a result, for underexposed data, the order relation can be enlarged to the interval $[0, 1/2]$ or $[1/2, 1]$, respectively.

Finally, using a biometric process based on intensity value measurements of input photos, the detection of faces was completed. The training images came from a number of sources, including the PSNR, SNR, and numerous structural aspects of the SSIM. The system worked effectively, detecting faces with an accuracy of 80%. This paper's software requirements are MATLAB software.

2 Preliminaries

A few fundamental definitions are discussed.

Definition 2.1 (GADFL). Let (R, A) be a fuzzy poset and $L(R, \wedge, \vee)$ be an algebra type $(2, 2)$. If $L(R, A)$ satisfies the following axioms, we call it a Generalized Almost Distributive Fuzzy Lattice.

- (i) $A((a \wedge b) \wedge c, a \wedge (b \wedge c)) = A(a \wedge (b \wedge c), (a \wedge b) \wedge c) = 1$;
- (ii) $A(a \wedge (b \vee c), (a \wedge b) \vee (a \wedge c)) = A((a \wedge b) \vee (a \wedge c), a \wedge (b \vee c)) = 1$;
- (iii) $A(a \vee (b \wedge c), (a \vee b) \wedge (a \vee c)) = A((a \vee b) \wedge (a \vee c), a \vee (b \wedge c)) = 1$;
- (iv) $A(a \wedge (a \vee b), a) = A(a, a \wedge (a \vee b)) = 1$;
- (v) $A((a \vee b) \wedge a, a) = A(a, (a \vee b) \wedge a) = 1$;
- (vi) $A((a \wedge b) \vee b, b) = A(b, (a \wedge b) \vee b) = 1$ for all $a, b, c \in R$.

Definition 2.2 (Erosion). An operator $\varepsilon : L \rightarrow L'$ is Erosion if it commutes with the infimum, for all $x_i \in L'$, $\varepsilon(S_i x_i) = S' \varepsilon(x_i)$, where S denotes the infimum associated with \geq and S' denotes the infimum associated with \geq' .

Definition 2.3 (Peak signal-to-noise ratio and Signal-to-noise ratio). The ratio of an image's maximum attainable power to the power of corrupting noise that degrades its quality of representation is called the peak signal-to-noise ratio, or PSNR. A picture's PSNR needs to be calculated by comparing it to the maximum power achievable, perfect clean image. The decibel ratio of signal to noise power in relation to background noise level is known as the signal-to-noise ratio, or SNR.

Definition 2.4 (Structural Similarity Index (SSIM)). A perceptual metric called the Structural Similarity Index (SSIM) is used to quantify the amount of image quality loss. It's a full reference metric that necessitates the use of two images: a reference image and a processed image, both of which must be captured from the same image capture.

3 Fuzzy Morphological Erosion of Generalized Almost Distributive Fuzzy Lattices

This section covers the notion of Fuzzy morphological erosion, as well as some theorems, in the context of Generalized Almost Distributive Fuzzy Lattices.

Definition 3.1. Fuzzy Morphological Erosion of GADFL with the binary operation infimum and its structuring and least elements are S and 0 . Moreover, let $\{X_i\}$ be a collection of elements from $L(R, A)$. An operator $\varepsilon_S : R \times R \rightarrow [0, 1]$ on L is defined by

$$\varepsilon_S A(x) = \inf_{y \in R} [S(y-x) * A(y), 0] > 0 \forall x \in R$$

Theorem 3.2. Let $L(R, A)$ be a GADFL. Then the fuzzy erosion $A_1 \geq A_2 \in L(R, A)$ which implies $\varepsilon_S(A_2, A_1) > 0$.

Proof. Given $L(R, A)$ be a GADFL. For every $x, y \in R$
Define the fuzzy erosion, S structuring element

$$\varepsilon_S A_1(x) = \inf_{y \in R} [S_1(y-x) * A_1(y), 0] > 0 \forall x \in R$$

And $\varepsilon_V A_2(x) = \inf_{y \in R} [S_1(y-x) * A_2(y), 0] > 0 \forall x \in R$,

To prove $A_1 \geq A_2 \Rightarrow \varepsilon_S(A_2, A_1) > 0$

Suppose $A_1 = S_1(y-x)$ and $A_2 = S_2(y-x) \forall x \in R$

Then $A_1 \geq A_2 \Rightarrow S_1(y-x) \geq S_2(y-x)$

$\Rightarrow \varepsilon_{S_1}(x) \geq \varepsilon_{S_2}(x) (\because (S_1, S_2) = S), \varepsilon_S(x) = S(y-x)$

$\Rightarrow \varepsilon(S_2(x), S_1(x)) > 0 [\because a \geq b \Rightarrow A(b, a) > 0]$

$\Rightarrow (\inf_{y \in R} [S_2(y-x) * A_2(y), 0], \inf_{y \in R} [S_1(y-x) * A_1(y), 0])$

$\Rightarrow (\inf_{y \in R} (\inf_{y \in R} [(S_2(y-x), S_1(y-x)) * (A_2(y), A_1(y)), 0]) > 0$

$\Rightarrow (\inf_{y \in R} [S(y-x) * (A_2(y), A_1(y)), 0] > 0 (\because (S_1, S_2) = S)$

$\Rightarrow \varepsilon_S(A_2, A_1) > 0$

Thus $A_1 \geq A_2 \Rightarrow \varepsilon_S(A_2, A_1) > 0$

Hence proved. \square

Theorem 3.3. If operation $*$ is distributes over arbitrary meets, then given a family of GADFL, fuzzy set $\{A_i | i \in I\} \subseteq L(R, A)$, it holds $\varepsilon_S(\bigwedge_{i \in I} A_i) = \bigwedge_{i \in I} (\varepsilon_S(A_i))$

Proof. Let family of fuzzy sets $\{A_i | i \in I\} \subseteq L(R, A)$ and $y \in R$.

We have $\varepsilon_S(\bigwedge_{i \in I} A_i)(y)$

$= \inf_{y \in R} [S(y-x) * \bigwedge_{i \in I} A_i(y), 0] > 0$

$= \inf_{y \in R} [\bigwedge_{i \in I} S(y-x) * A_i(y), 0] > 0$

$= \bigwedge_{i \in I} \inf_{y \in R} [S(y-x) * A_i(y), 0] > 0$

$= \bigwedge_{i \in I} (\varepsilon_S(A_i))$

Thus $\varepsilon_S(\bigwedge_{i \in I} A_i) = \bigwedge_{i \in I} (\varepsilon_S(A_i))$. Hence proved. \square

Theorem 3.4. Let S_1 and S_2 be two structuring elements and $\varepsilon_{S_i} : R \times R \rightarrow [0, 1] (i = 1, 2)$ the fuzzy erosion operator in $L(R, A)$ associated with them. If $*$ represents the usual composition, it is verified $(\varepsilon_{S_1} * \varepsilon_{S_2}) = \varepsilon_{\varepsilon_{S_1}(S_2)}$.

Proof. For all $A \in L(R, A)$

$(\varepsilon_{S_1} * \varepsilon_{S_2})(A) = \varepsilon_{S_1}(\varepsilon_{S_2}(A))$ Therefore $\forall x \in R$

$((\varepsilon_{S_1} * \varepsilon_{S_2})(A)(x) = \varepsilon_{S_1}(\varepsilon_{S_2}(A))(x)$

$= \inf_{y \in R} [S_1(y-x) * \varepsilon_{S_2}(A)(y), 0] > 0$

$= \inf_{y \in R} [S_1(y-x) * \inf_{z \in R} [S_2(z-y) * A(z), 0], 0] > 0$

$= \inf_{y \in R} [\inf_{z \in R} [S_1(y-x) * [S_2(z-y) * A(z), 0]] > 0$

$= \inf_{z \in R} [A(z) * \inf_{y \in R} [S_1(y-x) * S_2(z-y)], 0] > 0$

If we take $z-y=r$, then the last expression is equal to

$\inf_{z \in R} [A(z) * \inf_{r \in R} [S_1(z-r-x) * S_2(r)], 0] > 0$

$= \inf_{z \in R} [A(z) * \varepsilon_{S_1}(S_2)(z-x), 0] > 0$

and applying the commutativity of $*$, we have

$= \sup_{z \in R} [\varepsilon_{S_1}(S_2)(z-x) * A(z), 0] > 0$

$= (\varepsilon_{\varepsilon_{S_1}(S_2)}(A))(x)$

$\therefore ((\varepsilon_{S_1} * \varepsilon_{S_2})(A))(x) = (\varepsilon_{\varepsilon_{S_1}(S_2)}(A))(x)$

Thus $(\varepsilon_{S_1} * \varepsilon_{S_2}) = \varepsilon_{\varepsilon_{S_1}(S_2)}$

Hence proved. \square

Theorem 3.5. Let ε_S be a fuzzy erosion in GADFL $L(R, A)$. Then the following are equivalent.

- (i) $L(R, A)$ is an ADFL.
- (ii) For any structuring element 'U' of $L(R, A)$, ε_U is a fuzzy erosion on $L(R, A)$.
- (iii) ε_V is a fuzzy erosion on $L(R, A)$.

Proof. (i) (1) \Rightarrow (2)

Assume (1)

Let 'U' is a structuring element $L(R, A)$ Then ε_U is a fuzzy erosion on $L(R, A)$. Let $x, y \in R$.

$$\begin{aligned} \Rightarrow A(x) &= U(y-x) \quad \forall y \in R \\ &= U(y-x) > 0 \end{aligned}$$

$\therefore A(y-x, 0) > 0$

$\Rightarrow A(0, 0) > 0$ and $\varepsilon_U A(x) = \{x \in R \mid U(y-x) * A(y)\} \quad \forall y \in R$

$\Rightarrow A[(y-x) * A(y), 0] = A(x * y, 0) = 1$

$= A(0, 0) > 0$

$\Rightarrow \inf_{y \in R} [U(y-x) * A(y), 0] > 0$

$\Rightarrow \varepsilon_U A(x) > 0$

Thus $\varepsilon_U A(x) = \inf_{y \in R} \{U(y-x) * A(y), 0\} > 0$

Hence ε_U is a fuzzy erosion on $L(R, A)$. Hence (1) \Rightarrow (2)

ii) (2) \Rightarrow (3) It is obvious.

iii) (3) \Rightarrow (1)

Assume (3)

ε_V is a fuzzy erosion it is defined.

$\varepsilon_V A(x) = \inf_{y \in R} \{V(y-x) * A(y), 0\} > 0 \quad \forall x \in R$

Since $\inf_{y \in R} \{V(y-x) * A(y), 0\} > 0$

$\Rightarrow \varepsilon_V A(x) > 0$

Thus $x, y \in R$, since $\varepsilon_V A(x)$ is a fuzzy erosion on $L(R, A)$.

Thus $\varepsilon_V A(x) > 0$.

Therefore $L(R, A)$ be an ADFL. □

4 Algorithm:

Original photos and face identification approaches have been able to improve the PSNR, SNR, and different structural elements of the SSIM from training images using this algorithm.

- Step 1: Select a set of original photographs.
- Step 2: Find the binary images from the original photos.
- Step 3: On the binary set of images, do morphological erosion.
- Step 4: We determine the minimum and maximum value from the set values of the training images using the fuzzy function (write down the formula).
- Step 5: The information is saved for subsequent face detection processing.
- Step 6: Choose an input image that changes totally or somewhat from the originals.
- Step 7: Compare and take minimum value of the neighbor.
- Step 8: Set the pixel value to that minimum value.

- Step 9: Take all the neighbourhoods.
- Step 10: Take logical different SE (Structural Element).
- Step 11: Fuzzy erosion function for logical SE
- Step 12: Compare and take maximum value of the neighbor
- Step 13: And set the pixel value to that maximum value.
- Step 14: Take all the neighbourhoods.
- Step 15: Image plot original image and fuzzy Image.
- Step 16: The input image's face is recognized. PSNR, SNR, and SSIM are calculated from scratch and compared to database images. The most closely matching photograph is utilized to determine the person's name.

This algorithm utilizes MATLAB to enhance biometric identification by optimizing the PSNR, SNR, and key structural elements of the SSIM in the training images.

5 Face Recognition Techniques (GADFL) as a Fuzzy Morphological Erosion Process in MATLAB

The above algorithm, implemented using MATLAB, is effective for face recognition by optimizing PSNR, SNR, and structural similarity (SSIM). Figures 5.1 and 5.2 below demonstrate the application of face detection techniques using GADFL.

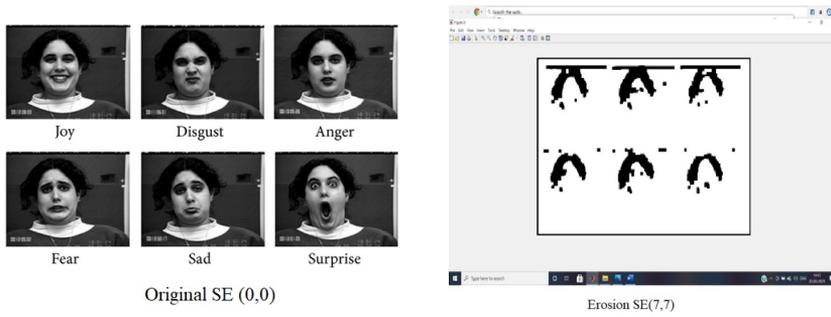


Figure 1. (a) and (b)



Figure 1. (c) and (d)



Figure 1. (e) and (f)

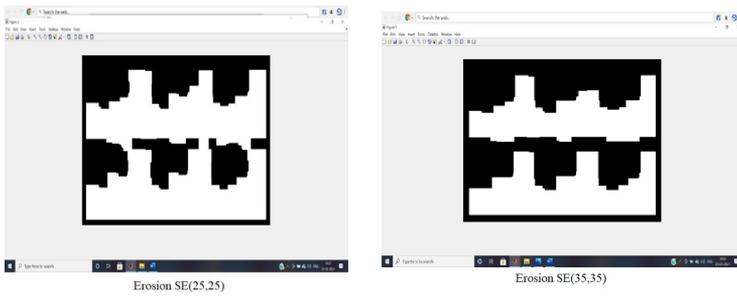


Figure 1. (g) and (h) FACE 1 – Original SE and Different SE.



(a)Before gray level equalization (b)After gray level equalization

Original SE (0,0)

Figure 2. (a)



Figure 2. (b) and (c)

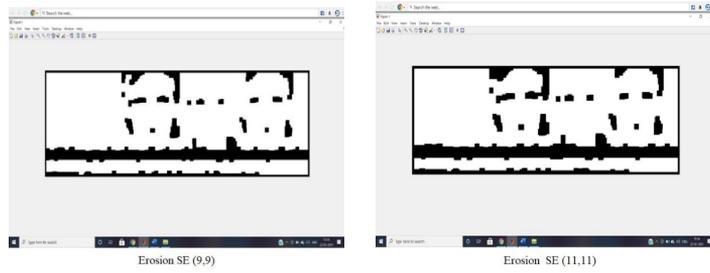


Figure 2. (d) and (e)



Figure 2. (f) and (g)

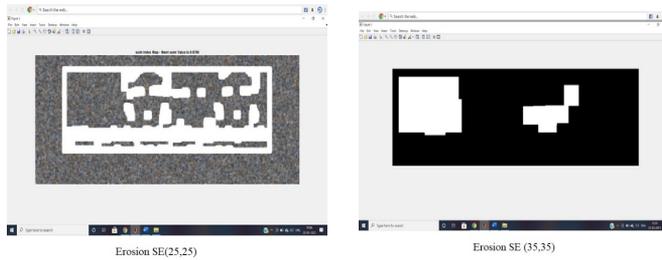


Figure 2. (h) and (i) FACE 2 – Original SE and Different SE.

6 Face Recognition (Noise Ratio) experimental table and Data Analysis

The face recognition problem was solved using fuzzy lattice theory. In the algorithm, we can locate a fuzzy morphological erosion point where the picture can be recognized. Tables 6.1 & 6.2 indicate the appropriate values for Peak-SNR, SNR, and SSIM for the fuzzy erosion approximation in face detection.

In MATLAB, you can calculate PSNR, SNR, and SSIM for a biometric image before and after morphological operations with different structural element (SE) sizes.

Table 1. Experimental result for Face 1 Erosion Structural Element (SE).

Values	Original SE (0,0)	SE (5,5)	SE (7,7)	SE (9,9)	SE (11,11)	SE (13,13)	SE (25,25)	SE (35,35)
PSNR	28.5512	28.6957	27.3216	27.5414	27.3969	27.5659	27.9405	28.1713
SNR	25.1758	27.0638	27.0651	27.0863	27.0754	27.0714	27.0701	27.0701
SSIM	0.7048	0.6322	0.4692	0.5243	0.4775	0.4954	0.5359	0.5599

The experimental table (Table 6.1) evaluates the performance of the face recognition system based on three key metrics:

PSNR (Peak Signal-to-Noise Ratio): Measures the quality of the image after the erosion operation. A higher PSNR indicates better image quality.

SNR (Signal-to-Noise Ratio): Evaluates the ratio between the useful signal (face features) and the noise in the image. Higher SNR values are indicative of cleaner data for recognition.

SSIM (Structural Similarity Index): Assesses the structural similarity between the processed and original images. A higher SSIM indicates that the image features, such as the face structure, are better preserved during processing.

Data Analysis for Face 1 Erosion Structural Element (SE).

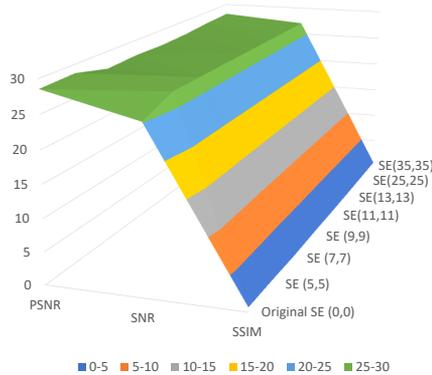


Figure 3. Data Analysis for Face 1 Erosion Structural Element (SE).

Table 2. Experimental results for Face 2 Erosion Structural Element (SE).

Values	Original SE	SE (5,5)	SE (7,7)	SE (9,9)	SE (11,11)	SE (13,13)	SE (25,25)	SE (35,35)
PSNR	27.9568	27.9163	27.9377	27.9317	27.3969	27.9472	27.9268	27.9408
SNR	27.1007	27.0601	27.0815	27.0755	27.0803	27.0910	27.0706	27.0846
SSIM	0.5788	0.5775	0.5784	0.5779	0.5781	0.5783	0.5780	0.5785

These values suggest that the different SE sizes used in morphological operations have a minimal impact on the overall image quality. The slight variations in PSNR, SNR, and SSIM across SE sizes imply that the morphological preprocessing does not significantly alter the core features of the biometric image, ensuring that the identification system can still perform accurately. The choice of SE size (such as 5x5, 7x7, etc.) may be optimized based on specific needs like noise reduction or feature enhancement.

Data Analysis for Face 2 Erosion Structural Element (SE).

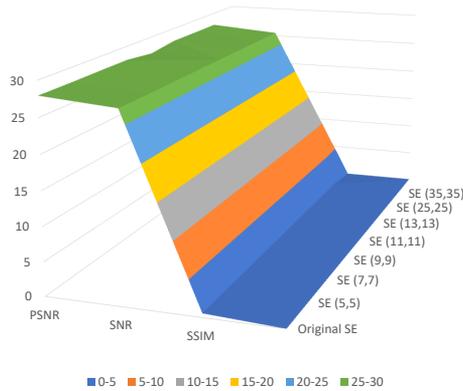


Figure 4. Data Analysis for Face 2 Erosion Structural Element (SE).

7 Conclusion

In this study, implemented a face recognition system using MATLAB that utilizes the fuzzy morphological erosion operator to biometric process various facial reaction images. The system

is built on the principles of Generalized Almost Distributive Fuzzy Lattices, where also derived several relevant theorems related to fuzzy morphological erosion. The algorithm is capable of evaluating critical performance metrics, including Peak Signal-to-Noise Ratio (PSNR), Signal-to-Noise Ratio (SNR), and the Structural Similarity Index (SSIM). These metrics are tabulated and analyzed, reflecting the structural framework of Generalized Almost Distributive Fuzzy Lattices. For future work, we propose investigating the application of fuzzy morphological operators such as opening and closing on color images in combination with face detection techniques. This approach aims to further refine and optimize the entire face recognition and biometric identification process within the Generalized Almost Distributive Fuzzy Lattices framework using MATLAB.

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