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4-MEAN E-CORDIAL LABELING

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Abstract Let $l : E(G) \to \{1, 2, 3, 4\}$ be a function that labels the edges of a given simple graph $G(V, E)$ with the induced letter $l^* : V(G) \to \{0,\ 1\}$ It is provided by the following formula for each vertex u to express $l^*(u) = \begin{cases} 0, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is even} \\ 1, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is odd} \end{cases}$, use the formula: $|v_l(0) - v_l(1)| \leq 1$ is satisfied by $v_l(0)$ and $v_l(1)$, the function l is referred to as 4-Mean E-Cordial Labeling. We refer to a graph as the 4-Mean E-Cordial graph when it takes this label.

1 Introduction

In this paper, we focus on perfectly connected simple undirected graphs defined as G. The concept of labels was first proposed by Rosa in 1967 [1], where the label should be a function of the numbers for vertices or edges, or in some cases both. Graph labeling has been extensively developed since notable contributions by Rosa and Golomb. Function $l : V(G) \to \{0, 1, 2, \ldots, q\}$ if l is injective and the derived function $l^*(e = uv) = l(u) - l(v)$ is bijective. Pictures that can be written beautifully are called beautiful pictures. We extend this idea to the sincere notation E; such that every vertex $v \to V$, $l(v) \to (l(uv)$ for $uv \to E)(G))$ $(mod\, 2)$. E-Cordial labeling E of G is characterized by the minimum difference and similarly holds for vertices labeled 0 and 1. According to this definition, we introduce a new labeling technique called 4-Mean E-Cordial labeling. This article examines the implementation of the 4-Mean E-Cordial notation for public and private images.

2 4-Mean E-Cordial Labeling

Consider a simple graph $G(V, E)$. For each vertex u, define $l^*(u)$ as follows: $l^*(u)$ is 0 if the sum of $l(uv)$ overall adjacent edges divided by the degree of u is even, and 1, if it is odd. We call l as a 4-Mean E-Cordial labeling if the absolute difference between the number of vertices labeled 0 and 1, denoted $v_l(0)$ and $v_l(1)$ respectively, is at most 1.

Example 2.1. Let us consider some random graph,

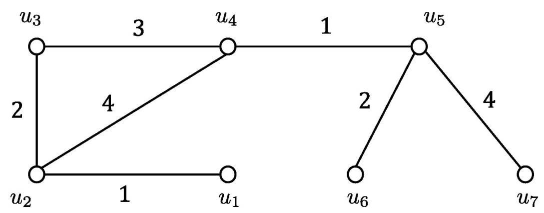

Figure 1. Random Graph

From the given graph, we observe that

$$l^*(u_1) = 1$$
$$l^*(u_2) = 1$$
$$l^*(u_3) = 0$$
$$l^*(u_4) = 0$$
$$l^*(u_5) = 1$$
$$l^*(u_6) = 0$$
$$l^*(u_7) = 0$$

Consequently, $v_l(0) = 4$ and $v_l(1) = 3$, indicating $|v_l(0) - v_l(1)| \leq 1$. Hence the given graph is 4-Mean E-Cordial graph.

3 4-Mean E-Cordial Labeling For Some Standard Graphs

Remark 3.1. The complete graphs are 4-Mean E-Cordial graph if $p \leq 4$.

Theorem 3.2. *If $p \leq 0, 2, 3(mod\, 4)$, then the star cluster $K_{1,p}$ is a 4-Mean E-Cordial graph.*

Proof. We examine the star graph $K_{1,p}$ with vertices denoted as $\{u, u_1, u_2, \ldots, u_p\}$ and edges $\{uu_1, uu_2, uu_3, \ldots, uu_p\}$.

For $p \equiv 0, 2, 3(mod\, 4)$, we define as follows:

$$l\,(uu_m) = \begin{cases} 1, & \text{if } m \equiv 1(mod\, 4) \\ 2, & \text{if } m \equiv 2(mod\, 4) \\ 3, & \text{if } m \equiv 3(mod\, 4) \\ 4, & \text{if } m \equiv 0(mod\, 4) \end{cases}$$

Additionally, we define l^* on $V(G)$ as:

$$l^*(u) = \begin{cases} 0, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is even} \\ 1, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is odd} \end{cases}$$

This labeling scheme encompasses all potential edge arrangements.

	$e_l(1)$	$e_l(2)$	$e_l(3)$	$e_l(4)$	$v_l(0)$	$v_l(1)$
$p \equiv 0$	$\frac{p}{4}$	$\frac{p}{4}$	$\frac{p}{4}$	$\frac{p}{4}$	$\frac{p}{2}+1$	$\frac{p}{2}$
$p \equiv 2$	$\frac{p}{4}+1$	$\frac{p}{4}+1$	$\frac{p}{4}$	$\frac{p}{4}$	$\frac{p}{2}$	$\frac{p}{2}+1$
$p \equiv 3$	$\frac{p}{4}+1$	$\frac{p}{4}+1$	$\frac{p}{4}+1$	$\frac{p}{4}$	$\frac{p}{2}$	$\frac{p}{2}$

Table 1.

This defined labeling satisfies $|v_l(0) - v_l(1)| \leq 1$ and $|e_l(m) - e_l(n)| \leq 1$, where $m, n \in \{1, 2, 3, 4\}$. Hence, the star graph exhibits a 4-Mean E-Cordial labeling, demonstrating its status as a 4-Mean E-Cordial graph. □

Example 3.3. Consider $K_{1,6}$

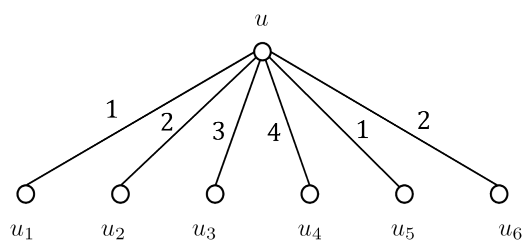

Figure 2. $K_{1,6}$

In the above graph,

$$\begin{aligned} l^*(t_1) &= 1 \\ l^*(t_2) &= 0 \\ l^*(t_3) &= 1 \\ l^*(t_4) &= 0 \\ l^*(t_5) &= 1 \\ l^*(t_6) &= 0 \\ l^*(t) &= 1 \end{aligned}$$

Also, $v_l(0) = 3,\ v_l(1) = 4$ this implies $\mid v_l(0) - v_l(1) \mid \leq 1$.
Hence $K_{1,6}$ is a 4-Mean E-Cordial graph.

Theorem 3.4. *A Path Graph P_p is a 4-Mean E-Cordial graph if p is odd.*

Proof. Consider a path graph P_p with vertices $V(P_p) = \{u_1, u_2, \ldots, u_p\}$ and edges $E(P_p) = \{u_1u_2, u_2u_3, u_3u_4, \ldots, u_{p-1}u_p\}$.
Let us assume p is an odd number.
To establish the 4-Mean E-Cordial property, we define:

$$l(u_m u_{m+1}) = \begin{cases} 1 & \text{if } m \equiv 1 (mod\, 4) \\ 2 & \text{if } m \equiv 2 (mod\, 4) \\ 3 & \text{if } m \equiv 3 (mod\, 4) \\ 4 & \text{if } m \equiv 0 (mod\, 4) \end{cases}$$

Now, let us define l^* on $V(G)$ by,

$$l^*(u) = \begin{cases} 0 & \text{if } \left\lceil \dfrac{\sum_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is even} \\ 1 & \text{if } \left\lceil \dfrac{\sum_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is odd} \end{cases}$$

This labeling scheme encompasses all possible edge arrangements.
We observe the following values for $e_l(1), e_l(2), e_l(3), e_l(4), v_l(0)$, and $v_l(1)$:

When $p \equiv 1(mod\, 4)$:

$$\begin{aligned} e_l(1) &= \frac{p-1}{4}, \\ e_l(2) &= \frac{p-1}{4}, \\ e_l(3) &= \frac{p-1}{4}, \\ e_l(4) &= \frac{p-1}{4}, \\ v_l(0) &= \frac{p-1}{2}, \\ v_l(1) &= \frac{p+1}{2} \end{aligned}$$

When $p \equiv 3(mod\, 4)$:

$$\begin{aligned} e_l(1) &= \frac{p+1}{4}, \\ e_l(2) &= \frac{p+1}{4}, \\ e_l(3) &= \frac{p-3}{4}, \\ e_l(4) &= \frac{p-3}{4}, \\ v_l(0) &= \frac{p-1}{2}, \\ v_l(1) &= \frac{p+1}{2} \end{aligned}$$

Thus, the defined labeling satisfies $|v_l(0) - v_l(1)| \leq 1$ and $|e_l(m) - e_l(n)| \leq 1$.

Therefore, the path graph P_p admits a 4-Mean E-Cordial labeling, establishing it as a 4-Mean E-cordial graph. □

Example 3.5. Consider P_5,

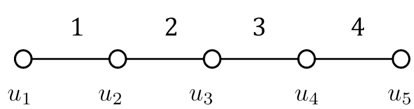

Figure 3. P_5

In the above graph,

$$\begin{aligned} l^*(u_1) &= 1 \\ l^*(u_2) &= 1 \\ l^*(u_3) &= 0 \\ l^*(u_4) &= 0 \\ l^*(u_5) &= 0 \\ &\text{and} \\ v_l(0) &= 3 \\ v_l(1) &= 2 \end{aligned}$$

This implies $| \ v_l(0) - v_l(1) \ | \leq 1$. Hence path P_p is a 4-Mean E-Cordial graph.

Theorem 3.6. *Cycle graph C_p is 4-Mean E-Cordial graph if p is odd.*

Proof. C_p of vertex set $V(C_p) = \{u_1, u_2, \ldots, u_p\}$ and edge set $E(C_p) = \{u_1u_2, u_2u_3, u_3u_4, \ldots, u_{n-1}u_p\}$.
Let p be an odd integer. We aim to show that C_p is a 4-Mean E-Cordial graph.
Define l^* on $V(G)$,

$$l^*(u) = \begin{cases} 0 & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is even} \\ 1 & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is odd} \end{cases}$$

This labeling covers all possible arrangements of edges.

	$e_l(1)$	$e_l(2)$	$e_l(3)$	$e_l(4)$	$v_l(0)$	$v_l(1)$
$p \equiv 1$	$\frac{p+3}{4}$	$\frac{p-1}{4}$	$\frac{p-1}{4}$	$\frac{p-1}{4}$	$\frac{p+1}{2}$	$\frac{p-1}{2}$
$p \equiv 3$	$\frac{p+1}{4}$	$\frac{p+1}{4}$	$\frac{p+1}{4}$	$\frac{p-3}{4}$	$\frac{p+1}{2}$	$\frac{p-1}{2}$

Table 2.

This labeling satisfies $|v_l(0) - v_l(1)| \leq 1$ and $|e_l(m) - e_l(n)| \leq 1$. Thus, the cycle C_p admits a 4-Mean E-Cordial labeling, making it a 4-Mean E-Cordial graph. □

Theorem 3.7. *Bistar $B(p,p)$ is 4-Mean E-Cordial graph if p is odd.*

Proof. Let $B_{p,p}$ denote the bistar graph with vertices $V(B_{p,p}) = \{u, v, u_m, v_m : 1 \leq l \leq p\}$ and edges $E(B_{p,p}) = \{uv, uu_m, vv_m : 1 \leq m \leq p\}$.
We define

$$l(uv) = 4$$

$$l(uu_m) = \begin{cases} 1, & \text{if } m \equiv 1, 3 (mod\, 4) \\ 3, & \text{if } m \equiv 0, 2 (mod\, 4) \end{cases}$$

$$l(vv_m) = \begin{cases} 2, & \text{if } m \equiv 1, 3 (mod\, 4) \\ 4, & \text{if } m \equiv 0, 2 (mod\, 4) \end{cases}$$

We define l^* on $V(B_{p,p})$ by:

$$l^*(u) = \begin{cases} 0, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is even} \\ 1, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is odd} \end{cases}$$

This labeling pattern covers all possible arrangements of edges.
We observe the following counts for the labels,

$$\begin{aligned} e_l(1) &= \frac{p+1}{2} \\ e_l(2) &= \frac{p+1}{2} \\ e_l(3) &= \frac{p-1}{2} \\ e_l(4) &= \frac{p+1}{2} \\ v_l(0) &= p+1 \\ v_l(1) &= p+1 \end{aligned}$$

Hence, the bistar graph $B_{p,p}$ admits a 4-Mean E-Cordial labeling.
Thus, $B_{p,p}$ is a 4-Mean E-Cordial graph. □

Example 3.8. Consider $B_{5,5}$

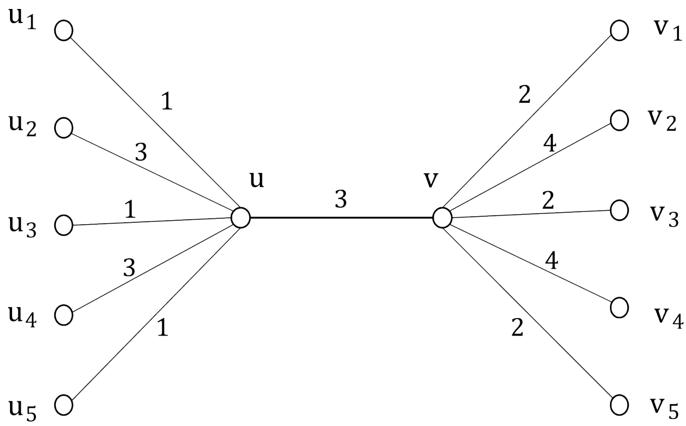

Figure 4. $B_{5,5}$

In the above graph,
$l^*(u) = 1 \quad l^*(u_1) = 1 \quad l^*(u_2) = 1 \quad l^*(u_3) = 1 \quad l^*(u_4) = 1 \quad l^*(u_5) = 1$
$l^*(v) = 0 \quad l^*(v_1) = 0 \quad l^*(v_2) = 0 \quad l^*(v_3) = 0 \quad l^*(v_4) = 0 \quad l^*(v_5) = 0$
Hence $v_l(0) = 6$ and $v_l(1) = 6$ and $\mid v_l(0) - v_l(1) \mid \leq 1$. Which implies that $B_{5,5}$ is a 4-Mean E-Cordial graph.

Theorem 3.9. *The wheel graph W_p is a 4-Mean E-Cordial graph if p is even.*

Proof. Now $V(W_p) = \{u, u_1, u_2, u_3, \ldots, u_p\}$ denote the vertices and $E(W_p) = \{uu_1, uu_2, uu_3, \ldots, uu_p, u_1u_2, u_2u_3, u_3u_4, \ldots, u_pu_1\}$ denote the edges.

$$l(uu_m) = \begin{cases} 1, & \text{if } m \equiv 1, 3 (mod\, 4) \\ 3, & \text{if } m \equiv 0, 2 (mod\, 4) \end{cases}$$
$$l(u_m u_{m+1}) = \begin{cases} 2, & \text{if } m \equiv 1, 3 (mod\, 4) \\ 4, & \text{if } m \equiv 0, 2 (mod\, 4) \end{cases}$$

Define l^* on $V(W_p)$,

$$l^*(u) = \begin{cases} 0, & \text{if } \left\lceil \dfrac{\sum_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is even} \\ 1, & \text{if } \left\lceil \dfrac{\sum_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is odd} \end{cases}$$

This labeling pattern covers all possible arrangements of edges. Hence, the wheel graph W_p admits a 4-Mean E-Cordial labeling, making it a 4-Mean E-Cordial graph. □

Example 3.10. Consider W_6

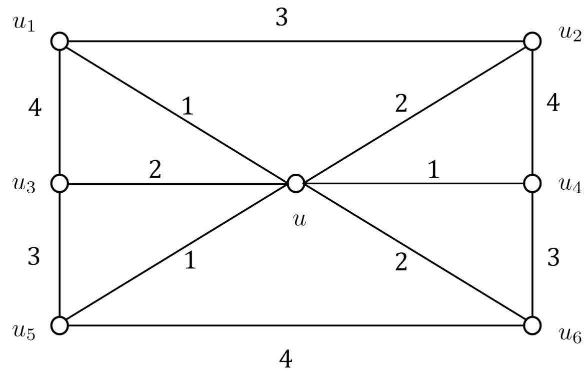

Figure 5. W_6

In the above graph

$l^*(u) = 1 \quad l^*(u_1) = 1 \quad l^*(u_2) = 1 \quad l^*(u_3) = 1 \quad l^*(u_4) = 1 \quad l^*(u_5) = 1$

$l^*(v) = 0 \quad l^*(v_1) = 0 \quad l^*(v_2) = 0 \quad l^*(v_3) = 0 \quad l^*(v_4) = 0 \quad l^*(v_5) = 0$

Hence $v_l(0) = 6, v_l(1) = 6$ this implies $\mid v_l(0) - v_l(1) \mid \leq 1$. Hence W_6 is a 4-Mean E-Cordial graph.

Theorem 3.11. *Subdivision of Star Graph $S(S_p)$ as a 4-Mean E-Cordial Graph.*

Proof. Consider the star image $S(S_p)$ of the vertex set $V[S(S_p)] = \{u, u_1, u_2, \ldots, u_p, v_1, v_2, \ldots, v_p\}$ and edge set $E[S(S_p)] = \{uu_1, uu_2, uu_3, \ldots, uu_p, u_1v_1, u_2v_2, \ldots, u_pv_p\}$.

Thus, $|V[S(S_p)]| = 2p + 1$ and $|E[S(S_p)]| = 2p$.

We define,

$$l(uu_m) = \begin{cases} 1, & \text{if } m \equiv 1, 3 (mod\, 4) \\ 3, & \text{if } m \equiv 0, 2 (mod\, 4) \end{cases}$$

$$l(u_mv_m) = \begin{cases} 2, & \text{if } m \equiv 1, 3 (mod\, 4) \\ 4, & \text{if } m \equiv 0, 2 (mod\, 4) \end{cases}$$

We define l^* on $V(S(S_p))$,

$$l^*(u) = \begin{cases} 0, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is even} \\ 1, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is odd} \end{cases}$$

This labeling pattern covers all possible edge arrangements. The table below illustrates the labeling for different cases:

Case	$e_l(1)$	$e_l(2)$	$e_l(3)$	$e_l(4)$	$v_l(0)$	$v_l(1)$
If p is odd	$\frac{p+1}{2}$	$\frac{p+1}{2}$	$\frac{p-1}{2}$	$\frac{p-1}{2}$	p	$p+1$
If p is even	$\frac{p}{2}$	$\frac{p}{2}$	$\frac{p}{2}$	$\frac{p}{2}$	$p+1$	p

Table 3.

Labeling meets $|v_l(0) - v_l(1)| \leq 1$ and $|e_l(m) - e_l(n)| \leq 1$. Thus, the subdivision of star graph $S(S_p)$ admits a 4-Mean E-Cordial labeling, proving it to be a 4-Mean E-Cordial graph. □

Example 3.12. Consider $S(S_4)$

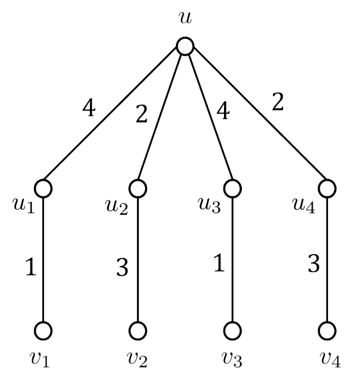

Figure 6. $S\,(S_4)$

In the above graph

$l^*(u) = 0 \quad l^*(u_1) = 0 \quad l^*(u_2) = 0 \quad l^*(u_3) = 0 \quad l^*(u_4) = 0$

$l^*(v_1) = 1 \quad l^*(v_2) = 1 \quad l^*(v_3) = 1 \quad l^*(v_4) = 1$

Hence $v_l(0) = 5$, $v_l(1) = 4$ this implies $\mid v_l(0) - v_l(1) \mid \leq 1$.

Hence $S\,(S_4)$ is a 4-Mean E-Cordial graph.

Theorem 3.13. *The Crown graph $C_p \odot K_2$ is a 4-Mean E-Cordial graph.*

Proof. Consider the Crown graph $C_p \odot K_2$, denoted by G, with vertex set $V(G) = \{u_1, u_2, u_3, \dots, u_p, u'_1, u'_2, u'_3, \dots, u'_p\}$ and edge set $E(G) = \{u_m u_{m+1}, u_m u'_m \mid 1 \leq m \leq n\}$, where indices are taken modulo p.

We define, if $p \equiv 0(mod\,4)$, then

$$l\,(u_m u_{m+1}) = \begin{cases} 3, & \text{if } m \equiv 1, 3(mod\,4) \\ 4, & \text{if } m \equiv 0, 2(mod\,4) \end{cases}$$

if $p \equiv 0(mod\,4)$, then

$$l\,(u_m u'_m) = \begin{cases} 1, & \text{if } m \equiv 1, 3(mod\,4),, \\ 2, & \text{if } m \equiv 0, 2(mod\,4) \end{cases}$$

Now, we define l^* on V(G) by:

$$l^*(u) = \begin{cases} 0, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is even} \\ 1, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is odd} \end{cases}$$

Where $N(u)$ represents the neighbors of vertex u.

The following table presents the values of $e_l(1), e_l(2), e_l(3), e_l(4), v_l(0)$, and $v_l(1)$:

Case	$e_l(1)$	$e_l(2)$	$e_l(3)$	$e_l(4)$	$v_l(0)$	$v_l(1)$
Odd p	$\frac{p+1}{2}$	$\frac{p-1}{2}$	$\frac{p+1}{2}$	$\frac{p-1}{2}$	p	p
Even p	$\frac{p}{2}$	$\frac{p}{2}$	$\frac{p}{2}$	$\frac{p}{2}$	p	p

Table 4.

This labeling satisfies $|v_l(0) - v_l(1)| \leq 1$ and $|e_l(m) - e_l(n)| \leq 1$, ensuring a 4-Mean E-Cordial labeling for the Crown graph $C_n \odot K_2$.

Thus, $C_n \odot K_2$ is a 4-Mean E-cordial graph. □

Example 3.14. Consider $C_4 \odot K_2$

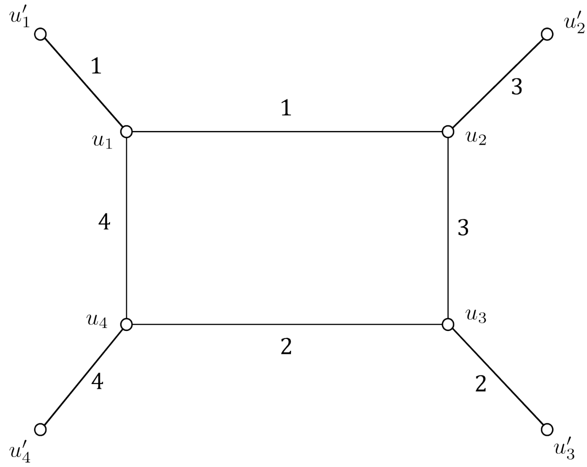

Figure 7. $C_4 \odot K_2$

In the above graph
$l^*(u_1) = 0 \quad l^*(u_2) = 1 \quad l^*(u_3) = 1 \quad l^*(u_4) = 0$
$l^*(u_1') = 1 \quad l^*(u_2') = 1 \quad l^*(u_3') = 0 \quad l^*(u_4') = 0$
Hence $v_l(0) = 4$, $v_l(1) = 4$ this implies $| v_l(0) - v_l(1) | \leq 1$.
Hence $C_4 \odot K_2$ is a 4-Mean E-Cordial graph.

Theorem 3.15. *The combo graph $P_p \odot K_2$ is a 4-Mean E-Cordial graph when p is even.*

Proof. Consider combo graph $P_p \odot K_2$, where $V(P_p \odot K_2) = \{u_m, u_m' \mid 1 \leq m \leq p\}$ represents the vertices and $E(P_p \odot K_2) = \{u_m u_{m+1}, uu_m' \mid 1 \leq m \leq p\}$ denotes edges.

We define,

$$l^*(u) = \begin{cases} 0, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is even} \\ 1, & \text{if } \left\lceil \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rceil \text{ is odd} \end{cases}$$

This labeling pattern exhausts all possible edge arrangements. We then observe that the defined labeling.

Hence, $P_p \odot K_2$ admits a 4-Mean E-Cordial labeling, establishing it as a 4-Mean E-Cordial graph. □

Example 3.16. Consider $P_5 \odot K_2$

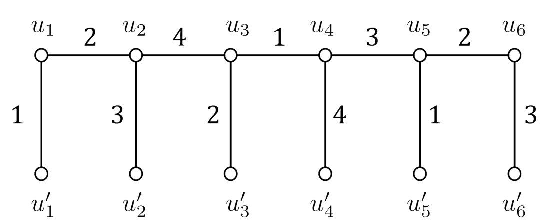

Figure 8. $P_5 \odot K_2$

In the above graph
$l(u_1) = 0 \quad l(u_2) = 0 \quad l(u_3) = 1 \quad l(u_4) = 1 \quad l(u_5) = 0 \quad l(u_6) = 0$
$l(u_1') = 1 \quad l(u_2') = 1 \quad l(u_3') = 0 \quad l(u_4') = 0 \quad l(u_5') = 1 \quad l(u_6') = 1$

Hence $v_l(0) = 6$, $v_l(1) = 6$ this implies $| v_l(0) - v_l(1) | \leq 1$.
Hence $P_5 \odot K_2$ is a 4-Mean E-Cordial graph.

Theorem 3.17. *The fan graph $F_{1,p}$ qualifies as a 4-Mean E-Cordial graph.*

Proof. Observe the vertex fan graph $F_{1,p}$, $V(F_{1,p}) = \{u, u_1, u_2, u_3, \ldots, u_p\}$ and edges $E(F_{1,p}) = \{u_1u_2, u_2u_3, u_3u_4, \ldots, u_{p-1}u_p, uu_1, uu_2, uu_3, \ldots, uu_p\}$.
l for the edges and l^* for the vertices as follows:
The labeling function l is defined as:
Function l^* on $V(F_{1,p})$ is defined as:

$$l^*(u) = \begin{cases} 0 & \text{if } \left\lfloor \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rfloor \text{ is even} \\ 1 & \text{if } \left\lfloor \dfrac{\sum\limits_{uv} l(uv)}{\deg(u)} \right\rfloor \text{ is odd} \end{cases}$$

This labeling pattern covers all possible edge arrangements. Next, we demonstrate that for p,

$$\begin{aligned} e_l(1) &= \frac{p+1}{2}, \\ e_l(2) &= \frac{p-1}{2}, \\ e_l(3) &= \frac{p-1}{2}, \\ e_l(4) &= \frac{p-1}{2}, \\ v_l(0) &= \frac{p+1}{2}, \\ v_l(1) &= \frac{p+1}{2} \end{aligned}$$

For even p,

$$\begin{aligned} e_l(1) &= \frac{p}{2}, \\ e_l(2) &= \frac{p}{2}, \\ e_l(3) &= \frac{p}{2}, \\ e_l(4) &= \frac{p}{2} - 1, \\ v_l(0) &= \frac{p}{2}, \\ v_l(1) &= \frac{p}{2} + 1 \end{aligned}$$

Finally, we verify that this labeling, thereby establishing that the fan graph $F_{1,p}$ is indeed a 4-Mean E-Cordial graph. □

Example 3.18. Consider a fan graph $F_{1,6}$

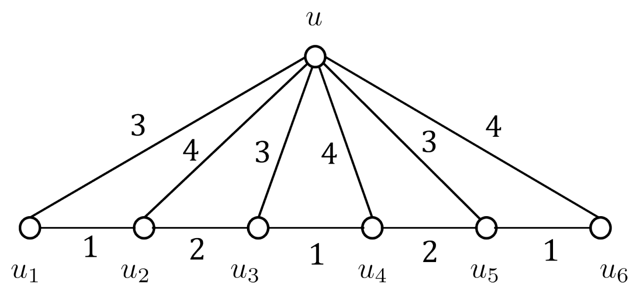

Figure 9. $F_{1,6}$

In the above graph

$l^*(u) = 1 \quad l^*(u_1) = 0 \quad l^*(u_2) = 1 \quad l^*(u_3) = 0 \quad l^*(u_4) = 1 \quad l^*(u_5) = 0 \quad l^*(u_6) = 1$

Hence $v_l(0) = 3$, $v_l(1) = 4$ this implies $\mid v_l(0) - v_l(1) \mid \leq 1$.

Hence $F_{1,6}$ is a 4-Mean E-Cordial graph.

Theorem 3.19. *The Jelly Fish graph $J(p, p)$ is a 4-Mean E-Cordial graph if p is even.*

Proof. Let us consider the Jelly Fish graph $J(p, p)$ with vertex set $V[J(p, p)] = \{x, y, u, v\} \cup \{u_1, u_2, \ldots, u_p\} \cup \{v_1, v_2, \ldots, v_p\}$ and edge set $[J(p, p)] = \{(u, x), (u, y), (v, x), (v, y), (u, v)\} \cup \{(uu_m) \mid 1 \leq m \leq p\} \cup \{(vv_m) \mid 1 \leq m \leq p\}$.

Order of $J(p, p)$ is $|V[J(p, p)]| = 2p + 4$ and $|E[J(p, p)]| = 2p + 5$.

If p is odd:

$$l(xy) = 3$$
$$l(ux) = 1$$
$$l(uy) = 3$$
$$l(vx) = 2$$
$$l(vy) = 4$$
$$l(uu_m) = \begin{cases} 1, & \text{if } m \equiv 1, 3(mod\ 4) \\ 2, & \text{if } m \equiv 0, 2(mod\ 4) \end{cases}$$
$$l(vv_m) = \begin{cases} 3, & \text{if } m \equiv 1, 3(mod\ 4) \\ 4, & \text{if } m \equiv 0, 2(mod\ 4) \end{cases}$$

This labeling pattern covers all possible arrangements of edges. Let's now verify that the labeling meets the requirements for a 4-Mean E-Cordial labeling:

When p is even, we have $e_l(1) = e_l(3) = e_l(4) = \dfrac{p+2}{2}$ and $e_l(2) = \dfrac{p+4}{2}$.

Also, $v_l(0) = v_l(1) = \dfrac{p+4}{2}$.

The defined labeling satisfies $|v_l(0) - v_l(1)| \leq 1$ and $|e_l(m) - e_l(n)| \leq 1$ for $1 \leq m, n \leq 4$. Thus, the Jelly Fish graph $J(p, p)$ admits a 4-Mean E-Cordial labeling, making it a 4-Mean E-Cordial graph. □

Example 3.20. Consider the graph $J_{6,6}$

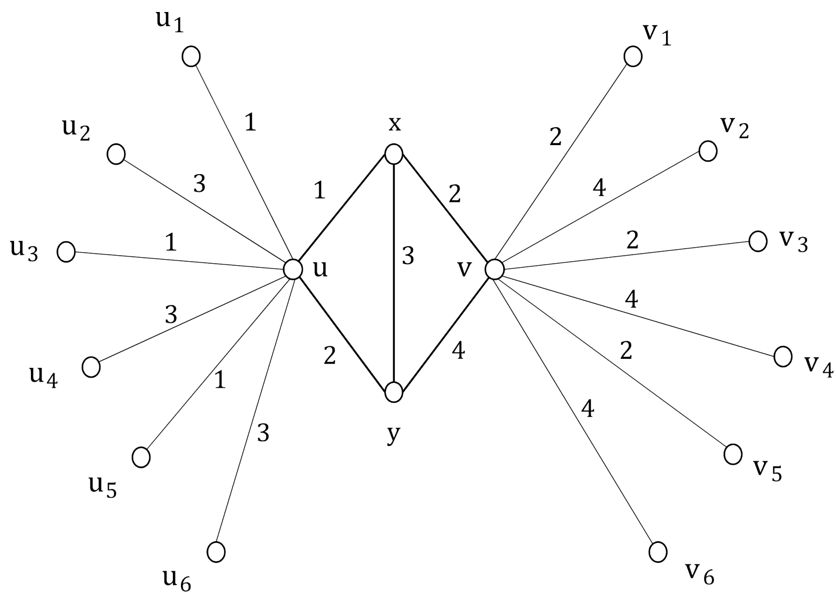

Figure 10. $J_{6,6}$

In the above graph

$l^*(u_1) = 1 \quad l^*(u_2) = 1 \quad l^*(u_3) = 1 \quad l^*(u_4) = 1 \quad l^*(u_5) = 1 \quad l^*(u_6) = 1$

$l^*(v_0) = 0 \quad l^*(v_2) = 0 \quad l^*(v_3) = 0 \quad l^*(v_4) = 0 \quad l^*(v_5) = 0 \quad l^*(v_6) = 0$

Hence $v_l(0) = 6$, $v_l(1) = 6$ this implies $\mid v_l(0) - v_l(1) \mid \leq 1$.

Hence $J_{6,6}$ is a 4-Mean E-Cordial graph.

Theorem 3.21. *The jewel graph J_p is a 4-Mean E-Cordial graph.*

Proof. Let G be the jewel graph denoted as J_p, with vertex set $V(J_p) = \{u, v, x, y, u_m : 1 \leq m \leq p\}$, and edge set $E(J_p) = \{ux, uy, xy, xv, yv, u_m, vu_m : 1 \leq m \leq p\}$. The cardinality of the vertex set is $|V(J_p)| = p + 4$, and the cardinality of the edge set is $|E(J_p)| = 2p + 5$.

We define the labeling function $l : E(J_p) \to \{0, 1\}$ as follows:

If p is odd:

$$l(xy) = 3$$
$$l(ux) = 1$$
$$l(uy) = 3$$
$$l(vx) = 2$$
$$l(vy) = 4$$
$$l(uu_m) = \begin{cases} 1, & \text{if } m \equiv 1, 3 (mod\, 4) \\ 2, & \text{if } m \equiv 0, 2 (mod\, 4) \end{cases}$$
$$l(vv_m) = \begin{cases} 3, & \text{if } m \equiv 1, 3 (mod\, 4) \\ 4, & \text{if } m \equiv 0, 2 (mod\, 4) \end{cases}$$

Define l^* on $V(J_p)$ accordingly. This labeling pattern covers all possible arrangements of edges.

Case	$e_l(1)$	$e_l(2)$	$e_l(3)$	$e_l(4)$	$v_l(0)$	$v_l(1)$
When p is even	$\frac{p+2}{2}$	$\frac{p+2}{2}$	$\frac{p+4}{2}$	$\frac{p+2}{2}$	$\frac{p+4}{2}$	$\frac{p+4}{2}$

Table 5.

Thus, the jewel graph admits a 4-Mean E-Cordial labeling, establishing it as a 4-Mean E-Cordial graph. □

Example 3.22. Consider J_6

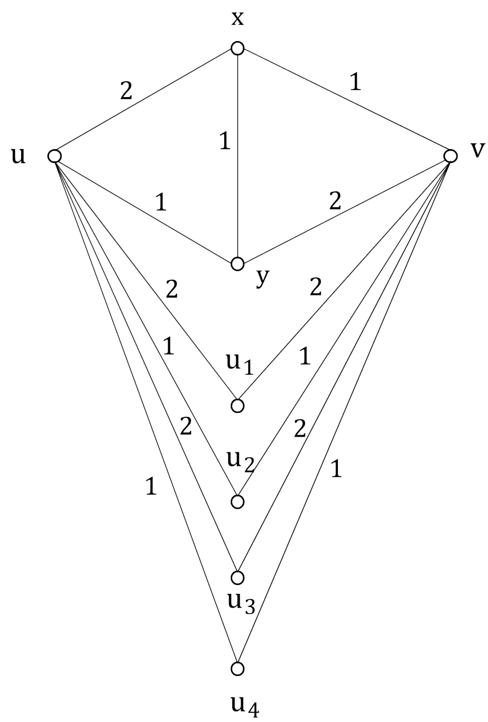

Figure 11. J_6

In the above graph
$l(x) = 0 \quad l(y) = 0 \quad l(u) = 1 \quad l(v) = 1$
$l(u_1) = 0 \quad l(u_2) = 1 \quad l(u_3) = 1 \quad l(u_4) = 0$
Hence $v_f(0) = 4$, $v_f(1) = 4$ this implies $\mid v_f(0) - v_f(1) \mid \leq 1$.
Jewel Graph J_6 is a 4-Mean E-Cordial graph.

4 Conclusion remarks

In this work, we provide the notion of 4-Mean E-Cordial graph as well as a novel labeling method known as 4-Mean E-Cordial labeling. By verifying its suitability for picture structures such stars, highways, circles, binary stars, star subdivisions, graphs, segment drawings, composite drawings, jellyfish drawings, and fine art drawings, we provide this registration approach. In the future, we want to add additional original photographs to this work.

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