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A study on the Skin friction Effects past a Vertical Plate in a Rotating Fluid with uniform Temperature and Variable Mass Diffusion

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Abstract: A study is done to analyse the effects of skin friction on an unsteady free convection and variable mass diffusion in a rotating fluid over a moving vertical plate. A complex velocity is considered to obtain the exact solution combining the axial and transverse components. The skin friction effects due to the Schmidt number, mass Grashof number, radiation, rotation, thermal Grashof number on the plate are examined and analyzed.

1 INTRODUCTION

In science and technology, coriolis force has wide applications. At high temperature, the radiation has a significant effect. For the design of reliable equipments in manufacturing industries the role of radiative heat transfer combined with mass transfer is important. As a method of energy transfer, thermal radiation has got great attention of researchers in the developments of combustion chambers, power plants, gas cooled nuclear reactors and hypersonic flight.

Arpaci [1] analyzed the heated vertical plate with effects of laminar convection in a stagnant radiating gas with thermal radiation. The causes of thermal radiation past a vertical plate with free convection flow was examined by Raptis and Perdikis [2]. .

Perdikis [3] experimented the flow past the vertical plate for mass transfer combining free convection effects. Howard [4] presented the fundamentals of rotating fluids. Vijayalakshmi A.R[5] analyzed the free-convective flow across the vertical plate started impulsively on a rotating fluid coping with radiation effects. Muthucumaraswamy and Lakshmi [6] examined the influences of thermal radiation with variable mass over the vertical plate accelerated exponentially. .

Muthucumarswamy and Ganeshan [7] experimented the impacts of radiation having variable temperature over the impulsively started vertical plate. Iranian et al.[8] experimented the radiative flow on a perpendicular plate with the influence of thermal conductivity using the method of finite difference analysis..

Jhansi Rani and Ramana murthy [9] analyzed the impacts of skin friction on unsteady convective transfer of heat and mass with radiation. Ravikumar.J et al. [10] examined the impacts of skin friction in a rotating fluid along the vertical plate having varying mass and temperature.

Arumugam.S and Neel Armstrong.A [11] experimented the impacts of Nusselt & Sherwood Number along the numerical analysis on skin friction on parabolic flow having uniform mass and temperature across a vertical plate.

Vijayalakshmi.A.R and Ravikumar.J [12] experimented the effects in a rotating fluid over a vertical plate having varying mass along the thermal radiation. In this paper, the various profiles for velocity, concentration, rotation, radiation and temperature effects were discussed in detail. But skin friction effect was not discussed in this paper. So the researcher aimed to examine the influences of skin friction in a rotating fluid having varying mass along a vertical plate.

2 BASIC EQUATIONS

As mentioned in Vijayalakshmi and Ravikumar [12], the domain, governing equations and boundary conditions are same, but for ready reference of current readers, they are mentioned again.Importance is given to the velocity vector q; from here onwards current work starts. A viscous incompressible rotating fluid flowing in three dimensions with variable mass is taken for the study past an infinite vertical plate started impulsively. The vertical upward direction along the plate is taken as x'-axis. z'-axis is normal to y' andy' is normal to x'-axis. With angular velocity Ω' about z'-axis the fluid,the plate are in rigid rotation. A non-scattering, absorbing, emitting radiation, gray fluid is considered. With concentration C'_∞ and temperature T'_∞ everywhere, initially the plate is at rest and so the fluid. Against the gravitational field with constant velocity u_0,in the vertical direction, the plate is given an impulsive motion at time $t' > 0$ in a fluid with thermal radiation. The concentration and the temperature is increased to C'_w andT'_w respectively at the same time and maintained as constant thereafter. The plane z'= 0 occupied by the plate is infinite, the physical quantities depending on z' and t'. The unsteady flow, represented by equations based on Boussinesq's approximation is given below.

$$\nu\frac{\partial^2 u'}{\partial z'^2} + g\beta(T' - T'_\infty) = \frac{\partial u'}{\partial t'} - g\beta^*(C' - C'_\infty) - 2\Omega' v' \tag{2.1}$$

$$2\Omega' u' = \nu\frac{\partial^2 v'}{\partial z'^2} - \frac{\partial v'}{\partial t'} \tag{2.2}$$

$$\frac{\partial q_r}{\partial z'} = k\frac{\partial^2 T'}{\partial z'^2} + \rho C_p\frac{\partial T'}{\partial t'} \tag{2.3}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial z'^2} \tag{2.4}$$

With a distance normal to the plate change in the radiative flux represented by $\frac{\partial q_r}{\partial z'}$ with the following conditions:

$$\begin{gathered} t' \leq 0 : C' = C'_\infty,\ T' = T'_\infty,\ u' = 0,\ \text{for all } z' \\ T' = T'_w + (T'_w - T'_\infty), u' = u_0 \text{when} t' > 0 \\ C' - C'_w = (C'_w - C'_\infty)t'A \text{ at } z' = 0 \\ \text{as} \quad z' \to \infty, C' \to C'_\infty,\ T \to T_\infty,\ u = 0, \end{gathered} \tag{2.5}$$

where $u_0^2 = A\nu$

By Rosseland approximation, the energy equation (2.3) reduces to

$$16a^*\sigma {T'_\infty}^3 (T'_\infty - T') = \rho C_p\frac{\partial T'}{\partial t'} - k\frac{\partial^2 T'}{\partial z'^2} \tag{2.6}$$

Using the dimensionless quantities

$$\begin{gathered} zv = z'u_0, \quad tv = t'u_0'^2, \quad \theta(T'_w - T'_\infty) = T' - T'_\infty \quad (u,v)u_0 = (u', v') \\ Gcu_0^3 = vg\beta^*(C'_w - C'_\infty), Gr{u'_0}^3 = g\beta v(T'_w - T'_\infty), C(C'_w - C'_\infty) = (C' - C'_\infty) \\ Rku_0'^2 = 16a^*v^2\sigma(T'_\infty)^3, \Omega u_0'^2 = \Omega' v, kPr = \mu C_p \end{gathered} \tag{2.7}$$

The equations from (2.1) to (2.5), of the problem reduces to

$$\frac{\partial q}{\partial t} + 2i\Omega = Gr\theta + GcC + \frac{\partial^2 q}{\partial z^2} \tag{2.8}$$

$$\frac{\partial \theta}{\partial t} + \frac{R}{Pr}\theta = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial z^2} \tag{2.9}$$

$$\frac{\partial^2 C}{\partial z^2} = Sc\frac{\partial C}{\partial t} \tag{2.10}$$

Here,$i = \sqrt{-1}$ & the complex velocity q = u + iv
The conditions (initial & boundary) of the dimensionless form are
when t ≤ 0 $z \leq 0, \quad \theta = 0, \quad C = 0, q = 0$
$t > 0$: at $z = 0 \quad , \theta = 1, \quad C = t, \quad q = 1,$ (2.11)
as z $\to \infty, \quad \theta \to 0, \quad C \to 0 \quad , q = 0,$

3 METHOD OF SOLUTION

All the variables (physical) appearing in the above section are given in the nomenclature. From equations (2.8) to (2.10) the solution is obtained employing Laplace transform with respect to the conditions (2.11). The solutions of θ, C & q are as follow

$$\theta = \frac{1}{2}\Big[e^{-H\sqrt{Pr\,a}}\,\text{erfc}\big(\eta\sqrt{Pr} - \sqrt{at}\big) + e^{2H\sqrt{Pr\,a}}\,\text{erfc}\big(\eta\sqrt{Pr} + \sqrt{at}\big)\Big] \tag{3.1}$$

$$C = t\big(1 + 2\eta^2 Sc\big)\,\text{erfc}\big(\eta\sqrt{Sc}\big) - 2\eta\Big(\frac{\sqrt{Sc}}{\pi}\Big)e^{-\eta^2 Sc} \tag{3.2}$$

$$\begin{aligned}
q = &\left[\frac{1}{2} - \frac{Gr}{2b(Pr-1)} - \frac{Gc}{2c^2(Sc-1)} - \frac{t \cdot Gc}{2c(Sc-1)}\right]\left[e^{-H\sqrt{m}}\,\text{erfc}(\eta - \sqrt{mt}) + e^{H\sqrt{m}}\,\text{erfc}(\eta + \sqrt{mt})\right] \\
&+ \frac{Gr \cdot e^{bt}}{2b(Pr-1)}\left[e^{-H\sqrt{(b+m)t}}\,\text{erfc}(\eta - \sqrt{(b+m)t}) + e^{H\sqrt{(b+m)t}}\,\text{erfc}(\eta + \sqrt{(b+m)t})\right] \\
&+ \frac{\sqrt{t} \cdot Gc}{2\sqrt{m}(Sc-1)}\left[e^{-H\sqrt{m}}\,\text{erfc}(\eta - \sqrt{mt}) - e^{H\sqrt{m}}\,\text{erfc}(\eta + \sqrt{mt})\right] \\
&+ \frac{Gc \cdot e^{ct}}{2c^2(Sc-1)}\left[e^{-H\sqrt{(c+m)t}}\,\text{erfc}(\eta - \sqrt{(c+m)t}) + e^{H\sqrt{(c+m)t}}\,\text{erfc}(\eta + \sqrt{(c+m)t})\right] \\
&+ \frac{Gr}{2b(Pr-1)}\left[e^{-H\sqrt{Pra}}\,\text{erfc}(\eta\sqrt{Pr} - \sqrt{at}) + e^{H\sqrt{Pra}}\,\text{erfc}(\eta\sqrt{Pr} + \sqrt{at})\right] \\
&- \frac{Gr \cdot e^{bt}}{2b(Pr-1)}\left[e^{-H\sqrt{Pr(a+b)}}\,\text{erfc}(\eta\sqrt{Pr} - \sqrt{(a+b)t}) + e^{H\sqrt{Pr(a+b)}}\,\text{erfc}(\eta\sqrt{Pr} + \sqrt{(a+b)t})\right] \\
&+ \frac{Gc}{c^2(Sc-1)}\,\text{erfc}(\eta\sqrt{Sc}) + \frac{Gc}{c(Sc-1)}\left[t(1 + 2\eta^2 Sc)\,\text{erfc}(\eta\sqrt{Sc}) - 2\eta\frac{\sqrt{Sc}}{\pi}(e^{-\eta^2 Sc})\right] \\
&- \frac{Gc \cdot e^{ct}}{2c^2(Sc-1)}\left[e^{-H\sqrt{cSc}}\,\text{erfc}(\eta\sqrt{Sc} - \sqrt{ct}) + e^{H\sqrt{cSc}}\,\text{erfc}(\eta\sqrt{Sc} + \sqrt{ct})\right]
\end{aligned} \tag{3.3}$$

where $aPr = R, b(1 - Pr) = R - m, c(Sc - 1) = m, z = 2\eta\sqrt{t}$, and $m = 2i\Omega$

In equation (3.3), the functions erf() and erfc() have complex arguments. The components u and v for velocity and skin-friction are derived using the Abramowitz and Stegun [12] formula.

4 SKIN FRICTION

The skin friction is obtained as

$$\tau = -\left[\frac{dq}{d\eta}\right]_{\eta=0} \tag{4.1}$$

is non dimensional form
where $\tau = \tau_x + i\tau_y$ Therefore, the skin friction from equations (3.3) and (4.1) is

$$
\begin{aligned}
\tau = & \left[\frac{1}{2} - \frac{Gr}{2b(Pr-1)} - \frac{Gc}{2c^2(Sc-1)} - \frac{t\cdot Gc}{2c(Sc-1)}\right]\left[e^{-H\sqrt{m}}\,\mathrm{erfc}(\eta-\sqrt{mt}) + e^{H\sqrt{m}}\,\mathrm{erfc}(\eta+\sqrt{mt})\right] \\
& + \frac{Gr\cdot e^{bt}}{2b(Pr-1)}\left[e^{-H\sqrt{(b+m)t}}\,\mathrm{erfc}(\eta-\sqrt{(b+m)t}) + e^{H\sqrt{(b+m)}}\,\mathrm{erfc}(\eta+\sqrt{(b+m)t})\right] \\
& + \frac{\sqrt{t}\cdot Gc}{2\sqrt{m}(Sc-1)}\left[e^{-H\sqrt{m}}\,\mathrm{erfc}(\eta-\sqrt{mt}) - e^{H\sqrt{m}}\,\mathrm{erfc}(\eta+\sqrt{mt})\right] \\
& + \frac{Gc\cdot e^{ct}}{2c^2(Sc-1)}\left[e^{-H\sqrt{(c+m)}}\,\mathrm{erfc}(\eta-\sqrt{(c+m)t}) + e^{H\sqrt{(c+m)}}\,\mathrm{erfc}(\eta+\sqrt{(c+m)t})\right] \\
& + \frac{Gr}{2b(Pr-1)}\left[e^{-H\sqrt{Pra}}\,\mathrm{erfc}(\eta\sqrt{Pr}-\sqrt{at}) + e^{H\sqrt{Pra}}\,\mathrm{erfc}(\eta\sqrt{Pr}+\sqrt{at})\right] \\
& - \frac{Gr\cdot e^{bt}}{2b(Pr-1)}\left[e^{-H\sqrt{Pr(a+b)}}\,\mathrm{erfc}(\eta\sqrt{Pr}-\sqrt{(a+b)t}) + e^{H\sqrt{Pr(a+b)}}\,\mathrm{erfc}(\eta\sqrt{Pr}+\sqrt{(a+b)t})\right] \\
& + \frac{Gc}{c^2(Sc-1)}\,\mathrm{erfc}(\eta\sqrt{Sc}) + \frac{Gc}{c(Sc-1)}\left[t(\mathrm{erfc}(\eta\sqrt{Sc}) - 2\eta\frac{\sqrt{Sc}}{\pi}(e^{-\eta^2 Sc})\right] \\
& - \frac{Gc\cdot e^{ct}}{2c^2(Sc-1)}\left[e^{-H\sqrt{cSc}}\,\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{ct}) + e^{H\sqrt{cSc}}\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{ct})\right].
\end{aligned}
\tag{4.2}
$$

where H=$2\eta\sqrt{t}$

5 ANALYSIS OF RESULTS

To understand the problem's physics, u and v values are obtained for the values of parameters like Sc (Schmidt number), Ω(rotation), Gc,Gr (Grashof numbers), R (radiation) and Gc (mass Grashof number) using equations (3.1), (3.2) ,(3.3) and skin-friction from equation (4.2).
$\tau = \tau_x + i\tau_y$ from equations (4.2).
The skin friction effects for various parameters are analyzed and the results are presented as follows.
The skin-friction profiles for τ_x(Primary) and τ_y(Secondary) are displayed here.

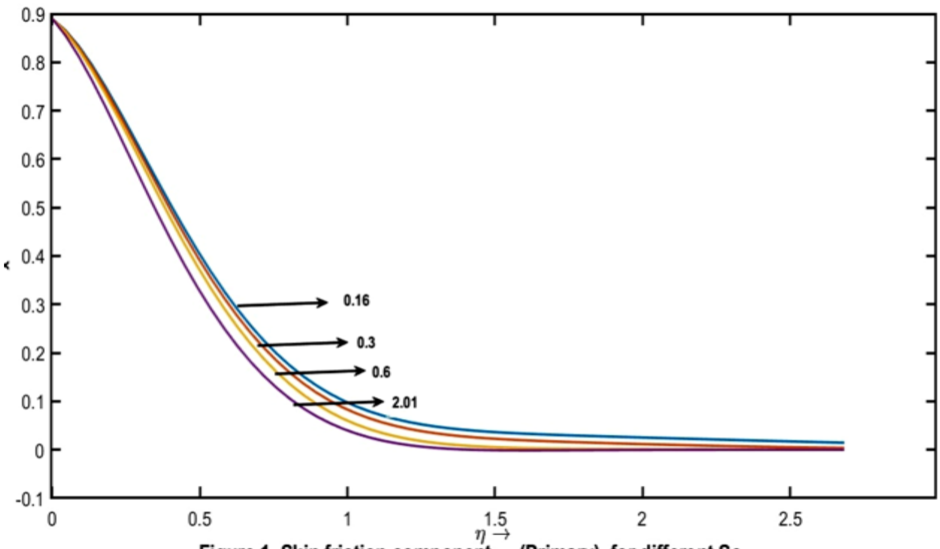

Figure 1. Skin friction component τ_x (Primary) for different Sc

The skin friction τ_x for the values of Sc (Schmidt number)= 2.01, 0.6, 0.3, 0.16 with Gr =5, Pr = 0.71,Gc = 5, rotation parameterΩ=2, R= 10, and t = 0.2 are displayed in the above Figure 1 The impacts of Sc (Schmidt number) is significant on primary component of skin friction τ_x

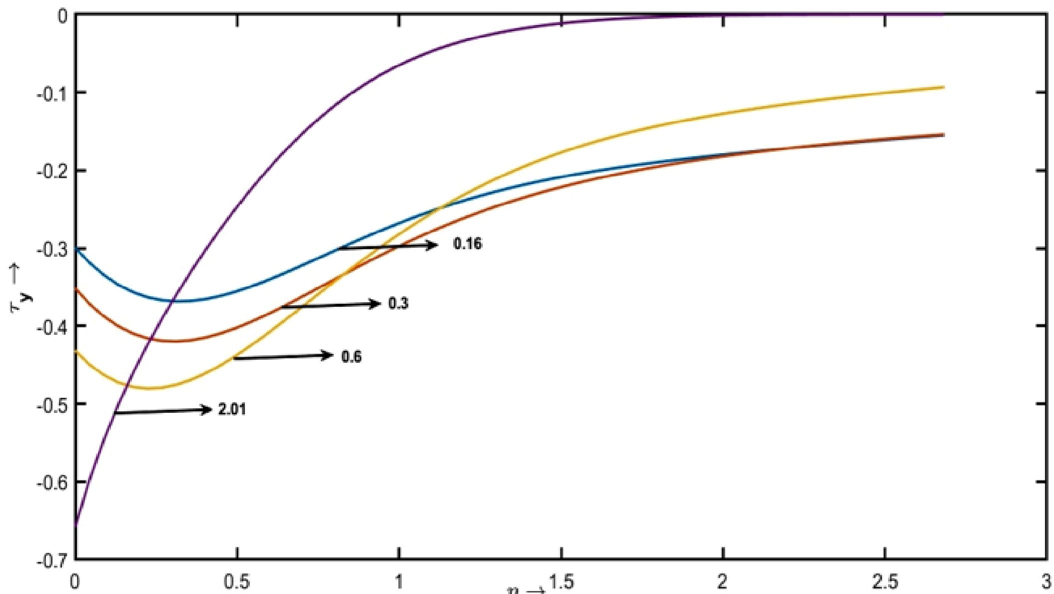

Figure 2. Skin friction component τ_y (Secondary) for different Sc

For values of Sc (Schmidt number)= 2.01, 0.6, 0.3, 0.16, the skin friction component τ_y with Gr =5, R= 10, Gc = 5, Pr = 0.71, t = 0.2 Ω = 2 are presented in Figure 2.
The trend is reversed for skin friction componentτ_y

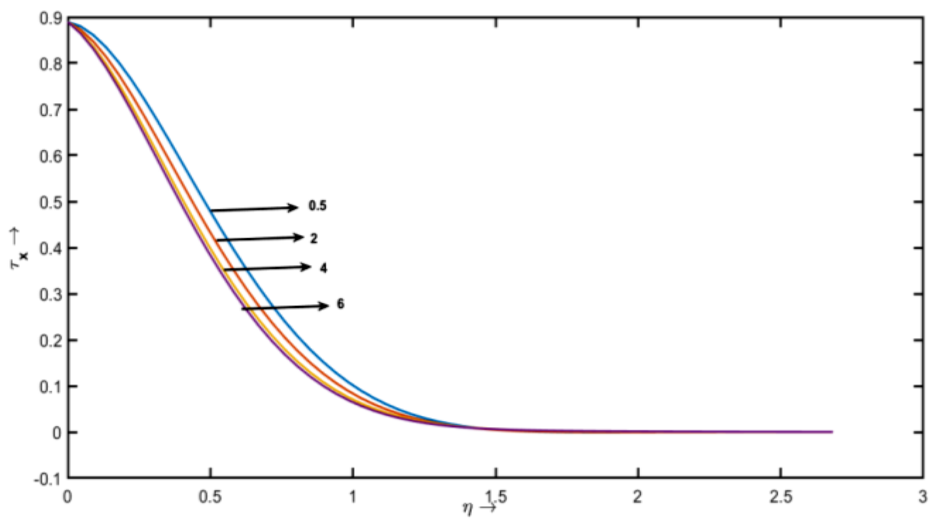

Figure 3. Skin friction component τ_x (Primary) for different R

The primary skin friction component τ_x for various values of radiation parameter (R= 0.5, 2, 4, 6) with rotation parameterΩ=2, t = 0.2 Sc = 0.6, Gr =5, Gc= 5, Pr = 0.7 are depicted in Figure 3. It is evident; there is an increase in skin friction when the thermal radiation parameter decreases.

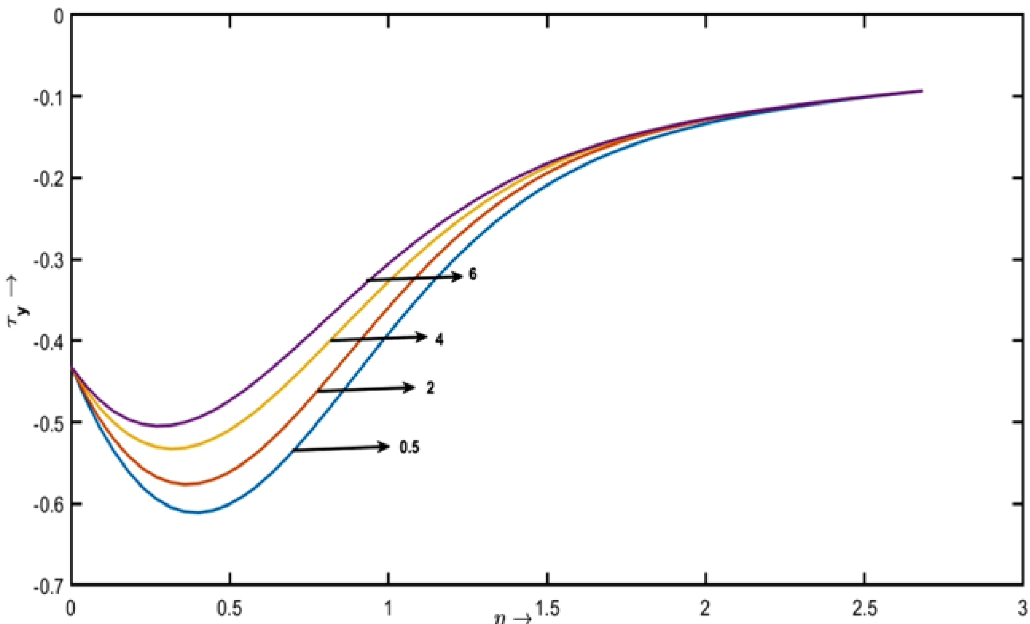

Figure 4. Skin friction component τ_y (Secondary) for different R

Secondary skin friction component τ_y for various values of radiation parameter R= 0.5, 2, 4, 6 with rotation parameter Ω = 2,t = 0.2, Gr =5,Sc = 0.6, , Gc = 5, Pr = 0.71 are shown in Figure 4. In the case of skin friction component τ_y , the skin friction increases when radiation decreases.

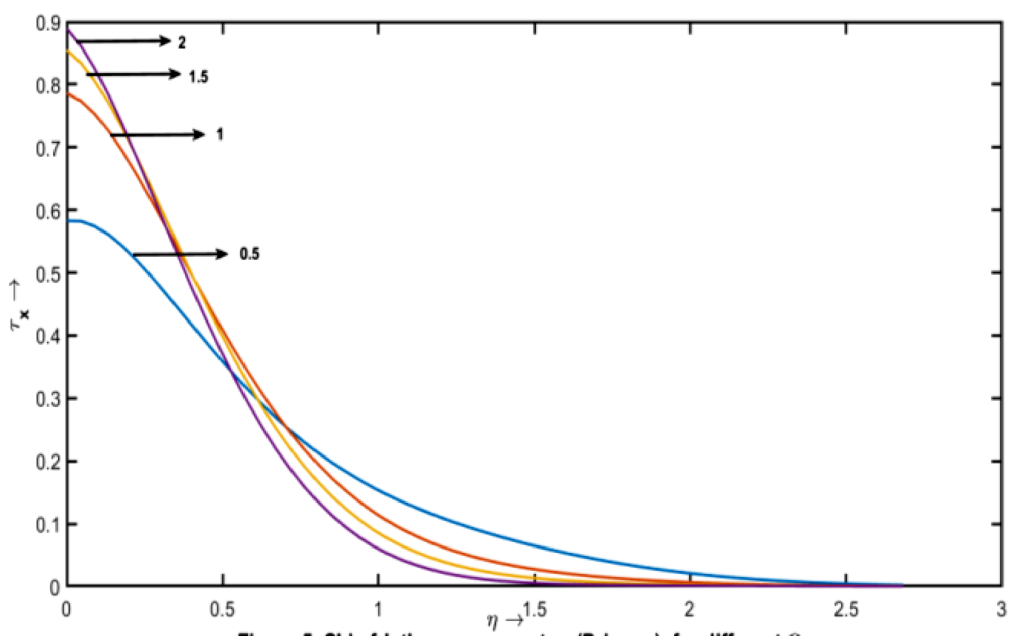

Figure 5. Skin friction component τ_x (Primary) for different Ω

For the following values of Ω(rotation parameter) = 0.5, 1, 1.5, 2 having Sc= 0.6,t= 0.2, Pr = 0.71, Gr =5, R= 2, Gc = 5 the skin friction component τ_x displayed in Figure 5.
The increase of rotation parameter causes an increase for τ_x

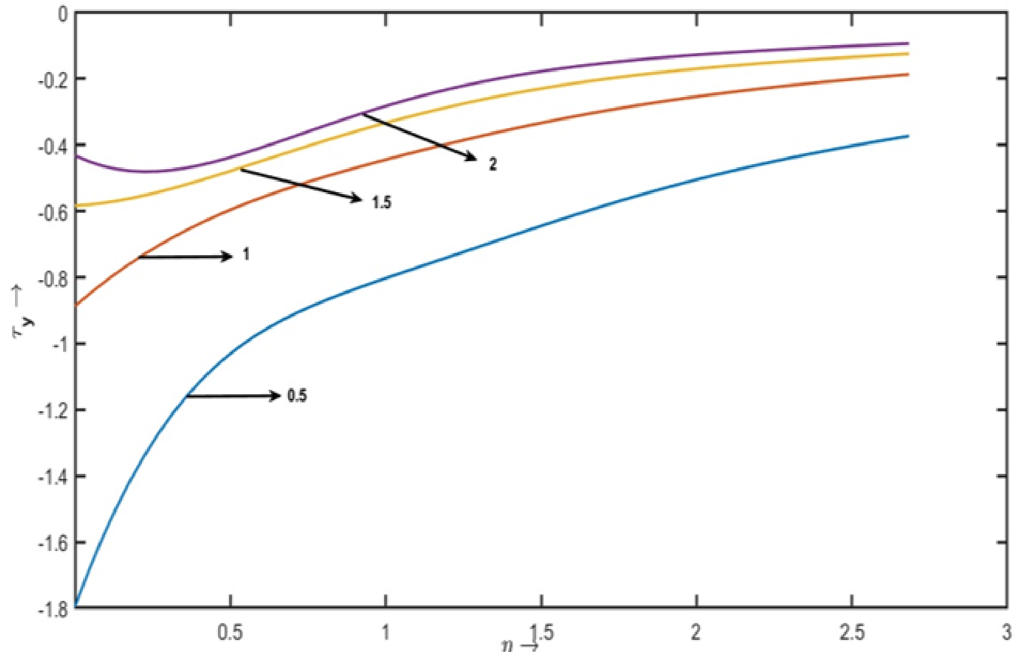

Figure 6. Skin friction component τ_y (Secondary) for different Ω

For the following values of Ω(rotation parameter) = 0.5, 1, 1.5, 2) having Sc= 0.6,t= 0.2, Pr = 0.71, Gr =5, R= 2, Gc = 5 the skin friction componentτ_x displayed in Figure 6.
The skin friction τ_y increases as the rotation parameter decreases.

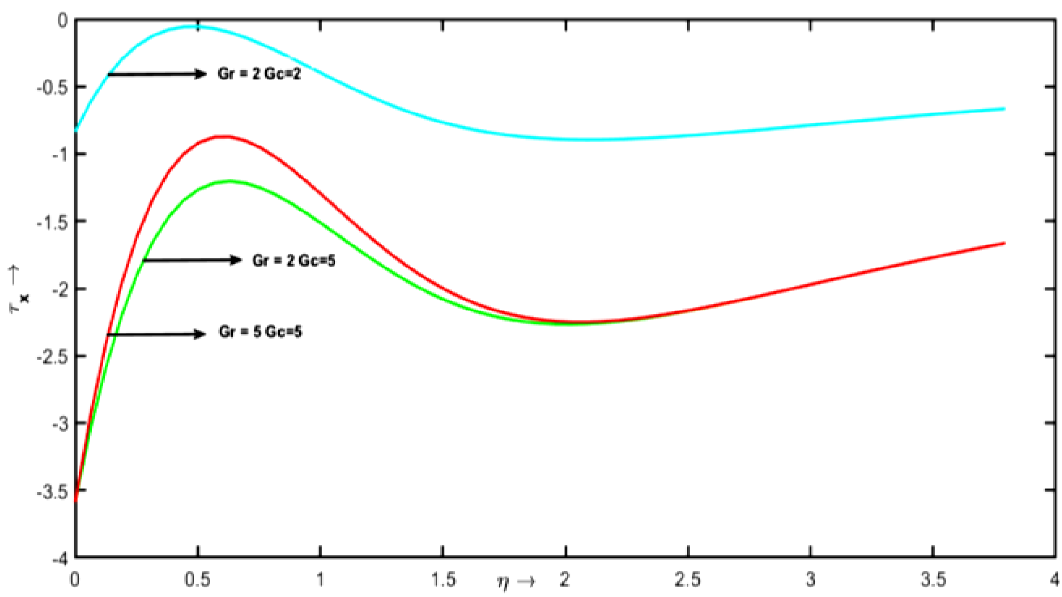

Figure 7. Skin friction component τ_x (Primary) for different Gr,Gc

Figure 7 shows the skin friction τ_x for Gr and Gc with different values
Here,$\Omega = 0.5$,Pr = 0.71, Sc = 0.16,t = 0.4,R=2 taking Gr as 2, 2, 5 and Gc as 2, 5, 5. raise in Gr, Gc results a raise in the skin friction component.

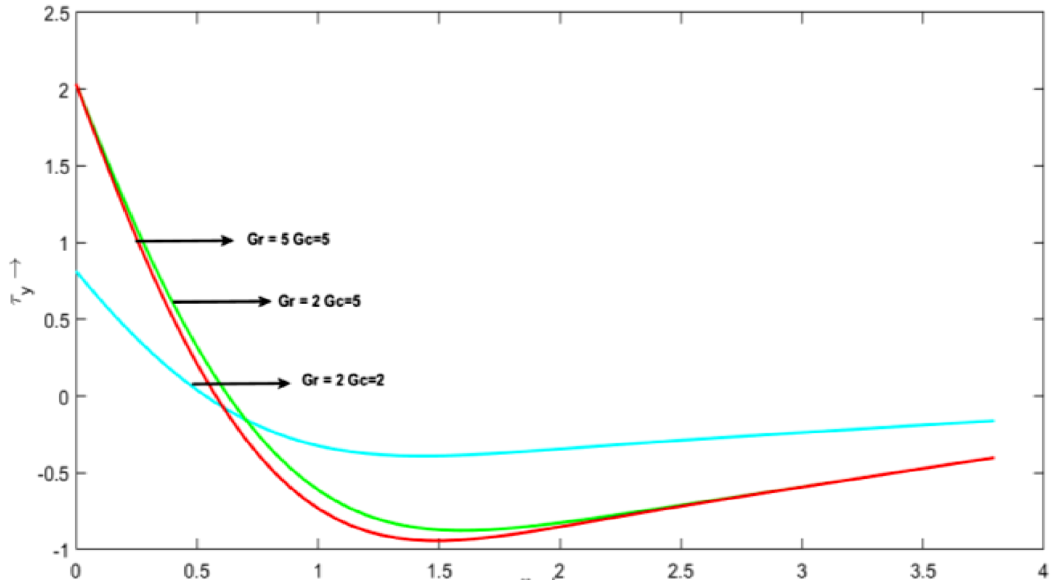

Figure 8. Skin friction component τ_y (Secondary) for different Gr,Gc

Figure 8 shows the skin friction τ_y for Gr and Gc with different values.
Here,Ω = 0.5 Pr = 0.71,Sc = 0.16,and t = 0.4,R=2 Gr as 2, 2, 5 and Gc as 2, 5,5.
The raise in Gr, Gc results a raise in the skin friction component

6 CONCLUSION

A theoretical analysis is performed. Laplace-transform method is used for solving the governing equations (dimensionless).

The observations of the above study are presented below.

As concentration increases the skin friction decreases for τ_x and the trend is reversed for τ_y
Skin friction increases when radiation decreases for both τ_x and τ_y

Skin friction decreases with increasing rotation parameter for both τ_x and τ_y.

The skin friction increases for both τ_x and τ_y due to the raise of thermal Grashof and mass Grashof number

7 NOMENCLATURE

a^*	"absorption coefficient"
C	"dimensionless concentration"
D	"mass diffusion coefficient"
Gr	"thermal Grashof number"
k	"thermal conductivity of the fluid"
q_r	"radiative heat flux in the y-direction"
Sc	"Schmidt number"
T'_w	"temperature of the plate"
t'	"time"
u'	"velocity of the fluid in the x'-direction"
u	"dimensionless velocity"
v	"dimensionless velocity"
z'	"coordinate axis normal to the plate"
C'	"concentration"
C_p	"specific heat at constant pressure"

g "acceleration due to gravity"
Gc "mass Grashof number"
Pr "Prandtl numbe"r
R "radiation parameter"
T'_∞ "temperature of the fluid far away from the plate
T' "temperature of the fluid near the plate"
t "dimensionless time"
u_0 "velocity of the plate"
v' "velocity of the fluid in the y'-direction"
v' "velocity of the fluid in the y'-direction"
y' "coordinate axis normal to x'-axis"
z "dimensionless coordinate axis normal to the plate"

Greek symbols

β' "volumetric coefficient of thermal expansion concentration"
ν "kinematic viscosity"
Ω "dimensionless rotation parameter"
τ "dimensionless skin-friction"
θ "dimensionless temperature"
β "volumetric coefficient of thermal expansion"
μ "coefficient of viscosity"
Ω' "rotation parameter"
ρ "density"
σ "Stefan-Boltzman constant"
erfc "complementary errorfunction"

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