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E-Super Vertex Magic Graceful Labeling Of Graphs

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Abstract A VMGL of a graph $Q(r, s)$ is a one to one and onto function $h{:}V(Q) \cup E(Q)$ to $\{1, \ldots, r+s\}$ includes a prerequisite that each $y \in V(Q)$, $h(u) - \sum\limits_{z \in N(y)} h(yz) = C$. This kind of labeling is called E-SVMGL if $h(E(Q)) = \{1, 2, \ldots, s\}$. In this paper presents and examines a few fundamental characteristics of E-SVMGL. We demonstrate the existence as well as non-existence of E-SVMGL for few families of graphs. Additionally, we determine the magic constant(Mc) E-SVMGL of graphs.

1 Introduction

Different types of labelings have been studied and investigated by numerous researchers [4] provides a comprehensive summary of graph labeling, see more[1, 2, 3, 5, 10, 18].

MacDougall et al. [7] was presented the SVMTL. A VMTL is referred to as super if $h(V(Q))$ $= \{1, \ldots, r\}$. A different labeling technique known as SVML was presented by Swaminathan and Jeyanthi [17]. A VMTL is considered exceptional if $h(E(Q)) = \{1, ..., s\}$. To prevent misunderstanding, a VMTL is E-super if $h(E(Q)) = \{1, ..., q\}$. If Q is a E-SVML, then it is referred to as E-SVM [8] . Lot of SVML have been specified and investigated see [6, 4, 9, 11, 12, 13, 14, 15, 16].

In this article, we introduce a new labeling is called E-SVMGL of graphs. A VMGL of a graph $Q(r, s)$ is a one to one and onto $h{:}V(Q) \cup E(Q)$ to $\{1, \ldots, r+s\}$ includes a prerequisite that each $y \in V(Q)$, $h(y) - \sum\limits_{z \in N(y)} h(yz) = C$. This kind of labeling is E-SVMGL if $h(E(Q)) = \{1, 2, \ldots, s\}$. In this paper presents and examines a few fundamental characteristics of E-SVMGL. We demonstrate the existence as well as non-existence of E-SVMGL for few families of graphs. Additionally, we determine the magic constant(Mc) E-SVMGL of graphs. Throughout this paper connected graph refer as CG.

2 E-SVMGL

We explore some of the basic features of E-SVMGL in this section.

Lemma 2.1. *Assume that Q is a CG with order $r(\geq 2)$. The graph Q does not accept E-SVMGL if, for some $t_1, t_2 \in V(Q)$ $(t_1 \neq t_2)$, $w(t_1) = w(t_2)$.*

Proof. Let Q be a CG. Assume $w(t_1) = w(t_2)$ for two different vertices t_1 and t_2 of Q. Then $h(t_1) - w(t_1) \neq h(t_2) - w(t_2)$ for any one to one and onto $h : V(Q) \cup E(Q) to \{1, \ldots, r+s\}$. Thus Q is not E-SVMG □

Lemma 2.2. *Let $Q(r, s)$ be a CG and Q is E-SVMG, then MC is $C = s + \frac{r+1}{2} - \frac{s(s+1)}{r}$.*

Proof. Assume h to be a E-SVMGL of Q with C as the MC.
Since $C = h(y) - \sum\limits_{z \in N(y)} h(yz)$ for all $y \in V$, we have

$$rC = \sum_{y \in V} h(y) - \sum_{y \in V} \sum_{z \in N(y)} h(yz)$$

$= \sum_{y\in V} h(y) - 2 \sum_{e\in E(Q)} h(e)$

(Because every edge is included twice in the total $\sum_{y\in V}\sum_{z\in N(y)} h(yz)$)

$= \left[(s+1) + \cdots + (s+r)\right] - 2\left[1 + 2 + \cdots + r\right]$

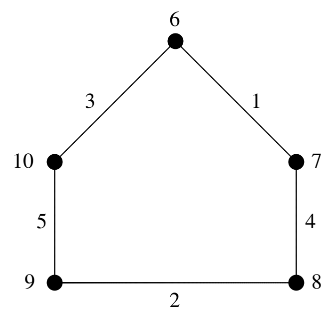
$= sr + \frac{r^2+r}{2} - 2\left[\frac{s(s+1)}{2}\right]$

$= s + \frac{r(r+1)}{2r} - \frac{s(s+1)}{2r}$ Thus $C = s + \frac{r+1}{2} - \frac{s(s+1)}{r}$. □

Theorem 2.3. *Assume that Q be a CG. If G is E-SVMG with MC, then*
1). $C \geq \frac{r+1}{2}$
2). $C = \frac{r-1}{2}$ if $s = r$.

Proof. (1). Since Q is CG, $s \geq r - 1$. From Lemma 2.2, we have $C = s + \frac{r+1}{2} - \frac{s(s+1)}{r} \geq \frac{r+1}{2}$.
(2). Since $s = r$. From Lemma 2.2, we have $C = s + \frac{r+1}{2} - \frac{s(s+1)}{r} = \frac{r-1}{2}$. □

Remark 2.4. Examine the following E-SVMGL for the cycle C_5. Here, $C = 2$. From Theorem 2.3, $C \geq \frac{r-1}{2} = 2$.



E-SVMGL of C_5

Corollary 2.5. *If $Q(r, s)$ is a CG. If r is even and $s = r - 1$, or r, then Q is not E-SVMG.*

Proof. Since G is CG, $s \geq r - 1$. From Theorem 2.3, we get $C \geq \frac{r+1}{2}$, which is not integer (since r is even).

Suppose $s = r$. From Theorem 2.3, we get, $C = \frac{r-1}{2}$, which is not integer (since r is even). □

3 E-SVMGL of some families of graphs

In this part, we examine a few families of graphs that permit E-SVMG, including Paths, Cycles, and Union of Cycles.

Theorem 3.1. *Let $r(\geq 3)$ be an integer. The path P_r admits E-SVMGL iff r is odd.*

Proof. Suppose $\exists$ a E-SVMGL h of P_r with MC. From Lemma 2.2, $C = \frac{r+1}{2}$, r has to be odd.

Presume that r is odd. Let $V(P_r) = \{y_i : 1, ..., r\}$ and $E(P_r) = \{y_i y_{i+1} : 1, ...r - 1\}$. Define $h : V(P_r) \cup E(P_r) \to \{1, \ldots, 2r - 1\}$ as

$$h(y_i) = \begin{cases} r & for\ i = 1 \\ 2r - (i - 1) & for\ i = 2, ..., r \end{cases}$$

$$h(y_i y_{i+1}) = \begin{cases} \frac{r-i}{2} & if\ i = 1, 3, ...r - 2 \\ \frac{2r-i}{2} & if\ i = 2, 4, ...r - 1. \end{cases}$$

Let $v \in V$.
Case 1: Suppose $v = y_i$ for $i = 2, 4, ..., r-1$.
Now $C = h(y_i) - h(y_{i-1}y_i) + h(y_iy_{i+1}) = [2r-(i+1)] - [\frac{r-(i-1)}{2}] - [\frac{2r-i}{2}] = \frac{r+1}{2}$.
Case 2: Suppose $v = y_i,\ i$ is odd $(3 \le i \le r-2)$.
Now, $C = h(y_i) - h(y_{i-1}a_i) - h(y_ia_{i+1}) = [2r-(i-1)] - [\frac{2r-(i-1)}{2}] - [\frac{r-i}{2}] = \frac{r+1}{2}$.
Case 3: Suppose $v = y_r$.
Here $C = h(y_r) - f(y_{r-1}y_r) = [2r-(r-1)] - [\frac{2r-(r-1)}{2}] = \frac{r+1}{2}$.
Case 4: Suppose $v = y_1$,
Here $C = h(y_1) - h(y_1y_2) = [2r-(r-1)] - [\frac{(r-1)}{2}] = \frac{r+1}{2}$. Thus h is a E-SVMGL. □

Theorem 3.2. *Let $r(\ge 3)$. The graph C_r is E-SVMGL iff r is odd.*

Proof. Suppose ∃ a E-SVMGL h of C_r with MC C. From Lemma 2.2, $C = \frac{r-1}{2}$, r has to be odd.

Assume that r is odd. Let $V(C_r) = \{y_i : 1 \le i \le r\}$ and $E(C_r) = \{y_ry_1\} \cup \{y_iy_{i+1} : 1 \le i \le r-1\}$. Define $h : V(C_r) \cup E(C_r) \to \{1, 2, \dots, 2r\}$ as
$h(x_i) = r+i$ for $1 \le i \le r$, $h(x_ix_{i+1}) = \frac{i+1}{2}$ when i is odd; $h(x_ix_{i+1}) = \frac{r+i+1}{2}$ when i is even and $h(x_nx_1) = \frac{r+1}{2}$. Let $v \in V$.
Case 1: Suppose $v = y_i,\ i$ is even $(2 \le i \le r-1)$.
Now $C = h(y_i) - h(y_{i-1}y_i) + h(y_iy_{i+1}) = [r+i] - \frac{i}{2} - \frac{r+i+1}{2} = \frac{r-1}{2}$.
Case 2: Suppose $v = y_i,\ i$ is odd $(3 \le i \le r-2)$
Now, $C = h(a_i) - h(a_{i-1}a_i) - h(a_ia_{i+1}) = [r+i] - [\frac{(r+i)+1}{2}] = [\frac{i+1}{2}] = \frac{r-1}{2}$.
Case 3: Suppose $v = y_n$,
Here $C = h(y_n) - h(y_{n-1}y_n) - f(y_ny_1) = 2r - [\frac{r-1}{2} + \frac{r_1+r-1}{2}] = \frac{r-1}{2}$.
Case 4: Suppose $v = x_1$,
Here $C = h(y_1) - h(y_ny_1) - h(y_1y_2) = [r+1] = [1] - [\frac{r+1}{2}] = \frac{r-1}{2}$.
Thus h is a E-SVMGL. □

Theorem 3.3. *Let $t \ge 1$. Then tC_r is E-SVMG iff t and r are odd.*

Proof. Suppose ∃ a E-SVMGL h of tC_r. Since $r = s = tr$ in Lemma 2.2, $C = tr + \frac{(tr)(tr+1)}{2} - \frac{tr(tr+1)}{tr} = \frac{tr-1}{2}$. When t or r are even, C cannot be an integer. Hence, t and r are both odd numbers.
Let t and r be odd integers. Let $V(tC_r) = V_1 \cup \ldots \cup V_t$, where $V_i = \{y_i^1, y_i^2, \dots, y_i^n\}$ for $i = 1, 2, \dots, t$. Let $E(tC_r) = E_1 \cup E_2 \cup \ldots \cup E_t$, where $E_i = \{e_i^1, e_i^2, \dots, e_i^r\}$ with $e_i^j = (y_i^j, y_i^{j \oplus_r 1})$ for $1 \le i]leqt$ and $1 \le j \le r$.
Define a function $h : V(tC_r) \cup E(tC_r) \to \{1, \dots, 2tr\}$ as
For $1 \le i \le \frac{t-1}{2}$,

$$h(y_i^j) = \begin{cases} tr + \frac{t-1}{2} - (i-1) & for\ j = 1 \\ tr + (j-1)t + 2i & for\ j = 2, 3, 4, \dots, r-1 \\ 2tr - (i-1) & for\ j = n \end{cases}$$

$$h(e_i^j) = \begin{cases} \frac{(j-1)t+2i}{2} & for\ j = 1, 3, \dots, r-2 \\ \frac{(r+j)t+1+2i}{2} & for\ j = 2, 4, \dots, r-1 \\ \frac{(r+1)t}{2} + 1 - 2i & for\ j = r. \end{cases}$$

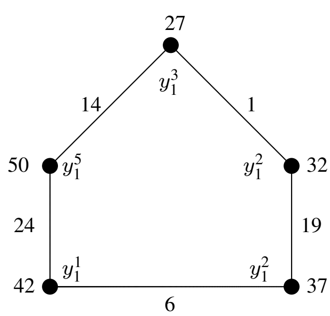
For $\frac{t+1}{2} \le i \le t$,

$$h(y_i^j) = \begin{cases} tr + t + \frac{t+1}{2} - i & for\ j = 1 \\ tr + (j-2)t + 2i & for\ j = 2, 3, 4, \dots, r-1 \\ 2tr - (i-1) & for\ j = r \end{cases}$$

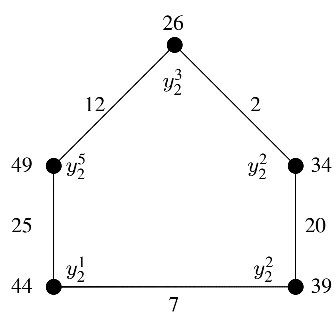
$$h(e_i^j) = \begin{cases} \frac{(j-1)t}{2} + i & for\ j = 1, 3, \dots, r-2 \\ \frac{(r+j-2)t}{2} + \frac{1}{2} + i & for\ j = 2, 4, \dots, r-1 \\ \frac{(r+3)t}{2} + 1 - 2i & for\ j = r. \end{cases}$$

To prove $C = \frac{tr-1}{2}$ for every vertex $y \in V(tC_r)$. Let $y \in V(tC_r)$.
Then $C = h(y_i^j) - h((y_i^{j-1}, y_i^j)) - h((y_i^j, y_i^{j+})) = h(y_i^j) - f(e_i^{j-1}) - f(e_i^{j+1})$.
Case A: Suppose $1 \leq i \leq \frac{r-1}{2}$.
Subcase 1: Suppose $j = 1$. Then
$C = h(y_i^1) - w(y_i^1) = h(y_i^1) - h(e_i^n) - h(e_i^1)$
$= [tr + \frac{t-1}{2} - (i-1)] - [\frac{(r+1)t}{2} + 1 - 2i] - [i]$
$= [\frac{2tr+t-1}{2}] - [\frac{tr+t}{2}] = \frac{tr-1}{2}$.
Subcase 2: Suppose $j = 2$. Then
$C = h(y_i^2) - w(y_i^2) = h(y_i^2) - h(e_i^1) - h(e_i^2)$
$= [tr + t + 2i] - [i] - [\frac{(r+2)t+1}{2} + i] = \frac{tr-1}{2}$.
Subcase 3: Suppose j is odd for $3 \leq j \leq r-2$. Then
$C = h(y_i^j) - w(y_i^j) = h(y_i^j) - h(e_i^{j-1}) - h(e_i^j)$
$= [tr + jt - t + 2i] - [\frac{(r+j-1)t+1}{2} + i] - [\frac{(j-1)t}{2} + i]) = \frac{tr-1}{2}$.
Subcase 4: Suppose j is even for $4 \leq j \leq r-1$. Then
$C = h(y_i - w(y_i^j)^j) = h(y_i^j) - h(e_i^{j-1}) - h(e_i^j)$
$= [tr + jt - t + 2i] - (\frac{(j-2)t}{2} + i) + (\frac{(r+j)t+1}{2} + i) = \frac{tr-1}{2}$.
Subcase 5: Suppose $j = r$. Then
$C = h(y_i^r) - w(y_i^n) = h(y_i^n) - h(e_i^{r-1}) - h(e_i^r)$
$= [2tr - (i-1)] - [\frac{(2r-1)t+1}{2} + i] - [\frac{(r+1)t}{2} + 1 - 2i] = \frac{tr-1}{2}$.
Similarly, we can prove that the magic constant for the remaining cases $\frac{t+1}{2} \leq i \leq t$.

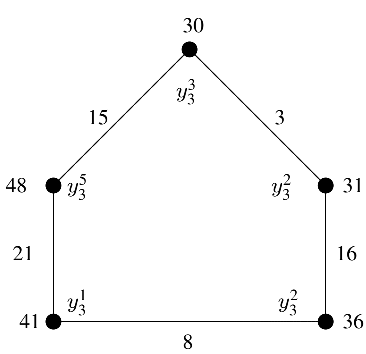
□



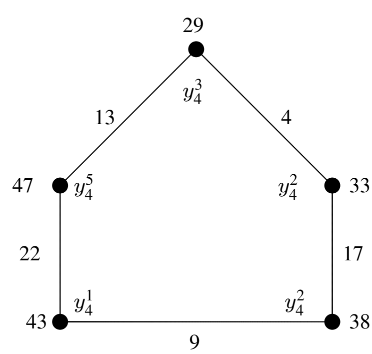
$i = 1$



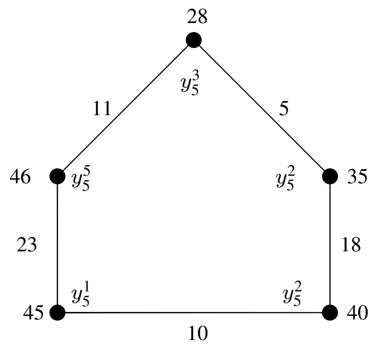
$i = 2$



$i = 3$



$i = 4$



$i = 5$

E-SVMGL of $5C_5$

4 Conclusion

In this paper, we study few elementary characteristics of E-SVMGL. Using these possessions, we explain the existence and nonexistence of E-SVMGL for few kins of graphs. It is natural to have the following problems. Find the E-SVMGL of distinguished graphs such as generalized prism graph, circulant digraphs and etc.

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