AN INQUISITION OF CORONA COVERING NUMBER FOR NOTABLE TYPES OF GRAPHS

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Abstract A vertex cover set $S \subseteq V(G)$ is a corona cover set if every vertex $v \in S$ such that d(v) = 1 or there exist a vertex $u \in S$ with d(u) = 1 and $uv \in E$. The least cardinality of a corona cover set is the corona covering number of a graph and it is expressed as τ_C . Through this analysis, we enhance the evaluation of τ_C for few notable types of graphs.

1 Introduction

The graphs under consideration are simple, finite, and undirected. Motivated by the corona domination and the vertex covering a concept of corona covering number was introduced [2]. A vertex cover set $S \subseteq V(G)$ is a corona cover set if every vertex $v \in S$ such that d(v) = 1 or there exist a vertex $u \in S$ with d(u) = 1 and $uv \in E$. The least cardinality of a corona cover set is the corona covering number of a graph and it is expressed as τ_C . Uniform t-ply graph $P_{t,s}(x,y)$ is obtained by adjoining each pendant vertex of each t copy of P_s to 2 copies of K_1 [6]. Evened olive tree eOT_n is accomplished by joining each i^{th} terminal vertex of a $K_{1,n}$ to any of the pendant vertices of a path P_{2i} by an edge[5]. Slanting ladder SL_n is accomplished by considering two copies of path P_n namely G_1 and G_2 and adjoining i^{th} vertex of G_1 to $(i+1)^{th}$ *vertex of* G_2 *where* $1 \le i \le n-1$ [4]. *In the context of a graph* G*,* G_1 *and* G_2 *are replicas of* Gand an empty graph of order |V(G)| respectively, then mycielskian graph $\mu(G)$ of G is obtained from $G_1 \cup G_2 \cup K_1$ with $V(G_1) = \{v_i : i \in \mathbb{N}\}, V(G_2) = \{u_i : i \in \mathbb{N}\}, V(K_1) = \{w\}, by$ joining v_i and u_k if $v_iv_k \in E(G)$ and join u_k and w, $1 \le i, k \le n.[3],[10],[11]$. The Coconut *Tree graph* $CT_{n,m}$ *is obtained from* $P_n \cup K_{1,m}$ *by pasting an end vertex of* P_n *on a centre vertex* of $K_{1,m}$ [7]. Tadpole graph $T_{n,m}$ is accomplished by adjoining the end vertex of path P_m with a vertex of a cycle C_n . The corona covering number for few special types of graphs will be discussed in the section 2. Throughout this paper The collection of white vertices denote the corona cover set of the given graph.

2 Corona Covering number of graphs

Theorem 2.1. For a Uniform t-ply graph, $q \in \mathbb{W}$, then $\tau_C(P_{t,p}(a,b)) = \begin{cases} t(2\lceil \frac{p}{3} \rceil) + 2 & \text{if } p = 3q, \\ t(2(\lfloor \frac{p}{3} \rfloor)) + 3 & \text{if } p = 3q + 1, \\ t(2\lceil \frac{p}{3} \rceil) - 1 & \text{if } p = 3q + 2. \end{cases}$

Proof. Let $V(P_{t,p}(a, b)) = \{a, b, v_g^h : 1 \le g \le p, 1 \le h \le t\}$ and $E(P_{t,p}(a, b)) = \{v_g^h v_{g+1}^h : 1 \le g \le p - 1, 1 \le h \le t\} \cup \{v_1^h a, v_t^h b : 1 \le h \le t\}$ Presume $S = \{a, v_g^h, b : g \equiv 0 \text{ or } 2 \pmod{3}, h \ge 2, 1 \le g \le p\} \cup \{v_g^1 : g \equiv 0 \text{ or } 1 \pmod{3}, 1 \le g \le p\}$ Then S is a Corona cover set of $P_{t,p}(a, b)$ and thus

$$\tau_C(P_{t,p}(a,b)) \le |S| = \begin{cases} t(2\lceil \frac{p}{3} \rceil) + 2 & if \ p \ = 3q, \\ t(2\lfloor \frac{p}{3} \rfloor) + 3 & if \ p \ = 3q + 1, \\ t(2\lceil \frac{p}{3} \rceil) - 1 & if \ p \ = 3q + 2. \end{cases}$$

Let N' be a corona cover set of $P_{t,p}(a, b)$. Suppose D is a corona cover set of order at most $\begin{cases} t(2\lceil \frac{p}{2} \rceil) + 1 & if \ p = 3q, \end{cases}$

$$N = \begin{cases} t(2\lfloor \frac{p}{3} \rfloor) + 2 & if \ p = 3q + 1, \\ t(2\lceil \frac{p}{3} \rceil) - 2 & if \ p = 3q + 2. \end{cases}$$

then either $\langle D \rangle$ has an vertex with degree zero or D is not a vertex cover set, Thus we have $\int t(2\lceil \frac{p}{3}\rceil) + 2$ if p = 3q,

$$|N'| \ge N + 1 = \begin{cases} (1 \le j) + 3 & \text{if } p = 3q + 1 \\ t(2\lfloor \frac{p}{3} \rfloor) + 3 & \text{if } p = 3q + 2 \end{cases}$$

Therefore, the result is obtained.

Example 2.2.

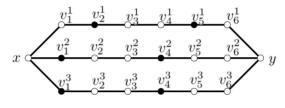


Figure 1. $P_{3.6}(a, b)$

In the graph in figure 1, The collection of white vertices is corona cover set of minimum order and hence $\tau_C(P_{3,6}(a,b)) = 14$.

 $\begin{aligned} \text{Theorem 2.3. For an evened Olive tree,} \\ \tau_{C}(eOT_{n}) &= \sum_{i=1}^{\left\lceil \frac{n-1}{3} \right\rceil} (2\lfloor \frac{6i-1}{3} \rfloor) \sum_{j=1}^{\left\lceil \frac{n+1}{3} \right\rceil} (2\lfloor \frac{6j-5}{3} \rfloor) + \sum_{k=1}^{\left\lceil \frac{n}{3} \right\rceil} (2\lfloor \frac{6k-3}{3} \rfloor) + \lceil \frac{n-1}{3} \rceil + 1 \\ \end{aligned} \\ \text{Proof. Let } V(eOT_{n}) &= \{v, v_{g}^{h} : 1 \leq h \leq n, 1 \leq g \leq 2h + 1\} \text{ and} \\ E((eOT_{n}) &= \{vv_{1}^{h} : 1 \leq j \leq n\} \cup \{v_{g}^{h}v_{g+1}^{h} : 1 \leq g \leq 2h, 1 \leq h \leq n\} \\ \end{aligned} \\ \text{Presume } S &= \begin{cases} v_{g}^{h} & \text{if } h \equiv 0 \pmod{3} \text{ and } g \equiv 0 \text{ or } 2 \pmod{3}, \\ v_{g}^{h} & \text{if } h \equiv 1 \pmod{3} \text{ and } g \equiv 0 \text{ or } 1 \pmod{3}, \\ v_{g}^{h} & \text{if } h \equiv 2 \pmod{3} \text{ and } g \equiv 0 \text{ or } 1 \pmod{3}. \\ \end{aligned} \\ \text{Then } S \text{ is a Corona cover set of } (eOT_{n}) \text{ and hence} \\ \tau_{C}(eOT_{n}) \leq |S| &= \sum_{i=1}^{\left\lceil \frac{n-1}{3} \right\rceil} (2\lfloor \frac{6i-3}{3} \rfloor) \sum_{j=1}^{\left\lceil \frac{n+1}{3} \right\rceil} (2\lfloor \frac{6j-5}{3} \rfloor) + \sum_{k=1}^{\left\lceil \frac{n}{3} \right\rceil} (2\lfloor \frac{6k-3}{3} \rfloor) + \lceil \frac{n-1}{3} \rceil + 1 \\ \texttt{Let } N' \text{ be a corona cover set of } eOT_{n}. \text{ Suppose } D \text{ is a corona cover set of order at most} \\ N &= \sum_{i=1}^{\left\lceil \frac{n-1}{3} \right\rceil} (2\lfloor \frac{6i-3}{3} \rfloor) \sum_{j=1}^{\left\lceil \frac{n+1}{3} \right\rceil} (2\lfloor \frac{6j-5}{3} \rfloor) + \sum_{k=1}^{\left\lceil \frac{n}{3} \right\rceil} (2\lfloor \frac{6k-3}{3} \rfloor) + \lceil \frac{n-1}{3} \rceil \\ \text{then either } < D > \text{has an vertex with degree zero or } D \text{ is not a vertex cover set, Thus we have} \\ |N'| &\geq N + 1 = \sum_{i=1}^{\left\lceil \frac{n-1}{3} \right\rceil} (2\lfloor \frac{6i-1}{3} \rfloor) \sum_{j=1}^{\left\lceil \frac{n+1}{3} \right\rceil} (2\lfloor \frac{6j-5}{3} \rfloor) + \sum_{k=1}^{\left\lceil \frac{n}{3} \right\rceil} (2\lfloor \frac{6k-3}{3} \rfloor) + \lceil \frac{n-1}{3} \rceil + 1 \\ \text{Therefore, the result is obtained.} \\ \square$

$$\textbf{Theorem 2.4. For a Slanting Ladder } SL_k, q \in \mathbb{W} \text{ then } \tau_C(SL_k) = \begin{cases} 8\lfloor \frac{k}{6} \rfloor & \text{for } k = 6q + 1, \\ 8\lceil \frac{k}{6} \rceil - 6 & \text{for } k = 6q + 2, \\ 8\lceil \frac{k}{6} \rceil - 5 & \text{for } k = 6q + 3, \\ 8\lceil \frac{k}{6} \rceil - 3 & \text{for } k = 6q + 4, \\ 8\lceil \frac{k}{6} \rceil - 2 & \text{for } k = 6q + 5. \end{cases}$$

 $\begin{array}{l} \textit{Proof. Let } V(SL_k) = \{v_g, u_g: 1 \leq g \leq k\} \textit{ and } E(SL_k) = \{v_g u_{g-1}: 2 \leq g \leq k\} \cup \{v_g v_{g+1}: 1 \leq g \leq k-1\} \cup \{u_g u_{g+1}: 1 \leq g \leq k-1\}. \textit{ Let } S_1 = \{v_g: g \equiv 1 \textit{ or } 2 \textit{ or } 3 \textit{ or } 5 \pmod{6}, 2 \leq g \leq k\}, S_2 = \{u_g: g \equiv 1 \textit{ or } 3 \textit{ or } 4 \textit{ or } 5 \pmod{6}, 1 \leq g \leq k\}, S_3 = \{v_g: g \equiv 0 \textit{ or } 2 \textit{ or } 3 \textit{ or } 4 \pmod{6}, 1 \leq g \leq k\}, S_4 = \{u_g: g \equiv 0 \textit{ or } 2 \textit{ or } 4 \textit{ or } 5 \pmod{6}, 1 \leq g \leq k\} \end{array}$

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$$\operatorname{Presume} S = \begin{cases} S_1 \cup S_2 & \text{for } k \equiv 0 \text{ or } 2 \text{ or } 4 \pmod{6} \\ S_3 \cup S_4 & \text{for } k \equiv 1 \pmod{6}, \\ (S_1 - \{v_k\}) \cup (S_2 \cup \{u_{k-1}\} - \{u_k\}) & \text{for } k \equiv 3 \pmod{6}, \\ S_1 \cup (S_2 - \{u_k\}) & \text{for } k \equiv 5 \pmod{6}, \\ S_1 \cup (S_2 - \{u_k\}) & \text{for } k \equiv 5 \pmod{6}, \end{cases}$$

$$\operatorname{Then} S \text{ is a corona cover set of } (SL_k) \text{ and hence} \\ \begin{cases} 8\binom{k}{6} - 1 & \text{for } k = 6q, \\ 8\lfloor \frac{k}{6} \rfloor & \text{for } k = 6q + 1, \\ 8\lceil \frac{k}{6} \rceil - 6 & \text{for } k = 6q + 2, \\ 8\lceil \frac{k}{6} \rceil - 5 & \text{for } k = 6q + 3, \\ 8\lfloor \frac{k}{6} \rceil - 2 & \text{for } k = 6q + 4, \\ 8\lceil \frac{k}{6} \rceil - 2 & \text{for } k = 6q + 4, \\ 8\lceil \frac{k}{6} \rceil - 2 & \text{for } k = 6q + 5. \end{cases}$$
Let N' be a corona cover set of SL_k . Suppose D is a corona cover set of order at most $\begin{cases} 8(\frac{k}{6}) - 2 & \text{for } k = 6q + 1, \\ 8\lceil \frac{k}{6} \rceil - 2 & \text{for } k = 6q + 4, \\ 8\lceil \frac{k}{6} \rceil - 1 & \text{for } k = 6q + 1, \\ 8\lceil \frac{k}{6} \rceil - 7 & \text{for } k = 6q + 1, \\ 8\lceil \frac{k}{6} \rceil - 7 & \text{for } k = 6q + 2, \\ 8\lceil \frac{k}{6} \rceil - 6 & \text{for } k = 6q + 2, \\ 8\lceil \frac{k}{6} \rceil - 6 & \text{for } k = 6q + 2, \\ 8\lceil \frac{k}{6} \rceil - 6 & \text{for } k = 6q + 3, \\ 8\lceil \frac{k}{6} \rceil - 3 & \text{for } k = 6q + 4, \\ 8\lceil \frac{k}{6} \rceil - 3 & \text{for } k = 6q + 5. \end{cases}$
then either $< D >$ has an vertex with degree zero or D is not a vertex cover set, Thus we $\begin{cases} 8(\frac{k}{6}) - 1 & \text{for } k = 6q, \\ 8\lfloor \frac{k}{6} \rceil - 1 & \text{for } k = 6q, \end{cases}$

$$|N'| \ge N+1 = \begin{cases} 8\lfloor \frac{k}{6} \rfloor & \text{for } k = 6q+1, \\ 8\lfloor \frac{k}{6} \rfloor - 6 & \text{for } k = 6q+2, \\ 8\lfloor \frac{k}{6} \rceil - 5 & \text{for } k = 6q+3, \\ 8\lfloor \frac{k}{6} \rceil - 3 & \text{for } k = 6q+4, \\ 8\lfloor \frac{k}{6} \rceil - 2 & \text{for } k = 6q+5. \end{cases}$$

Therefore, the result is obtained.

Example 2.5.

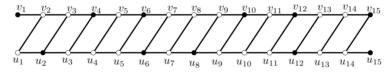


Figure 2. SL₁₅

In the graph in figure 1, The collection of white vertices is corona cover set of minimum order and hence $\tau_C(SL_{15}) = 19$.

Theorem 2.6. For a cycle, C_{4k} , then $\tau_c(\mu(C_{4k})) = 5k + 1$

Proof. Let $V(\mu(C_{4k})) = \{v_g, u_g, w\}$ and $E(\mu(C_{4k})) = \{v_g v_{g+1}, v_1 v_{4k} : 1 \le g \le 4k - 1\} \cup \{v_g u_{g+1} : 1 \le g \le 4k - 1\} \cup \{v_g u_{g-1} : 2 \le g \le n\} \cup \{v_1 u_4 k, v_{4k} u_1\} \cup \{u_g w\}$ Presume $S = \{v_g : g \equiv 0 \pmod{2}, 1 \le g \le n\} \cup \{u_g : g \equiv 0 \text{ or } 2 \text{ or } 3(\mod 9), 1 \le g \le n\} \cup \{w\}$ Then S is a Corona cover set of (C_{4k}) and hence

 $\tau_C(\mu(C_{4k})) \leq |S| = 5k + 1$ Let N' be a corona cover set of $\mu(C_{4k})$. Suppose D is a corona cover set of order at most N = 5k then either $\langle D \rangle$ has an vertex with degree zero or D is not a vertex cover set, Thus we have $|N'| \ge N + 1 = 5k + 1$ Therefore, the result is obtained.

Theorem 2.7. For a Coconut tree $CT_{n,m}$, $n \ge 4$ and $q \in \mathbb{W}$ $\tau_C(\mu(CT_{n,m})) = \begin{cases} 5\lceil \frac{n}{4} \rceil + 1 & \text{for } n = 4q \text{ or } 4q + 3, \\ 5\lceil \frac{n}{4} \rceil - 1 & \text{otherwise.} \end{cases}$

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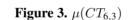
have

 $\begin{array}{l} Proof. \ \text{Let} \ V(\mu(CT_{n,m})) = \{v_g, v_h', u_g, u_h', w: 1 \le g \le n, 1 \le h \le m\} \ \text{and} \\ E(\mu(CT_{n,m})) = \{v_g v_{g+1}, w_n v_h': 1 \le g \le n - 1, 1 \le h \le m\} \cup \{u_g v, u_h' w: 1 \le g \le n, 1 \le h \le m\} \cup \{v_g u_{g+1}: 1 \le g \le n - 1\} \cup \{v_{g-1} u_g: 2 \le g \le n\} \cup \{v_h' u_n: 1 \le h \le m\}. \ \text{Let} \\ S_1 = \{v_g: g \equiv 0 \ (mod \ 2), 1 \le g \le n\}, \ S_2 = \{u_g, w: g \equiv 0 \ or \ 2 \ or \ 3 \ (mod \ 4), 1 \le h \le n)\}, \\ S_3 = \{v_g: g \equiv 1 \ (mod \ 2), 1 \le g \le n\}, \ S_4 = \{u_g, w: g \equiv 1 \ or \ 2 \ or \ 3 \ (mod \ 4), 1 \le g \le n\} \\ Presume \ S = \begin{cases} S_1 \cup S_2 & for \ n \equiv 0 \ (mod \ 4), \\ S_3 \cup S_4 \cup \{u_m'\} & for \ n \equiv 1 \ (mod \ 4), \\ S_3 \cup S_4 & for \ n \equiv 3 \ (mod \ 4). \end{cases} \end{array}$ *Proof.* Let $V(\mu(CT_{n,m})) = \{v_g, v'_h, u_g, u'_h, w : 1 \le g \le n, 1 \le h \le m\}$ and $\begin{cases} S_3 \cup S_4 & \text{for } n \equiv 3 \pmod{4}. \end{cases}$ Then S is a corona cover set of $\mu(CT_{n,m})$ and hence $\tau_C(\mu(CT_{n,m})) \leq |S| = \begin{cases} 5\lceil \frac{n}{4} \rceil + 1 & \text{for } n = 4qor4q + 3, \\ 5\lceil \frac{n}{4} \rceil - 1 & \text{otherwise.} \end{cases}$ Let N' be a corona cover set of $\mu(CT_{n,m})$. Suppose D is a corona cover set of order at most $N = \begin{cases} 5\lceil \frac{n}{4} \rceil & \text{for } n = 4q \text{ or } 4q + 3, \\ 5\lceil \frac{n}{4} \rceil - 2 & \text{otherwise.} \end{cases}$ then either $\langle D \rangle$ has an vertex with degree zero or D is not a vertex cover set, Thus we have $|N'| \geq N + 1 = \begin{cases} 5\lceil \frac{n}{4} \rceil + 1 & \text{for } n = 4q \text{ or } 4q + 3, \\ 5\lceil \frac{n}{4} \rceil - 1 & \text{otherwise.} \end{cases}$ Therefore, the result is obtained. \Box

Corollary 2.8. *For a path*
$$P_n, \tau_c(\mu(CT_{n,1})) = \tau_c(\mu(P_{n+1})).$$

Proof. Since $CT_{n,1} \cong P_{n+1}$ the result follows.

Example 2.9.

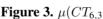


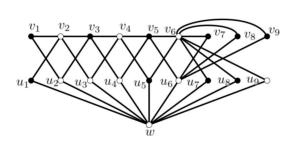
In the graph in figure 1, The collection of white vertices is corona cover set of minimum order and hence $\tau_{C}(\mu(CT_{6,3})) = 9$.

Theorem 2.10. For a Tadpole graph $T_{4k,m}$, k and q are positive integers, then

 $\tau_C(\mu(T_{4k,m})) = \begin{cases} 5(\frac{m}{4}) + 5k + 1 & \text{for } m = 4q, \\ 5\lceil \frac{m}{4} \rceil + (5k - 4) & \text{for } m = 4q + 1, \\ 5\lceil \frac{m}{4} \rceil + (5k - 2) & \text{for } m = 4q + 2, \\ 5\lceil \frac{m}{4} \rceil + (5k - 1) & \text{for } m = 4q + 3. \end{cases}$

Proof. Let $V(\mu(T_{4k,m}) = \{v_g, u_g, v : 1 \le g \le 4k + m\}$ and $\begin{array}{l} Frog. \ \text{Let } v\left(\mu(1_{4k,m}) - \{v_g, u_g, v: 1 \le g \le 4k + m\} \text{ and } \\ E(\mu(T_{4k,m}) = \{v_g v_{g+1}, v_g u_{g+1}, v_1 v_{4k}, v_1 u_{4k}, v_{4k} u_1: 1 \le g \le 4k + m - 1\} \cup \{u_g v: 1 \le g \le 4k + m\} \\ 4k + m\} \cup \{v_g u_{g-1}, 2 \le g \le 4k + m\}. \ \text{Let } S_1 = \{vv_g: g \equiv 0 \pmod{2}, 1 \le g \le 4k + m\}, \\ S_2 = \{u_g: g \equiv 0 \text{ or } 2 \text{ or } 3 \pmod{4}, 1 \le g \le 4k + m\}, \\ S_3 = \{vv_g: g \equiv 1 \pmod{2}, 1 \le g \le 4k + m\}, \\ S_4 = \{u_g, v: g \equiv 0 \text{ or } 1 \text{ or } 3 \pmod{4}, 1 \le g \le 4k + m\} \\ \text{Presume } S = \begin{cases} S_3 \cup S_4 & \text{for } m \equiv 2 \pmod{4}, \\ S_1 \cup S_2 & \text{Otherwise.} \end{cases} \end{array}$ Then S is a Corona cover set of $(\mu(T_{4k,m}))$ and hence





$$\tau_C(\mu(T_{4k,m}) \le |S|) = \begin{cases} 5(\frac{m}{4}) + 5k + 1 & \text{for } m = 4q, \\ 5\lceil \frac{m}{4} \rceil + (5k - 4) & \text{for } m = 4q + 1, \\ 5\lceil \frac{m}{4} \rceil + (5k - 2) & \text{for } m = 4q + 2, \\ 5\lceil \frac{m}{4} \rceil + (5k - 1) & \text{for } m = 4q + 3. \end{cases}$$

Let N' be a corona cover set of $\mu(T_{4k,m})$. Suppose D is a corona cover set of order at most

$$N = \begin{cases} 5(\frac{m}{4}) + 5k & \text{for } m = 4q, \\ 5\lceil \frac{m}{4} \rceil + (5k - 5) & \text{for } m = 4q + 1, \\ 5\lceil \frac{m}{4} \rceil + (5k - 3) & \text{for } m = 4q + 2, \\ 5\lceil \frac{m}{4} \rceil + (5k - 2) & \text{for } m = 4q + 3. \end{cases}$$

then either $\langle D \rangle$ has an vertex with degree zero or D is not a vertex cover set. Thus we have $\int (5(\frac{m}{2}) + 5k + 1) = for m - 4a$

$$|N'| \ge N+1 = \begin{cases} 5(\frac{4}{4}) + 5k + 1 & \text{for } m = 4q, \\ 5[\frac{m}{4}] + (5k - 4) & \text{for } m = 4q + 1, \\ 5[\frac{m}{4}] + (5k - 2) & \text{for } m = 4q + 2, \\ 5[\frac{m}{4}] + (5k - 1) & \text{for } m = 4q + 3. \end{cases}$$

Therefore, the result is obtained.

Theorem 2.11. For a Tadpole graph $T_{4k+2,m}$, k and q are positive integers, then

$$\tau_C(\mu(T_{4k+2,m})) = \begin{cases} 5\lceil \frac{m}{4} \rceil + (5(\lceil \frac{k+1}{2} \rceil) - 6) & \text{for } m = 4q+1, \\ 5\lceil \frac{m}{4} \rceil + (5(\lceil \frac{k+1}{2} \rceil) - 4) & \text{if } m \neq 4q, 4q+1. \end{cases}$$

Proof. Let $V(\mu(T_{4k+2,m}) = \{v_i, u_i, v : 1 \le i \le 4k + m + 2\}$ and $E(\mu(T_{4k+2,m}) = \{v_i v_{i+1}, v_i u_{i+1}, v_1 v_{4k+2}, v_1 u_{4k+2}, v_{4k+2} u_1 : 1 \le i \le 4k + m + 1\} \cup \{u_i v : 1 \le i \le 4k + m + 1\} \cup \{u_i v : 1 \le i \le 4k + m + 1\} \cup \{u_i v : 1 \le i \le 4k + m + 1\} \cup \{u_i v : 1 \le i \le 4k + m + 1\} \cup \{u_i v : 1 \le k + 1\} \cup \{u_i v : 1 \le k + 1\} \cup \{u_i v : 1 \le k + 1\} \cup \{u_i v : 1 \le k + 1\} \cup \{u_i v : 1 \le k + 1\} \cup \{u_i v : 1$ $i \leq 4k + m + 2$ $\cup \{v_i u_{i-1}, 2 \leq i \leq 4k + m + 2\}$. Let $S_{=}\{vv_i : i \equiv 0 \pmod{2}, 1 \leq i \leq 1 \}$ 4k + m + 2 \cup { $u_i : i \equiv 0 \text{ or } 2 \text{ or } 3 \pmod{4}, 1 \leq i \leq 4k + m + 2$ } Then S is a corona cover set of $(\mu(T_{4k+2,m}))$ and hence

 $\tau_C(\mu(T_{4k+2,m}) \le |S| = \begin{cases} 5\lceil \frac{m}{4} \rceil + (5(\lceil \frac{k+1}{2} \rceil) - 6) & for \ m = 4q+1, \\ 5\lceil \frac{m}{4} \rceil + (5(\lceil \frac{k+1}{2} \rceil) - 4) & for \ m \ne 4q, 4q+1. \end{cases}$ Let N' be a corona cover set of $\mu(T_{4k+2,m})$. Suppose D is a corona cover set of order at most

 $N = \begin{cases} 5\lceil \frac{m}{4} \rceil + (5(\lceil \frac{k+1}{2} \rceil) - 7) & \text{for } m = 4q + 1, \\ 5\lceil \frac{m}{4} \rceil + (5(\lceil \frac{k+1}{2} \rceil) - 5) & \text{for } m \neq 4q, 4q + 1. \end{cases}$ then either < D > has an vertex with degree zero or D is not a vertex cover set, Thus we have

 $|N'| \ge N+1 = \begin{cases} 5\lceil \frac{m}{4} \rceil + (5(\lceil \frac{k+1}{2} \rceil) - 6) & for \ m = 4q+1, \\ 5\lceil \frac{m}{4} \rceil + (5(\lceil \frac{k+1}{2} \rceil) - 4) & if \ m \neq 4q, 4q+1. \end{cases}$ Therefore, the result is obtained

Therefore, the result is obt

Remark 2.12. If m = 4q then the corona cover set of $\mu(T_{4k+2,m})$ does not exists.

Example 2.13.

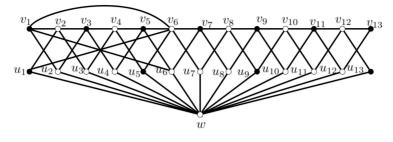


Figure 4. $\mu(T_{6,7})$

In the graph in figure 1, The collection of white vertices is corona cover set of minimum order and hence $\tau_C(\mu(T_{6,7})) = 15$.

3 Conclusion remarks

Corona covering number has already been discussed for some standard graphs. Throughout this paper, we have obtained corona covering number for few special types of graphs and mycielskian graph of some standard graphs.

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