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EXPLICITING CCD-NUMBER FOR MIDDLE GRAPH OF SOME NOTABLE GRAPHS AND JAHANGIR GRAPH

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Abstract Just a while ago, the idea of complementary corona domination number of a graph was first suggested by G. Mahadevan et al[5]. A dominating set S of a graph U is said to be a complementary corona dominating set (CCD-set) if every vertex in $< V - S >$ is either a pendent vertex or support vertex. The smallest cardinality of a CCD-set is called the CCD-number and is denoted by $\gamma_{CCD}(U)$. In this article, we obtain CCD number for middle graph of some notable graphs and Jahangir graph is given below. Further, we prove that the smallest cardinality of CCD-number .

1 Introduction

This article contains graphs such as finite, non-trivial, simple, and undirected [1],[2]. Haynes and colleagues [3] were the first to propose the concept of a domination number. The idea of corona domination of a graph and the complementary corona domination number of a graph initially by G. Mahadevan et al. [4], [5].

The wheel graph [6] $W_{1,g}$ is defined to be the graph $K_1 + C_g$. By finding g copies of the cycle C_3 at a shared vertex, the friendship graph [7], represented by F_g, can be built. The crown $C_g \odot K_1$ is obtained by adjoining a pendent edge to each vertex of cycle C_g[8]. Let P_g be a path on g vertices. The comb graph is defined as $P_g \odot K_1$. It has $2g$ vertices and $2g - 1$ edges. Tadpole[9] $T_{g,h}$ is a graph in which path P_h is attached to any one vertex of cycle C_g. The graph $P_2 \times P_g$ is a ladder graph $L_g(U)$[10]. The Middle graph is discussed in Weigen Yan [11]. By adding a new vertex to each edge and connecting by edges the pairs of these new vertices that are located on neighboring edges of G are known as $M(U)$. The Jahangir graph [12] A graph with $C_{g,h}$ cycle and a vertex g that is close to the vertices of $C_{g,h}$ at a vertex distance of g is called $J_{g,h}$. Throughout, this paper set of lightened vertices is a CCD set of graphs.

2 CCD- Number for middle graph of some notable graphs

Theorem 2.1. *If $g \geq 3$ and $q \in W$, then $\gamma_{CCD}(M(W_g)) = \lceil \frac{g}{2} \rceil + g + 1$.*

Proof. Let $V(M(W_g)) = \{x, x_1, x_2, ..., x_g, x'_1, x'_2, ..., x'_g, x''_1, x''_2, ..., x''_g\}$.
$E(M(W_g)) = \{xx''_i, x_ix''_i, x''_jx''_{j+1}, x'_jx'_{j+1}, x'_ix''_i, x'_{k-1}x''_k, x_ix'_i, x_kx'_{k-1}, x''_1x''_g, x'_1x'_g, x''_1x'_g, x_1x'_g : 1 \leq i \leq g; 1 \leq j \leq g-1; 2 \leq k \leq g\}$. Let $A_1 = \{x, x'_j \ : \ j \ = 2q+1\} \cup \{x''_k : 1 \leq k \leq g\}$.

Assume $A = \begin{cases} A_1 & if \ g \ = 2q \\ \{A_1 - \{x'_g\}\} \cup \{x''_g\} & if \ g \ = 2q+1. \end{cases}$

Then A is a CCD-set of $M(W_g)$ and hence $\gamma_{CCD}\ (M(W_g)) \leq |A| = \lceil \frac{g}{2} \rceil + g + 1$. Since any dominating set B of cardinality $c = \lceil \frac{g}{2} \rceil + g$, contains atleast vertex degree one $< V - B >$, we have $|A'| \geq c + 1 = \lceil \frac{g}{2} \rceil + g + 1$. Therefore the proof. □

Example 2.2.

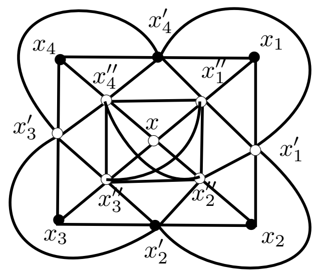

Figure 1. $CCD(M(W_4)) = 7$

Illustration

In the above graph the lightened vertices of CCD-set and hence $CCD(M(W_4)) = 7$.

Theorem 2.3. *If $g \geq 2$, then $\gamma_{CCD}(M(F_g)) = 2g$.*

Proof. Let $V(M(F_g)) = \{x_0, x_{1j}, x_{2j}, y_{1j}, y_{2j}, y_{3j} : j = 1, 2, \ldots g\}$.
$E(M(F_r)) = \{x_0y_{2j}, x_0y_{3j}, x_{1j}y_{1j}, x_{2j}y_{1j}, y_{1j}y_{2j}, y_{1j}y_{3j}, x_{1j}y_{2j}, x_{2j}y_{3j}, y_{3k}y_{2(k+1)}, y_{21}y_{3g} : 1 \leq j \leq g; 1 \leq k \leq g-1\}$. Assume $A_1 = \{y_{2j}, y_{3j} \ : \ j = 1, 2, .., g\} - \{y_{3g}\} \cup \{x_{2g}\}$.
Then A is a CCD-set of $M(F_g)$ and hence $\gamma_{CCD}\ (M(F_g)) \leq |A| = 2g$. Since any dominating set B of cardinality $c = 2g - 1$, contains atleast vertex degree one $< V - B >$, we have $|A'| \geq c + 1 = 2g$. Therefore the proof. □

Example 2.4.

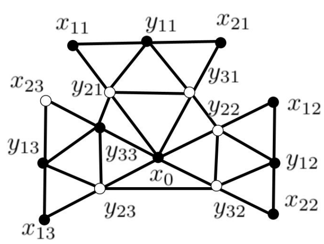

Figure 2. $CCD(M(F_3)) = 6$

Illustration

In the above graph the lightened vertices of CCD set and hence $CCD(M(F_3)) = 6$.

Theorem 2.5. *If $C_g \odot K_1$ is a crown graph, then $\gamma_{CCD}(M(C_g \odot K_1)) = 2g$.*

Proof. Let $V(M(C_g \odot K_1)) = \{u_1, u_2, ..., u_g, v_1, v_2, ..., v_g, x_1, x_2, ..., x_g, y_1, y_2, ..., y_g\}$.
$E(M(C_g \odot K_1)) = \{v_ix_i, x_iy_i, x_ix_{i+1}, x_iv_{i+1}, x_iy_{i+1}, y_iu_i, v_iy_i \ : \ 1 \leq i \leq g\}$. Assume $A_1 = \{x_i \ : \ 1 \leq i \leq g\} \cup \{u_i \ : \ 1 \leq i \leq g\}$. Then A is a CCD-set of $M(C_g \odot K_1)$ and hence $\gamma_{CCD}\ (M(C_g \odot K_1)) \leq |A| = 2g$. Since any dominating set B of cardinality $c = 2g - 1$. contains atleast vertex degree one $< V - B >$, we have $|A'| \geq c + 1 = 2g$. Therefore the proof. □

Example 2.6.

Illustration

In the above graph the lightened vertices of CCD set and hence $CCD(M(C_4 \odot k_1))) = 8$.

Theorem 2.7. *If $P_g \odot K_1$ is a comb graph, then $\gamma_{CCD}(M(P_g \odot K_1)) = (g-1) + g$.*

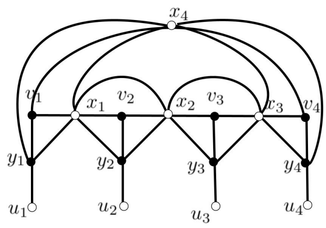

Figure 3. $CCD(M(C_4 \odot k_1))) = 8$

Proof. Let $V(M(P_g \odot K_1)) = \{u_1, u_2, ..., u_g, v_1, v_2, ..., v_g, x_1, x_2, ..., x_h, y_1, y_2, ..., y_h\}$. $E(M(P_g \odot K_1)) = \{v_i x_i, x_i y_i, x_i x_{i+1}, x_i v_{i+1}, x_i y_{i+1} : 1 \leq i \leq g-1\} \cup \{y_i u_i, v_i y_i : 1 \leq i \leq g\}$. Assume $A_1 = \{x_i \ : 1 \leq i \leq g\} \cup \{u_i : 1 \leq i \leq g\}$. Then A is a CCD-set of $M(P_g \odot K_1)$ and hence $\gamma_{CCD}\ (M(P_g \odot K_1)) \leq |A| = (g-1) + g$. Since any dominating set B of cardinality $c = (g-1) + g - 1$. contains atleast vertex degree one $< V - B >$, we have $|A'| \geq c + 1 = (g-1) + g$. Therefore the proof. □

Example 2.8.

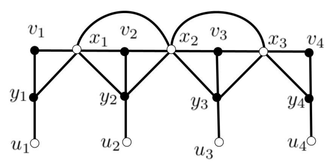

Figure 4. $CCD(M(P_4 \odot k_1))) = 7$

Illustration

In the above graph the lightened vertices of CCD set and hence $CCD(M(P_4 \odot k_1))) = 7$.

Theorem 2.9. *If* $g \geq 2$, *then* $\gamma_{CCD}(M(L_g)) = 2g - 2$.

Proof. Let $V(M(L_g)) = \{v_1, v_2, ..., v_g, u_1, u_2, ..., u_r, w_1, w_2, ..., w_g, x_1, x_2, ..., x_g, e_1, e_2, ..., e_g\}$. $E(M(L_g)) = \{v_i u_i, u_i v_{i+1}, x_i e_i, e_i w_{i+1}, u_j u_{j+1}, e_j e_{j+1}, v_k w_k, w_k x_k, u_i w_i, u_i w_{i+1}, e_i w_i \ : 1 \leq i \leq g-1; 1 \leq j \leq g-2 : 1 \leq k \leq g\}$. Assume $A = \{u_i \ : \ 1 \leq i \leq g\} \cup \{e_i : 1 \leq i \leq g\}$. Then A is a CCD-set of $M(L_g)$ and hence $\gamma_{CCD}\ (M(L_g)) \leq |A| = 2g - 2$. Since any dominating set B of cardinality $c = 2g - 3$. contains atleast vertex degree one $< V - B >$, we have $|A'| \geq c + 1 = 2g - 2$. Therefore the proof.

Example 2.10.

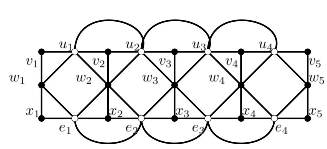

Figure 5. $CCD(M(L_5))) = 8$

Illustration

In the above graph the lightened vertices of CCD set and hence $CCD(M(L_5))) = 8$. □

3 CCD-Number for middle graph of tadpole graph

Let $V(M(T_{g,h})) = \{u_1, u_2, ..., u_g, x_1, x_2, ..., x_g, v_1, v_2, ..., v_h, y_1, y_2, ..., y_h\}$.
$E(M(T_{g,h})) = \{x_i x_{i+1}, u_i x_i, x_i u_{i+1}, v_i y_i, u_1 v_1, x_g v_1, x_1 v_1 : 1 \leq i \leq g\} \cup \{v_i v_{i+1}, y_i v_{i+1} : 1 \leq i \leq h-1\}$ where $q = \{0, 1, .., h\}$

Theorem 3.1. *If* $g \equiv 0\ (mod\ 2)$ *and* $g \geq 3$*, then* $\gamma_{CCD}(M(T_{g,h})) = \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil + 1$.

Proof. Assume $A = \{u_i\ :\ i\ \equiv 0\ (mod\ 2)\} \cup \{v_i\ :\ i\ \equiv 1\ (mod\ 2)\} \cup \{y_h\}$
Then A is a CCD-set of $M(T_{g,h})$ and hence $\gamma_{CCD}\ (M(T_{g,h})) \leq |A| = \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil + 1$. Since any dominating set B of cardinality $c = \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil$ contains atleast vertex degree one $< V - B >$, we have $|A'| \geq c + 1 = \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil + 1$. Therefore the proof. □

Theorem 3.2. *If* $g \geq 3$ *and* $g \equiv 1\ (mod\ 2)$*, then*
$$\gamma_{CCD}(M(T_{g,h})) = \begin{cases} \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil + 1 & if h\ \equiv\ 0\ \ (mod\ 2) \\ \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil & if\ h\ \equiv\ 1\ (mod\ 2). \end{cases}$$

Proof. Let $A_1 = \{u_i\ :\ i\ \equiv 1\ (mod\ 2)\} \cup \{v_i\ :\ i\ \equiv 0\ (mod\ 2)\}$
Assume $S = \begin{cases} A_1 \cup \{y_h\} & if\ h\ \equiv\ 0\ or\ 1\ (mod\ 2). \end{cases}$
Then A is a CCD-set of $M(T_{g,h})$ and hence
$$\gamma_{CCD}\ (M(T_{g,h})) \leq |A| = \begin{cases} \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil + 1 & if\ h\ \equiv\ 0\ \ (mod\ 2) \\ \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil & if\ h\ \equiv\ 1\ (mod\ 2). \end{cases}$$
Since any dominating set B of cardinality $c = \begin{cases} \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil & if\ h =\ 2q \\ \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil - 1 & if\ h =\ 2q + 1. \end{cases}$
contains atleast vertex degree one $< V - B >$, we have
$$|A'| \geq c + 1 = \begin{cases} \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil + 1 & if\ h =\ 2q \\ \lceil \frac{g}{2} \rceil + \lceil \frac{h}{2} \rceil & if\ h =\ 2q + 1. \end{cases}$$
Therefore the proof. □

Example 3.3.

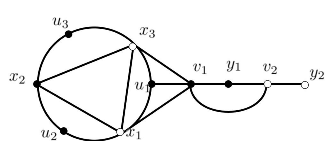

Figure 6. $CCD(M(T_{3,2}))) = 4$

Illustration

In the above graph the lightened vertices of CCD set and hence $CCD(M(T_{3,2}))) = 4$.

4 CCD- Number for Jahangir graph

Let $V(J_{g,h}) = \{x, x_1, x_2, ..., x_{gh}\}$ and $E(J_{g,h}) = \{x x_{i+1} x x_j, x_1 x_{gh}\ :\ 1 \leq i \leq gh - 1; j \equiv 1(mod\ h)\}$ where $q = 0, 1, ..., r$.

Theorem 4.1. *If* $h \geq 3$ *and* $g \equiv 0\ (mod\ 3)$*, then* $\gamma_{CCD}(J_{g,h}) = \lceil \frac{gh}{3} \rceil + 1$.

Proof. Assume $A = \{x_i\ :\ i = 3q\} \cup \{v\}$. Then S is a CCD-set of $(J_{g,h})$ and hence $\gamma_{CCD}\ (J_{g,h}) \leq |A| = \lceil \frac{gh}{3} \rceil + 1$. Since any dominating set B of cardinality $c = \lceil \frac{gh}{3} \rceil$. contains atleast vertex degree one $< V - B >$, we have $|A'| \geq c + 1 = \lceil \frac{gh}{3} \rceil + 1$. Therefore the proof. □

Theorem 4.2. *If $h \geq 3$ and $g \equiv 1\ (mod\,3)$, then* $\gamma_{CCD}(J_{g,h}) = \begin{cases} \lfloor \frac{gh}{3} \rfloor & if\ h\ \equiv\ 1\ (mod\,3) \\ \lceil \frac{gh}{3} \rceil & Otherwise. \end{cases}$

Proof. Assume $A_1 = \{x_i\ :\ i \equiv 3(k+l)\ (mod\ g)\ where\ k = 1, l = \{0, 1, .., \frac{g-1-3k}{3}\} \cup \{v\}$.
Then A is a CCD-set of $(J_{g,h})$ and hence $\gamma_{CCD}\ (J_{g,h}) \leq |A| = \begin{cases} \lfloor \frac{gh}{3} \rfloor & if\ h\ \equiv\ 1\ (mod\,3) \\ \lceil \frac{gh}{3} \rceil & Otherwise. \end{cases}$
Since any dominating set B of cardinality $c = \begin{cases} \lfloor \frac{gh}{3} \rfloor - 1 & if\ h\ \equiv\ 1\ (mod\,3) \\ \lceil \frac{gh}{3} \rceil - 1 & Otherwise. \end{cases}$
contains atleast vertex degree one $< V - B >$, we have
$|A'| \geq c + 1 = \begin{cases} \lfloor \frac{gh}{3} \rfloor & if\ h\ \equiv\ 1\ (mod\,3) \\ \lceil \frac{gh}{3} \rceil & Otherwise. \end{cases}$
Therefore the proof. □

Theorem 4.3. *If $h \geq 3$ and $g \equiv 2\ (mod\,3)$, then* $\gamma_{CCD}(J_{g,h}) = \begin{cases} \lceil \frac{gh}{3} \rceil & if\ h\ = 2q \\ \lceil \frac{gh}{3} \rceil + 1 & if\ h\ = 2q + 1. \end{cases}$

Proof. Assume $A_1 = \{x_i\ :\ i \equiv 3(k+l)\ (mod\,2g)\ where\ k = 1, l = \{0, 1, .., \frac{2g-1-3k}{3}\} \cup \{v\}$.
Assume $s = \begin{cases} A_1 & if\ h =\ 2q \\ A_1 \cup \{x_{g-1}\} & if\ h =\ 2q + 1. \end{cases}$
Then A is a CCD-set of $(J_{g,h})$ and hence $\gamma_{CCD}\ (J_{g,h}) \leq |A| = \begin{cases} \lceil \frac{gh}{3} \rceil & if\ \ h\ = 2q \\ \lceil \frac{gh}{3} \rceil + 1 & if\ \ h\ = 2q + 1. \end{cases}$
Since any dominating set B of cardinality $c = \begin{cases} \lceil \frac{gh}{3} \rceil - 1 & if\ h\ = 2q \\ \lceil \frac{gh}{3} \rceil & if\ h\ = 2q + 1\ . \end{cases}$
contains atleast vertex degree one $< V - B >$, we have
$|A'| \geq c + 1 = \begin{cases} \lceil \frac{gh}{3} \rceil & if\ h\ = 2q \\ \lceil \frac{gh}{3} \rceil + 1 & if\ h\ = 2q + 1\ . \end{cases}$
Therefore the proof. □

Example 4.4.

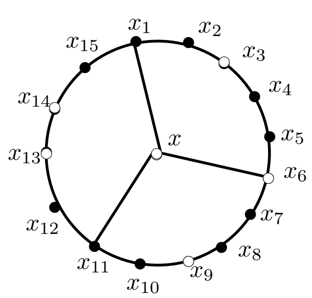

Figure 7. $CCD(J_{5,3}) = 6$

Illustration

In the above graph the lightened vertices of CCD set and hence $CCD(J_{5,2}) = 4$ and hence $CCD(J_{5,3}) = 6$.

5 Conclusion remarks

In this article, we found the CCD-number for $M(W_r)$, $M(F_r)$, $M(C_r \odot K_1)$, $M(P_r \odot K_1)$, $M(T_{r,s})$, $M(L_r)$ and $J_{r,s}$. For a future work, we extend our results for product related graphs,tree and comparing another parameters.

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