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A Numerical Approach to the Natural Convective Flow of MHD Dissipative Fluid with Radiative Isothermal Heat Flux and Heat Absorption/Generation Across an Inclined Infinite Plate

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Abstract This article delves into the fascinating realm of laminar natural convective hydromagnetic dissipated flow over an infinite plate, illuminating the interplay between radiated isothermal heat flux, heat absorption, and generation. Employing the esteemed Crank-Nicholson approach, this article derives the consequential expressions of the governing equations that govern this physical phenomenon. Through this method, the nondimensional expressions are skilfully transformed into tridiagonal expressions, enabling a deeper understanding of the system's behaviour. Furthermore, to enhance comprehension and offer a visual representation of the obtained numerical data for momentum and temperature, captivating graphical sketches are skilfully employed. These graphical depictions vividly showcase the intricate relationship between the Prandtl number (Pr), Magnetic parameter (M), heat absorption and generation (Δ), Eckert number (Ec), and Radiation (Rd), allowing for a comprehensive analysis of the system under various constraints.

1 Introduction

Modern technological advancements required considerable changes in the discipline of temperature transportation. Analysing engineering problems using MHD is a viable option for many problems. It is essential that engineers employ MHD principles when designing heat exchangers, nuclear reactor coolants, flow meters, magnetic endoscopy, confinement schemes, metallurgical industries, pumps as well as power-generating systems. Magnetohydrodynamics attempts to modify the flow in an intended path by figuring out the formulation of the boundary layer. Consequently, it seems more reliable and flexible to alter the fluid flow utilising MHD. In 1960, Soundalgekar et al.[1] implemented a thorough examination of radiative naturally convective impacts on the flow of gas through a semi-infinite plate. Expanding on this research, Hossain et al.[2] inquired into how radiation influences mixed convection over an isothermal vertical plate. The scientific community delved into a substantial body of work focused on magnetohydrodynamics (MHD) and explored various fluid properties through both analytical and experimental approaches using a vertical plate [3, 4, 5, 6, 7]. Building upon these investigations, Das [8] executed comprehensive studies exploring the effects of partial slip, MHD radiative fluid flow across a flat plate. The investigation considered convective heat flux across the non-isothermal boundary region under the heat source/sink. Notably, the current research endeavours focus primarily on the investigation into heat and mass transmission across a vertical infinite plate.

Viscous dissipation pertains to the irreversibility process by which the fluid motion caused by shear stresses is transformed into heat. During the motion inside fluid particles, viscous dissipation is acknowledged. Israel-Cookey et al. [9] embarked upon a thorough examination of the interplay between dissipative and radiative MHD natural convection along an infinite plate of the porosity medium. Balamurugan and Sreelatha [10] developed a perturbation approach for the investigation of mixed convective thermal and mass transmission. Their attention was drawn to a Newtonian fluid that was incompressible and electrically conducting as it passed over a permeable, infinite vertical plate. First-order chemical reactions, radiation, a heat source with a temperature gradient, and a magnetic field are considered. Thermal radiation on an electrically conducting fluid of the revolving system combined with Hall current on a vertical surface of the plate when there is uniform mass and the non-isothermal temperature was inquired by Manjula and Muthucumaraswamy [11]. A non-dimensional version of coupled PDEs has an accurate solution through the Laplace transform method. The comprehensive investigation carried out by Sami et al. [12] centred on the discussion of the time-dependent naturally convective magnetohydrodynamic flow of a viscous fluid. The study specifically examined the radiative influence of homogeneous thermal flux on an infinite vertical surface of a plate enclosed by a porosity surface. Notably, the researchers employed Laplace transform techniques to discover viable solutions to the complex dynamics of this system.

A vacuum is found to be more effective than a medium for the transmission of heat by radiation, as the radiation does not require any medium. Radiation appears to play a crucial role in commercial and technical applications, for instance, activities requiring very high operating temperatures at very modest convective heat transfer coefficients. The consideration of thermal radiation performs a vital role in the design of fins, gas turbines, Medical Imaging, Energy Generation, spacecraft, satellites, and missiles. The understanding and analysis of heat transmission with the existence of radiation ensure the optimal performance of these technological advancements. Santhi Kumari et al. [13] performed a Galerkin finite element analysis for how the Soret phenomina signifies MHD free convective motion of a radiative and dissipative liquid towards a chemically interactive infinite plate placed within a porous medium. Muthucumaraswamy and Vijayalakshmi [14] proposed the MHD flow across a vertically moving infinite plate that incorporates temperature flux and mass distribution. Ramachandra Prasad et al. [15] implemented the finite difference technique to analyse the intricate interplay of free convection in an unsteady radiative flow along a vertical plate in motion. Marneni et al. [16] delved into the consequences of an unsteady free convective flow within MHD, heat generation, radiation, chemical reactions, and the soret effects with the oscillating temperature in an infinite plate with porosity. Ganesan and Palani [17] carried out a computer analysis concentrating on the transient nature of magnetohydrodynamic flow across a semi-infinite inclined surface of a plate. Their study considered an incompressible viscous fluid and analysed the implications of non-isothermal temperature and mass flux. The pioneering work by Shankar and Dharmendar [18] focused on the intricate implications of thermal consumption and radiation in a magnetohydrodynamic time-dependent natural convective flow. Their study specifically examined the dynamics of a Newtonian fluid over a moving vertical infinite porous plate. Osman et al. [19] discussed the MHD free convection flow through an infinitely inclined plate. An investigation of the two-fold impacts of radiation and dissipation through an unsteady infinite inclined plate with an unvarying wall heat flux is presented by Sambath et al.[20]. Vara Prasad et al. [21] considered the significace of a chemical reaction over a dissipative unsteady semi-infinite inclined plate with porosity. This comprehensive examination contributes to our understanding of the complex dynamics involved in these phenomena.

According to the available literature review, there are only a few investigations on natural convective flow across an inclined infinite plate in addition to multiple thermophysical effects. The current endeavour has been undertaken with the intention of looking at free convective hydromagnetic dissipative fluids with heat transfer through an inclined infinite plate that incorporates heat source/sinks in the existence of radiative isothermal surface heat flux. Graphs are used to display numerous physical characteristics such as velocity, temperature, skin friction, and Nusselt number. The current analysis may potentially be applied to a variety of industries, including biomedicine, metallurgy, environmental engineering, pharmaceuticals, food processing, and chemical engineering, among others.

2 Mathematical Framework

We delve into a phenomenon of the two-dimensional naturally convective and dissipative unsteady flow adjacent to an inclined infinite plate while considering the heat generation and absorption, MHD, radiation, and homogeneous heat flux. Within this context, let us designate a point denoted by ϕ, positioned between the plate surfaces and inclined with a centre. To facilitate our understanding, we orient the x-axis to run vertical direction towards the plate, while the y-axis stands perpendicular to it. A powerful magnetic field B_0 is considered into account on the y-axis. The temperature is maintained at $T_\infty^{'}$ for the time being $t^{'} \leq 0$. For $t^{'} > 0$ the temperature raised uniformly to q on the plate. Considering the infinite length of the plate, all the terms in equations are not dependent on x.

The physical situation is governed by underlying equations through leveraging Boussinesq's approximation:

$$\frac{\partial u}{\partial t'} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T^{'} - T_\infty^{'})\cos\phi - \frac{\sigma B_0^2 u}{\rho}, \tag{2.1}$$

$$\frac{\partial T^{'}}{\partial t'} = \alpha \frac{\partial^2 T^{'}}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho C_p}(T_w^{'} - T_\infty^{'}) \tag{2.2}$$

Where u and $T^{'}$stands for velocity and thermal energy factors corresponding to fluid, $T_\infty^{'}$ stands for the temperature located far away from the plate. $B_0, g, t^{'}$stands for the magnetic impact, gravitational acceleration, and time consequently. The symbol β represents the volumetric coefficients that describe the extent to which a substance expands or contracts in response to changes in temperature. The variables ν, σ, ρ, C_p and αsymbolize various properties: kinematic viscosity, electrical conductivity, density, specific capacity, and thermal conductivity, accordingly. These properties pertain to distinct characteristics of a substance. Q_0 serves as a dimensional parameter that characterizes the heat source/sink.

The relevant boundary conditions:

$$\begin{array}{llll} t^{'} \leq 0: & u = 0 & T^{'} = T_\infty^{'} & \forall \text{ x and y} \\ t^{'} > 0: & u = 0 & \frac{\partial T^{'}}{\partial y} = -\frac{q}{K} & \text{at } y = 0 \\ & u \to 0 & T^{'} \to T_\infty^{'} & \text{as } y \to \infty \end{array} \tag{2.3}$$

As per the governing equations, the temperature of the plate is presumed to generate a radiative heat flux, which is derived by utilizing the Rosseland approximation.

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^{'4}}{\partial y} \tag{2.4}$$

Here the mean absorption coefficient is k*, and the Stefan-Boltzmann constant is represented as $\sigma*$. By employing the Taylor series and expanding $T^{'4}$ about $T_\infty^{'}$, while not taking into terms of higher order, we arrive at the below outcome:

$$T^{'4} \cong 4T_\infty^{'3}T - 3T_\infty^4 \tag{2.5}$$

Then the Rosseland approximation becomes,

$$q_r = -\frac{16\sigma^* T_\infty^{'3}}{3K^*}\frac{\partial^2 T^{'}}{\partial Y^2} \tag{2.6}$$

Furthermore, non-dimensional expressions are provided by

$$\begin{array}{l} \mathrm{Y} = \frac{\mathrm{y}}{L} Gr_L^{\frac{1}{5}}, \quad \mathrm{t} = \frac{\mathrm{t}^{'}\nu}{L^2} Gr_L^{\frac{2}{5}}, \quad \mathrm{T} = \frac{\mathrm{T}^{'} - T_\infty^{'}}{\left(\frac{qL}{K}\right)} Gr_L^{\frac{1}{5}}, \quad \mathrm{U} = \frac{u}{\nu} L Gr_L^{\frac{-2}{5}}, \quad \mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu C_p}{K}, \\ \mathrm{Gr}_L = \frac{g\beta q L^4 \cos\phi}{k\nu^2}, \quad \mathrm{M} = \frac{\sigma \mathrm{B}_0^2 L^2}{\mu} Gr_L^{\frac{-2}{5}}, \quad \mathrm{Ec} = \frac{\nu^2 k Gr_L}{qL^3 C_p}, \quad R_d = \frac{kk^*}{4\sigma^* T_\infty^{'3}}, \quad \Delta = \frac{\mathrm{Q}_0 L^2}{\rho C_p \nu} Gr_L^{\frac{-2}{5}} \end{array} \tag{2.7}$$

The non-dimensional form of (2.1) to (2.3) results as follows:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Y^2} + T - MU \tag{2.8}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial Y^2}\left(1 + \frac{4}{3Rd}\right) + Ec\left(\frac{\partial U}{\partial Y}\right)^2 + \Delta T \tag{2.9}$$

The non-dimensional boundary conditions are as follows:

$$
\begin{array}{llll}
t \le 0: & U = 0 & T = 0 & \forall \ \text{Y and t} \\
t > 0: & U = 0 & \frac{\partial T}{\partial Y} = -1 & \text{at} \quad Y = 0 \\
 & U \to 0 & T \to 0 & \text{as} \quad Y \to \infty
\end{array}
\tag{2.10}
$$

Where M-magnetic field, Pr-Prandtl number, Rd-radiation, Ec-Eckert number, Δ-heat source or sink parameter.

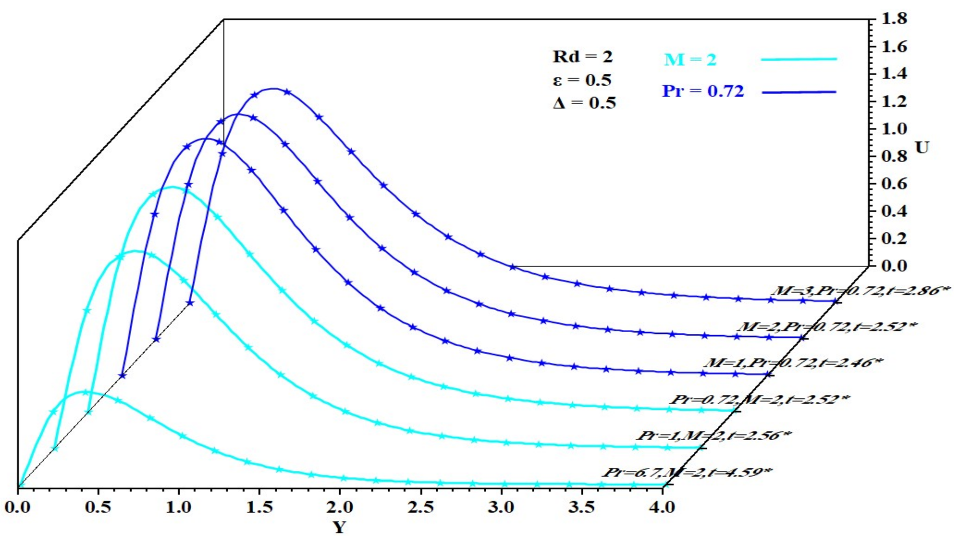
3 Numerical Procedure

To tackle the dynamic coupled non-linear PDEs encapsulated within equations (2.8) and (2.9), as well as equation (2.10), the Crank-Nicolson technique is used. These equations are first converted into non-dimensional expressions and subsequently converted into a set of tri-diagonal equations. To obtain the desired solution, we utilize the widely recognized Thomas method, known for its rapid convergence and ability to provide an unconditionally stable solution. Furthermore, it is crucial to consider the order in which the final and preliminary requirements are fulfilled. It has been observed through careful analysis that an impressive precision level up to 10^{-5} has been achieved. The meshing range has been meticulously determined, with a step size increase of Δt = 0.01, while ΔX and ΔY has been set to 0.05. It is important to note that as Δt, and ΔY approach infinitesimal values, the truncation error denoted by O ($\Delta t^2 + \Delta Y^2$) tends to approach negligible levels. This leads us to a compelling conclusion that a well-thought-out and systematic course of action, supported by the calculations, computations, and approximations discussed earlier, demonstrates the existence of a viable solution.

4 Results and discussion

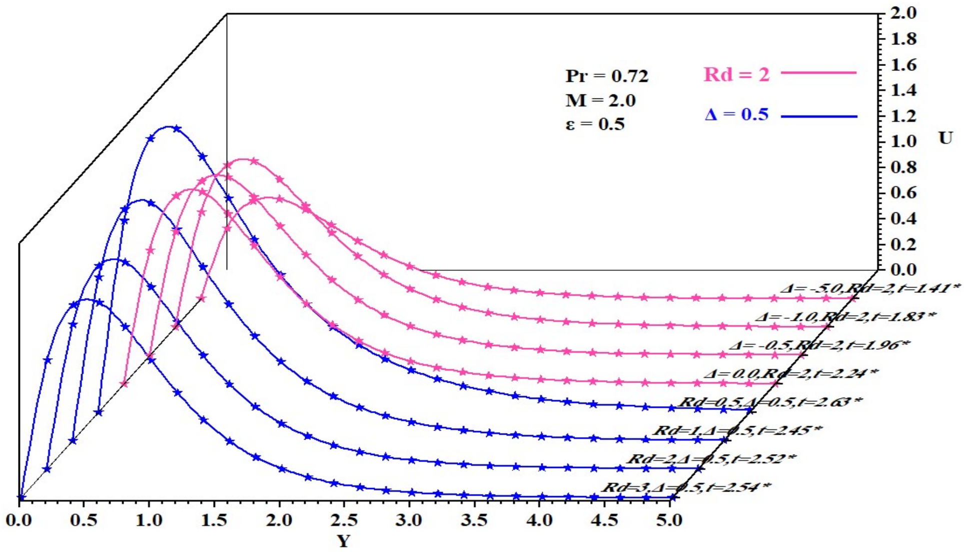
Velocity Profiles

Figure.1 illustrates the dynamic momentum boundary layers, exhibiting different behaviour as we vary the values of Pr while keeping M=2, Rd=2, Δ=0.5, and ϵ=0.05 constant. It has been discovered that the velocity flow rate drops with improving Pr Values. Furthermore, when we consider different values of M, increasingly 1.0, 2.0, and 3.0, the velocity profile undergoes a decrease for a fixed Pr of 0.72, ϵ of 0.5, and Rd of 2.0. All the values considered here are the steady-state denoted by (*).



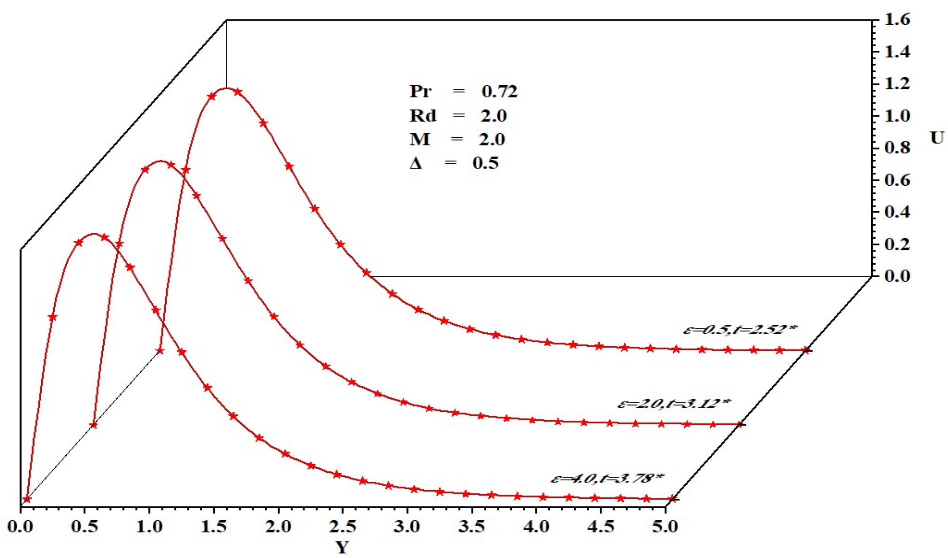
$Figure$: 1 $Plot\ of\ velocity\ for\ various\ Pr\ and\ M$

Figure.2 presents a visual representation of the instantaneous velocity, considering specific values such as Pr = 0.72, M = 2, and $\epsilon = 0.05$ while exploring different heat generation/absorption constraints and radiation parameters. Upon closer observation, it becomes apparent that as heat is generated, the buoyancy force gradually strengthens. However, it is beneficial to know that the momentum boundary layer experiences growth as Δ levels up. Furthermore, the velocity phenomenon is influenced by the radiation parameter, which is characterized by various values such as 0.5, 1.0, 2.0, and 3.0. Specifically, for $\Delta = 0.5$, $\epsilon = 0.05$, Pr = 0.72, and M = 2.0, it was plainly noted that different accelerated values of Rd result in a drop with fluid velocity.



$Figure: 2\ Plot\ of\ velocity\ for\ various\ Rd\ and\ \Delta$

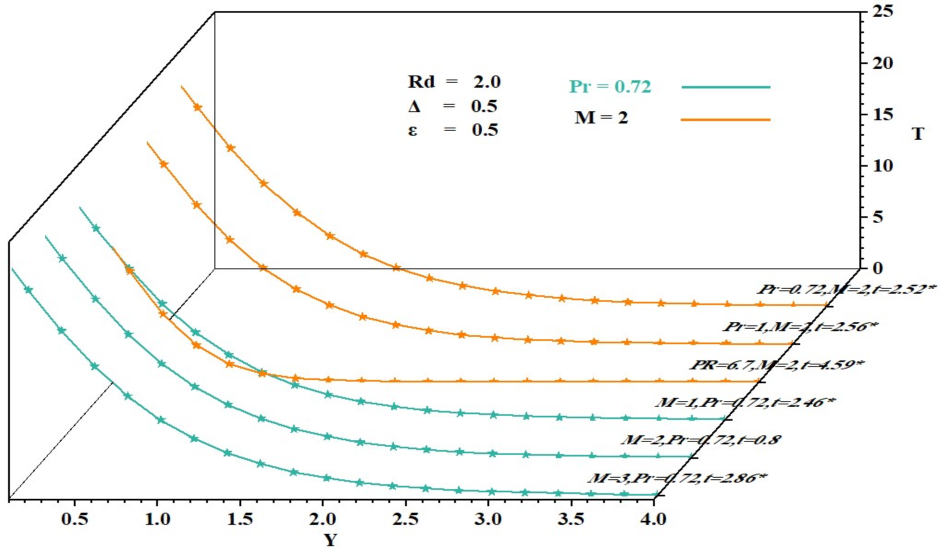
Figure.3 depicts the impacts of viscous dissipation on velocity. The plot demonstrates that raised values of ϵ cause the velocity to increase as the consequences of energy are absorbed by the fluid. This observation holds true when the parameters Pr = 0.72, Rd = 2, M = 2, and $\Delta = 0.5$ remain fixed.



$Figure: 3\ Plot\ of\ velocity\ for\ various\ \epsilon$

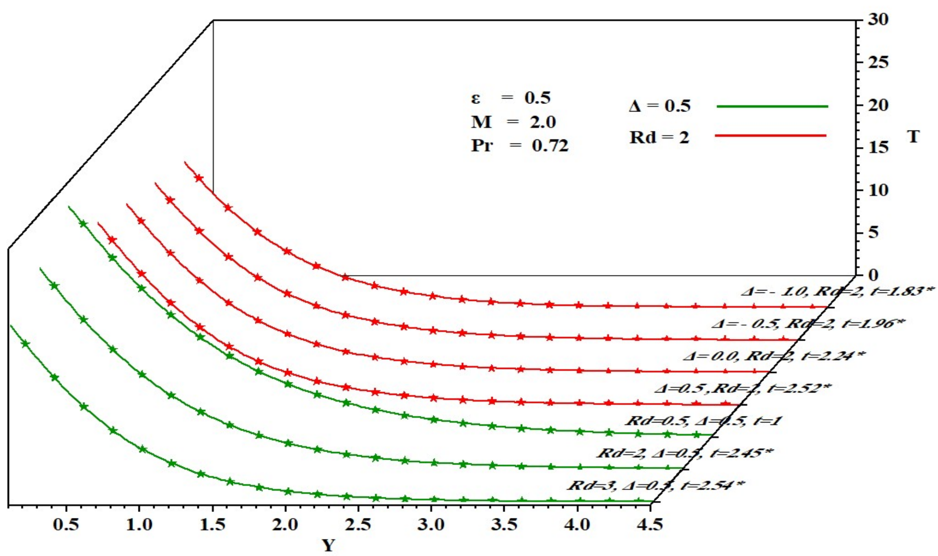
Temperature Profiles

Figure.4 illustrates the dynamic thermal profile, considering different combinations of Pr and M while keeping $\Delta = 0.05$, Rd = 2, and $\epsilon = 0.5$ constant. Notably, as the M values increase from 1 to 2 and 3, with a fixed Pr value of 0.72, the surface temperature exhibits a noticeable rise. It was found that the temperature was demonstrated to decrease with the impact of Pr from 0.72, 1.0, and 6.7 while maintaining a magnetic parameter of 2.



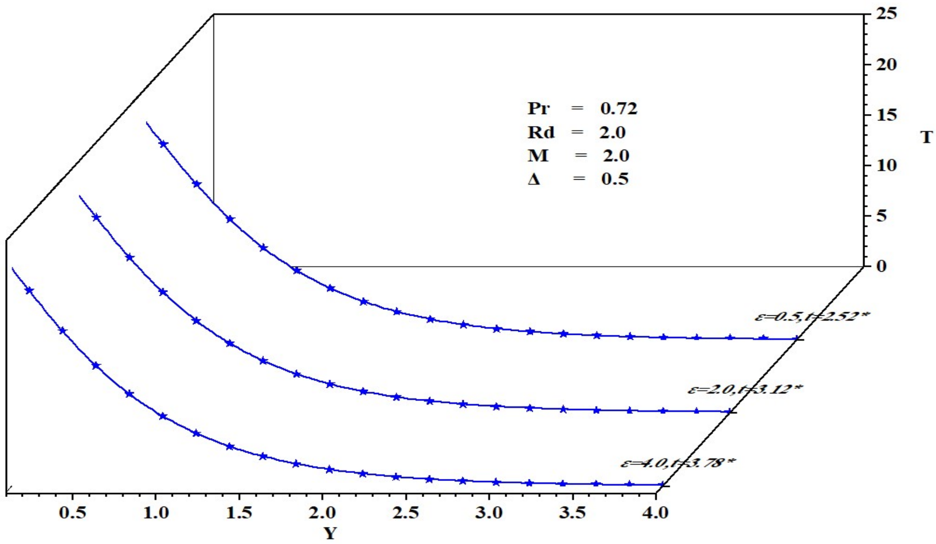
$Figure: 4\ Plot\ of\ temperature\ for\ various\ Pr\ and\ M$

Figure.5 presents the impact of the radiation constraint (Rd) and the parameter (Δ) on the distribution of constant temperature along the Y coordinate. As we observe from the graph, it becomes apparent that as the emission constraint (Rd) increases, the fluid expands in width. The presence of a stronger radiation constraint causes the thermal gradients to become more concentrated, resulting in a reduction of thickness in the boundary layer. An improvement in heat absorption/generation parameters makes the fluid's temperature to be enhanced.



$Figure: 5\ Plot\ of\ temperature\ for\ various\ Rd\ and\ \Delta$

Figure.6 examines the significance of dissipation on temperature. As the Eckert value decreases, it indicates that the convective heat transfer rate is relatively low compared to the internal energy of the fluid. This implies that the fluid can absorb more heat before experiencing a significant temperature rise. Consequently, the temperature boundary layer becomes thinner because the fluid can absorb more heat without experiencing a substantial temperature change, leading to a diminished thickness of the boundary region.



$Figure$: 6 $Plot\ of\ temperature\ for\ various\ \epsilon$

5 Conclusion

The current investigation delves into the dynamic nature of magnetohydrodynamic free convection, as it encounters an infinite inclined plate by an isothermal heat flux. The comprehensive model takes into account the cumulative influence of diverse factors, including the MHD, radiation, heat source/sink, and viscosity dissipation. To surmount the challenge, the (PDEs) are effectively resolved through the implementation of the esteemed crank-Nicholson technique. Consequently, the flow behaviours pertaining to the velocity and temperature fields are acquired under an analysis of the parameters considered. Herein lie the findings gleaned from this pioneering endeavour:

(i) As the Prandtl number (Pr) and Reynolds number (Rd) enhance, the distributions of temperature and velocity undergo a gradual decline.

(ii) Conversely, a remarkable increase in the values of Δ acts as a stimulant, infusing the system and giving the intricate patterns of motion and energy exchange which gives rise to velocity and temperature.

(iii) An increment in the value of ϵ enhances the velocity and it thins the thickness of the temperature boundary layer for reduced ϵ.

(iv) The magnetic field's increasing strength exerts a profound influence on the fluid motion along the wall, leading to a deceleration in its flow. As a consequence, the velocity drops while the temperature undergoes simultaneous growth.

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