Palestine Journal of Mathematics

Vol 14(Special Issue II)(2025) , 200–206

NEIGHBOURHOOD TOTAL 2 - OUT DEGREE EQUITABLE DOMINATION NUMBER

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MSC 2010 Classifications: 05C69.

Keywords and phrases: Two Out Degree, Isolated Vertices, Dominating Set, Equitable, Neighborhood.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper.

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Abstract The Out degree of u with respect to a set D is defined by $od_D(u) = |N(u) \cap ((V - D))|$ and denoted by $od_D(u)$. Based on the concept of out degree we introduce a new domination called neighborhood total 2 - out degree equitable domination (ODED) number. Here the proposed domination number verified for some general and special graphs.

1 Introduction

The graphs defined here are from [3]. In a graph $G = (V, E)$, V is the set of vertices and E is the set of edges. The cardinality of vertices is denoted by n and edges is denoted by m. The degree of a vertex is defined as number of edges incident on that vertex and it is represented by deg. If a degree of vertex is zero then the vertex is called an isolated vertex.

The domination number concept was introduced by Ore [6] and C. Berge [2]. A set D is said to be dominating set of V if every vertex $u \in (V - D)$ there some vertex in $v \in D$ such that $uv \in E(G)$. The minimum number of vertices of dominating set is called domination number and it is represented by $\gamma(G)$. S. Arumugam and C.Sivagnanam [7] introduce the neighborhood total domination number. A dominating set D is called a neighborhood total dominating set of V (ntd-set) if induced sub graph $< N(D) >$ has no isolated vertices. The minimum number of vertices of neighborhood total dominating set is called neighborhood total domination number and it is represented by $\gamma_{nt}(G)$. Ali Sahal and V.Mathad[1] introduce the 2 - out degree equitable domination number . A dominating set D is said to be two out degree equitable dominating set if for any two vertices $u, v \in D$ such that $|od_D(u) - od_D(v)| \leq 2$ where $od_D(u) = |N(u) \cap (V - D)|$. The minimum represented vertices in two out degree equitable dominating set is called 2 - out degree equitable domination number (2 - ODED) and it is denoted by $\gamma_{2oe}(G)$. M.S.Mahesh and P.Namsivayam [4, 5] introduce some new domination based on two out degree. Based on above domination number, here we introduce a new domination parameter called neighborhood total 2 - out degree equitable domination ($\gamma_{nt2oe}(G) - set$) number and we find this domination number for some general graphs, special graphs and some derived graphs.

2 Neighborhood Total 2 - Out Degree Equitable Domination Number

Definition 2.1. A dominating set D is called the neighborhood total 2 - ODED set, if for any two vertices $u, v \in D$ such that $|od_D(u) - od_D(v)| \leq 2$ and the induced sub graph $< N(D) >$ has no vertices of degree zero. The minimum cardinality of vertices of a neighborhood total 2 - ODED set ($\gamma_{nt2oe}(G) - set$) is called neighborhood total 2 - ODED number of G and is represented by $\gamma_{nt2oe}(G)$.

Example 2.2. Let us consider a set $D = \{v_4, v_5\}$ and $(V - D) = \{v_1, v_2, v_3, v_6\}$.

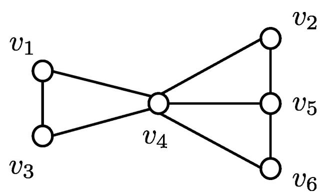

Figure 1. Example of $\gamma_{nt2oe}(G)$

$$od_D(v_4) = |N(v_4) \cap \{v_1, v_2, v_3, v_6\}| = 4.$$
$$od_D(v_5) = |N(v_5) \cap \{v_1, v_2, v_3, v_6\}| = 2.$$

Then, Clearly, for any $u, v \in D$ such that $|od_D(u) - od_D(v)| \leq 2$.
Thus $D = \{v_4, v_5\}$ is 2 out degree equitable dominating set and $< N(D) >$ has no isolated vertices and D is a minimum ($\gamma_{nt2oe}(G) - set$).

Hence, $\gamma_{nt2oe}(G) = 2$.

Remark 2.3. Any neighborhood total 2 - ODED set contains all the pendant vertex of a graph.

Example. From the figure 2, clearly $D = \{u, s, x, v\}$ is the minimum ($\gamma_{nt2oe}(G) - set$) and it contains all pendant vertices u and v.

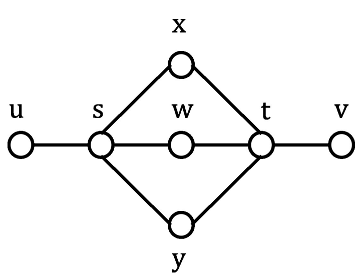

Figure 2. Example for remark 2.3

Remark 2.4. Every neighborhood total 2 - ODED set is a neighborhood total dominating set but not conversely.

Example. In the figure 3, $D = \{v_1, v_2, v_3, v_4\}$ is a $\gamma_{nt2oe}(G) - set$ and is also a neighborhood total dominating set. But $H = \{v_1, v_2\}$ is a neighborhood total dominating set but not a $\gamma_{nt2oe}(G) - set$.

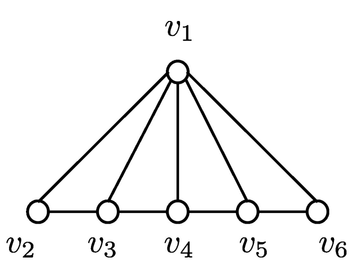

Figure 3. Example for remark 2.4

3 Neighborhood Total 2 - Out Degree Equitable Domination ($\gamma_{nt2oe}(G)$) Number for Some Standard Graphs

Here the neighborhood total 2 - ODED number is obtained for some standard graphs.

Corollary 3.1. *For all graph G with n vertices,* $2 \leq \gamma_{nt2oe}(G) \leq n-2$

Theorem 3.2. *For any star graph* $K_{1,n},\ \gamma_{nt2oe}(K_{1,n}) = n-2$.

Proof. We have $V(G) = \{v, v_1, v_2, \dots, v_n\}$ be the vertices of $K_{1,n}$.

Take the dominating set $D = \{v, v_1, v_2 \dots v_{n-2}\}$ and $(V-D) = \{v_{n-1}, u_n\}$.

Now, $od_D(v) = |N(v) \cap (V-D)| = |\{v_{n-1}, v_n\}| = 2$.

Now, $v_i \in D$, then $od_D(v_i) = |N(v_i) \cap (V-D)| = |\emptyset| = 0$.

Then $|od_D(v) - od_D(v_i)| = 2$.

For any $v_i \in D$, D is a 2 - ODED set and $N(D) = V$. Since $< V >$ has no isolated vertices then $< N(D) >$ has no vertices of degree zero. So D is a minimum neighborhood total 2 - ODED set.

Hence $\gamma_{nt2oe}(K_{1,n}) = n-2$. □

Theorem 3.3. *For any complete graph* K_n*,* $\gamma_{nt2oe}(K_n) = 2$.

Proof. We have $V = \{v_1, v_2, \dots, v_n\}$ be the vertices of K_n, and take a dominating set $D = \{v_1, v_2\}$ and $(V-D) = \{v_3, v_4, \dots, v_n\}$

Now $v_1 \in D$ then $od_D(v_1) = |N(v_1) \cap (V-D)| = |\{v_3, v_4, \dots, v_n\}| = n-2$.

Similarly, $od_D(v_2) = n-2$. Then $|od_D(v_1) - od_D(v_2)| \leq 2$. Hence D is a 2 - ODED.

Also for all $v_i \in D$ then $N(D) = N(v_1) \cup N(v_2) = \{v_1, v_2, \dots, v_n\} = V$, then $< N(D) >= V$ and $< V >$ has no vertices of degree zero in K_n.

Hence $\gamma_{nt2oe}(K_n) = 2$. □

Theorem 3.4. *For any complete bipartite graph,*

$$\gamma_{nt2oe}(K_{p,q}) = \begin{cases} 2, & if\ |p-q| \leq 2 \\ not\ exists, & otherwise \end{cases}$$

Proof. Let $V(K_{p,q}) = \{u_1, u_2, u_3, \dots, u_p, u_{p+1}, u_{p+2}, u_{p+3}, \dots, u_{p+q}\}$ be the vertices of $K_{p,q}$ and $\{u_1, u_2, u_3, \dots, u_p\}$ and $\{u_{p+1}, u_{p+2}, u_{p+3}, \dots, u_{p+q}\}$ be two subset of $V(G)$.

When $|\mathbf{p} - \mathbf{q}| \leq \mathbf{2}$

Take a dominating set $D = \{u_i, u_{p+i}\}$ and

$(V-D) = \{u_1, u_2, u_3, \dots,\ u_{i-1}, u_{i+1}, - \dots, u_p, u_{p+1}, u_{p+2}, \dots, u_{p+i-1}, u_{p+i+1} \dots, u_{p+q}\}$.

For any $u_i \in D$, then $od_D(u_i) = |N(u_i) \cap (V-D)| = q-1$.

If $v_j \in D$, then $od_D(v_j) = |N(v_j) \cap (V-D)| = p-1$.

Then $|od_D(u_i) - od_D(v_j)| \leq 2$.

For any $u_i, v_j \in D$, D is a 2 - ODED set. Here $N(D) = V(G)$ then $< N(D) >$ has no vertices of degree zero in $K_{p,q}$.

Hence $\gamma_{nt2oe}(K_{p,q}) = 2$. □

Theorem 3.5. *For any cycle* C_n*,*

$$\gamma_{nt2oe}(C_n) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor, & n \equiv 3(mod 4) \\ \left\lceil \frac{n}{2} \right\rceil, & otherwise \end{cases}$$

Proof. Let $V(C_n) = \{v_1, v_2, \dots v_n\}$ be the vertex set and D be a 2 - ODED set of C_n.

Let $D_1 = \begin{cases} D, & \text{if } n \equiv 0(mod 4) \\ D \cup \{v_n\}, & \text{if } n \equiv 1 or 2(mod 4) \\ D \cup \{v_{n-1}\}, & \text{if } n \equiv 3(mod 4) \end{cases}$

Clearly, D_1 is a neighborhood total 2 - ODED set of C_n,

$$< N(D) >= \begin{cases} C_n & if n \equiv 0(mod 4) \\ p_{n-1} & otherswise. \end{cases}$$

Then $< N(D) >$ has no isolated vertices.

Hence, $\gamma_{nt2oe}(C_n) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor & n \equiv 3(mod 4) \\ \left\lceil \frac{n}{2} \right\rceil & otherwise. \end{cases}$. □

Theorem 3.6. *For any Path P_n, $\gamma_{nt2oe}(P_n) = \left\lceil \frac{n}{2} \right\rceil$.*

Proof. Let $P_n = \{v_1, v_2, v_3, \dots, v_n\}$.

Case 1: $\mathbf{n \not\equiv 1(mod4)}$

Then $D = \{v_j,\ j = 2k,\ 2k+1$ and k is odd$\}$. Since G is a path, then $deg(v) \leq 2$, clearly D is a ODED set and $N < D >$ has no verticesof degree zero.

Hence D is a neighborhood total 2 - ODED set.

Case 2: $\mathbf{n \equiv 1(mod4)}$

Then $D_1 = D \cup v_{n-1}$ is a neighborhood total 2 - ODED set.

Hence $\gamma_{nt2oe}(P_n) \leq \lceil \frac{n}{2} \rceil$. Since, $\gamma_{nt}(G) = \lceil \frac{n}{2} \rceil$ and $\gamma_{nt}(G) \leq \gamma_{nt2oe}(G)$.

Thus $\gamma_{nt2oe}(P_n) = \lceil \frac{n}{2} \rceil$. □

Theorem 3.7. *For the double star, $S_{p,q}$, $\gamma_{nt2oe}(S_{p,q}) = p + q$.*

Proof. Consider $D = \{u_1, u_2, u_3 \dots, u_m, v_1, v_2, v_3, \dots, v_m\}$ and $(V - D) = \{u, v\}$. Since each vertex of D are pendant vertices, which are adjacent with one vertex in $(V - D)$. Clearly D is a 2 - ODED set and $N(u_i) = u$ and $N(v_i) = v$ for all i.

Now, $< N(D) >=< u, v >=< (V - D) >$ has no isolated vertices.

Hence $\gamma_{nt2oe}(S_{p,q}) = p + q$. □

Theorem 3.8. *For Combo graph $P_n{}^+$, $\gamma_{nt2oe}\left(P_n{}^+\right) = n$.*

Proof. Let $V\left(P_n{}^+\right) = \{v_1, v_2 v_3, \dots, v_n, v_{n1}, v_{n2}, v_{n3}, \dots, v_{nn}\}$ be the vertex set.

Here $\{v_1, v_2, v_3, \dots, v_n\}$ be the vertices of the path, $\{v_{n1}, v_{n2}, v_{n3}, \dots, v_{nn}\}$ be the pendant vertex which is adjacent with the vertex of the path $\{v_1, v_2, v_3, \dots, v_n\}$.

Take $D = \{v_{n1}, v_{n2}, v_{n3}, \dots, v_{nn}\}$ be a minimal dominating set and $(V - D) = \{v_1, v_2, v_3, \dots, v_n\}$. Each vertices of path v_{ni} is adjacent with one vertex in D and other in $(V - D)$.

Then $od_D(v_{ni}) = |N(v_{ni}) \bigcap (V - D)| = 1$, for all $i = 1, 2, 3, \dots, n$, and $|od_D(v_{ni}) - od_D(v_j)| \leq 2$. Then D is a 2 - ODED set and $< N(D) > = (V - D)$ has no isolated vertices.

Hence D is a minimal neighborhood total 2 - ODED set.

Hence, $\gamma_{nt2oe}\left(P_n{}^+\right) = |D| = n$. □

Theorem 3.9. *For any crown graph C_n^+ of order n, $\gamma_{nt2oe}(C_n^+) = n$.*

Proof. Let $V(G) = \{v_1, v_2, v_3, \dots, v_n, v_1', v_2', v_3', \dots, v_n'\}$ be the vertices of crown graph C_n^+.

Here $\{v_1, v_2, v_3, \dots, v_n\}$ be the vertices of cycle C_n, $\{v_1', v_2', v_3' \dots\dots .v_n'\}$ be the pendant vertices which is adjacent to $\{v_1, v_2, v_3, \dots, v_n\}$ respectively.

Take $D = \{v_{n+1}, v_{n+2} \dots, v_{2n}\}$ be the minimal dominating set and $(V - D) = \{v_1', v_2', v_3', \dots, v_n'\}$.

Now $od_D(v_{n+i}) = |N(v_{n+i}) \cap (V - D)| = |\{v_i\}| = 1$. Then, $|od_D(v_i) - od_D(v_j)| = 0 < 2$.

Then D is a 2 - ODED set and the sub graph $< N(D) >= C_n$ has no isolated vertices. Then D is a minimal neighborhood total 2 - ODED set.

Hence $\gamma_{nt2oe}(C_n^+) = |D| = n$. □

Theorem 3.10. *For any path $P_2 \times P_n$, $\gamma_{nt2oe}(P_2 \times P_n) = n$, for $n \geq 2$.*

Proof. The vertex set $V(P_2 \times P_p) = \begin{cases} (v1, v1), (v1, v2), (v1, v3), & \dots, (v1, vn) \\ (v2, v1), (v2, v2), (v2, v3), & \dots, (v2, vn) \end{cases}$ containing $2n$ vertices. Let dominating set $D = \{(v_1, v_1), (v_1, v_2), (v_1, v_3), \dots, (v_1, v_n)\}$ and $(V - D) = \{(v_2, v_1), (v_2, v_2), (v_2, v_3), \dots, (v_2, v_n)\}$.

Let $u = (v_1, v_j) \in D,\ j = 1, 2, 3, \dots, n$

If $u = (v_1, v_1)$,

$$od_D(u) = |N(u) \cap (V - D)| = |(v_2, v_1)| = 1$$

If $u = (v_1, v_n)$,

$$od_D(u) = |N(v_1, v_n) \cap (V - D)| = |\ (v_2, v_m)| = 1$$

If $u = (v_1, v_j)$; $j = 2, 3, \dots, n-1$, then $od_D(v_1, v_j) = |N(v_1, v_j) \cap (V - D)| = |(v_1, v_{j-1}), (v_1, v_{j+1}), (v_2, v_j) \cap (V - D)| = |(v_2, v_j)| = 1$. Then $|od_D(u) - od_D(v)| \leq 2$ and $< N(D) >$ has no isolated vertices and D is a minimal neighborhood total 2 - ODED set.

Hence $\gamma_{nt2oe}(P_2 \times P_n) = n$ for $n \geq 2$.

□

Corollary 3.11. *For any path $P_n \times P_2$, $\gamma_{nt2oe}(P_n \times P_2) = n$ for $n \geq 2$.*

Theorem 3.12. *For any Fan graph $F_{1,n-1}$*

$$\gamma_{nt2oe}(F_{1,n-1}) = \begin{cases} 2, & \text{if } n = 2,3 \\ n-2, & \text{if } n \geq 4. \end{cases}$$

Proof. Let the vertex set of Fan graph $F_{1,n-1}$. be $V = \{v, v_1, v_2, \dots, v_{n-1}\}$.

Case 1: $\mathbf{n = 2, 3}$

If $n = 2$, it is clear that, $\gamma_{nc2oe}(F_{1,1}) = 2$.

If $n = 3$, let the vertices of Fan graph $F_{1,2}$ be $V = \{u, v_1, v_2\}$.

Take $D = \{v, v_1\}$ and $(V - D) = \{v_2\}$.

Clearly, $od_D(u) = 1$ and $od_D(v_1) = 1$.

Then $|od_D(u) - od_D(v_1)| = 0 < 2$, so D is a minimal 2 - ODED set and $< N(D) >$ has no isolated vertices. Then $\gamma_{nt2oe}(F_{1,2}) \leq 2$ and $2 \leq \gamma_{nt2oe}(F_{1,2})$

Hence $\gamma_{nt2oe}(F_{1,2}) = 2$.

Case 2: If $\mathbf{n \geq 4}$,

Take $D = \{u, v_1, v_2, \dots v_{n-3}\}$ and $(V - D) = \{v_{n-2}, v_{n-1}\}$.

Now $od_D(u) = |N(u) \cap (V - D)| = |\{v_{n-2}, v_{n-1}\}| = 2$.

For $1 \leq i \leq n-4$, $od_D(v_i) = |N(v_i) \cap (V - D)| = 0$. Also $od_D(v_{n-3}) = |N(v_{n-3}) \cap (V - D)| = 1$. Clearly, $|od_D(u) - od_D(v)| \leq 2$, for all $u, v \in D$. Here D is a minimum 2 - ODED set and $< N(D) >$ has no isolated vertices.

Hence $\gamma_{nt2oe}(F_{1,n-1}) = n - 2$. □

Theorem 3.13. *For any triangular snake graph T_n^+, $\gamma_{nt2oe}(T_n^+) = n - 1$.*

Proof. The graph T_n^+ contains $2n - 1$ vertices and $n - 1$ triangles. The upper vertices are labeled from v_1 to v_{n-1} and the lower vertices are labeled form v_n to v_{2n-1}. Let $D = \{v_1, v_2, \dots, v_{n-1}\}$ and $(V - D) = \{v_n, v_{n+1}, v_{n+2}, v_{n+3}, \dots, v_{2n-2}, v_{2n-1}\}$.

The vertex v_1 is adjacent to $\{v_n, v_{n+1}\}$. The vertex v_2 is adjacent to $\{v_{n+1}, v_{n+2}\}$ and the vertex v_{n-1} is adjacent to $\{v_{2n-2}, v_{2n-1}\}$. Hence the set $\{v_{n+1}, v_{n+3}, \dots, v_{2n-2}\}$ is a minimum dominating set.

Now,

$$od_D(v_1) = |N(v_1) \cap (V - D)| = |\{v_n, v_{n+1}\}| = 2,$$

$$od_D(v_2) = |N(v_2) \cap (V - D)| = |\{v_{n+1}, v_{n+2}\}| = 2$$

and

$$od_D(v_{n-1}) = |N(v_{n-1}) \cap (V - D)| = |\{v_{2n-2}, v_{2n-1}\}| = 2.$$

Hence $|od_D(u) - od_D(v)| \leq 2 for\ all\ u, v \in D$, and D is a minimum 2 - ODED set. Clearly $< N(D) >= < (V - D) >$ and $(V - D)$ has no isolated vertices. Thus D is a minimum neighborhood total 2 - ODED set.

Hence $\gamma_{nt2oe}(T_n^+) = n - 1$. □

Theorem 3.14. *For any double triangular snake graph $D(T_n^+)$, $\gamma_{nt2oe}(D(T_n^+)) = n + 1$.*

Proof. Let $V(D(T_n^+)) = \{v_1, v_2, v_3, \dots, v_{n+1}, v_1', v_2', v_3', \dots, v_n', v_1'', v_2'', v_3'' \dots\dots v_n''\}$ be the vertex set.

Here $\{v_1, v_2, v_3, \dots, v_{n+1}\}$ be the vertices of path P_n. From path P_n, join v_i and v_{i+1} to a new vertex v_i' by edges $v_i v_i'$ and $v_{i+1} v_i'$, for $i = 1, 2, 3, \dots, n$ and join v_i and v_{i+1} to a new vertex v_i'' by edges $v_i v_i''$ and $v_{i+1} v_i''$, for $i = 1, 2, 3 \dots, n$.

Take the vertices of path P_n, $D = \{v_1, v_2, v_3 \dots\dots v_{n+1}\}$ and $(V - D) = \{v_1', v_2', v_3', \dots v_n', v_1'', v_2'', v_3'' \dots\dots v_n''\}$. Clearly D is a dominating set.

Now

$$od_D(v_i) = |N(v_i) \cap (V - D)| = |\{v_i', v_i''\}| = 2, \text{ for } i = 1, 2, \dots, n + 1$$

and

$$od_D(v_i) = |N(v_i) \cap (V - D)| = |\{v_{i-1}', v_i', v_{i-1}'', v_i''\}| = 4, \text{ for } i = 2 \dots n.$$

Hence, $|od_D(v_i) - od_D(v_j)| \leq 2$ for all $v_i, v_j \in D$ and D is a minimum 2 - ODED set. Clearly $< N(D) >=< V >$ and $< N(D) >$ has no isolated vertices. So D is a minimum neighborhood total 2 - ODED set.

Hence $\gamma_{nt2oe}(D(T_n^+)) = n + 1$. □

Theorem 3.15. *For any bistar graph $B^2_{p,q}$, $\gamma_{nt2oe}\left(B^2_{p,q}\right) = 2$.*

Proof. Consider Bistar $B_{p,q}$ with vertices $\{v, v_1, v_2, v_3, \ldots, v_p, v', v_1', v_2', v_3', \ldots, v_q'\}$, where v_i, v_i' are pendant vertices, v and v' are connected, v_i, v_i' are adjacent to v and v' respectively.

Take $D = \{v, v'\}$ and $(V - D) = \{v_1, v_2, v_3 \ldots\ldots v_p, v_1', v_2', v_3' \ldots .v_q'\}$. Clearly D is a dominating set.

Now

$$od_D(v) = |N(v) \cap (V - D)| = p + q$$

and

$$od_D(v') = |N(v') \cap (V - D)| = p + q.$$

Hence, $|od_D(u) - od_D(v)| = 0 \leq 2$ and D is a minimum 2 - ODED set . Clearly $< N(D) >=< V > \; and \; < N(D) >$ has no isolated vertices. So D is a minimum neighborhood total 2 - ODED set.

Hence $\gamma_{nt2oe}\left(B^2_{p,q}\right) = 2$. □

Theorem 3.16. *For any semi total point of path $T_2(P_n), \gamma_{nt2oe}(T_2(P_n)) = n$.*

Proof. Let the path P_n having the vertices $v_1, v_2, v_3 \ldots, v_n$ and the edges $e_1, e_2, e_3 \ldots, e_{n-1}$.

To construct $T_2(P_n)$ from path P_n join v_i and v_{i+1} to a new vertex w_i by edges $e'_{2i-1} = v_i w_i$ and $e'_{2i} = v_{i+1} v_i'$, for $i = 1$ to n.

Take $D = \{v_1, v_2, \ldots, v_n\}$ and $(V - D) = \left\{v_1', v_2', \ldots, v'_{n-1}\right\}$. The vertex v_1 is adjacent to $\{v_2, v_1'\}$ and the vertex v_i is adjacent to $\{v_{i-1}, v_i'\}$ and the vertex v_n is adjacent to $\{v_{n-1}, v_n'\}$. Hence the set D is a minimum dominating set.

Now,

$$od_D(v_1) = |N(v_1) \cap (V - D)| = |\{v_1'\}| = 1$$

$$od_D(v_i) = |N(v_2) \cap (V - D)| = \left|\left\{v'_{i-1}, v_i'\right\}\right| = 2$$

and

$$od_D(v_n) = |N(v_n) \cap (V - D)| = |\{v_n'\}| = 2.$$

Hence $|od_D(u) - od_D(v)| \leq 2$ for all $u, v \in D$ and D is a minimum 2 - ODED se set. Clearly $< N(D) >=< (V - D) >$and $(V - D)$ has no isolated vertices.

Therefore D is a minimum neighborhood total 2 - ODED set. Thus $\gamma_{nt2oe}(T_2(P_n)) = n$. □

Theorem 3.17. *For any semi total point of cycle $T_2(C_n)$, $\gamma_{nt2oe}(T_2(C_n)) = n$.*

Proof. Let the cycle C_n having the vertices $v_1, v_2, v_3 \ldots v_n$ and the edges $e_1, e_2, e_3 \ldots e_{n-1}$.

To construct $T_2(C_n)$ from cycle C_n join v_i and v_{i+1} to a new vertex v_i' by edges $e'_{2i-1} = v_i v_i'$ and $e'_{2i} = v_{i+1} v_i'$, for i from 1 to n.

Take $D = \{v_1', v_2', \ldots, v_n'\}$ and $(V - D) = \{v_1, v_2, \ldots, v_n\}$. The vertex w_i is adjacent to v_i, v_{i+1}. Hence, the set D is a minimum dominating set.

Now, $od_D(v_i') = |N(v_i') \cap (V - D)| = |\{v_i, v_{i+1}\}| = 1$, for i from 1 to n.

Hence, $\left|od_D(v_i') - od_D\left(v_j'\right)\right| = 0 \leq 2$ for all $v_i', v_j' \in D$ and D is a minimum 2 - ODED set. Clearly, $< N(D) >= < C_n >$ and C_n has no isolated vertices.

Thus D is a minimum neighborhood total 2 - ODED set.

Hence $\gamma_{nt2oe}(T_2(C_n)) = n$. □

Theorem 3.18. *For any semi total point of star $T_2(K_{1,n}), \gamma_{nt2oe}(T_2(K_{1,n})) = n$.*

Proof. Let the star $K_{1,n}$ having the vertices $v, v_1, v_2, v_3 \ldots v_n$ and the edges $e_1, e_2, e_3 \ldots e_{n-1}$. To construct $T_2(K_{1,n})$ from star $K_{1,n}$ join v and v_i to a new vertex v_i' by edges $e'_{2i-1} = v v_i'$ and $e'_{2i} = v_i v_i'$, for i from 1 to n.

Take $D = \{v_1', v_2', \ldots, v_n',\}$ and $(V - D) = \{v, v_1, v_2, v_3, \ldots, v_n\}$. The vertex w_i is adjacent to $\{v, v_i\}$. Hence the set D is a minimum dominating set.

Now $od_D(w_i) = |N(v_i') \cap (V - D)| = |\{v, v_i\}| = 1$, for i from 1 to n. Hence $|od_D(v_i') - od_D(v_j')| = 0 \leq 2$ for all $v_i', v_j' \in D$, and D is a minimum 2 - ODED set. Clearly $< N(D) >=< K_{1,n} >$ and $K_{1,n}$ has no isolated vertices.

Thus D is a minimum neighborhood total 2 - ODED set. Hence $\gamma_{nt2oe}(T_2(K_{1,n})) = n$.

□

4 Conclusion remarks

In this paper, we introduce a new domination number called the neighbourhood total 2 - ODED number. Also, we investigate the proposed domination number for some general graphs. We would like to extend our research work to include an additional set of graphs and real life applications, as well as to investigate the limitations of the neighbourhood total 2 - out degree equitable domination number.

References

[1] Sahal. A and Mathad.V , *Two-out degree equitable Domination in graphs*, Transactions on Combinatorics, Vol. 2, No. 3, 13–19, (2013).

[2] Berge C, *Theory of graph and its applications*. Methuen London, (1962).

[3] Harray.F, *Graph theory*, Addison-Wesly Reading MA, (1969).

[4] M.S. Mahesh and P. Namasivayam, *Neighborhood Connected two out degree equitable domination number in different graphs*, International Journal of Mathematical Archive, Vol. 6, No. 2, 156–163, (2015).

[5] M.S. Mahesh and P. Namasivayam, *Bounds Of Neighborhood Connected Two Out Degree Equitable Domination Number*, International Journal of Statistics and Applied Mathematics, Vol. 2, No. 3, 6–9, (2017).

[6] Ore O, *Theory of graphs*, Amerarica. Math. Soc. Collog. Publications 38. Providence, (1962).

[7] S. Arumugam, C. Sivagnanam, *Some Integrals Involving -Functions and Laguerre Polynomials*, Opuscula Math., **31**, 519–531, (2011).

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