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ANALYZATION OF INVENTORY REPLENISHMENT APPROACH WITH FOCUS ON LEAD TIME

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Abstract The foremost approach of this work is based on cost components such as holding cost and shortage cost, it examines a probabilistic inventory model with many inventory items where the product demand follows a normal distribution and crashing cost follows an exponential distribution. Reorder points are determined by inventory level because lead times and demand might vary, it is common to maintain safety stock. Lead time is recognized as a stochastic variable or an established constant that may not be controlled. The primary contribution is establishing replenishment tactics, for both single and joint replenishment policies, which result in the least overall cost, based on lead time variation.

1 Introduction

All stock in all manufacturing stages is referred to as "Inventory" in accounting terms, which are current assets. Retailers and manufacturers alike can keep selling or producing goods by holding onto inventory. Most businesses view inventory as a significant asset on their balance sheets, but having too much of it can be a burden. Top-level inventory kinds are merchandise and supplies, finished goods, work-in-progress (WIP), and raw materials. Items that businesses have in stock or may require in the future can be categorized and tracked using these four primary categories. To assist businesses manage their inventory more precisely and effectively, the major categories can be further subdivided.

Lead time can help coordinate production decisions of a supplier and ordering decisions of a retailer. However, the main difficulties for the coordination mechanism of lead times are twofold: First, frequent deviations from the actual duration and the quoted lead time distorts coordination by introducing feedback uncertainty [3].

Lead time management is the act of efficiently scheduling and tracking the amount of time needed to finish or satisfy an order. It entails measuring, analyzing, and cutting down on the amount of time that passes between an order being placed and when the customer receive it [8].

A paradigm change in statistical estimating is represented by maximum likelihood estimation, which provides a method based on probability theory to determine parameter values that maximize the likelihood of observed data. The basis of MLE is the idea of likelihood, which functions as a quantitative indicator of how plausible a set of parameter values is in light of the available data [1].

Inventory control is crucial for many different types of enterprises. By creating regulations that accomplish these goals, the businesses want to obtain the best possible returnon their inventory investment. As a result, the rate of return will be maximized and liquidity and business risk will be decreased. Put another way, cutting down on inventory will result in cheaper costs and higher profitability [7]. Regular and on-going reviews are the two most popular types of inventory replenishment policies. An inventory level check is conducted at regular intervals of T under a periodic-review (R,T) policy. Ordering a quantity R - y raises the inventory level to R if it is currently at level y. The inventory level can be replenished for a quantity Q under a continuous-review (r,Q) policy when it drops below a predetermined level, sometimes referred to as the reorder point r. Many businesses handle multiple item inventories [5]. The main difficulty in maintaining multi-item inventory systems is that certain expenses are incurred collectively. Specifically, for numerous items that are part of any given production batch, purchase order, or shipment, the setup costs in purchasing, production, and transportation are frequently spent jointly.

Joint setups present both an opportunity and a challenge since, by properly coordinating the replenishment of numerous products, scale economies can be used to lower setup costs, cycle inventories, or both. Determining the inventory replenishment strategy for several goods that have a common configuration is known as the joint replenishment problem. When replacing a single item in several locations, joint replenishment is also important [2].

Finding the point at which inventory needs to be added in order to keep it at the ideal level is essential. This point is frequently referred to as Return on Point (ROP) [10]. Lead time refers to the time taken - or allowed for - between the start and completion of an operation or project [9]. The term is commonly used in supply chain management and manufacturing fields. Generally, when the theoretical lead times exhibit an exponential distribution with parameter λ, the long-term mean practical lead time approaches the theoretical lead time's expectation, or $1/\lambda$ [6]. There are differences in the standard deviation between the theoretical and practical lead times. The most popular technique for estimating an exponential distribution's parameter has been Maximum Likelihood Estimation [4].

Regarding the current work, demand is distributed normally, lead time crashing cost is distributed exponentially, and several inventory items are associated with a probabilistic model. Verifying the optimal replenishment strategy that minimises overall costs is the primary contribution. Holding and shortage costs are also the primary emphases; these costs alone should be taken into consideration when determining the optimum replenishment policy

2 Notations and Assumptions

This model espouses the following notations.

D	Average Demand
Q	Optimal Order Quantity
O	Order Cost per Order
P	Purchase Cost per unit
C	Holding Cost Fraction per unit time
S	Shortage Cost per unit
m	Transportation Cost
CC	Crashing cost
$1/\lambda$	Mean Lead Time
K	Safety Stock
T	Total Time Period
r	Reorder Point
TIC	Total Inventory Cost

The following presumptions form the basis of the mathematical model.

- ➢ Demand rate is constant.
- ➢ Safety stock is considered.
- ➢ Replenishment is infinite.
- ➢ Demand follows normal distribution.
- ➢ Crashing cost follows an exponential distribution.
- ➢ Total Time period is fixed.
- ➢ Lead time is varying from product to product.

3 Model Formation

The calculation for every expense is listed below.

- Purchase cost = $P \times D$
- Ordering cost = $O \times \frac{D}{Q}$
- Holding cost = $P \times C \times [\frac{Q}{2} + r - E(x)]$
- Shortage cost = $\frac{D}{Q} \times \int_r^\infty (X - r) f(x) dx$
- Transportation cost = $\times \frac{D}{Q} \times m$
- Crashing cost = $\frac{D}{Q} \times \frac{1}{\lambda}$

$$TIC = (P \times D) + \left(O \times \frac{D}{Q}\right) + PC\left[\frac{Q}{2} + r - E(x)\right] + \frac{D}{Q}\int_r^\infty (X - r) f(x) dx + \left(\frac{D}{Q} \times m\right) + \left(\frac{D}{Q} \times \frac{1}{\lambda}\right) \tag{3.1}$$

The equation above shows the total cost of inventory, including the costs associated with orders, purchases, storage, transportation, shortages, and crashes.

Based on the desired service level and the expected demand during lead time, the reorder point is established in a probabilistic inventory model. The following algorithm should be used before entering the computation to determine the quantity and reorder point.

3.1 Computational Algorithm for finding Quantity & Reorder point

Step 1: Find order quantity using by EOQ formulae.
Step 2: Calculate mean lead time demand.
Mean lead time demand = Average demand per unit $\times$ Lead time.
Step 3: Estimate average demand per unit time.
Average demand per unit time = Total demand / Total time period
Step 4: Calculate Safety stock.
Safety stock = $Z \times \sigma$
Step 4: Determine reorder point.
Reorder point = Mean lead time demand + Safety stock

In this specific instance, Safety stock is defined as a buffer stock held to accommodate for fluctuation in lead time or demand, whereas mean lead time demand is the average demand throughout the lead period. Using a probabilistic model, ascertain the reorder point, calculate the mean lead time demand, and choose the right safety stock level based on the required service level and lead time demand variability. Ascertaining the rate parameter λ of the exponential distribution is the first step to take if the crashing cost is distributed exponentially. It is crucial to remember that if the distribution of crashing costs differs among products, then the value of λ needs to be unique to each product. With the MLE method, the rate parameter λ for the exponential distribution of crashing costs can be found.

3.2 Algorithm for finding crashing cost

Step 1: To calculate the rate parameter, using the MLE technique.
Step 2: Calculating the mean lead time $1/\lambda$ after obtaining the MLE estimate (λ)
Step 3: Determine the cost value of the crash.

4 Numerical Example

This section provides a numerical example of how to use the TIC equation (3.1) from section 3 to get the total inventory cost for both single and joint replenishment policies. This outcome demonstrates which replenishment results in the lowest overall cost. In this example, three products whose replenishment policies minimize costs while also adhere to certain requirements: in a single replenishment policy, each product has a different lead time and reorder point; in a joint replenishment strategy, these parameters are the same for all products, and each product has the same mean and standard deviation demand over a 100-day period. There is an exponential distribution for the crashing cost.

The rate parameter of this distribution is estimated using the MLE and varies depending on the product. Each product has a unique demand, quantity, ordering, holding, purchasing, transportation, and crashing costs. As a result, the mean lead time demand varies depending on the product under a single replenishment policy but remains the same under a joint replenishment policy. The detail information is shown in following Table 1.

Table 1. Inventory Requirements with Product Description

	Product 1	**Product 2**	**Product 3**
Demand	580	330	260
Quantity	96	70	67
Ordering cost per unit	20	15	13
Purchase cost per unit	10	8	6
Holding cost per unit	2.5	2	1.5
Shortage cost per unit	1.79	1.44	1.08
Transportation cost per unit	4	3.2	2.4
Crashing cost per unit	6	4.8	3.6
Mean demand	390	390	390
Standard deviation demand	137.35	137.35	137.35
Reorder point (Single)	32	41	48
Reorder point (Joint)	39	39	39
Lead time (Singe)	3	5	6
Lead time (Joint)	3	3	3
Mean Lead Demand (Single)	17.4	16.5	15.6
Mean Lead Demand (Joint)	35	35	35

The results for holding costs, shortage costs, buy costs, ordering costs, transportation costs, and crashing costs for a specific lead time are shown in the Table 2 below, which also provides the overall inventory costs for single and combined replenishment policies.

Table 2. Result for Product Cost

	Product 1	**Product 2**	**Product 3**	**Total**
Holding cost (Single)	1565	952	593	3110.1
Holding cost (Joint)	1300	624	337.5	2261.5
Shortage cost (Single)	8.7256	5.3007	3.1901	17.2164
Shortage cost (Joint)	8.5061	5.3394	3.2964	17.1419
purchase cost (Single & Joint)	5800	2640	1560	10000
Ordering cost (Single & Joint)	120.8333	70.7142	50.4477	241.9952
Transportation cost (Single & Joint)	24.1666	15.0857	9.3134	48.5657
Crashing cost (Single & Joint)	36.2645	22.6321	13.969	72.8656
TIC (Single)				13490.74
TIC (Joint)				12642.06

The holding costs of single and joint replenishment policies are compared in Figure 1, which indicates that when the demand and standard deviation are the same for all three products, the joint replenishment gives minimum holding costs than single replenishment.

The comparison of the total inventory costs for single and joint replenishment policies is shown in Figure 2, which also demonstrates that the joint replenishment policy provides mini-

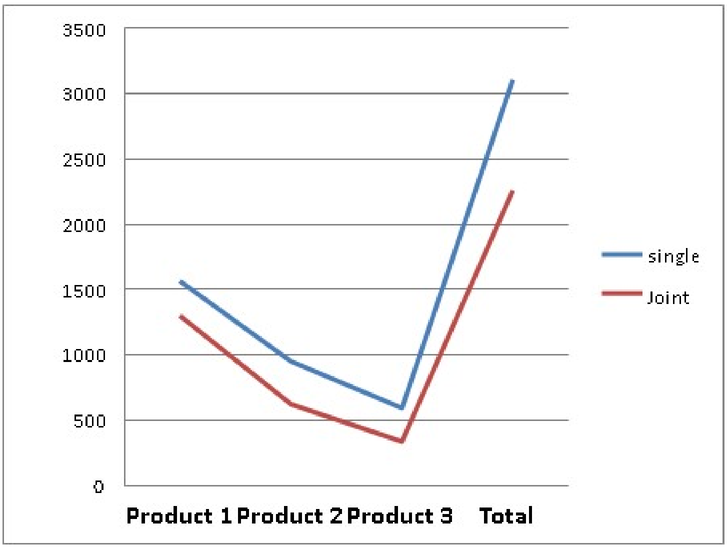

Figure 1. Comparison of Holding Cost

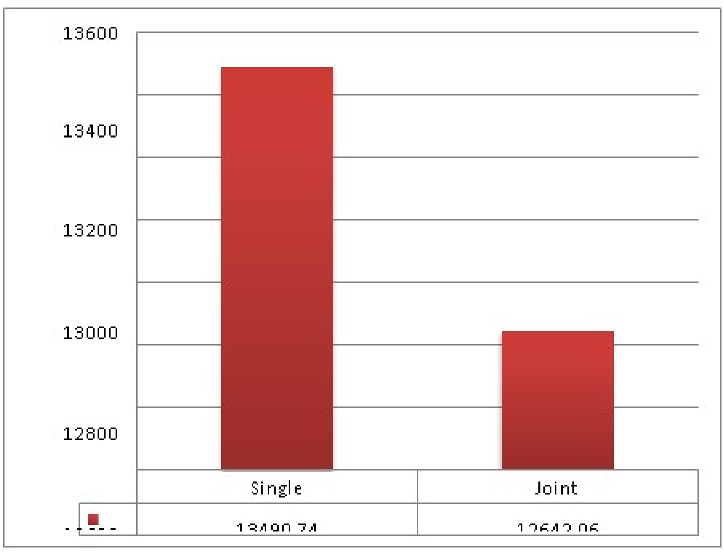

Figure 2. Comparison of Total Inventory Cost

mum total cost than the single replenishment policy.

5 Conclusion

This study considers a probabilistic inventory model that uses several inventory items to assess the replenishment strategy with a lead time focus. The combined replenishment policy minimizes total cost and also minimizes holding and shortage costs based on the numerical result obtained.

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