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Few Results on Fixed Point Theorems in Multiplicative 2-Metric Space

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Abstract In this article, we construct a unique idea of Multiplicative 2-metric space and we broaden few results involving fixed points using asymptotically regular mappings within the context of complete multiplicative 2-metric space.

1 Introduction

Gahler initiated[1] the approach of 2-metric space and several authors employed this space in many directions like asymptotically regular mapping, compatible mapping, weakly commuting mapping, and many more mappings. Browder and Petryshyn [2] were initially presented the idea of asymptotically regular mapping at an element in a space. Bashirov explained the potentiality of multiplicative calculus (exponential calculus) over Newtonian calculus in 2011. Thus, they revealed that multiplicative calculus has high capability than Newtonian calculus for deriving various problems in multiple fields, which leads to provide foundation for multiplicative metric spaces [3]. In 2012, Luc Florack and Hans Van Assen have illustrated the intense usefulness of multiplicative calculus in the field of biomedical image analysis [5].
Our aim is to define a new metric space called Multiplicative 2-metric space (M2MS) in which the existence of unique fixed point (UFP) is proved. Now we recall some preliminaries, which is necessary to establish our key finding.

2 Preliminaries

Definition 2.1. [1] A function $\sigma : M \times M \times M \to [0, \infty)$ upon a non-empty set M is known as 2-metric such that the below conditions holds:

(i) There exists a point $\omega \in M$ for any distinct pairs κ and ζ such that $\sigma(\kappa, \zeta, \omega) \neq 0$

(ii) $\sigma(\kappa, \zeta, \omega) = 0$ if two or more points κ, ζ, ω coincide.

(iii) $\sigma(\kappa, \zeta, \omega) = \sigma(\kappa, \omega, \zeta) = \sigma(\zeta, \omega, \kappa) = \sigma(\zeta, \kappa, \omega), \forall\, \kappa, \zeta, \omega \in M.$

(iv) $\sigma(\kappa, \zeta, \omega) \leq \sigma(\kappa, \zeta, \xi) + \sigma(\kappa, \xi, \omega) + \sigma(\xi, \zeta, \omega), \forall\, \kappa, \zeta, \omega, \xi \in M.$

Then σ is 2-metric and the ordered pair (M, σ) is referred to as 2-metric space.

Definition 2.2. [3] A multiplicative metric $\sigma_m : M \times M \to [1, \infty)$ is a function in a non-empty set M that satisfies the below conditions :

(i) $\sigma_m(\mu, \nu) \geq 1$, for all $\mu, \nu \in M.$

(ii) $\sigma_m(\mu, \nu) = \sigma_m(\nu, \mu)$, for all $\mu, \nu \in M.$

(iii) $\sigma_m(\mu, \nu) \leq \sigma_m(\mu, \eta).\sigma_m(\eta, \nu)$ for all $\mu, \nu, \eta \in M$

Hence the ordered pair (M, σ_m) is called Multiplicative metric space.

Now we define a new extension of metric space called M2MS.

3 Main Results

Definition 3.1. Given a nonempty set M. Define a mapping $\sigma^* : M \times M \times M \to [1, \infty)$ such that the below conditions holds:

(i) For any two elements μ and ν $(\mu \neq \nu)$, $\exists$ a point $\xi \in M$ such that $\sigma^*(\mu, \nu, \xi) \neq 1$.

(ii) $\sigma^*(\mu, \nu, \xi) = 1$ if two or more points μ, ν, ξ coincide.

(iii) $\sigma^*(\mu, \nu, \xi) = \sigma^*(\mu, \xi, \nu) = \sigma^*(\nu, \xi, \mu) = \sigma^*(\nu, \mu, \xi), \forall \mu, \nu, \xi \in M$.

(iv) $\sigma^*(\mu, \nu, \xi) \leq \sigma^*(\mu, \nu, \kappa).\sigma^*(\mu, \kappa, \xi).\sigma^*(\kappa, \nu, \xi), \forall \mu, \nu, \xi, \kappa \in M$.

Then the ordered pair (M, σ^*) is called M2MS.

Example 3.2. Let $M = \{1, 2, 3, 4\}$ and $\sigma^*(\zeta, \eta, \tau) = e^{min\{|\zeta-\eta|, |\eta-\tau|, |\tau-\zeta|\}}$ for all $\zeta, \eta, \tau \in M$. Then σ^* is Multiplicative 2-metric and thus the ordered pair (M, σ^*) is M2MS.

Definition 3.3. Let (M, σ^*) be M2MS. Then a sequence $\{\kappa_n\}$ in M is called 2-multiplicative Cauchy sequence $\Leftrightarrow \lim_{m,n\to\infty} \sigma^*(\kappa_n, \kappa_m, \eta) = 1$.

Definition 3.4. A mapping $T : M \to M$ of M2MS (M, σ^*) is defined to be asymptotically regular at an element $x \in M$, if $\lim_{n\to\infty} \sigma^*(T^n x, T^{n+1} x, z) = 1$, for all $z \in M$.

Definition 3.5. Let (M, σ^*) be M2MS. A mapping $T : M \to M$ is called 2-multiplicative contraction, if $\exists$ a real number $\gamma \in [0, 1)$ that satisfies $\sigma^*(Tx, Ty, z) \leq [\sigma^*(x, y, z)]^\gamma$.

The relation between the 2-metric space and M2MS is given below.

Theorem 3.6. If (M, σ^*) is M2MS along with the mapping $\sigma : M \times M \times M \to [0, \infty)$ defined by $\sigma(x, y, z) = ln(\sigma^*(x, y, z))$, then (M, σ) is a 2-metric space.

Proof. The proof follows from the properties of logarithmic functions. □

Theorem 3.7. Suppose (M, σ) represents a 2-metric space and if we define $\sigma^* : M \times M \times M \to [1, \infty)$ by $\sigma^*(x, y, z) = e^{\sigma(x,y,z)}$, then (M, σ^*) is M2MS.

Proof. The proof follows from the properties of exponential functions. □

Theorem 3.8. A sequence $\{x_n\}$ is a 2-multiplicative Cauchy sequence in M2MS (M, σ^*) iff $\{x_n\}$ is a Cauchy sequence in the corresponding 2-metric space (M, σ).

Proof. Applying the logarithmic function in 2-multiplicative contraction inequality defined in definition (3.5), we obtain
$\sigma(Tx, Ty, z) = ln(\sigma^*(Tx, Ty, z)) \leq \gamma ln(\sigma^*(x, y, z)) \leq \gamma\sigma(x, y, z)$
Note that if (M, σ^*) is a complete M2MS, then the corresponding 2-metric space (M, σ) is also complete. □

Theorem 3.9. Suppose (X, σ^*) is a M2MS, which is complete. If the self mapping $\mathcal{J}$, that is asymptotically regular in some element of the set X holding the condition

$$\sigma^*(\mathcal{J}x, \mathcal{J}y, z) \leq [\sigma^*(x, y, z)]^p.[\sigma^*(x, \mathcal{J}x, z)]^q.[\sigma^*(y, \mathcal{J}y, z)]^r, \tag{3.1}$$

for each values of x, y, z belongs to the set X with $0 \leq p, q, r < 1$, then $\mathcal{J}$ possess UFP in (X, σ^*).

Proof. Given $\mathcal{J}$ is asymptotically regular at an element $x_0 \in X$.
Let $\{\mathcal{J}^n x_0\}$ be a sequence in X defined by $x_n = \{\mathcal{J}^n x_0\}$. Then we obtain

$$\begin{aligned}
\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z) &\leq [\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^{n-1}x_0, z)]^p.[\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^m x_0, z)]^q \\
&\quad .[\sigma^*(\mathcal{J}^{n-1}x_0, \mathcal{J}^n x_0, z)]^r \\
&\leq [\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^{n-1}x_0, \mathcal{J}^m x_0).\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^m x_0, z) \\
&\quad .\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1}x_0, z)]^p.[\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^m x_0, z)]^q \\
&\quad .[\sigma^*(\mathcal{J}^{n-1}x_0, \mathcal{J}^n x_0, z)]^r \\
&\leq [\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^{n-1}x_0, \mathcal{J}^m x_0).\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^m x_0, z) \\
&\quad .\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1}x_0, \mathcal{J}^n x_0).\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z).\sigma^*(\mathcal{J}^n x_0, \mathcal{J}^{n-1}x_0, z)]^p \\
&\quad .[\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^m x_0, z)]^q.[\sigma^*(\mathcal{J}^{n-1}x_0, \mathcal{J}^n x_0, z)]^r
\end{aligned}$$

$$\begin{aligned}
[\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z)]^{(1-p)} &\leq [\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^m x_0, z)]^{(p+q)}.[\sigma^*(\mathcal{J}^{n-1}x_0, \mathcal{J}^n x_0, z)]^{(p+r)} \\
&\quad .[\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^{n-1}x_0, \mathcal{J}^m x_0)]^p.[\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1}x_0, \mathcal{J}^n x_0)]^p
\end{aligned}$$

$$\begin{aligned}
[\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z)] &\leq [\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^m x_0, z)]^{\left(\frac{p+q}{1-p}\right)}.[\sigma^*(\mathcal{J}^{n-1}x_0, \mathcal{J}^n x_0, z)]^{\left(\frac{p+r}{1-p}\right)} \\
&\quad .[\sigma^*(\mathcal{J}^{m-1}x_0, \mathcal{J}^{n-1}x_0, \mathcal{J}^m x_0)]^{\left(\frac{p}{1-p}\right)}.[\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1}x_0, \mathcal{J}^n x_0)]^{\left(\frac{p}{1-p}\right)}
\end{aligned}$$

Since $\mathcal{J}$ is asymptotically regular at x_0, $\lim\limits_{m,n\to\infty} \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z) = 1$. Thus $\{\mathcal{J}^n x_0\}$ is a Cauchy sequence. Then, $\exists u \in X$ such that $\lim\limits_{n\to\infty} \{\mathcal{J}^n x_0\} = u$, because (X, σ^*) is complete. If $\mathcal{J}u \neq u$, then by applying Equation (3.1), we get

$$\begin{aligned}
\sigma^*(\mathcal{J}u, u, z) &\leq \sigma^*(\mathcal{J}u, u, \mathcal{J}^n x_0).\sigma^*(\mathcal{J}u, \mathcal{J}^n x_0, z).\sigma^*(\mathcal{J}^n x_0, u, z) \\
&\leq \sigma^*(\mathcal{J}u, u, \mathcal{J}^n x_0).\sigma^*(\mathcal{J}^n x_0, u, z).[\sigma^*(u, \mathcal{J}^{n-1}x_0, z)]^p.[\sigma^*(u, \mathcal{J}u, z)]^q \\
&\quad .[\sigma^*(\mathcal{J}^{n-1}x_0, \mathcal{J}^n x_0, z)]^r.
\end{aligned}$$

Applying the limit as letting $n \to \infty$, we attain $\sigma^*(\mathcal{J}u, u, z) = [\sigma^*(u, \mathcal{J}u, z)]^q$,
which contradicts $q < 1$ except in the case that $u = \mathcal{J}u$. Hence $\mathcal{J}$ holds the fixed point u. In the case concerning fixed point of $\mathcal{J}$, to demonstrate it's uniqueness, consider $\mathcal{J}$ has two different fixed points d and e. (i.e.,) $\mathcal{J}d = d$ and $\mathcal{J}e = e$. Then by Equation (3.1), we have

$$\begin{aligned}
\sigma^*(d, e, z) &\leq \sigma^*(\mathcal{J}d, \mathcal{J}e, z) \\
&\leq [\sigma^*(d, e, z)]^p.[\sigma^*(d, \mathcal{J}d, z)]^q.[\sigma^*(e, \mathcal{J}e, z)]^r \\
&\leq [\sigma^*(d, e, z)]^p.[\sigma^*(d, d, z)]^q.[\sigma^*(e, e, z)]^r.
\end{aligned}$$

Thus, we get $\sigma^*(d, e, z) \leq [\sigma^*(d, e, z)]^p$, which is contradiction to our assumption. Since $p < 1$, it implies $d = e$. Therefore, the mapping $\mathcal{J}$ holds UFP in a complete M2MS.

□

Theorem 3.10. Given a complete M2MS (X, σ^*). Consider the self mapping $\mathcal{J}$, which is asymptotically regular in some element of the set X satisfying the condition

$$\sigma^*(\mathcal{J}x, \mathcal{J}y, z) \leq [\sigma^*(x, y, z)]^a.[\sigma^*(x, \mathcal{J}x, z)]^b.[\sigma^*(y, \mathcal{J}y, z)]^c.[\sigma^*(x, \mathcal{J}y, z).\sigma^*(\mathcal{J}x, y, z)]^d, \tag{3.2}$$

for each values of x, y, z belongs to the set X with $0 \leq a, b, c, d, a + 2d \leq 1$.
Then $\mathcal{J}$ has a UFP in a complete M2MS (X, σ^*).

Proof. Given $\mathcal{J}$ is asymptotically regular at an element $x_0 \in X$. Let $\{\mathcal{J}^n x_0\}$ be a sequence in X defined by $x_n = \mathcal{J}^n x_0$. Thus we obtain

$$
\begin{aligned}
\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z) &\leq \left[\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^{n-1} x_0, z)\right]^a . \left[\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^m x_0, z)\right]^b \\
&\quad . \left[\sigma^*(\mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0, z)\right]^c \left[\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^n x_0, z) . \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, z)\right]^d \\
&\leq [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^m x_0) . \sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^m x_0, z) \\
&\quad . \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, z)]^a . \left[\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^m x_0, z)\right]^b \\
&\quad . \left[\sigma^*(\mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0, z)\right]^c . \left[\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^n x_0, z) . \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, z)\right]^d \\
&\leq [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^m x_0) . \sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^m x_0, z) \\
&\quad . \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0) . \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z) . \sigma^*(\mathcal{J}^n x_0, \mathcal{J}^{n-1} x_0, z)]^a \\
&\quad . \left[\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^m x_0, z)\right]^b . \left[\sigma^*(\mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0, z)\right]^c \\
&\quad . \left[\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^n x_0, z) . \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, z)\right]^d
\end{aligned}
$$

$$
\begin{aligned}
[\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z)]^{(1-a)} &\leq [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^m x_0, z)]^{(a+b)} . [\sigma^*(\mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0, z)]^{(a+c)} \\
&\quad . [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^m x_0)]^a . [\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0)]^a \\
&\quad . [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^n x_0, z) . \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, z)]^d .
\end{aligned}
$$

$$
\begin{aligned}
[\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z)]^{(1-a)} &\leq [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^m x_0, z)]^{(a+b)} . [\sigma^*(\mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0, z)]^{(a+c)} \\
&\quad . [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^m x_0)]^a . [\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0)]^a \\
&\quad . [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^n x_0, \mathcal{J}^m x_0) . \sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^m x_0, z) \\
&\quad . \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z) . \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0) \\
&\quad . \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z) . \sigma^*(\mathcal{J}^n x_0, \mathcal{J}^{n-1} x_0, z)]^d
\end{aligned}
$$

$$
\begin{aligned}
[\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z)]^{(1-a-2d)} &\leq [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^m x_0, z)]^{(a+b+d)} . [\sigma^*(\mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0, z)]^{(a+c+d)} \\
&\quad . [\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0)]^{(a+d)} \\
&\quad . [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^m x_0)]^a . [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^n x_0, \mathcal{J}^m x_0)]^d
\end{aligned}
$$

$$
\begin{aligned}
[\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z)] &\leq [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^m x_0, z)]^{\left(\frac{a+b+d}{1-(a+2d)}\right)} . [\sigma^*(\mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0, z)]^{\left(\frac{a+c+d}{1-(a+2d)}\right)} . \\
&\quad . [\sigma^*(\mathcal{J}^m x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0)]^{\left(\frac{(a+d)}{1-(a+2d)}\right)} \\
&\quad . [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^{n-1} x_0, \mathcal{J}^m x_0)]^{\left(\frac{a}{1-(a+2d)}\right)} \\
&\quad . [\sigma^*(\mathcal{J}^{m-1} x_0, \mathcal{J}^n x_0, \mathcal{J}^m x_0)]^{\left(\frac{d}{1-(a+2d)}\right)} .
\end{aligned}
$$

Since $\mathcal{J}$ is asymptotically regular at x_0, $\lim\limits_{m,n\to\infty} \sigma^*(\mathcal{J}^m x_0, \mathcal{J}^n x_0, z) = 1$. Thus $\{\mathcal{J}^n x_0\}$ is a Cauchy sequence. Then, $\exists w \in X$ such that $\lim\limits_{n\to\infty} \{\mathcal{J}^n x_0\} = w$, because (X, σ^*) is complete. Applying Equation (3.2) with the assumption that w is not a fixed point of $\mathcal{J}$, we get

$$
\begin{aligned}
&\sigma^*(\mathcal{J}w, w, z) \leq \sigma^*(\mathcal{J}w, w, \mathcal{J}^n x_0) . \sigma^*(\mathcal{J}w, \mathcal{J}^n x_0, z) . \sigma^*(\mathcal{J}^n x_0, w, z) \\
&\sigma^*(\mathcal{J}w, w, z) \leq \sigma^*(\mathcal{J}w, w, \mathcal{J}^n x_0) . \sigma^*(\mathcal{J}^n x_0, w, z) . [\sigma^*(w, \mathcal{J}^{n-1} x_0, z)]^a . [\sigma^*(w, \mathcal{J}w, z)]^b \\
&\qquad . [\sigma^*(\mathcal{J}^{n-1} x_0, \mathcal{J}^n x_0, z)]^c . [\sigma^*(w, \mathcal{J}^n x_0, z) . \sigma^*(\mathcal{J}w, \mathcal{J}^{n-1} x_0, z)]^d .
\end{aligned}
$$

Applying the limit as letting $n \to \infty$, we get $\sigma^*(\mathcal{J}w, w, z) = [\sigma^*(\mathcal{J}w, w, z)]^{b+d}$ which contradicts $b, d < 1$ unless $w = \mathcal{J}w$. Hence w is the fixed point of $\mathcal{J}$. Inorder to determine the uniqueness, consider $\mathcal{J}$ has two different fixed points r and s. (i.e.,) $\mathcal{J}r = r$ and $\mathcal{J}s = s$. Then, by Equation (3.2), we have

$$\begin{aligned}\sigma^*(r, s, z) &\leq \sigma^*(\mathcal{J}r, \mathcal{J}s, z)\\ &\leq [\sigma^*(r, s, z)]^a.[\sigma^*(r, \mathcal{J}r, z)]^b.[\sigma^*(s, \mathcal{J}s, z)]^c.[\sigma^*(r, \mathcal{J}s, z).\sigma^*(\mathcal{J}r, s, z)]^d\\ &\leq [\sigma^*(r, s, z)]^a.[\sigma^*(r, r, z)]^b.[\sigma^*(s, s, z)]^c.[\sigma^*(r, s, z).\sigma^*(r, s, z)]^d.\end{aligned}$$

Thus, $\sigma^*(r, s, z) \leq [\sigma^*(r, s, z)]^{a+2d}$, which is a contradiction to our assumption. Since $a + 2d < 1$, it implies $r = s$. Therefore, the mapping holds the fixed point which is unique. So, the mapping $\mathcal{J}$ holds UFP in a complete M2MS. □

4 Conclusion

In this article, we have defined a unique notion of Multiplicative 2 - metric space (M2MS) and obtained few fixed point results involving the uniquely existed fixed point, using self mapping asymptotically regular function, in the complete M2MS. Thus, various fixed point results can be obtained in the space of M2MS, where any contraction mapping consistently possess the existence together with uniqueness of fixed point, in the case of complete M2MS.

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