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EXTREME DISCONNECTEDNESS OF $\mathfrak{F}_{\mathfrak{p}}^*$ MIXED SPACE

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Abstract *The concept of frame is a generalisation of the concept of category of open subsets of topological space. As a result, each frame acts as an open set in this context and the Pythagorean fuzzy set is defined as a frame. The primary goal of this research unit is to study extremally disconnected spaces in a $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space. To define $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space, Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^*$ structure is introduced using Pythagorean fuzzy frames. This $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space is the combination of Pythagorean fuzzy topological spaces and Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^*$ structure. Various properties of the $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space are discussed in this article.*

1 Introduction

One of the most significant changes in science and mathematics in the twenty-first century has been the notion of uncertainty. This scientific movement marks a progressive departure from the traditional perspective, which views uncertainty as undesirable in research and advocates for its avoidance at all costs. Instead, it embraces an alternative viewpoint that is accepting of uncertainty, recognizing that science cannot entirely eliminate it. In 1965, Zadeh invented the concept of fuzzy sets to mathematically express ambiguity and attempted to tackle such difficulties by assigning a specified grade of membership to each member of a given set.A fuzzy set is mathematically defined by assigning a value to each possible member of the universal set, representing the extent to which that member belongs to the fuzzy set. Subsequently, Atanassov [2] introduced the non-membership function. As an extension of intuitionistic fuzzy set Yager [23]introduced Pythagorean fuzzy sets and Pythagorean fuzzy topological spaces were introduced by Olgun et al [15]. In this article, these Pythagorean fuzzy sets are said to be frame using few conditions. A frame is analogous to a category of open subsets in a space that is potentially more general than a topological space. This is effectively defined as anything with a collection of open subsets that operates essentially like topological space open subsets. So far, research has highlighted the notion of frame category. The Pythagorean fuzzy set is used in this endeavour to envision frames. In this novel method, frames are seen as open sets in Pythagorean fuzzy logic. Using these frames Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^*$ structure space is defined. Further, $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space is defined using Pythagorean fuzzy topological space and Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^*$ structure space. This study is to investigate $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected space in $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space. Various features and relationships between specified spaces are examined and proven using frames.

2 Review of Literature

The term frame was introduced by Duffin and Schaeffer [5] in non-harmonic fourier series. Later Dowker and Papert [4] first studied frames in topology. He defined the complete lattice as the open subsets of a topological space[3]. They also established that in frame theory, or topology without points, the non-tautological assertion of point-set topology may be proven. Structured *frames has been studied by Frith[8]. He established the category of uniform frames and quasi uniform frames. He also investigated the links between various frames. Later paracompactness is studied using frames by Pultr and Ulehla [18]. This study defined frames as paracompact and properties of paracompact frames were examined. This study also proved the frames are normal. Pultr [19] also studied the characterisation of paracompactness. Closure and compactness of frames were also studied by Masuret[13]. Rajesh and Thrivikraman [20] investigated frames in fuzzy and intuitionisitcs fuzzy contexts. Various properties of fuzzy frames and intuitionistic frames were discussed in this study. Later Lattice valued fuzzy frames(L-Frames) were discussed by El-Saady[6]. In this research, the notion of L-fuzzy sub-frames within a particular ordinary frame was introduced, drawing a parallel to the definition of L-fuzzy topological spaces as it relates to L-topological spaces. Some L-fuzzy sub-frame characteristics are studied. This work suggested the existence of L-fuzzy sub-frames of a given ordinary frame, in the same manner as L-fuzzy topological spaces were defined in relation to L-topological spaces. Some L-fuzzy sub-frame characteristics are evaluated. Later fuzzy frames were studied via fuzzy posets Yao [24]. Yao's intention was to define an L-frame using an L-ordered set that included more restrictions. All of these works illustrate how frames have been investigated in a variety of circumstances. Frames are explored in fraction dense space in this article. Various authors investigated about the extremally disconnected spaces Noiri [14] studied on extremally disconnected spaces using locally s-closed and weakly hausdorff spaces. Then Ghosh [9] extremally disconnected spaces using fuzzy sets. Petricevic [17] studied extremally disconnected spaces using nets and filters in fuzzy topology. Keskin et al [10] studied extremally disconnected spaces in ideal topological spaces. Revathi et al. [21] devised ordered (r,s) intuitionistic fuzzy quasi-uniform regular* G_δ *sets and also demonstrated the establishment of an extremally disconnected space within them. Likewise many studies [1] [22] were undergone in extremally disconnected spaces. Similarly many studies were undergone on Pythagorean fuzzy topological spaces. Connectedness is also [7] has been studied in Pythagorean fuzzy topological spaces. As a new approach extremally disconnected space in* $\mathfrak{F}_{p^*}$ *mixed space. The property* $\mathfrak{B}$ *was introduced by Maki et al [12] generalised open and pre open sets.*

3 Motivation of the study

Expansion	***Abbreviation***
Pythagorean fuzzy set	*PFS*
Pythagorean fuzzy topological space	*PFTS*
Pythagorean fuzzy open set	*PFOS*
Pythagorean fuzzy closed set	*PFCS*
Pythagorean fuzzy frame	*PFF*
Pythagorean fuzzy $\mathfrak{F}_p^*$ *structure*	$PF\mathfrak{F}_p^*S$
Pythagorean fuzzy $\mathfrak{F}_p^*$ *structure space*	$PF\mathfrak{F}_p^*SS$
Pythagorean fuzzy $\mathfrak{F}_p^*$ *open set*	$PF\mathfrak{F}_p^*OS$
Pythagorean fuzzy $\mathfrak{F}_p^*$ *closed set*	$PF\mathfrak{F}_p^*CS$

Table 1: Nomenclature of this study

This study introduces a novel approach to extreme disconnectedness by defining it within the mixed space $\mathfrak{F}_p^*$.

(i) Pythagorean fuzzy frames is defined. Then Pythagorean fuzzy $\mathfrak{F}_\mathfrak{p}^*$ *structure space (*$PF\mathfrak{F}_\mathfrak{p}^*SS$*) is established.*

(ii) $\mathfrak{F}_\mathfrak{p}^*$ *mixed space is introduced and it is the combination of PFTS and* $PF\mathfrak{F}_\mathfrak{p}^*SS$.

(iii) Extreme disconnected $\mathfrak{F}_\mathfrak{p}^*$ *mixed space is defined. Then various properties of* $\mathfrak{F}_\mathfrak{p}^*$ *mixed space is explored.*

4 Preliminaries

The basic definition for this study is discussed in this section.

Definition 4.1. [11] Let $\mathfrak{Y}$ be a universal set and $\mathfrak{P}(\mathfrak{Y})$ be the power set of the partially ordered elements is called lattice that can be ordered by the set inclusion S in which the meet (greatest lower bound, infimum) and join (least upper bound, supremum) of any pair of sets $P, Q \in \mathfrak{P}(\mathfrak{Y})$ is given by $P \cap Q$ and $P \cup Q$, respectively.

Definition 4.2. [3] A set $\mathcal{L}$ is a partially ordered set(poset) with a relation $\leqq$, such that

(i) if $u \leqq v$ and $v \leqq u$ then $u \leqq v$ and

(ii) if $u \leqq v$ and $v \leqq w$ then $u \leqq w$ where $u, v, w \in P$ and P is the subset of the lattice $\mathcal{L}$

Definition 4.3. [3] A complete lattice is the poset, if every subset P of $\mathcal{L}$ has the least upper bound which is unique and generally termed as the join of P and represented as $\bigvee P$. In terms of components, $\bigvee p_\alpha$ or $p_1 \vee p_2$.

Definition 4.4. [18] A frame constitutes a complete lattice $\mathcal{L}$ that follows the distributivity law: $(\bigvee \mathcal{P}) \wedge q = \bigvee\{p \wedge q | p \in P\}$ for every subset $\mathcal{P} \subseteq \mathcal{L}$ and any $q \in \mathcal{L}$.

Definition 4.5. [15] A Pythagorean fuzzy set (PFS) R of $\mathfrak{Y} \neq 0$ is a pair (μ_R, ν_R) where μ_R and ν_R are fuzzy sets of $\mathfrak{Y}$ in which ${\mu_R}^2\,(x) +\ {\nu_R}^2\,(x) \leq 1$ for any $x \,\epsilon\, \mathfrak{Y}$ where the fuzzy set μ_R, ν_R is the degree of belongingness and non-belongingness respectively.

Definition 4.6. [15] Let τ be a family of PFS of $\mathfrak{Y} \neq \emptyset$. If

(i) $0_{\mathfrak{Y}},\ 1_{\mathfrak{Y}} \,\epsilon\, \tau$

(ii) $R_i \subset\ \tau$, we have $\bigcup\ R_i \,\epsilon\, \tau$ where I is an index set .

(iii) $R_1, R_2 \,\epsilon\, \tau$, we have $R_1 \bigcap\ R_2 \,\epsilon\, \tau$, where $0_Y\ =\ (0, 1)$ and $1_Y = (1, 0)$, then τ is called a Pythagorean fuzzy topology(PFT) on $\mathfrak{Y}$. Then $(\mathfrak{Y}, \tau)$ is the Pythagorean fuzzy topological space (PFTS). Each member in the PFTS is the Pythagorean fuzzy open set (PFOS). The complement of PFOS is called Pythagorean fuzzy closed set (PFCS).

Definition 4.7. [15] Let $S = (\mu_S, \nu_S)$ and $R = (\mu_R, \nu_R)$ be PFSs of a set $\mathfrak{Y}$. Then,

(i) $R \bigcup S = (max(\mu_R, \mu_S), min(\nu_R, \nu_S))$

(ii) $R \bigcap S = (min(\mu_R, \mu_S), max(\nu_R, \nu_S))$

(iii) $R^c = (\nu_R, \mu_R)$

(iv) $R \subset S$ or $S \supset R$ if $\mu_R \leq\ \mu_S$ and $\nu_R \geq \nu_S$.

Definition 4.8. [15] Let $(\mathfrak{Y}, \tau)$ be a PFTS and $R = (\mu_R, \nu_R)$ be a PFS in $(\mathfrak{Y}, \tau)$. Then the Pythagorean fuzzy interior and Pythagorean fuzzy closure are specified by,

(i) $int(R)$= $\bigcup\{G|G$ is a PFOS in $(\mathfrak{Y}, \tau\}$ and $G \subseteq R\}$

(ii) $cl(R)$= $\bigcap\{K|K$ is a PFCS in $(\mathfrak{Y}, \tau)\}$ and $R \subseteq K\}$

Definition 4.9. [16] Let $(\mathfrak{Y}, \tau)$ be a PFTS and $\mathcal{V}, \mathcal{V}_1, \mathcal{V}_2$ be PFSs over $(\mathfrak{Y}, \tau)$. Then it possesses the subsequent characteristics:

(i) $int(\mathcal{V}) \subseteq \mathcal{V}$

(ii) $int(int(\mathcal{V})) = int(\mathcal{V})$

(iii) $\mathcal{V}_1 \subseteq \mathcal{V}_2 \Rightarrow int(\mathcal{V}_1) \subseteq int(\mathcal{V}_2)$

(iv) $int(\mathcal{V}_1 \cap \mathcal{V}_2) = int(\mathcal{V}_1) \cap int(\mathcal{V}_2)$

(v) $int(1_{\mathfrak{Y}}) = 1_{\mathfrak{Y}}, int(0_{\mathfrak{Y}}) = 0_{\mathfrak{Y}}$

Definition 4.10. [16] Let $(\mathfrak{Y}, \tau)$ be a PFTS and $\mathcal{T}, \mathcal{T}_1, \mathcal{T}_2$ be PFSs over $(\mathfrak{Y}, \tau)$. Then it possesses the subsequent characteristics:

(i) $\mathcal{T} \supseteq cl(\mathcal{T})$

(ii) $cl(cl(\mathcal{T})) = cl(\mathcal{T})$

(iii) $\mathcal{T}_1 \subseteq \mathcal{T}_2 \Rightarrow cl(\mathcal{T}_1) \subseteq cl(\mathcal{T}_2)$

(iv) $cl(\mathcal{T}_1 \cup \mathcal{T}_2) = cl(\mathcal{T}_1) \cup cl(\mathcal{T}_2)$

(v) $cl(1_{\mathfrak{Y}}) = 1_{\mathfrak{Y}}, cl(0_{\mathfrak{Y}}) = 0_{\mathfrak{Y}}$

5 Pythagorean fuzzy frame (PFF)

In this section PFF and Pythagorean fuzzy $\mathfrak{F}_{p^*}$ structure space (PF$\mathfrak{F}_{\mathfrak{p}}^*$SS) are defined using PFFs and the properties are explored.

Definition 5.1. Let $\mathfrak{F}$ be the frame in $\mathfrak{Y}$, then the PFS $P = (\mu_P(y), \nu_P(y), y \in \mathfrak{F})$ is said to be PFF in $\mathfrak{F}$, if it fulfills the subsequent conditions:

(i) $\mu_P(\vee S) \supseteq inf\{\mu_P(p)|p \in S\}$
$\nu_P(\vee S) \subseteq sup\{\nu_P(p)|p \in S\}$ for every arbitrary $S \subset \mathfrak{F}$.

(ii) $\mu_P(p \wedge q) \supseteq min\{\mu_P(p), \nu_P(p)\}$
$\nu_P(p \wedge q) \subseteq max\{\mu_P(p), \nu_P(p)\}$for every $p, q \in \mathfrak{F}$.

(iii) $\mu_P(e_{\mathfrak{F}}) = \mu_P(O_{\mathfrak{F}}) \supseteq \mu_P(p)$
$\nu_P(e_{\mathfrak{F}}) = \nu_P(O_{\mathfrak{F}}) \subseteq \nu_P(p)$ for all $p \in \mathfrak{F}$ where $e_{\mathfrak{F}}$ and $O_{\mathfrak{F}}$ are unit and zero element of the frame $\mathfrak{F}$.

Example 5.2. Let $\mathfrak{F} = \{\mathfrak{Y}, \emptyset, \{m, n\}, \{n, l\}, \{l\}\}$ on $\mathfrak{Y}$ where $\mathfrak{Y} = \{l, m, n\}$ be the frame and $P = (\mu_P(y), \nu_P(y), y \in \mathfrak{F})$ where $\mu_P(\mathfrak{Y}) = \mu_P(\emptyset) = 1_{\mathfrak{Y}}, \mu_P(\{m, n\}) = 0.2, \mu_P(\{n, l\}) = 0.4, \mu_P(\{n\}) = 0.5$, $\nu_P(\mathfrak{Y}) = \nu_P(\emptyset) = 0_{\mathfrak{Y}}, \nu_P(\{m, n\}) = 0.7, \mu_P(\{n, l\}) = 0.7, \mu_P(\{n\}) = 0.3$ is a PFF of $\mathfrak{F}$.

Definition 5.3. Let $\mathfrak{F}$ be the frame of any $\mathfrak{Y} \neq 0$ and let $\mathfrak{F}_{\mathfrak{p}}^*$ be a collection of PFFs. If this collection satisfies the following axioms

(i) $0_{\mathfrak{Y}}$,$1_{\mathfrak{Y}} \in \mathfrak{F}_{\mathfrak{p}}^*$

(ii) for any $\varrho_1, \varrho_2 \in \mathfrak{F}_{\mathfrak{p}}^*$, where have $\varrho_1 \cap \varrho_2 \in \mathfrak{F}_{\mathfrak{p}}^*$

(iii) for any $\{\varrho_i\}_{i \in I} \in \mathfrak{F}_{\mathfrak{p}}^* \cup \varrho_i \in \mathfrak{F}_{\mathfrak{p}}^*$, then $(\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^*)$ is called Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^*$structure space (PF$\mathfrak{F}_{\mathfrak{p}}^*$SS). Each member in $(\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^*)$ is Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^*$ open set(PF$\mathfrak{F}_{\mathfrak{p}}^*$OS) and its complement is called Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^*$ closed set(PF$\mathfrak{F}_{\mathfrak{p}}^*$CS).

Example 5.4. Consider the frame $\mathfrak{F} = \{\mathfrak{Y}, \emptyset, \{m, n\}, \{n, l\}, \{l\}\}$. The PFFs $\mathcal{P}, \mathcal{Q}, \mathcal{R}$ are defined as $\mathcal{P} = \{(\mu_{\mathcal{P}}(y), \nu_{\mathcal{P}}(y))|y \in \mathfrak{F}\}$, $\mathcal{Q} = \{(\mu_{\mathcal{Q}}(y), \nu_{\mathcal{Q}}(y))|y \in \mathfrak{F}\}$, $\mathcal{R} = \{(\mu_{\mathcal{R}}(y), \nu_{\mathcal{R}}(y))|y \in \mathfrak{F}\}$
where,$\mu_{\mathcal{P}}(\mathfrak{Y}) = \mu_{\mathcal{P}}(\emptyset) = 1_{\mathfrak{Y}}, \mu_P(\{m, n\}) = 0.2, \mu_P(\{c, a\}) = 0.4, \mu_P(\{c\}) = 0.5$
$\nu_{\mathcal{P}}(\mathfrak{Y}) = \nu_{\mathcal{P}}(\emptyset) = 0_{\mathfrak{Y}}, \nu_P(\{m, n\}) = 0.7, \nu_P(\{n, l\}) = 0.7, \nu_P(\{n\}) = 0.3$
$\mu_{\mathcal{Q}}(\mathfrak{Y}) = \mu_{\mathcal{Q}}(\emptyset) = 1_{\mathfrak{Y}}, \mu_Q(\{b, c\}) = 0.2, \mu_Q(\{n, l\}) = 0.5, \mu_Q(\{n\}) = 0.3$
$\nu_{\mathcal{Q}}(\mathfrak{Y}) = \nu_{\mathcal{Q}}(\emptyset) = 0_{\mathfrak{Y}}, \nu_Q(\{b, c\}) = 0.7, \nu_Q(\{n, l\}) = 0.5, \nu_Q(\{n\}) = 0.6$
$\mu_{\mathcal{R}}(\mathfrak{Y}) = \mu_{\mathcal{R}}(\emptyset) = 1_{\mathfrak{Y}}, \mu_R(\{b, c\}) = 0.6, \mu_R(\{n, l\}) = 0.4, \mu_R(\{n\}) = 0.3$
$\nu_{\mathcal{R}}(\mathfrak{Y}) = \nu_{\mathcal{R}}(\emptyset) = 0_{\mathfrak{Y}}, \nu_R(\{m, n\}) = 0.3, \nu_R(\{n, l\}) = 0.5, \nu_R(\{n\}) = 0.3$.
Therefore the collection of PFFs $\mathfrak{F}_{\mathfrak{p}}^* = \{0_{\mathfrak{Y}}, 1_{\mathfrak{Y}}, \mathcal{P}, \mathcal{Q}, \mathcal{R}\}$. Then the structure space $(\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^*)$ is a PF$\mathfrak{F}_{\mathfrak{p}}^*$SS.

Definition 5.5. Let $(\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^*)$ is a PF$\mathfrak{F}_{\mathfrak{p}}^*$SS.Then Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^*$ closure and Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^*$ interior of a Pythagorean fuzzy set is specified by,
$cl_{\mathfrak{F}_{\mathfrak{p}}^*}(M) = \bigcap\{N : M \subseteq N; N$ is $PF\mathfrak{F}_{\mathfrak{p}}^*$CS in $(\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^*)\}$
$int_{\mathfrak{F}_{\mathfrak{p}}^*}(M) = \bigcup\{K : K \subseteq M; K$ is $PF\mathfrak{F}_{\mathfrak{p}}^*$OS in $(\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^*)\}$

Proposition 5.6. *Let $\beth$ be a PFF of a PF$\mathfrak{F}_{\mathfrak{p}}^{*}$S $(\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})$. Then it possesses the subsequent characteristics:*

(i) $[int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\beth)]^{c} = cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\beth)^{c}$.

(ii) $[cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\beth)]^{c} = int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\beth)^{c}$.

Proof. (i) $[int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\beth)]^{c} = [\bigcup\{\beta : \beta^{c} \in (\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})\beta \subseteq \beth\}]^{c}$
$= \bigcap\{\beta^{c} \in (\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})\beta \subseteq \beth\}$
$= \{\alpha : \alpha \in (\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})\alpha \supseteq \beth^{c}\}$
$= cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\beth^{c})$ where $\alpha = \beta^{c}$.

(ii) $[cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\beth)]^{c} = [\subset \{\beta : \beta^{c} \in (\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})\beta \supseteq \beth\}]^{c}$
$= \bigcup\{\beta^{c} \in (\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})\beta \supseteq \beth\}$
$= \{\alpha : \alpha \in (\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})\alpha \subseteq \beth^{c}\}$
$= int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\beth^{c})$ where $\alpha = \beta^{c}$. □

Proposition 5.7. *Let $(\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})$ be a PF$\mathfrak{F}_{\mathfrak{p}}^{*}$S. For PFF $\mathcal{K}$ and $\mathcal{Q}$ in $(\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})$, then it possesses the subsequent characteristics*

(i) If $\mathcal{K}^{c} \in (\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})$ then, $cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K}) = \mathcal{R}$ and if $\mathcal{K} \in (\mathfrak{Y}, \mathfrak{F}_{\mathfrak{p}}^{*})$ then, $int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K}) = \mathcal{K}$.

(ii) $cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\emptyset) = \emptyset$, $cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathfrak{Y}) = \mathfrak{Y}$ and $int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\emptyset) = \emptyset$, $int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathfrak{Y}) = \mathfrak{Y}$.

(iii) If $\mathcal{K} \subset \mathcal{Q}$ then $cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K}) \subset cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K})$ and $int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K}) \subset int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{Q})$.

(iv) $\mathcal{K} \subset cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K})$ and $int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K}) \subset \mathcal{K}$.

(v) $cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K})) = \mathcal{K}$ and $int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K})) = \mathcal{K}$.

Proof. The proof is simple. □

Definition 5.8. A Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^{*}$ structure (PF$\mathfrak{F}_{\mathfrak{p}}^{*}$S) on a non-empty set has characteristic $\mathfrak{B}$ if the union of any of subsets in the PF$\mathfrak{F}_{\mathfrak{p}}^{*}$S belong to PF$\mathfrak{F}_{\mathfrak{p}}^{*}$S.

Definition 5.9. Let $\mathfrak{Y} \neq 0$ and $\mathfrak{F}_{\mathfrak{p}}^{*}$ be a PF$\mathfrak{F}_{\mathfrak{p}}^{*}$S on $\mathfrak{Y}$. For any subset $\mathcal{K}$ on $\mathfrak{Y}$, then it possesses the subsequent characteristics

(i) $\mathcal{K} \in \mathfrak{F}_{\mathfrak{p}}^{*}$ if and only if $int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K}) = \mathcal{K}$.

(ii) $\mathcal{K}$ is PF$\mathfrak{F}_{\mathfrak{p}}^{*}$CS if and only if $cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K}) = \mathcal{K}$.

(iii) $int_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K}) \in \mathfrak{F}_{\mathfrak{p}}^{*}$ and $cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K})$ is PF$\mathfrak{F}_{\mathfrak{p}}^{*}$CS.

A PFTS $(\mathfrak{Y}, \varsigma)$ with a $\mathfrak{F}_{\mathfrak{p}}^{*}$ structure on $\mathfrak{Y}$ is called a $\mathfrak{F}_{\mathfrak{p}}^{*}$ mixed space and it is denoted by $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^{*})$.

6 Characterizations of $\mathfrak{F}_{\mathfrak{p}}^{*}$ extremally disconnected space

In this section extremally disconnected is defined in the Pythagorean fuzzy topological space and in $\mathfrak{F}_{\mathfrak{p}}^{*}$ mixed space. Various the properties are examined.

Definition 6.1. A PFTS $(\mathfrak{Y}, \varsigma)$ is called extremally disconnected if the closure of every PFOS $\mathcal{P}$ of $(\mathfrak{Y}, \varsigma)$ is PFOS.

Definition 6.2. A PF$\mathfrak{F}_{\mathfrak{p}}^{*}$S is called Pythagorean fuzzy $\mathfrak{F}_{\mathfrak{p}}^{*}$ hyperconnected if every non-empty PF$\mathfrak{F}_{\mathfrak{p}}^{*}$OS is Pythagorean fuzzy dense in PF$\mathfrak{F}_{\mathfrak{p}}^{*}$S.

Definition 6.3. A $\mathfrak{F}_{\mathfrak{p}}^{*}$ mixed space $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^{*})$ is called $\mathfrak{F}_{\mathfrak{p}}^{*}$ extremally disconnected (resp.$\mathfrak{F}_{\mathfrak{p}}^{*}$ hyperconnected) if $cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K}) \in \varsigma$ (resp.,$cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(\mathcal{K}) = 1_{\mathfrak{Y}}$) for each $\mathcal{K} \in \varsigma$.

Example 6.4. Consider the mixed space $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^{*})$ where $\mathfrak{X} = \{a, b, c\}$. Let $\mathcal{K}, \mathcal{Q}, \mathcal{R}$ be the Pythagorean fuzzy sets defined by $\mathcal{P} = \{(0.3, 0.4), (0.5, 0.6), (0.4, 0.4)\}$,
$\mathcal{Q} = \{(0.6, 0.3), (0.6, 0.2), (0.7, 0.4)\}$ and $(\mathfrak{Y}, \varsigma) = \{0_{\mathfrak{X}}, 1_{\mathfrak{Y}}, \mathcal{K}, \mathcal{Q}\}$ be the PFTS on $\mathfrak{Y}$. Let PF$\mathfrak{F}_{\mathfrak{p}}^{*}$S=$\{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathcal{R}, \mathcal{S}, \mathcal{T}\}$ where $\mathcal{R}, \mathcal{S}, \mathcal{T}$ be the PFF on the PF$\mathfrak{F}_{\mathfrak{p}}^{*}$ and it is defined by $\mathcal{R} = \{(0.4, 0.3), (0.6, 0.5), (0.4, 0.4)\}$,$\mathcal{S} = \{(0.3, 0.6), (0.2, 0.6), (0.4, 0.7)\}$,
$\mathcal{T} = \{(0.2, 0.7), (0.8, 0.2), (0.7, 0.2)\}$. $\mathcal{K}$ is PFOS in $(\mathfrak{Y}, \varsigma)$. Then $cl_{\mathfrak{F}_{\mathfrak{p}}^{*}}(K)$ is PFOS. Hence $\mathfrak{Y}$ is $\mathfrak{F}_{\mathfrak{p}}^{*}$ extremally disconnected.

Proposition 6.5. *Let $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^*)$ be a $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space. Then the subsequent characteristics hold*

(i) If $\mathfrak{Y}$ is hyperconnected, then $\mathfrak{Y}$ is $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected.

(ii) If $\mathfrak{F}_{\mathfrak{p}}^*$=PFOS, then $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^*)$ is $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected.

(iii) Let $(\mathfrak{Y}, \varsigma)$ be extremally disconnected. If $\mathfrak{F}_{\mathfrak{p}}^*$=PFOS, then $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^*)$ is $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected.

Proof. (i) Let $\mathfrak{Y}$ be $\mathfrak{F}_{\mathfrak{p}}^*$ hyperconnected. Consider a PFOS $\mathcal{K}$ in $\mathfrak{Y}$. According to the definition of $\mathfrak{F}_{\mathfrak{p}}^*$ hyperconnected $\mathcal{K}$ is Pythagorean fuzzy dense in $\mathfrak{Y}$. Therefore $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K}) = 1_{\mathfrak{Y}}$. Hence $\mathfrak{Y}$ is $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected.
(ii) Let PF$\mathfrak{F}_{\mathfrak{p}}^*$S= PFOS and consider a PFOS $\mathcal{K}$ then the $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K})$ is PFOS, then $(\mathfrak{X}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^*)$ is $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected.
(iii) Let $(\mathfrak{Y}, \varsigma)$ be a extremally disconnected. Let PF$\mathfrak{F}_{\mathfrak{p}}^*$S= PFOS, then $cl(\mathcal{K})$ is PF$\mathfrak{F}_{\mathfrak{p}}^*$OS then $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^*)$ is $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected. □

Proposition 6.6. *Let $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^*)$ be a $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space. Then the subsequent characteristics are equivalent*

(i) $\mathfrak{Y}$ is $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected.

(ii) $int_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K})$ is PFCS for every $\mathcal{K}$ of $\mathfrak{Y}$.

(iii) $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(int(\mathcal{K})) \subseteq int(cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K}))$ for every subset of $\mathcal{K}$ of $\mathfrak{Y}$.

Proof. $(i) \Rightarrow (ii)$ Consider $\mathcal{K}$ be a PFCS in $\mathfrak{Y}$. Then $\mathcal{K}^c$ is PFOS. By (i) $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{P}^c) = [int_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{P})]^c$ By proposition 5.6 $int_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K})$ is closed.
$(ii) \Rightarrow (iii)$ Consider any PFS $\mathcal{K}$ in $\mathfrak{Y}$. And $[int(\mathcal{K})]^c$ is PFCS in $\mathfrak{Y}$ by (ii) $int_{\mathfrak{F}_{\mathfrak{p}}^*}(int(\mathcal{K}))^c$ is closed. Therefore $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(int(\mathcal{K}))$ is PFOS in $\mathfrak{Y}$. Hence $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(int(\mathcal{K})) \subseteq int(cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K}))$.
$(iii) \Rightarrow (i)$ $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(int(\mathcal{K})) \subseteq int(cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K}))$. This implies $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K}) \subseteq int(\mathcal{K})$ by proposition 5.9. Then $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K}) \subseteq \mathcal{K}$. Therefore $\mathfrak{Y}$ is $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected. □

Proposition 6.7. *Let $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^*)$ be a $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space and $\mathfrak{F}_{\mathfrak{p}}^*$ have the property $\mathfrak{B}$. Then the subsequent characteristics are equivalent*

(i) $\mathfrak{Y}$ is $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected.

(ii) $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K}) \in \varsigma$ for any $\mathcal{K} \in \varsigma$ and $\mathcal{Q} \in PF\mathfrak{F}_{\mathfrak{p}}^*S$ such that $\mathcal{K} \cap \mathcal{Q} = 0_{\mathfrak{Y}}$ there exists disjoint PF$\mathfrak{F}_{\mathfrak{p}}^*$CS $\mathcal{S}$ and Pythagorean fuzzy closed set $\mathcal{T}$ such that $\mathcal{K} \subseteq \mathcal{S}$ and $\mathcal{Q} \subseteq \mathcal{T}$.

(iii) $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S}) \cap cl(\mathcal{T}) = 0_{\mathfrak{Y}}$ for every $\mathcal{S} \in \varsigma$ and $\mathcal{T} \in PF\mathfrak{F}_{\mathfrak{p}}^*S, \mathcal{S} \cap \mathcal{T} = 0_{\mathfrak{Y}}$.

(iv) $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(int(cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S}))) \cap cl(\mathcal{T}) = 0_{\mathfrak{X}}$ for every $\mathcal{S} \subseteq \mathfrak{Y}$ and $\mathcal{T} \in PF\mathfrak{F}_{\mathfrak{p}}^*S$ with $\mathcal{S} \cap \mathcal{T} = 0_{\mathfrak{Y}}$.

Proof. $(i) \Rightarrow (ii)$ Let $\mathfrak{Y}$ is extremally disconnected. Let $\mathcal{P}$ and $\mathcal{Q}$ be two disjoint PFOS and PF$\mathfrak{F}_{\mathfrak{p}}^*$OS respectively. Then $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K})$ and $[cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K})]^c$ are disjoint PF$\mathfrak{F}_{\mathfrak{p}}^*$CS and PFCS containing $\mathcal{P}$ and $\mathcal{Q}$ respectively.
$(ii) \Rightarrow (iii)$ Let $\mathcal{S} \in \varsigma$ and $\mathcal{T} \in PF\mathfrak{F}_{\mathfrak{p}}^*S$ with $\mathcal{S} \cap cl(\mathcal{T}) = 0_{\mathfrak{X}}$ By (ii), there exists disjoint PF$\mathfrak{F}_{\mathfrak{p}}^*$CS $\mathcal{U}$ and PFCS $\mathcal{V}$ in which $\mathcal{S} \subseteq \mathcal{U}$ and $\mathcal{T} \subseteq \mathcal{V}$. This implies $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S}) \cap cl(\mathcal{T}) \subseteq \mathcal{U} \cap \mathcal{V} = 0_{\mathfrak{Y}}$. Thus, $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S}) \cap cl(\mathcal{T}) = 0_{\mathfrak{Y}}$.
$(iii) \Rightarrow (iv)$ Let $\mathcal{S} \subseteq \mathfrak{Y}$ and $\mathcal{T} \in PF\mathfrak{F}_{\mathfrak{p}}^*S$ with $\mathcal{S} \cap \mathcal{T} = 0_{\mathfrak{Y}}$. Since $int(cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S})) \in \varsigma$ and $int(cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S})) \cap \mathcal{T} = 0_{\mathfrak{Y}}$, by (iii) $cl_{\mathfrak{F}_{\mathfrak{p}}^*} int(cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S})) \cap cl(\mathcal{T}) = 0_{\mathfrak{Y}}$.
$(iv) \Rightarrow (i)$ Let $\mathcal{S}$ be any PFOS. Then $[cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S})]^c \cup \mathcal{S} = 0_{\mathfrak{Y}}$. Since PF$\mathfrak{F}_{\mathfrak{p}}^*$S has the property $\mathfrak{B}$, $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S}) \in PF\mathfrak{F}_{\mathfrak{p}}^*S$ by (iv) $cl_{\mathfrak{F}_{\mathfrak{p}}^*} int(cl_{\mathfrak{F}_{\mathfrak{p}}^*}\mathcal{S}) \cap cl(cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S})) = 0_{\mathfrak{Y}}$. Since $\mathcal{S} \in \varsigma$, Then $cl_{\mathfrak{F}_{\mathfrak{p}}^*}\mathcal{S} \cap [int(cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S}))]^c = 0_{\mathfrak{Y}}$. Therefore $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S}) \subseteq int(cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S}))$ and $cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{S})$ is open. This shows that $\mathfrak{Y}$ is $\mathfrak{F}_{\mathfrak{p}}^*$ extremally disconnected. □

Definition 6.8. A subset $\mathcal{K}$ of $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space is called R$\mathfrak{F}_{\mathfrak{p}}^*$ open set if $\mathcal{K} = int((cl_{\mathfrak{F}_{\mathfrak{p}}^*}(\mathcal{K}))$. The complement of R$\mathfrak{F}_{\mathfrak{p}}^*$ open set (R$\mathfrak{F}_{\mathfrak{p}}^*$OS) is said to R $\mathfrak{F}_{\mathfrak{p}}^*$ closed set (R$\mathfrak{F}_{\mathfrak{p}}^*$CS).

Proposition 6.9. *Let $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathfrak{p}}^*)$ be a $\mathfrak{F}_{\mathfrak{p}}^*$ mixed space and $PF\mathfrak{F}_{\mathfrak{p}}^*S$ have property $\mathfrak{B}$. Then the subsequent characteristics are equivalent* *(i)* $\mathfrak{Y}$ *is* $\mathfrak{F}_{\mathrm{p}}^{*}$ *extremally disconnected.*

(ii) Every R$\mathfrak{F}_{\mathrm{p}}^{*}$*OS of* $\mathfrak{Y}$ *is PF*$\mathfrak{F}_{\mathrm{p}}^{*}$*CS in* $\mathfrak{Y}$.

(iii) Every R$\mathfrak{F}_{\mathrm{p}}^{*}$*CS of* $\mathfrak{Y}$ *is PF*$\mathfrak{F}_{\mathrm{p}}^{*}$*OS in* $\mathfrak{Y}$.

Proof. $(i) \Rightarrow (ii)$ Let $\mathfrak{Y}$ be $\mathfrak{F}_{\mathrm{p}}^{*}$ extremally disconnected. Let $\mathcal{K}$ be an R $\mathfrak{F}_{\mathrm{p}}^{*}$ open set of $\mathfrak{Y}$. Then $\mathcal{K} = int(cl_{\mathfrak{F}_{\mathrm{p}}^{*}}\mathcal{K}) = cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{K})$ and hence $\mathcal{K}$ is PF$\mathfrak{F}_{\mathrm{p}}^{*}$CS.
$(ii) \Rightarrow (i)$ Suppose that every R$\mathfrak{F}_{\mathrm{p}}^{*}$OS of $\mathfrak{Y}$ is PF$\mathfrak{F}_{\mathrm{p}}^{*}$CS in $\mathfrak{Y}$. Let $\mathcal{K}$ be a PFOS in $\mathfrak{Y}$. Since $int(cl_{\mathfrak{F}_{\mathrm{p}}^{*}}((\mathcal{K})))$ is R$\mathfrak{F}_{\mathrm{p}}^{*}$OS, then it is PF$\mathfrak{F}_{\mathrm{p}}^{*}$CS. This implies that $cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{K}) \subseteq cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(int(cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{K})) = int(cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{K}))$. As $\mathcal{K} \subseteq int(cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{K}))$. Thus $cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{K})$ is open and hence $\mathfrak{Y}$ is $\mathfrak{F}_{\mathrm{p}}^{*}$ extremally disconnected.
$(ii) \Leftrightarrow (iii)$. It is simple and obvious.

□

Proposition 6.10. *Let* $(\mathfrak{Y}, \varsigma, \mathfrak{F}_{\mathrm{p}}^{*})$ *be a* $\mathfrak{F}_{\mathrm{p}}^{*}$ *mixed space and PF*$\mathfrak{F}_{\mathrm{p}}^{*}$ *have property* $\mathfrak{B}$. *Then* $\mathfrak{Y}$ *is* $\mathfrak{F}_{\mathrm{p}}^{*}$ *extremally disconnected if and only if every PFOS* $\mathcal{G}$ *and every PF*$\mathfrak{F}_{\mathrm{p}}^{*}$*CS* $\mathcal{F}$ *with* $\mathcal{G} \subseteq \mathcal{F}$, *there exists of PFOS* $\mathcal{H}$ *and PF*$\mathfrak{F}_{\mathrm{p}}^{*}$*CS* $\mathcal{F}$ *such that* $\mathcal{G} \subseteq \mathcal{K} \subseteq \mathcal{H} \subseteq \mathcal{F}$.

Proof. Suppose $\mathfrak{Y}$ is $\mathfrak{F}_{\mathrm{p}}^{*}$ is extremally disconnected. Let $\mathcal{G}$ be PFOS and $\mathcal{F}$ be a PF$\mathfrak{F}_{\mathrm{p}}^{*}$CS in $\mathfrak{Y}$ in which $\mathcal{G} \subseteq \mathcal{F}$. Then $\mathcal{G} \cap \mathcal{F}^c = 0_{\mathfrak{Y}}$ By Proposition 6.7 $cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{G}) \cap cl(\mathcal{F}^c) = 0_{\mathfrak{Y}}$ that is $cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{G}) \subseteq [cl(\mathcal{F}^c)]^c$. Using $[cl(\mathcal{F}^c)]^c \subseteq \mathcal{F}$, then $cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{G}) = \mathcal{K}, [cl(\mathcal{F}^c)]^c = \mathcal{H}$. Therefore $\mathcal{G} \subseteq \mathcal{K} \subseteq \mathcal{H} \subseteq \mathcal{F}$.
Conversely, let the condition hold, Let $\mathcal{S}$ and $\mathcal{T}$ be a PF$\mathfrak{F}_{\mathrm{p}}^{*}$CS $\mathcal{F}$ such that $\mathcal{S} \subseteq \mathcal{F} \subseteq \mathcal{G} \subseteq \mathcal{T}^c$. This implies that $cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{S}) \cap [int(\mathcal{T}^{\rfloor})]^c = 0_{\mathfrak{Y}}$. But $[int(\mathcal{T}^{\rfloor})]^c = cl(\mathcal{T})$. That is $cl_{\mathfrak{F}_{\mathrm{p}}^{*}}(\mathcal{S}) \cap cl(\mathcal{T}) = 0_{\mathfrak{Y}}$ by Proposition 6.7. $\mathfrak{Y}$ is $\mathfrak{F}_{\mathrm{p}}^{*}$ extremally disconnected. □

7 Conclusion

Among the various extremally disconnected spaces this study gives the different approach to extremally through a mixed space. Here the $\mathfrak{F}_{\mathrm{p}}^{*}$ *mixed space is the combination of PF*$\mathfrak{F}_{\mathrm{p}}^{*}$*S and PFTS and* $\mathfrak{F}_{\mathrm{p}}^{*}$ *extremally disconnected spaces in studied in this* $\mathfrak{F}_{\mathrm{p}}^{*}$ *mixed space. Various characterizations of* $\mathfrak{F}_{\mathrm{p}}^{*}$ *extremally disconnected space is studied. In future, using this* $\mathfrak{F}_{\mathrm{p}}^{*}$ *mixed space topological variants, asymmetry can be investigated. This concept can be extended to the medical field as Devi N [23] studied the Pharamaceutical application using Neutrosophic sets. It is possible to study the Pharmaceutical application using* $\mathfrak{F}_{\mathrm{p}}^{*}$ *mixed space.*

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