Palestine Journal of Mathematics

Vol 14(Special Issue II)(2025) , 318–322

A NOTE ON $\tau^*\beta_I$-OPEN SETS IN IDEAL TOPOLOGICAL SPACES

S. Rajakumar, J. Sebastian Lawrence and B. Ananda Priya

MSC 2010 Classifications: 54C40, 14E20.

Keywords and phrases: β_I-open sets, $\tau^*\beta_I$-open sets, ideal topological spaces, $\tau^*\beta_I$-continuous, $\tau^*\beta_I$-connectedness.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper.

Corresponding Author: S. Rajakumar

Abstract *This paper aims to define and investigate a new class of open sets in ideal topological spaces called $\tau^*\beta_I$-open sets. We have obtained some fresh results accompanied by examples. Examples are provided to demonstrate independent connection with more generalized open sets. In addition, we study and characterize the continuous mappings and connectedness in topological spaces with respect to $\tau^*\beta_I$-open sets.*

1 Introduction

Jankovic and Hamlett [1, 2] developed the idea of ideal topological spaces in 1962. Dunham [3] proposed the idea of $\mathscr{C}\ell^*$ and τ^* in 1990. β-open is a concept that was introduced by Glaisa T. Catalan et al. [4]. Chalice Boonpok [5, 6, 7, 8, 9, 10, 11, 12, 13, 14], Ferit yalaz, Aynur keskin [5, 6, 7, 8, 9, 10, 11, 12, 13, 14] have recently contributed their novel ideas on local and multi continuous functions in topolological spaces. In topological spaces, Pushpalatha et al. [15] created the τ^*g-closed sets and mappings. Approximations of some near open sets in ideal topological spaces have been examined by Nawar, A.S. [16]. The authors explained the latest open set in ideal topological space, known as $\tau^*\beta_I$-open set, using these terms. Using the $\tau^*\beta_I$-open set, a novel method is developed to investigate connectedness, continuous path and independent outcomes in topological spaces.

2 Preliminaries

Readers require a previously specified definition that follows.

Definition 2.1. [1] A non empty family of subsets of a set X is said to be an ideal $\mathscr{I}$ if it satisfies (i) If $\mathscr{I}_1$ and $\mathscr{I}_2$ belongs to $\mathscr{I}$ then $\mathscr{I}_1 \cup \mathscr{I}_2 \in \mathscr{I}$ and (ii) If $\mathscr{I}_1 \in \mathscr{I}$ and $\mathscr{I}_2 \subseteq \mathscr{I}_1$ then $\mathscr{I}_2 \in \mathscr{I}$.

In every part of this paper, the ideal topological space $(X, \tau, \mathscr{I})$ represented as ITS, open set of X as $o_s(X)$, closed set of X as $c_s(X)$, interior of a set A as $int(A)$, closure of a set A as $cl(A)$, cl^* as $\mathscr{C}\ell^*$, $\mathscr{C}\ell^*(int(\mathscr{C}\ell^*(\Gamma))$ as $\mathscr{B}$, $\mathscr{C}\ell^*(int(\mathscr{C}\ell^*(L))$ as $\mathscr{H}$, continuous as Cs, connected as Cd and mapping as Mpg.

Remark 2.2. The definition of β_I-open sets and topology τ^* utilized in this paper and be found in [3] and [4].

3 On $\tau^*\beta_I$-Open Sets in Ideal Topological Spaces

This section examined the idea of $\tau^*\beta_I$-open sets.

Definition 3.1. A subset A of a ITS is called $\tau^*\beta_I$-open if $\exists\ o_s(\Gamma)$ such that

(i) $\Gamma\backslash A \in \mathscr{I}$

(ii) $A\backslash\mathscr{C}\ell^*(int(\mathscr{C}\ell^*(\Gamma))) \in \mathscr{I}$.

Remark 3.2. $\overline{\tau^*\beta_I - o_s(X)} = \tau^*\beta_I - c_s(X)$.

Example 3.3. Let $X = \{l_1, l_2, l_3\}$ with the topology
$\tau = \{\emptyset, \{l_1\}, \{l_3\}, \{l_1, l_3\}, X\}$ and the ideal $\mathscr{I} = \{\emptyset, \{l_3\}\}$.
Here $\tau^* = \{\emptyset, \{l_1\}, \{l_3\}, \{l_1, l_2\}, \{l_1, l_3\}, X\}$.
Then the sets $\{l_1\}, \{l_3\}, \{l_1, l_2\}, \{l_1, l_3\}, X$ and $\emptyset$ are $\tau^*\beta_I - o_s(X)$.

Theorem 3.4. *Every $o_s(X)$ in a ITS is a $\tau^*\beta_I - o_s(X)$.*

Proof. In ITS, let G be any $o_s(X)$. Since every $o_s(X)$ is $\tau^* - o_s(X)$, G is a $\tau^* - o_s(X)$. To prove G is $\tau^*\beta_I$-open set, it is enough to find a $o_s(\Gamma)$ satisfying

(1) $\Gamma\backslash G \in \mathscr{I}$ (2) $G\backslash\mathscr{B} \in \mathscr{I}$. Since G is itself an $\tau^* - o_s(X)$.
Let us take $\Gamma = G$. Also $G = \Gamma\backslash\Gamma = \emptyset \in \mathscr{I}$.
Now $\Gamma \subseteq \mathscr{C}\ell^*(\Gamma) \Rightarrow int(\Gamma) \subseteq int(\mathscr{C}\ell^*(\Gamma))$. Then $\Gamma \subseteq int(\Gamma) \subseteq int(\mathscr{C}\ell^*(\Gamma)) \Rightarrow \mathscr{C}\ell^*(\Gamma) \subseteq \mathscr{B} \Rightarrow \Gamma \subseteq \mathscr{C}\ell^*(\Gamma) \subseteq \mathscr{B}$. So $\Gamma \subseteq \mathscr{C}\ell^*(int(\mathscr{C}\ell^*(M))$. Now $G\backslash\mathscr{B} = \Gamma\backslash\mathscr{B} = \emptyset \in \mathscr{I}$. So, both the conditions are satisfied. Hence G is $\tau^*\beta_I - o_s(X)$. □

Theorem 3.5. *In ITS if $G \in \mathscr{I}$, then G is a $\tau^*\beta_I - o_s(X)$.*

Proof. Let $G \subseteq X$ such that $G \in \mathscr{I}$. Let us take $\Gamma = \emptyset$. Obviously Γ is τ^*-open set. Also $\Gamma\backslash G = \emptyset \in \mathscr{I}$ and $G\backslash\mathscr{B} = G\backslash\emptyset = G \in \mathscr{I}$. Hence G is an $\tau^*\beta_I - o_s(X)$. □

Theorem 3.6. *In ITS if $\mathscr{I} = \{\emptyset\}$, then L is a $\beta_I - o_s(X)$ iff L is a $\tau^*\beta_I - o_s(X)$.*

Proof. Let L be a $\beta_I - o_s(X)$ in X and $\mathscr{I} = \{\emptyset\}$, $\exists\ o_S(\Gamma)$ such that (i) $\Gamma\backslash G \in \mathscr{I}$ and (ii) $G\backslash\mathscr{B} \in \mathscr{I}$. Since $\mathscr{I} = \{\emptyset\}$, $\mathscr{C}\ell^*(L) = \mathscr{C}\ell(L)$. Then $G\backslash\mathscr{H} = G\backslash\mathscr{C}\ell(int(\mathscr{C}\ell(L)))$. By (ii) $G\backslash\mathscr{H} \in \mathscr{I}$. Therefore L is a $\tau^*\beta_I - o_s(X)$.
Conversely, suppose L is a $\tau^*\beta_I - o_s(X)$, $\exists\ o_S(\Gamma)$ such that (i) $\Gamma\backslash G \in \mathscr{I}$ and (ii) $G\backslash\mathscr{B} \in \mathscr{I}$. By the above arguments, $G\backslash\mathscr{B} = G\backslash\mathscr{B}$. From these arguments, we can say that L is a $\tau^*\beta_I - o_s(X)$. □

Theorem 3.7. *If $\mathscr{P}, \mathscr{Q} \in \tau^*\beta_I - o_s(X)$, then $\mathscr{P} \cup \mathscr{Q}$ is a $\tau^*\beta_I - o_s(X)$.*

Proof. Let $\mathscr{P}, \mathscr{Q} \in \tau^*\beta_I - o_s(X)$. Then $\exists$ a $o_S(\Gamma)$ s.t
$\Gamma\backslash\mathscr{P} \in \mathscr{I}$, $\mathscr{P}\backslash\mathscr{B} \in \mathscr{I}$ and $\Gamma\backslash\mathscr{Q} \in \mathscr{I}$, $\mathscr{Q}\backslash\mathscr{B} \in \mathscr{I}$. Since, $\Gamma\backslash\mathscr{P}\cup\mathscr{Q} \subseteq \Gamma\backslash\mathscr{P}\cup\Gamma\backslash\mathscr{Q} \in \mathscr{I}\cup\mathscr{I} \in \mathscr{I}$.
$\mathscr{P}\cup\mathscr{Q}\backslash\mathscr{B} \subseteq \mathscr{P}\backslash\mathscr{B}\cup\mathscr{Q}\backslash\mathscr{B} \in \mathscr{I}\cup\mathscr{I} \in \mathscr{I}$
$\mathscr{P}\cup\mathscr{Q}$ is an $\tau^*\beta_I - o_s(X)$. □

Theorem 3.8. *If $\mathscr{P}, \mathscr{Q} \in \tau^*\beta_I - o_s(X)$, then $\mathscr{P} \cap \mathscr{Q}$ is a $\tau^*\beta_I - o_s(X)$.*

Proof. Let $\mathscr{P}, \mathscr{Q} \in \tau^*\beta_I - o_s(X)$. Then $\exists$ a $o_S(\Gamma)$ s.t
$\Gamma\backslash\mathscr{P} \in \mathscr{I}$, $\mathscr{P}\backslash\mathscr{B} \in \mathscr{I}$ and $\Gamma\backslash\mathscr{Q} \in \mathscr{I}$, $\mathscr{Q}\backslash\mathscr{B} \in \mathscr{I}$. Since $\Gamma\backslash\mathscr{P}\cap\mathscr{Q} \subseteq \Gamma\backslash\mathscr{P}\cap\Gamma\backslash\mathscr{Q} \in \mathscr{I}\cap\mathscr{I} \in \mathscr{I}$
$\mathscr{P}\cap\mathscr{Q}\backslash\mathscr{B} \subseteq \mathscr{P}\backslash\mathscr{B}\cap\mathscr{Q}\backslash\mathscr{B} \in \mathscr{I}\cap\mathscr{I} \in \mathscr{I}$ □

Corollary 3.9. *If $A_1, A_2, \ldots, A_n$ are $\tau^*\beta_I - o_s(X)$ then*

(i) $A_1 \cup A_2 \cup \cdots \cup A_n$ is a $\tau^*\beta_I - o_s(X)$

(ii) $A_1 \cap A_2 \cap \cdots \cap A_n$ is a $\tau^*\beta_I - o_s(X)$

Remark 3.10. In example 3.3, the set $A = \{l_1\}$ and $B = \{l_3\}$ gives their union $A \cup B = \{l_1, l_3\}$ is a $\tau^*\beta_I - o_s(X)$.

Remark 3.11. In example 3.3, the set $A = \{l_1, l_2\}$ and $B = \{l_1, l_3\}$ gives their intersection $A \cap B = \{l_1\}$ is a $\tau^*\beta_I - o_s(X)$.

Example 3.12. Let $X = \{l_1, l_2, l_3\}$, $\tau = \{\emptyset, \{l_1\}, X\}$ and $\mathscr{I} = \{\emptyset, \{l_3\}\}$. Here the set $\{l_1, l_3\}$ is $\tau^*\beta_I - o_s(X)$ but not $\tau^* - o_s(X)$.
Consider another ITS $X = \{l_1, l_2, l_3\}$, $\tau = \{\emptyset, \{l_1\}, X\}$, $\mathscr{I} = \{\emptyset, \{l_1\}\}$. Here the set $\{l_2, l_3\}$ is $\tau^* - o_s(X)$ but not $\tau^*\beta_I - o_s(X)$. Hence $\tau^* - o_s(X)$ and $\tau^*\beta_I - o_s(X)$ are independent.

Example 3.13. Let $X = \{l_1, l_2, l_3\}$, $\tau = \{\emptyset, \{l_1\}, X\}$ and $\mathscr{I} = \{\emptyset, \{l_3\}\}$. Here the set $\{l_1, l_3\}$ is $\tau^*\beta_I - o_s(X)$ but not $\beta_I - o_s(X)$.
Consider another ITS $X = \{l_1, l_2, l_3\}$, $\tau = \{\emptyset, \{l_1\}, \{l_3\}, \{l_1, l_3\}, X\}$, $\mathscr{I} = \{\emptyset, \{l_3\}\}$. Here the set $\{l_2\}$ is $\beta_I - o_s(X)$ but not $\tau^*\beta_I - o_s(X)$.

Example 3.14. Let $X = \{l_1, l_2, l_3\}$, $\tau = \{\emptyset, \{l_1\}, X\}$ and $\mathscr{I} = \{\emptyset, \{l_3\}\}$. Here the set $\{l_3\}$ is $\tau^*\beta_I - o_s(X)$ but not $\beta_I - o_s(X)$.
Consider another ITS $X = \{l_1, l_2, l_3\}$, $\tau = \{\emptyset, \{l_1\}, \{l_3\}, \{l_1, l_3\}, X\}$, $\mathscr{I} = \{\emptyset, \{l_3\}\}$. Here the set $\{l_2\}$ is $\beta_I - o_s(X)$ but not $\tau^*\beta_I - o_s(X)$. Hence $\beta_I - o_s(X)$ and $\tau^*\beta_I - o_s(X)$ are independent.

4 $\tau^*\beta_I$-Continuous Maps in ITS

Throughout this section, same ideal for both domain and co domain in a mapping to be considered.

Definition 4.1. A Mpg $h : (X, \tau_1, \mathscr{I}) \to (Y, \tau_2, \mathscr{I})$ is called $\tau^*\beta_I$-Cs at $x_0 \in X$ iff for each τ^*-open set G containing $h(x_0)$ in $(Y, \tau_2, \mathscr{I})$, $\exists$ an $\tau^*\beta_I - o_s(X)$ in $\mathscr{I} \supseteq x_0$ in $(X, \tau_1, \mathscr{I})$, such that $f(\mathscr{I}) \subseteq G$.

Theorem 4.2. *A Mpg $h : (X, \tau_1, \mathscr{I}) \to (Y, \tau_2, \mathscr{I})$ is $\tau^*\beta_I$-Cs iff every $\tau^* - o_s(Y)$ is $\tau^*\beta_I - o_s(X)$ in $(X, \tau_1, \mathscr{I})$.*

Proof. Suppose $h : (X, \tau_1, \mathscr{I}) \to (Y, \tau_2, \mathscr{I})$ is $\tau^*\beta_I$-Cs. Let x_0 be any element in $(X, \tau_1, \mathscr{I})$ and G be a $\tau^* - o_s(Y)$ containing $h(x_0)$ in $(Y, \tau_2, \mathscr{I})$. Since h is continuous, h is continuous at x_0. Then $\exists$ $\tau^*\beta_I - o_s(\mathscr{I})$ in $\mathscr{I} \supseteq x_0$ in $(X, \tau_1, \mathscr{I})$ such that $h(\mathscr{I}) \subseteq G$. Since $\mathscr{I}$ is $\tau^*\beta_I - o_s(X)$, choose $h^{-1}(G) = \mathscr{I}$. Hence, $h^{-1}(G)$ is $\tau^*\beta_I - o_s(X)$.
Conversely, suppose every $\tau^* - o_s(Y)$ is $\tau^*\beta_I - o_s(X)$. If $x \in X$ then G be a part of $\tau^* - o_s(Y) \supseteq h(x)$. By hypothesis $h^{-1}(G)$ is $\tau^*\beta_I - o_s(X)$, where $h(x) \in G$, $x \in h^{-1}(G)$. Then $h^{-1}(G)$ is $\tau^*\beta_I - o_s(X) \subseteq X$. Also $h(h^{-1}(G)) \subseteq G$. Hence h is $\tau^*\beta_I$-Cs at all points. So h is $\tau^*\beta_I$-Cs. □

Theorem 4.3. *A Mpg $q : (X, \tau_1, \mathscr{I}) \to (Y, \tau_2, \mathscr{I})$ is $\tau^*\beta_I$-Cs iff every $\tau^* - c_s(X)$ and K in $(Y, \tau_2, \mathscr{I})$ is $\tau^*\beta_I - c_s(X)$ in $(X, \tau_1, \mathscr{I})$.*

Proof. Suppose $q : (X, \tau_1, \mathscr{I}) \to (Y, \tau_2, \mathscr{I})$ is $\tau^*\beta_I$-Cs. If K be a part of $\tau^* - c_s(Y)$. Then $Y \backslash K$ is $\tau^* - o_s(Y)$. Since q is $\tau^*\beta_I$-Cs and by Theorem 4.2, $q^{-1}(Y \backslash K)$ is $\tau^*\beta_I - o_s(X)$ in X. That is, $q^{-1}(Y \backslash K) = q^{-1}(Y) \backslash q^{-1}(K) = X \backslash q^{-1}(K)$ is $\tau^*\beta_I - o_s(X)$ in X. Then $q^{-1}(K)$ is $\tau^*\beta_I - c_s(X)$ in X.
Conversely, suppose every $\tau^* - c_s(X)$ is $\tau^*\beta_I - c_s(X)$. Let G be any $\tau^* - o_s(Y)$. Then let $B = Y \backslash G$ is $\tau^*\beta_I - c_s(X)$ in X. But $q^{-1}(B) = q^{-1}(Y \backslash G) = q^{-1}(Y) \backslash q^{-1}(G) = X \backslash q^{-1}(G)$. Then $X \backslash q^{-1}(G)$ is $\tau^*\beta_I - c_s(X)$. This implies, $q^{-1}(G)$ is $\tau^*\beta_I - o_s(X)$. Therefore, we get, every $\tau^* - o_s(Y)$ in Y is $\tau^*\beta_I - o_s(X)$ in X. So, by Theorem 4.2, q is $\tau^*\beta_I$-Cs. □

Theorem 4.4. *Every Cs Mpg is $\tau^*\beta_I$-Cs Mpg.*

Proof. The proof is straight forward by the following theorems.

(1) h is Cs iff $h^{-1}(o_s(X))$ is $o_s(X)$.

(2) Every $o_s(X)$ is a $\tau^* - o_s(X)$.

(3) Every $o_s(X)$ in ITS is a $\tau^*\beta_I - o_s(X)$. □

5 Connectedness with respect to $\tau^*\beta_I - o_s(X)$

In this section, our discussion is about the connectedness, separated sets in ITS with respect to $\tau^*\beta_I - o_s(X)$. Consider the ITS, where τ is any topology and $\mathscr{I}$ is any ideal.

Definition 5.1. Let $\mathscr{P}, \mathscr{B} \subseteq X$. If $\mathscr{P}, \mathscr{B}$ are said to be $\tau^*\beta_I$-Separated sets then $\mathscr{C}\ell^*(\mathscr{P}) \cap \mathscr{B} = \emptyset = \mathscr{P} \cap \mathscr{C}\ell^*(\mathscr{B})$

Definition 5.2. Let $\mathscr{P}, \mathscr{B} \subseteq X$. If $\mathscr{P} \neq$ union of two disjoint $\tau^*\beta_I - o_s(X)$, then $\mathscr{P}$ is said to be $\tau^*\beta_I$-Cd.

Theorem 5.3. *Consider ITS. If $A_1(\neq \emptyset)$, $A_2(\neq \emptyset) \subseteq X$ and $A_1 \cap A_2 = \emptyset$ such that A_1 and A_2 are $\tau^*\beta_I - o_s(X)$, then A_1, A_2 are $\tau^*\beta_I$-Separated sets.*

Proof. Let $A_1(\neq \emptyset)$, $A_2(\neq \emptyset)$ be two disjoint $\tau^*\beta_I$-Open subsets of X. To prove A_1 and A_2 are $\tau^*\beta_I$-Separated sets, by using the definition, it is enough to prove that either $\mathscr{C}\ell^*(A_1) \cap A_2 = \emptyset$ or $A_1 \cap \mathscr{C}\ell^*(A_2) = \emptyset$. Since A_1 is $\tau^*\beta_I$-Open, A_1^C is $\tau^*\beta_I$-Closed. Also, since $A_1 \cap A_2 = \emptyset$, $A_1 \subseteq A_2^C$. This implies $\mathscr{C}\ell^*(A_1) \subseteq A_2^C$. As $A_2^C \cap A_2 = \emptyset$, $\mathscr{C}\ell^*(A_1) \cap A_2 = \emptyset$ by using the $\tau^*\beta_I$-Open set A. □

Theorem 5.4. *If X is $\tau^*\beta_I$-Cd in a ITS then (X, τ) is Cd.*

Proof. Let X is not Cd and $X_1(\neq \emptyset)$, $X_2(\neq \emptyset)$ be two disjoint $o_s(X)$ such that $X = X_1 \cup X_2$. As every open set is $\tau^*\beta_I$-Open, X_1 and X_2 are both $\tau^*\beta_I - o_s(X)$. Since $X = X_1 \cup X_2$, $X_1 = X_2^C$ and $X_2 = X_1^C$, X_1 and X_2 are also both $\tau^*\beta_I$-Closed. Thus $X_1 = \mathscr{C}\ell^*(X_1)$ and $X_2 = \mathscr{C}\ell^*(X_2)$, $\mathscr{C}\ell^*(X_1) \cap X_2 = X_1 \cap X_2 = \emptyset$ and $X_1 \cap \mathscr{C}\ell^*(X_2) = X_1 \cap X_2 = \emptyset$. Therefore, ITS is not Cd, $\Rightarrow\Leftarrow$. □

Theorem 5.5. *Consider ITS and $\mathscr{K}$ be $o_s(X)$. If $\mathscr{H}$ is a $\tau^*\beta_I \subseteq X$, then $\mathscr{H} \cap \mathscr{K}$ is $\tau^*\beta_I$-Open $\subseteq \mathscr{K}$.*

Proof. Let $\mathscr{H} \in \tau^*\beta_I - o_s(X)$. Then $\exists$ an $o_s(X)$ $\mathscr{J}'$ such that $\mathscr{J}' \backslash \mathscr{H} \in \mathscr{I}$ and $\mathscr{H} \backslash \mathscr{C}\ell^*(int(\mathscr{C}\ell^*(\mathscr{J}'))) \in \mathscr{I}$.
Let $\mathscr{J} = \mathscr{J}' \cap \mathscr{K}$. Then

$$\begin{aligned}
\mathscr{J}' \backslash (\mathscr{H} \cap \mathscr{K}) &= \mathscr{J}' \cap (\mathscr{H} \cap \mathscr{K})^C \\
&= (\mathscr{J}' \cap \mathscr{K}) \cap (\mathscr{H}^C \cup \mathscr{K}^C) \\
&= (\mathscr{J}' \cap \mathscr{K} \cap \mathscr{H}^C) \cup (\mathscr{J}' \cap \mathscr{K} \cap \mathscr{K}^C) \\
&= \mathscr{J}' \cap \mathscr{K} \cap \mathscr{H}^C \\
&= (\mathscr{J}' \backslash \mathscr{H}) \cap \mathscr{K} \in \mathscr{I}.
\end{aligned}$$

Also,

$$\begin{aligned}
(\mathscr{H} \cap \mathscr{K}) \backslash \mathscr{C}\ell^*(int(\mathscr{C}\ell^*(\mathscr{J})) &= (\mathscr{H} \cap \mathscr{K}) \backslash \mathscr{C}\ell^*(int(\mathscr{C}\ell^*(\mathscr{J}' \cap \mathscr{K}))) \\
&= (\mathscr{H} \cap \mathscr{K}) \backslash \mathscr{C}\ell^*(int(\mathscr{C}\ell^*(\mathscr{J}'))) \cap \mathscr{K} \\
&= [\mathscr{H} \backslash \mathscr{C}\ell^*(int(\mathscr{C}\ell^*(\mathscr{J}'))) \cap \mathscr{K}] \cap \mathscr{K} \in \mathscr{I}
\end{aligned}$$

Hence, $\mathscr{H} \cap \mathscr{K}$ is $\tau^*\beta_I$-open subset of $\mathscr{K}$. □

6 Conclusion Remarks

In this article a new type of open sets called $\tau^*\beta_I$-open sets is formed in Ideal Topological spaces. The present work is explained about definitions of $\tau^*\beta_I$-open sets with examples and some new theorems with proofs. The result works well in the domain of continuous maps. Finally various forms of connected related results are obtained. The reader may extend the results in Bi topological spaces, Fuzzy Topological spaces, Nano topological spaces, Soft topological spaces and Rough topological spaces in future.

References

[1] *T.R. Hamlett and D. Jankovic,* Ideals in General Topology, *General Topology and Applications, Middletown, CT, 115–125, (1988) SE: Lecture Notes in Pure & Appl. Math.,* **123**, *(1990), Dekker, New York.*

[2] *D. Jankovic and T.R. Hamlett,* New topologies from old via ideals, *The American Mathematical Monthly,* ***97***, *295–310, (1990).*

[3] *W. Dunham,* A new closure operator for non-T1 topologies, *Kyungpook Math. J.,* ***22****(1), 55–60, (1982).*

[4] *Glaisa T. Catalan, Roberto N. Padua and Michael P. Baldado Jr.,* On β-Open Sets and Ideals in Topological Spaces, *European Journal of Pure and Applied Mathematiccs,* ***12****(3), 893–905 (2019).*

[5] *Ferit yalaz and Aynur Keskin,* A new local function and a new compatibility type in ideal topological spac AIMS, *Mathematics,* ***8****(3), 7097–7114 (2023).*

[6] *C. Boonpok,* On Continuous Multifunctions in Ideal Topological Spaces, *Lobachevskii J Math.,* ***40***, *24–35 (2019).*

[7] *Chalice Boonpok,* Upper and lower $\beta(*)$-continuity, *Heliyon,* ***7****(1), e05986 (2021).*

[8] *Chalice Boonpok,* On Characterizations of $*$-Hyperconnected Ideal Topological Spaces, *Journal of Mathematics, 2020, 7 pages, Article ID9387601 (2020).*

[9] *Wadei AL-Omeri and Takashi Noiri,* On semi$*-I$-open sets, pre$*-Ii$-open sets and $e-I$-open sets in ideal topological spaces, ***41***, *1–8, (2023). in press.*

[10] *Powar, P. L T. Noiri and Shikha Bhadauria,* On β-local Functions in ideal topological spaces, *European Journal of Pure and Applied Mathematics,* ***13****(4), 758–765, (2020).*

[11] *Hariwan Z. Ibrahim,* Micro β-open sets in Micro topology, *Gen. Lett. Math.,* ***8****(1), 8–15, (2020).*

[12] *A. Vadivel and C. John Sundar,* Nncβ-open sets, *Advances in Mathematics: Scientific Journal,* ***9****(4), 2203–2207, (2020).*

[13] *A. Vadivel and C. John Sundar,* Some Types of Continuous Function Via N-Neutrosophic Crisp Topological Spaces, *Applications and Applied Mathematics,* ***18****(1), Article 12, 18 pages (2023).*

[14] *A. Vadivel, C. John Sundar and P. Thangaraja,* Nncβ-Continuous maps, *South East Asian Journal of Mathematics & Mathematical Sciences,* ***18****(2), 275–288 (2022).*

[15] *Pushpalatha, S. Easwaran and P. Rajarubi,* τ^*-generalized closed sets in topological spaces, *Proceedings of World Congress & Engineering, London, UK,* ***5*** *(2009).*

[16] *A.S. Nawar,* Approximations of some near open sets in ideal topological spaces, *J Egypt Math Soc.,* ***28****(5), 1–11, (2020).*

Author information

S. Rajakumar, Sri Manakula Vinayagar Engineering College (Autonomous), Puducherry - 605107, India.
E-mail: srkumar277@gmail.com

J. Sebastian Lawrence, SRM TRP Engineering College, Irungalur, Tiruchirappalli - 621 105, Tamil nadu, India.
E-mail: lawrence.seba@gmail.com and sebastian.j@trp.srmtrichy.edu.in

B. Ananda Priya, Ayya Nadar Janaki Ammal College (Autonomous), Sivakasi - 626 123, India.
E-mail: priyaraghul1331@gmail.com