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FINDING THE FUZZY CRITICAL PATH OF THE PROJECT NETWORK IN MULTIPLE PATHS FROM LINEAR PROGRAMMING USING OCTAGONAL FUZZY NUMBERS

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Abstract When fuzzy linear programming (FLPP) is utilized to ascertain unknown vital routes and completion deadlines, project planning becomes more intricate and precise. One effective method for handling uncertainty in project management is FLPP. Project network's decisions are all defined by octagonal fuzzy numbers (OCFN). To reduce the constraints in the FLPP, A different description of the octagonal fuzzy number is presented. The concept of this paper is mainly related to octagonal fuzzy numbers, the formation of LP & technique of fuzziness rating. In this research, we employ a tree with roots to explain an experimental circuit with fuzziness because it facilitates the communication of the process timings with each task using octagonal fuzzy integers and makes it easier to find the early start times. This novel rank technique may take into account variables like task dependencies, resource availability, and job length uncertainty to find not one, but several significant paths that could have a major impact on project completion. A relevant instance would be helpful in understanding the real application of the plan of action. By following a specific situation, readers may comprehend the implications of the new ranking system for project management decision-making. A computational instance is used to illustrate the suggested strategy, and the outcomes reveal how effective and adaptable it is for locating total floats. The outcomes of a comparison analysis of the approaches discussed in this paper are provided.

1 Introduction

It appears that the suggested approach expands on the Critical Path Method's (CPM) framework to tackle issues with project predictable operating times. Shortest project duration, is found using the tried-and-true CPM method of project scheduling. It does, however, make the assumption that task durations are predictable and deterministic. The CPM project management technique finds the longest path of dependent activities in a project network. This longest path required to complete the task. The CPM, and rest of conventional deterministic approaches might not be sufficient because they depend on precise time estimations for every activity. To get past this obstacle, project managers usually resort to techniques that can handle uncertainty and unpredictability more effectively. Fuzzy sets and fuzzy numbers are used to express inaccurate or vague information regarding activity durations instead of precise values. This gives project managers more flexibility when modelling uncertainty and enables them to base their decisions on a range of potential outcomes as opposed to precise forecasts. An intriguing application is to use OCFN in conjunction with the CPM to manage projects efficiently when activity times are known and deterministic. A flexible framework for expressing uncertainty is offered by octagonal fuzzy numbers, even in situations when deterministic values are accessible. Other forms of uncertainty or imprecision that may arise in project scheduling, such as resource availability, task dependencies, or subjective estimations of activity durations, can be incorporated into the schedule using OCFN in this situation. Through the use of OCFNs, project managers can portray these aspects in a way that allows for a wider variety of potential outcomes or outcomes. CPM is widely recognized as a helpful tool for organizing and controlling complex operations in many engineering and management applications, especially where activity times are well-established and predictable.

By [2] provided a strategy for assessing important paths in fuzzy project networks using the extending notion and a formulation of linear programming (LPP). Fuzzy CPM issues are more realistic than crisp ones. S.P. Both [3] established an accessible method for solving them by combining a fuzzy number ranking algorithm with a LPP. A new fuzzy LPP model was proposed by [5] using trapezoidal fuzzy numbers. To determine the critical path, then [7, 8] suggested and employed a novel form for OCFN. The difficulties in estimating potential values for an activity's early, and latest start length in fuzzy project networks with short time intervals and ambiguous durations provided by fuzzy intervals or fuzzy numbers are covered by [4]. The fuzzy arithmetic operational model was expanded by [1] to calculate the most recent start every duration in task. A novel technique for identifying a critical path from a variety of paths in the project network was discovered by [4]. An effective graphical method for project attribute estimation was developed by [9].

The preceding issues are covered in the parts that follow, while sections 2 and 3 offer an illustrated LP model of the fuzzy CPM. In section 4, we proposed to extract a critical path from a set of paths by using an OCFN. Section 5 contains a numerical example.

2 Preliminaries

Objective of introduction of research is to provide readers with a basic understanding themes, terminology that will be covered throughout. These are a few common, basic concepts that could be covered in discussion.

Definition 2.1. Assume X is any non-empty set. The membership value of $x \in X$ in a fuzzy set $\tilde{A}$ in X is given by $\mu_{\tilde{A}} : X \to [0,1]$, which transformation is said to be membership value of $x \in X$ in $\tilde{A}$

Definition 2.2. The fuzzy no. $\tilde{A}$ is fuzzy set if $\mu_{\tilde{A}}$ is (a) A fuzzy set of the universe of discourse X is convex (b) $\tilde{A}$ is normal if $\exists x_i \in X, \mu_{\tilde{A}}(x_i) = 1 (c) \mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 2.3. The normal OCFN $\tilde{A}$ is symbolized by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ with $\mu_{\tilde{A}}(x)$ standing for real numbers and its membership function.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a_1 \\ k\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ k, & a_2 \leq x \leq a_3 \\ k+(1-k)\left(\dfrac{x-a_3}{a_4-a_3}\right), & a_3 \leq x \leq a_4 \\ 1, & a_4 \leq x \leq a_5 \\ k+(1-k)\left(\dfrac{a_6-x}{a_6-a_5}\right), & a_5 \leq x \leq a_6 \\ k, & a_6 \leq x \leq a_7 \\ k\left(\dfrac{a_8-x}{a_8-a_7}\right), & a_7 \leq x \leq a_8 \\ 0, & x \geq a_8 \end{cases}, \text{ where } 0 < k < 1.$$

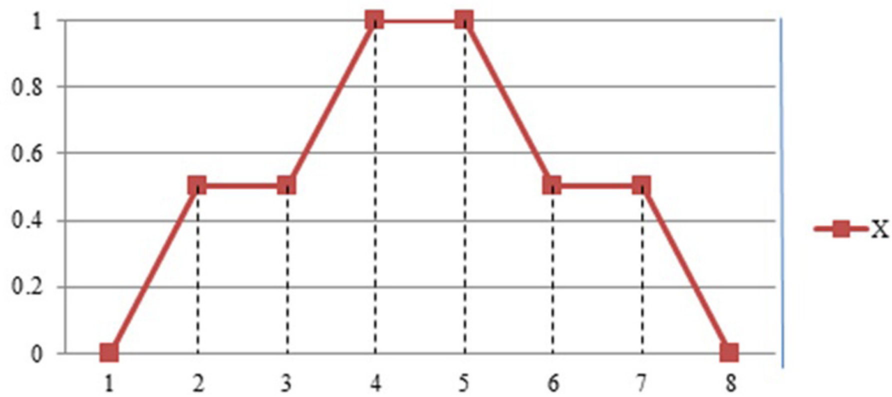

Figure 1. Octagonal Fuzzy Number for $k = 0.5$

Result 2.1. OCFN becomes to trapezoidal fuzzy no. (a_3, a_4, a_5, a_6) and (a_1, a_4, a_5, a_8) for $k = 0$ and $k = 1$.

Definition 2.4. A zero is presumed an OCFN $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ if and only if $a_1 = 0, a_2 = 0, a_3 = 0, a_4 = 0, a_5 = 0, a_6 = 0, a_7 = 0, a_8 = 0$.

Definition 2.5. Two OCFN $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ is said to be equal. i.e., $\tilde{A} = \tilde{B}$ if and only if $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4, a_5 = b_5, a_6 = b_6, a_7 = b_7, a_8 = b_8$.

Definition 2.6. The new representation of an OCFN $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) = (a^L - \alpha, a^L - \beta, a^L - \gamma, a^L, a^U, a^U + \alpha', a^U + \beta', a^U + \gamma\prime)$ is defined by $\tilde{A} = (x, y, \alpha, \beta, \gamma, \alpha', \beta', \gamma')$, where $x = a^L - \alpha, y = a^U + \gamma'$.

Definition 2.7. Two octagonal fuzzy number $\tilde{A} = (x_1, y_1, \alpha_1, \beta_1, \gamma_1, \alpha_1', \beta_1', \gamma_1')$, and $\tilde{B} = (x_2, y_2, \alpha_2, \beta_2, \gamma_2, \alpha_2', \beta_2', \gamma_2')$, are equal $(\tilde{A} = \tilde{B})$ iff $x_1 = x_2, y_1 = y_2, \alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2, \alpha_1' = \alpha_2', \beta_1' = \beta_2', \gamma_1' = \gamma_2'$.

Definition 2.8. If $\tilde{A} = (x_1, y_1, \alpha_1, \beta_1, \gamma_1, \alpha_1', \beta_1', \gamma_1')$, and $\tilde{B} = (x_2, y_2, \alpha_2, \beta_2, \gamma_2, \alpha_2', \beta_2', \gamma_2')$, are two OCFN then the addition operation defined as $\tilde{A} + \tilde{B} = (x_1 + x_2, y_1 + y_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2, \gamma_1 + \gamma_2, \alpha_1' + \alpha_2', \beta_1' + \beta_2', \gamma_1' + \gamma_2'$.

Definition 2.9. If $\tilde{A} = (x, y, \alpha, \beta, \gamma, \alpha', \beta', \gamma')$ be OCFN then the new ranking function of $\tilde{A}$ is defined as $\Re(\tilde{A}) = \frac{(x+y)}{2} + \frac{\alpha+\beta+\gamma-\alpha'-\beta'-\gamma'}{8}$.

Definition 2.10. If $\tilde{A} = (x, y, \alpha, \beta, \gamma, \alpha', \beta', \gamma')$ be OCFN then the divergence function of $\tilde{A}$ is stipulated as $Div\tilde{A} = y - x$.

Definition 2.11. If $\tilde{A} = (x, y, \alpha, \beta, \gamma, \alpha', \beta', \gamma')$ be OCFN then the mode of $\tilde{A}$ is provided as Mode $\tilde{A} = \frac{(x+y)}{2}$.

Definition 2.12. If $\tilde{A} = (x, y, \alpha, \beta, \gamma, \alpha', \beta', \gamma')$ be OCFN then the left spreads is $\tilde{A} = \alpha$ and right spreads is $\tilde{A} = \gamma'$.

3 Approaching Linear Programming for finding CPM Strategy [6]

Project activities and their dependencies are shown using a directed graph called a fuzzy project network. It is explained in the explanation you provided.

Examine the computational system $G(V, E)$ with a limited number V of nodes and a collection of arcs with varying activity lengths. Every edge is represented by an ordered pair (i, j) with $i, j \in V$ and $i \neq j$.

Take $x_{i,j}$ be the deciding variable that represents a flow quantity (i, j). With n nodes, the critical path problem is written as, The objective function $Max \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} t_{ij} \ni \sum_{j=1}^{n} x_{ij} = 1; \sum_{j=1}^{n} x_{ij} = \sum_{k=1}^{n} x_{kj}, i = 1, 2, \ldots, n-1; \sum_{k=1}^{n} x_{kn} = 1; x_{ij} = 0$ or $1, (i, j) \in A$.

A LP model that reduces the overall duration while meeting the scope of the network's restrictions can be created to optimize the activity's time interval from the first node to node n in a project network. Consider the network $N = \langle V, A, \tilde{T} \rangle$ along fuzzy duration. With the exception of the fact that $\tilde{T} : A \to Fn(R^*)$ determines, approximates the task time.

Determined $\tilde{T} : A \to Fn(R^*)$, V and A are the same as in the crisp example, where the set of fuzzy non-negative numbers is denoted by $Fn(R^*)$ indicate fuzzy duration $\tilde{T}_{i,j}$.

In order to create a new representation for octagonal fuzzy numbers,

i) Earliest start event $\tilde{E}_j = Max(\tilde{E}_i \oplus \tilde{t}_{ij})$, when $\tilde{E}_1 = 0$

ii) Latest finish event $\tilde{L}_i = Min(\tilde{L}_j \oplus (-\tilde{t}_{ij}))$, when $\tilde{L}_k = \tilde{E}_k$

iii) Total float Latest finish event $\tilde{T}_{ij} = \tilde{L}_j \oplus (-\tilde{E}_i) \oplus (-\tilde{t}_{ij})$

4 Procedure for finding a critical path by the proposed method

4.1 Method-I

Step 1: Construct a linear programming problem out of a critical path problem.

Step 2: Determine the linear programming approach. If the problem is solved, the critical path can be inferred in the outcome.

Step 3: The total float of each activity may be found, and the critical path can be computed so that floats in all the duration in a route add up to 0 if there are other options.

Step 4: The solution is found if there is a unique critical path.

Step 5: If not, find the total float's mode, measure the paths $\exists$ critical paths have the highest mode.

Step 6: Find the divergence of the paths if the modes of all the paths are identical, The critical path will be the one with the greatest divergence.

Step 7: Determine the spreads on the left and right paths, and the road with the largest spread will be the crucial path if the paths' divergence is equal.

4.2 Method-II

Algorithm:- [9]

Find every pathway and the fuzzy float of every action by a rooted tree.

I) Make a fuzzy project network and assign a number to every node.

II) Create a rooted tree of the fuzzy project network by designating the lower number node as the left child, the higher number node as the right child, and the beginning node as the root. For every activity, assign the linked edge's weight in the rooted tree to its fuzzy activity time.

III) Locate every path from the network's root to its final node inside the rooted tree.

IV) Add up all of the activity weights along each path to determine its length. Calculate the longest path.

V) Determine each path's metric distance rank and rank the paths in ascending order based on it by subtracting each road's length from the longest path's length (let's say $\tilde{p_v}$).

VI) Choose the route with the greatest rank and convert all total float in every activity along route to an equivalent value. Ignore the activities that have already been assigned, and instead select the path with the second-highest rank and pay comparable as total float of every durations along route.

Continue until all float has been awarded to every activity.

Ranking function:-

If Let $(o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8)$ be the OCFN then the ranking function is given by

$$R(\breve{A}) = \frac{1}{2}\left[\frac{(o_8 + 4(o_5 + o_6 + o_7)) + (o_1 + 4(o_2 + o_3 + o_4)}{(o_8 + 4(o_5 + o_6 + o_7) - (o_1 + 4(o_2 + o_3 + o_4))}\right]$$

4.3 Method-III (Haar Ranking)

HAAR Ranking procedure for Octagonal Fuzzy Number [10]

Assume $(o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8)$ be OCFN. The following is a list of the formulas for calculating the wavelet and scaling coefficients—the detailed and average coefficients of the OCFN.

(i) Match the provided OCFNs. i.e., $[o_1, o_2], [o_3, o_5], [o_5, o_6], [o_7, o_8]$

(ii) Replace the first four elements of $\check{O}$ with half of the difference between these pairs (approximation coefficients) and the last four elements of $\check{O}$ with the average of these pairs (detailed coefficients).
The first four elements of $\check{O}$ should be substituted for half of the difference between these pairs (approximation coefficients), and the final four elements of $\check{O}$ should be substituted for the average of these pairs (detailed coefficients).
$\therefore$ There is another way to write $\check{O}_1 = (\delta_1, \delta_2, \delta_3, \delta_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$
Where,

$$\delta_1 = \left(\frac{o_1 + o_2}{2}\right), \delta_2 = \left(\frac{o_3 + o_4}{2}\right), \delta_3 = \left(\frac{o_5 + o_6}{2}\right), \delta_4 = \left(\frac{o_7 + o_8}{2}\right)$$

$$\gamma_1 = \left(\frac{o_1 - o_2}{2}\right), \gamma_2 = \left(\frac{o_3 - o_4}{2}\right), \gamma_3 = \left(\frac{o_5 - o_6}{2}\right), \gamma_4 = \left(\frac{o_7 - o_8}{2}\right).$$

(iii) It is best to group the two $\check{O}$ approximation coefficients together. Next, ascertain the updated approximation coefficients $[\delta_1, \delta_2], [\delta_3, \delta_4]$ and detailed coefficients for the pair of $\check{O}$ approximation coefficients.

$$\alpha_1 = \left(\frac{\delta_1 + \delta_2}{2}\right), \alpha_2 = \left(\frac{\delta_3 + \delta_4}{2}\right), \beta_1 = \left(\frac{\delta_1 - \delta_2}{2}\right), \beta_2 = \left(\frac{\delta_3 - \delta_4}{2}\right)$$

After then, $\check{O}_1$ became $\check{O}_2 = (\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$.

(iv) Find the two approximation coefficients in $\check{O}_2$. Next, obtain the updated approximation and detailed coefficient $[\alpha_1, \alpha_2]$ for the pair of $\check{O}_2$ approximation coefficients.

$$\omega_1 = \left(\frac{\alpha_1 + \alpha_2}{2}\right), \omega_2 = \left(\frac{\alpha_1 - \alpha_2}{2}\right).$$

After then, $\check{O}_2$ became $H(\check{O}) = (\omega_1, \omega_2, \beta_1, \beta_2, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$.

(V) Check out the rankings.
$\check{O}, \check{P}$ in the event that the first element of $H(\check{O})$'s tuple is less than the first element of $H(\check{P})$'s tuple.If the first element in both $H(\check{O})$ and $H(\check{P})$ is the same, compare the second element, and so on, to the last element. Under these conditions, $\check{O} = \check{P}$.

5 Illustration

This project problem serves as an example of the suggested fuzzy critical route analysis computing method. The new method is demonstrated by examining a mathematical approach used in the study, and the conclusions of the alternative technique are found to be identical to those of the previous method.

Assume that a project network has $N = 1, 2, 3, 4, 5$ nodes and that each activity's fuzzy time is represented as an OCFN, as illustrated in the diagram. The objective is to estimate the fuzzy earliest starting & finish, fuzzy CP finish time of the project network (as indicated in the picture) by using the OCFN below to represent the fuzzy normal time of each activity.

Table 1. Events of Conference

Activity	Description	Predecessor	Duration
I	Conference holding	–	2,2.5,3,3.5,4.5,5,5.5,6
II	Communicate date to participants	–	9,10,11,12,14,15,16,17
III	Agenda preparation	I	7,7.5,8,8.5,9.5,10,10.5,11
IV	Conference hall arrangement	I	12,14,16,18,20,22,24,26
V	Refreshment	II,III	6,7,8,9,11,12,13,14
VI	Journal communication	IV	8,9,10,11,13,14,15,16

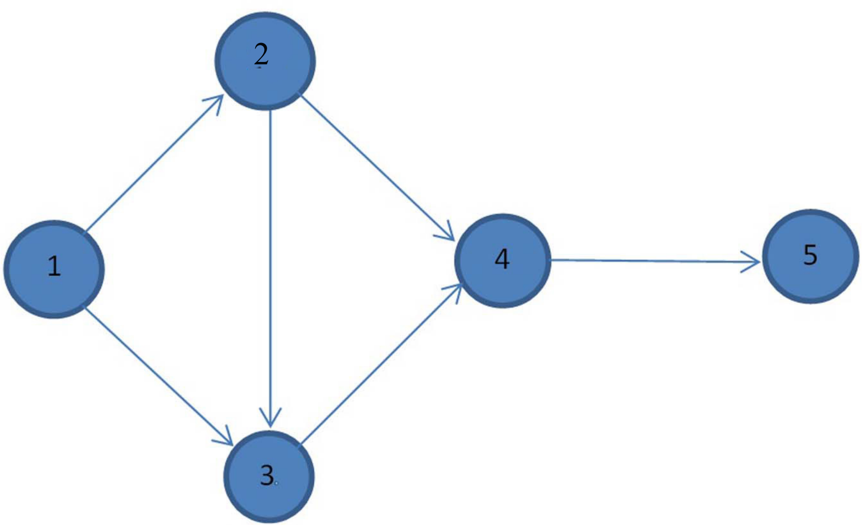

Figure 2. Project Network

The Linear Programming problem is

$$Max \left[\begin{array}{l} (2,6,1.5,1,0.5,0.5,1,1.5)x_{12} + (9,17,3,2,1,1,2,3)x_{13} + (7,11,1.5,1,0.5,0.5,1,1.5) \\ x_{23} + (12,26,6,4,2,2,4,6)x_{24} + (6,14,3,2,1,1,2,3)x_{34} + (8,16,3,2,1,1,2,3)x_{45} \end{array} \right]$$

Subject to

$$x_{12} + x_{13} = 1;$$
$$x_{12} - x_{23} - x_{24} = 0;$$
$$x_{13} + x_{23} - x_{34} = 0;$$
$$x_{24} + x_{34} = 0;$$
$$x_{45} = 1$$
$$\text{for } x_{12}, x_{13}, x_{23}, x_{24}, x_{34}, x_{45} = 0 \text{ or} 1$$

The aforementioned linear programmed are solved using TORA, a mathematical programming toolset.
The critical pathways are as follows:

a) 1-2-4-5 b) 1-3-4-5 c) 1-2-3-4-5

Method-I (Modified form)

Table 2. Duration of activities

Activity		Duration	
Name	**i-j**	**Fuzzy activity time**	**Modified form**
I	1-2	2,2.5,3,3.5,4.5,5,5.5,6	2,6,1.5,1,0.5,0.5,1,1.5
II	1-3	9,10,11,12,14,15,16,17	9,17,3,2,1,1,2,3
III	2-3	7,7.5,8,8.5,9.5,10,10.5,11	7,11, 1.5,1,0.5,0.5,1,1.5
IV	2-4	12,14,16,18,20,22,24,26	12,26,6,4,2,2,4,6
V	3-4	6,7,8,9,11,12,13,14	6,14,3,2,1,1,2,3
VI	4-5	8,9,10,11,13,14,15,16	8,16,3,2,1,1,2,3

Table 3. Earliest Start and Finish

Activity	Earliest	
	Start	**Finish**
1-2	0,0,0,0,0,0,0,0	2,6,1.5,1,0.5,0.5,1,1.5
1-3	0,0,0,0,0,0,0,0	9,17,3,2,1,1,2,3
2-3	2,6,1.5,1,0.5,0.5,1,1.5	9,17,3,2,1,1,2,3
2-4	2,6,1.5,1,0.5,0.5,1,1.5	14,32,7.5,5,2.5,2.5,5,7.5
3-4	9,17,3,2,1,1,2,3	15,31,6,4,2,2,4,6
4-5	15,31,6,4,2,2,4,6	23,47,9,6,3,3,6,9

Table 4. Latest Start and Finish

Activity	Latest	
	Start	Finish
1-2	-16,16,0,0,0,0,0,0	-19,27,0,0,0,0,0,0
1-3	-24,24,0,0,0,0,0,0	-7,33,3,2,1,1,2,3
2-3	-18,26,1.5,1,0.5,0.5,1,1.5	-7,33,3,2,1,1,2,3
2-4	-19,27,0,0,0,0,0,0	7,39,6,4,2,2,4,6
3-4	-7,33,3,2,1,1,2,3	7,39,6,4,2,2,4,6
4-5	7,39,6,4,2,2,4,6	23,47,9,6,3,3,6,9

Table 5. Total Float

Activity	Total Float
1-2	-25,25,6,4,2,2,4,6
1-3	-24,24,6,4,2,2,4,6
2-3	-24,24,6,4,2,2,4,6
2-4	-25,25,6,4,2,2,4,6
3-4	-24,24,6,4,2,2,4,6
4-5	-24,24,6,4,2,2,4,6

Table 6. Rank–Mode-Divergence

Paths	Total Float	Rank	Mode	Divergence
1-2-4-5	-74,74,18,12,6,612,18	0	0	148
1-3-4-5	-72,72,18,12,6,612,18	0	0	144
1-2-3-4-5	-97,97,18,12,6,612,18	0	0	194

The above data reveals that the network's critical path is 1-2-3-4-5.

Method-I (LR Fuzzy activities)

Table 7. Duration of LR fuzzy activities

Activity		Duration	
Name	i-j	Fuzzy activity time	LR-Fuzzy activity duration
I	1-2	2,2.5,3,3.5,4.5,5,5.5,6	3.5,4.5,1.5,1,0.5,1.5,1,0.5
II	1-3	9,10,11,12,14,15,16,17	12,14,3,2,1,3,2,1
III	2-3	7,7.5,8,8.5,9.5,10,10.5,11	8.5,9.5,1.5,1,0.5,1.5,1,0.5
IV	2-4	12,14,16,18,20,22,24,26	18,20,6,4,2,6,4,2
V	3-4	6,7,8,9,11,12,13,14	9,11,3,2,1,3,2,1
VI	4-5	8,9,10,11,13,14,15,16	11,13,3,2,1,3,2,1

Table 8. Earliest Start and Finish-LR fuzzy

Activity	Earliest	
	Start	Finish
I	0,0,0,0,0,0,0,0	3.5,4.5,1.5,1,0.5,1.5,1,0.5
II	0,0,0,0,0,0,0,0	12,14,3,2,1,3,2,1
III	3.5,4.5,1.5,1,0.5,1.5,1,0.5	12,14,3,2,1,3,2,1
IV	3.5,4.5,1.5,1,0.5,1.5,1,0.5	21.5,24.5,7.5,5,2.5,7.5,5,2.5
V	12,14,3,2,1,3,2,1	15,31,6,4,2,2,4,6
VI	21.5,24.5,7.5,5,2.5,7.5,5,2.5	32.5,37.5,10.5,7,3.5,10.5,7,3.5

Table 9. Latest Start and Finish-LR fuzzy

Activity	Latest	
	Start	Finish
I	-5,5,0,0,0,0,0,0	-0.5,8.5,1.5,1,0.5,1.5,1,0.5
II	-5.5,5.5,1.5,1,0.5,1.5,1,0.5	8.5,17.5,4.5,3,1.5,4.5,3,1.5
III	-1,9,3,2,1,3,2,1	8.5,17.5,4.5,3,1.5,4.5,3,1.5
IV	-0.5,8.5,1.5,1,0.5,1.5,1,0.5	19.5,26.5,7.5,5,2.5,7.5,5,2.5
V	8.5,17.5,4.5,3,1.5,4.5,3,1.5	19.5,26.5,7.5,5,2.5,7.5,5,2.5
VI	19.5,26.5,7.5,5,2.5,7.5,5,2.5	32.5,37.5,10.5,7,3.5,10.5,7,3.5

Table 10. Total Float

Activity	Total Float
1-2	-25,25,6,4,2,2,4,6
1-3	-24,24,6,4,2,2,4,6
2-3	-24,24,6,4,2,2,4,6
2-4	-25,25,6,4,2,2,4,6
3-4	-24,24,6,4,2,2,4,6
4-5	-24,24,6,4,2,2,4,6

Table 11. Path Ranking

Paths	Total Float	Rank	Mode	Divergence	Spread
1-2-4-5	-74,74,18,12,6,612,18	0	0	148	NA
1-3-4-5	-72,72,18,12,6,612,18	0	0	144	NA
1-2-3-4-5	-97,97,18,12,6,612,18	0	0	194	NA

The highest divergence path is critical path i.e., 1-2-3-4-5.

Method II (Rooted tree)

Table 12. Duration of fuzzy activities

Activity		Duration
Name	**i-j**	**Fuzzy activity time**
I	1-2	2,2.5,3,3.5,4.5,5,5.5,6
II	1-3	9,10,11,12,14,15,16,17
III	2-3	7,7.5,8,8.5,9.5,10,10.5,11
IV	2-4	12,14,16,18,20,22,24,26
V	3-4	6,7,8,9,11,12,13,14
VI	4-5	8,9,10,11,13,14,15,16

Table 13. Earliest Start and Finish

Activity	Earliest	
	Start	**Finish**
I	0,0,0,0,0,0,0,0	2,2.5,3,3.5,4.5,5,5.5,6
II	0,0,0,0,0,0,0,0	9,10,11,12,14,15,16,17
III	2,6,1.5,1,0.5,0.5,1,1.5	9,13.5,9.5,9.5,10,10.5,11.5,12.5
IV	2,6,1.5,1,0.5,0.5,1,1.5	14,20,17.5,19,21,22.5,25,27.5
V	9,17,3,2,1,1,2,3	15,24,11,11,12,13,15,17
VI	15,31,6,4,2,2,4,6	23,40,16,15,15,16,19,22

Table 14. Latest Start and Finish-LR fuzzy

Activity	Latest	
	Start	**Finish**
I	-25,-5,-25,-23,-18,-13,-7,-1	-19,1,-20,-18,-14,-10,4.5,1
II	-24,-4,-25,-23,-17,-13,-7,-1	-7,12,-10,-9,-5,-2,3,8
III	-18,1.5,-20,-19,-14,-10,4.5,1	-7,12,-10,-9,-5,-2,3,8
IV	-19,1,-20,-18,-14,-10,4.5,1	7,25,2,2,4,6,10,14
V	-7,12,-10,-9,-5,-2,3,8	7,25,2,2,4,6,10,14
VI	7,25,2,2,4,6,10,14	23,40,16,15,15,16,19,22

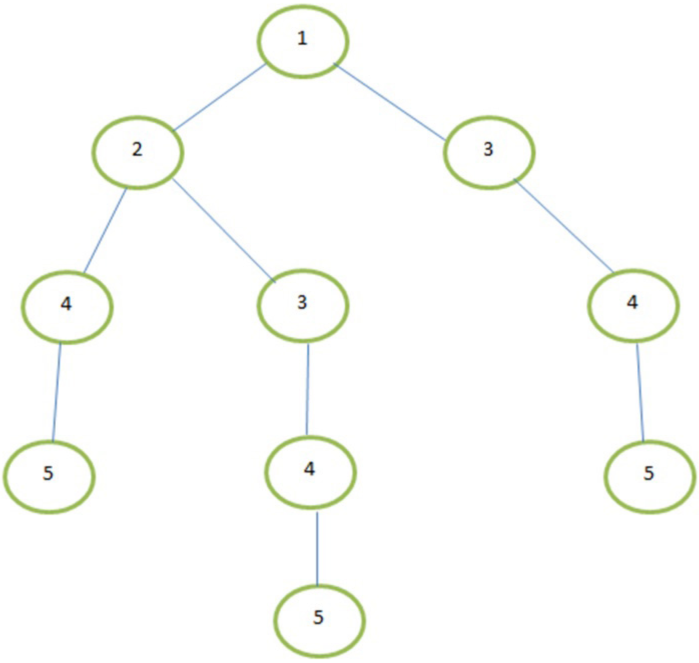

Figure 3. Rooted tree

Table 15. Total Float

Activity	Total Float
1-2	-25,-4.5,-25,-22.5,-17.5,-13,-7,-1
1-3	-24,-4,-25,-23,-17,-13,-7,-1
2-3	-19.5,0.5,-21,-19,-14.5,-12,-10.5,-1
2-4	-20.5,0,-21,-18.5,-15,-12,-10,0
3-4	-10,10,-11,-10,-7,-5,-14,-1
4-5	1,21,0,0,0,0,-21,-1

Table 16. Path Ranking

Paths	Path Length	$\tilde{p_v}$
1-2-4-5	-44.5,16.5, -46,-41,32.5, -25,-38,-2	-25,-18.5,-12, -5.5,5.5,12,18.5,25
1-3-4-5	-33,27,-36,-33, -24,-18,-42,-3	-24,-18,-9,-6,6,12, 18,24
1-2-3-4-5	-53.5,27,-57, -51.5,-39,-30, - 52.5,-4	-24,-18,-9,-6,6,12, 18,24

Table 17. Fuzzy Float, Rank

Paths	Total Fuzzy	Float Rank
1-2-4-5	-74,-55,-33,-17,17,36,55,74	0.003
1-3-4-5	-73,-54.5,-30,-18, -17.5,36,54.5,73	0.005
1-2-3-4-5	-97,-72.5,-39, -24,23.5,48,72.5,97	0.006

This technique is valid when all durations, important pathways are examined. The highest rank value is 0.006.

Therefore, the critical path is 1-2-3-4-5.

Method III (HAAR Ranking)

Table 18. Duration of Haar fuzzy activities

Activity		Duration	
Name	i-j	Fuzzy activity time	Haar-Fuzzy activity duration
I	1-2	2,2.5,3,3.5,4.5,5,5.5,6	4,-1.25,-0.5,-0.5,-0.25,-0.25,-0.25,-0.25
II	1-3	9,10,11,12,14,15,16,17	13,-2.5,-1,-1,-0.5,-0.5,-0.5,-0.5
III	2-3	7,7.5,8,8.5,9.5,10,10.5,11	9,-1.25,-0.5,-0.5,-0.25,-0.25,-0.25,-0.25
IV	2-4	12,14,16,18,20,22,24,26	19,-4,-2,-2,-1,-1,-1,-1
V	3-4	6,7,8,9,11,12,13,14	10,-2.5,-1,-1,-0.5,-0.5,-0.5,-0.5
VI	4-5	8,9,10,11,13,14,15,16	12,-2.5,-1,-1,-0.5,-0.5,-0.5,-0.5

Table 19. Earliest Start and Finish-Haar fuzzy activities

Earliest	
Start	Finish
0,0,0,0,0,0,0,0	4,-1.25,-0.5,-0.5,-0.25,-0.25,-0.25,-0.25
0,0,0,0,0,0,0,0	13,-2.5,-1,-1,-0.5,-0.5, -0.5,-0.5
4,-1.25,-0.5,-0.5,-0.25, -0.25,-0.25,-0.25	13,-2.5,-1,-1,-0.5,-0.5,-0.5,-0.5
4,-1.25,-0.5,-0.5,-0.25, -0.25,-0.25,-0.25	23,-5,-2,-2,-1,-1,-1,-1
13,-2.5,-1,-1,-0.5,-0.5, -0.5,-0.5	23,-5,-2,-2,-1,-1,-1,-1
23,-5,-2,-2,-1,-1,-1,-1	35,-7.5,-.3,-3,-1.5,-1.5,-1.5

Table 20. Latest Start and Finish-LR fuzzy

Latest	
Start	**Finish**
0,-2.5,-1,-1,-0.5, -0.5,-0.5,-0.5	4,-3.75,-1.5,-1.5,-0.75, -0.75,-0.75,-0.75
0,-2.5,-1,-1,-0.5,-0.5,-0.5,-0.5	13,-5,-2,-2,-1,-1,-1,-1
4,-3.75,-1.5,-1.5, -0.75, -0.75,-0.75,-0.75	13,-5,-2,-2,-1,-1,-1,-1
4,-1,0,0,0,0,0,0	23,-5,-2,-2,-1,-1,-1,-1
13,-2.5,-1,-1,-0.5,-0.5,-0.5,-0.5	23,-5,-2,-2,-1,-1,-1,-1
23,-5,-2,-2,-1,-1,-1,-1	35,-7.5,-3,-3,-1.5,-1.5,-1.5,-1.5

Table 21. Total Float

Activity	Total Float
1-2	0,-2.5,-1,-1,-0.5,-0.5,-0.5,-0.5
1-3	0,-2.5,-1,-1,-0.5,-0.5,-0.5,-0.5
2-3	0,-2.5,-1,-1,-0.5,-0.5,-0.5,-0.5
2-4	0,0.25,0.5,0.5,0.25,0.25,0.25,0.25
3-4	0,0,0,0,0,0,0,0
4-5	0,0,0,0,0,0,0,0

The critical path is 1-3-4-5 and 1-2-3-4-5

Method III (HAAR –LR fuzzy number)

Table 22. Duration of LR fuzzy activities

Activity		Duration	
Name	**i-j**	**Fuzzy activity time**	**LR Fuzzy activity duration**
I	1-2	2,2.5,3,3.5,4.5,5,5.5,6	3.5,4.5,1.5,1,0.5,1.5,1,0.5
II	1-3	9,10,11,12,14,15,16,17	12,14,3,2,1,3,2,1
III	2-3	7,7.5,8,8.5,9.5,10,10.5,11	8.5,9.5,1.5,1,0.5,1.5,1,0.5
IV	2-4	12,14,16,18,20,22,24,26	18,20,6,4,2,6,4,2
V	3-4	6,7,8,9,11,12,13,14	9,11,3,2,1,3,2,1
VI	4-5	8,9,10,11,13,14,15,16	11,13,3,2,1,3,2,1

Table 23. Duration of Haar fuzzy activities

Activity		Duration	
Name	**i-j**	**Fuzzy activity time**	**Haar Fuzzy activity duration**
I	1-2	3.5,4.5,1.5,1,0.5,1.5,1,0.5	4,-0.5,1.5,1,0.5,1.5,1,0.5
II	1-3	12,14,3,2,1,3,2,1	13,-1,3,2,1,3,2,1
III	2-3	8.5,9.5,1.5,1,0.5,1.5,1,0.5	13,-1,3,2,1,3,2,1
IV	2-4	18,20,6,4,2,6,4,2	19,-1,6,4,2,6,4,2
V	3-4	9,11,3,2,1,3,2,1	10,-1,3,2,1,3,2,1
VI	4-5	11,13,3,2,1,3,2,1	12,-1,3,2,1,3,2,1

Table 24. Earliest Start and Finish-Haar fuzzy activities

Earliest	
Start	**Finish**
0,0,0,0,0,0,0,0	4,-0.5,1.5,1,0.5,1.5,1,0.5
0,0,0,0,0,0,0,0	13,-1,3,2,1,3,2,1
4,-0.5,1.5,1,0.5,1.5,1,0.5	13,-1,3,2,1,3,2,1
4,-0.5,1.5,1,0.5,1.5,1,0.5	23,-1.5,7.5,5,2.5,7.5,5,2.5
13,-1,3,2,1,3,2,1	23,-2,6,4,2,6,4,2
23,-1.5,7.5,5,2.5,7.5,5,2.5	35,-2.5,10.5,7,3.5,10.5,7,3.5

Table 25. Latest Start and Finish-LR fuzzy

Latest	
Start	**Finish**
0,-1,0.5,-0.5,0,0.5,-0.5,5.5	4,0.5,1.5,1,0.5,1.5,1,5
0,-2.5,9.5,5,3.5,10.5,5,5.5	13,-1.5,11.5,8,4.5,12.5,8,4.5
4,-3.4.5,0.5,1,4.5,0.5,8.5	13,-2.5,5.5,2,1.5,5.5,2,8
4,-0.5,1.5,1,0.5,1.5,1,5	23,-1.5,7.5,5,2.5,7.5,5,7
13,-2.5,5.5,2,1.5,5.5,2,8	23,-1.5,7.5,5,2.5,7.5,5,7
23,-1.5,7.5,5,2.5,7.5,5,2.5	35,-2.5,10.5,7,3.5,10.5,7,3.5

Table 26. Total Float

Activity	Total Float
1-2	0,0,0,0,0,0,0,4.5
1-3	0,-0.5,8.5,6,3.5,9.5,6,3.5
2-3	0,-1.5,2.5,0,0.5,2.5,0,7
2-4	0,0,0,0,0,0,0,4.5
3-4	0,0.5,1.5,1,0.5,1.5,1,5
4-5	0,0,0,0,0,0,0,0

In this technique, the critical path is 1-2-4-5

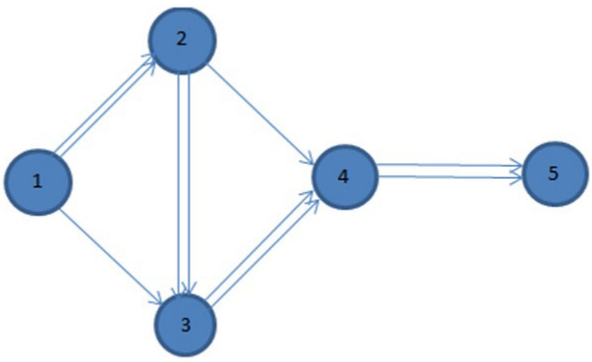

Figure 4. Critical Path

Table 27. Comparison Table

S.No.	Method	Duration type	Critical Path
1.	Method-I	Modified form	1-2-3-4-5
2.	Method-I	LR form	1-2-3-4-5
3.	Method-II	Actual form	1-2-3-4-5
4.	Method-III	Haar ranking	1-3-4-5 1-2-3-4-5
5.	Method-III	Haar ranking (LR fuzzy)	1-2-4-5

Observation

Integrating a ranking algorithm based on total floats into construction scheduling software for fuzzy projects could indeed offer significant benefits compared to the traditional CPM, especially when dealing with uncertainty in activity duration, ranking algorithms to effectively manage fuzzy projects, improving scheduling accuracy and project outcomes in the face of uncertainty.

6 Conclusion

An alternative approach to computing and debating fuzzy project lengths is presented in this article. The proposed approach makes the effort of building models easier by concentrating on fuzzy networks and streamlining their implementation. Many academics have examined the CPM with strategy networks along ambiguous task durations. We present an alternative approach based on octagonal fuzzy numbers to determine a critical path among several critical paths. The fundamental premise is derived from a definition of linear programming. A suitable illustration is provided to illustrate the idea. We would like to provide an alternative type of OCFN in a subsequent study to address critical path issues and practical applications.

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