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LEVEL OPERATORS ON BIPOLAR INTUITIONISTIC FUZZY α -IDEAL OF A BP-ALGEBRA

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Abstract The bipolar intuitionistic fuzzy α-ideal and bipolar intuitionistic anti fuzzy α-ideal are novel algebraic structures in BP-algebra that utilize level operators. The goal of this research is to apply fuzzy set theory and ideal theory to a BP-algebra. Bipolar intuitionistic fuzzy α-ideal and bipolar intuitionistic anti-fuzzy α-ideal are linked by the level operator operations.

1 Introduction

Lofti A. Zadeh [12] first proposed the idea of a fuzzy set in 1965 as a way to deal with the ambiguities that arise in everyday life. For a wide range of applied mathematics, information science, and decision-making challenges, fuzzy sets are incredibly helpful. Bipolar valued fuzzy sets are an extension of fuzzy sets that were first described by K.M. Lee [7] While bipolar fuzzy sets provide both a positive membership degree belonging to the interval [0, 1] and a negative membership degree belonging to the interval [-1, 0], fuzzy sets only provide the degree of membership of an element in a given set. The membership degrees range for bipolar fuzzy sets is expanded from the interval [0, 1] to the interval [-1, 1]. Following that, a number of fuzzy set generalizations are given, including intuitionistic fuzzy sets [1] and interval valued fuzzy sets [13]. Y.B. Jun and S.Z. Song [5] and K.J. Lee [6] presented sub algebras and ideals of BCK/BCI-algebras based on bipolar valued fuzzy sets using the concept of bipolar valued fuzzy sets. The notion of fuzzy subgroups was first introduced by A. Rosenfeld in 1977, which marked the beginning of the research of fuzzy algebraic structures [9]. The notion of fuzzy sets was expanded upon by K.T. Atanassov [2] in 1986 to create intuitionistic fuzzy sets. O.G. Xi [11] introduced the idea of fuzzy sets to BCK algebras in 1991. Y.B. Jun and K.H. Kim [4] investigated the fuzzy ideals of BCK algebras that are intuitionistic. BCH algebras are a broad class of abstract algebra that were introduced by Q.P. Hu and X. Li [3]. The idea of βalgebras was first suggested in 2002 by J. Neggers and H.S. Kim [8]. The concept of BP-Algebras was first developed by Sun Shin Ahn and Jeong Soon Han in 2012 [10]. In a variety of fields, including group theory, semigroup theory, ring theory, semiring theory, graph theory, engineering, physics, statics, medical science, social science, artificial intelligence, computer networks, expert systems, and decision making, the theory of bipolar fuzzy sets has recently become a hot topic of study. This paper investigates a number of theorems connected to the bipolar fuzzy α-ideal of BP algebra.

2 Preliminaries

Definition 2.1. Given any two bipolar intuitionistic fuzzy sets, let $A = (\chi_A^+, \chi_A^-, \psi_A^+, \psi_A^-)$ and $B = (\chi_B^+, \chi_B^-, \psi_B^+, \psi_B^-)$ in Γ, we define

(i) $A \cap B = \{(\kappa, minim(\chi_A^+(\kappa), \chi_B^+(\kappa)), maxim(\chi_A^-(\kappa), \chi_B^-(\kappa)), maxim(\psi_A^+(\kappa), \psi_B^+(\kappa)), minim(\psi_A^-(\kappa), \psi_B^-(\kappa)) \mid \kappa \in \Gamma\}$

(ii) $A \cup B = \{(\kappa, maxim(\chi_A^+(\kappa), \chi_B^+(\kappa)), minim(\chi_A^-(\kappa), \chi_B^-(\kappa)), minim(\psi_A^+(\kappa), \psi_B^+(\kappa)), maxim(\psi_A^-(\kappa), \psi_B^-(\kappa)) \mid \kappa \in \Gamma\}$

(iii) $\overline{A} = \{(\kappa, \psi_A^+(\kappa), \psi_A^-(\kappa), \chi_A^+(\kappa), \chi_A^-(\kappa) \mid \kappa \in \Gamma\}$

Definition 2.2. A bipolar intuitionistic fuzzy set $A = \{\chi_A^+, \chi_A^-, \psi_A^+, \psi_A^- \mid \kappa \in \Gamma\}$,in BP-algebra.If Γ meets the following requirements, it is referred to as a bipolar intuitionistic fuzzy α-ideal of Γ:

(i) $\chi_A^+(0) \geq \chi_A^+(\kappa)$ and $\chi_A^-(0) \leq \chi_A^-(\kappa)$

(ii) $\chi_A^+(\lambda * \mu) \geq minim\{\chi_A^+(\kappa * \mu), \chi_A^+(\kappa * \lambda)\}$

(iii) $\chi_A^-(\lambda * \mu) \leq maxim\{\chi_A^-(\kappa * \mu), \chi_A^-(\kappa * \lambda)\}$

(iv) $\psi_A^+(0) \leq \psi_A^+(\kappa)$ and $\psi_A^-(0) \geq \chi_A^-(\kappa)$

(v) $\psi_A^+(\lambda * \mu) \leq maxim\{\psi_A^+(\kappa * \mu), \psi_A^+(\kappa * \lambda)\}$

(vi) $\psi_A^-(\lambda * \mu) \geq minim\{\psi_A^-(\kappa * \mu), \chi_A^-(\kappa * \lambda)\}$ for all $\kappa, \lambda, \mu \in \Gamma$

Definition 2.3. BP-algebra's bipolar intuitionistic fuzzy set $A = \{\chi_A^+, \chi_A^-, \psi_A^+, \psi_A^- \mid \kappa \in \Gamma\}$, If Γ meets the following criteria, it is referred to as a bipolar intuitionistic anti fuzzy α-ideal of Γ:

(i) $\chi_A^+(0) \leq \chi_A^+(\kappa)$ and $\chi_A^-(0) \geq \chi_A^-(\kappa)$

(ii) $\chi_A^+(\lambda * \mu) \leq maxim\{\chi_A^+(\kappa * \mu), \chi_A^+(\kappa * \lambda)\}$

(iii) $\chi_A^-(\lambda * \mu) \geq minim\{\chi_A^-(\kappa * \mu), \chi_A^-(\kappa * \lambda)\}$

(iv) $\psi_A^+(0) \geq \psi_A^+(\kappa)$ and $\psi_A^-(0) \leq \chi_A^-(\kappa)$

(v) $\psi_A^+(\lambda * \mu) \geq minim\{\psi_A^+(\kappa * \mu), \psi_A^+(\kappa * \lambda)\}$

(vi) $\psi_A^-(\lambda * \mu) \leq maxim\{\psi_A^-(\kappa * \mu), \chi_A^-(\kappa * \lambda)\}$ for all $\kappa, \lambda, \mu \in \Gamma$

Definition 2.4. If A is a bipolar fuzzy set of intuitionistic Γ, then the level operator ! is determined by

$$!A = \{(\kappa, maxim(\tfrac{1}{2}, \chi_A^+(\kappa)), minim(\tfrac{-1}{2}, \chi_A^-(\kappa)), minim(\tfrac{1}{2}, \psi_A^+(\kappa)), maxim(\tfrac{-1}{2}, \psi_A^-(\kappa))) \mid \kappa \in \Gamma\} = L_1$$

Definition 2.5. If A is a bipolar fuzzy set of intuitionistic Γ, then the definition of the level operator ? is as follows:

$$?A = \{(\kappa, minim(\tfrac{1}{2}, \chi_A^+(\kappa)), maxim(\tfrac{-1}{2}, \chi_A^-(\kappa)), maxim(\tfrac{1}{2}, \psi_A^+(\kappa)), minim(\tfrac{-1}{2}, \psi_A^-(\kappa))) \mid \kappa \in \Gamma\} = L_2$$

3 Level Operators on Bipolar Intuitionistic Fuzzy α-Ideal

Theorem 3.1. *If A is a fuzzy α-ideal of Γ that is bipolar and intuitionistic, then $!A$ is the bipolar intuitionistic fuzzy α-ideal of Γ.*

Proof. Given A is bipolar intuitionistic fuzzy α-ideal of Γ. Consider $0, \kappa, \lambda, \mu \in A$.

(i) Now, $\chi_{!A}^+(0) = maxim(\frac{1}{2}, \chi_A^+(0)) \geq maxim(\frac{1}{2}, \chi_A^+(\kappa)) = \chi_{!A}^+(\kappa)$

Therefore $\chi_{!A}^+(0) \geq \chi_{!A}^+(\kappa)$

Now, $\chi_{!A}^-(0) = minim(\frac{-1}{2}, \chi_A^-(0)) \leq minim(\frac{-1}{2}, \chi_A^-(\kappa)) = \chi_{!A}^-(\kappa)$

Therefore $\chi_{!A}^-(0) \leq \chi_{!A}^-(\kappa)$

(ii) Now, $\chi^{+}_{!A}(\lambda * \mu) = maxim(\frac{1}{2}, \chi^{+}_{A}(\lambda * \mu))$

$$\geq maxim(\tfrac{1}{2}, minim\{\chi^{+}_{A}(\kappa * \mu), \chi^{+}_{A}(\kappa * \lambda)\})$$

$$= minim\{maxim(\tfrac{1}{2}, \chi^{+}_{A}(\kappa * \mu)), maxim(\tfrac{1}{2}, \chi^{+}_{A}(\kappa * \lambda))\}$$

$$= minim\{\chi^{+}_{!A}(\kappa * \mu), \chi^{+}_{!A}(\kappa * \lambda)\}$$

Therefore $\chi^{+}_{!A}(\lambda * \mu) \geq minim\{\chi^{+}_{!A}(\kappa * \mu), \chi^{+}_{!A}(\kappa * \lambda)\}$

(iii) Now, $\chi^{-}_{!A}(\lambda * \mu) = minim(\frac{-1}{2}, \chi^{-}_{A}(\lambda * \mu))$

$$\leq minim(\tfrac{-1}{2}, maxim\{\chi^{-}_{A}(\kappa * \mu), \chi^{-}_{A}(\kappa * \lambda)\})$$

$$= maxim\{minim(\tfrac{-1}{2}, \chi^{-}_{A}(\kappa * \mu)), minim(\tfrac{-1}{2}, \chi^{-}_{A}(\kappa * \lambda))\}$$

$$= maxim\{\chi^{-}_{!A}(\kappa * \mu), \chi^{-}_{!A}(\kappa * \lambda)\}$$

Therefore $\chi^{-}_{!A}(\lambda * \mu) \leq maxim\{\chi^{-}_{!A}(\kappa * \mu), \chi^{-}_{!A}(\kappa * \lambda)\}$

(iv) Now, $\psi^{+}_{!A}(0) = minim(\frac{1}{2}, \psi^{+}_{A}(0)) \leq minim(\frac{1}{2}, \psi^{+}_{A}(\kappa)) = \psi^{+}_{!A}(\kappa)$

Therefore $\psi^{+}_{!A}(0) \leq \psi^{+}_{!A}(\kappa)$

Now, $\psi^{-}_{!A}(0) = maxim(\frac{-1}{2}, \psi^{-}_{A}(0)) \geq maxim(\frac{-1}{2}, \psi^{-}_{A}(\kappa)) = \psi^{-}_{!A}(\kappa)$

Therefore $\psi^{-}_{!A}(0) \geq \psi^{-}_{!A}(\kappa)$

(v) Now, $\psi^{+}_{!A}(\lambda * \mu) = minim(\frac{1}{2}, \psi^{+}_{A}(\lambda * \mu))$

$$\leq minim(\tfrac{1}{2}, maxim\{\psi^{+}_{A}(\kappa * \mu), \psi^{+}_{A}(\kappa * \lambda)\})$$

$$= maxim\{minim(\tfrac{1}{2}, \psi^{+}_{A}(\kappa * \mu)), minim(\tfrac{1}{2},^{+}_{A}(\kappa * \lambda))\}$$

$$= maxim\{\psi^{+}_{!A}(\kappa * \mu), \psi^{+}_{!A}(\kappa * \lambda)\}$$

Therefore $\psi^{+}_{!A}(\lambda * \mu) \leq maxim\{\psi^{+}_{!A}(\kappa * \mu),^{+}_{!A}(\kappa * \lambda)\}$

(vi) Now, $\psi^{-}_{!A}(\lambda * \mu) = maxim(\frac{-1}{2}, \psi^{-}_{A}(\lambda * \mu))$

$$\geq maxim(\tfrac{-1}{2}, minim\{\psi^{-}_{A}(\kappa * \mu), \psi^{-}_{A}(\kappa * \lambda)\})$$

$$= minim\{maxim(\tfrac{-1}{2}, \psi^{-}_{A}(\kappa * \mu)), maxim(\tfrac{-1}{2}, \psi^{-}_{A}(\kappa * \lambda))\}$$

$$= minim\{\psi^{-}_{!A}(\kappa * \mu), \psi^{-}_{!A}(\kappa * \lambda)\}$$

Therefore $\psi^{-}_{!A}(\lambda * \mu) \geq minim\{\psi^{-}_{!A}(\kappa * \mu), \psi^{-}_{!A}(\kappa * \lambda)\}$

Therefore $!A$ is the bipolar intuitionistic fuzzy α-ideal of Γ. □

Theorem 3.2. *In the event that A and B represent the bipolar intuitionistic fuzzy α-ideal of Γ, then $!(A \cap B) = !A \cap !B$ bipolar intuitionistic fuzzy α-ideal of Γ.*

Proof. Let A and B be bipolar intuitionistic fuzzy α-ideal of Γ. Consider $0, \kappa, \lambda, \mu \in A \cap B$, then $0, \kappa, \lambda, \mu \in A$ and $0, \kappa, \lambda, \mu \in B$

(i) Now, $\chi^{+}_{!(A \cap B)}(0) = maxim(\frac{1}{2}, \chi^{+}_{A \cap B}(0))$

$$\geq maxim(\tfrac{1}{2}, minim\{\chi^+_A(\kappa), \chi^+_B(\kappa)\})$$

$$= minim\{\chi^+_{!A}(\kappa), \chi^+_{!B}(\kappa)\} = \chi^+_{!A\cap !B}(\kappa)$$

Therefore $\chi^+_{!(A\cap B)}(0) \geq \chi^+_{!A\cap !B}(\kappa)$

Now, $\chi^-_{!(A\cap B)}(0) = minim(\frac{-1}{2}, \chi^-_{A\cap B}(0))$

$$\leq minim(\tfrac{-1}{2}, maxim\{\chi^-_A(\kappa), \chi^-_B(\kappa)\})$$

$$= maxim\{\chi^-_{!A}(\kappa), \chi^-_{!B}(\kappa)\} = \chi^-_{!A\cap !B}(\kappa)$$

Therefore $\chi^-_{!(A\cap B)}(0) \leq \chi^-_{!A\cap !B}(\kappa)$

(ii) Now, $\chi^+_{!(A\cap B)}(\lambda * \mu) = maxim(\frac{1}{2}, \chi^+_{A\cap B}(\lambda * \mu))$

$$\geq maxim(\tfrac{1}{2}, minim\{minim\{\chi^+_A(\kappa * \mu), \chi^+_A(\kappa * \lambda)\},$$

$$minim\{\chi^+_B(\kappa * \mu), \chi^+_B(\kappa * \lambda)\}\})$$

$$= minim\{maxim(\tfrac{1}{2}, minim\{\chi^+_A(\kappa * \mu), \chi^+_B(\kappa * \mu)\}),$$

$$maxim(\tfrac{1}{2}, minim\{\chi^+_A(\kappa * \lambda), \chi^+_B(\kappa * \lambda)\})\}$$

$$= minim\{minim\{\chi^+_{!A}(\kappa * \mu), \chi^+_{!B}(\kappa * \mu)\},$$

$$minim\{\chi^+_{!A}(\kappa * \lambda), \chi^+_{!B}(\kappa * \lambda)\}\}$$

$$= minim\{\chi^+_{!A\cap !B}(\kappa * \mu), \chi^+_{!A\cap !B}(\kappa * \lambda)\}$$

Therefore $\chi^+_{!(A\cap B)}(\lambda * \mu) \geq minim\{\chi^+_{!A\cap !B}(\kappa * \mu), \chi^+_{!A\cap !B}(\kappa * \lambda)\}$

(iii) Now, $\chi^-_{!(A\cap B)}(\lambda * \mu) = minim(\frac{-1}{2}, \chi^-_{A\cap B}(\lambda * \mu))$

$$\leq minim(\tfrac{-1}{2}, maxim\{maxim\{\chi^-_A(\kappa * \mu), \chi^-_A(\kappa * \lambda)\},$$

$$maxim\{\chi^-_B(\kappa * \mu), \chi^-_B(\kappa * \lambda)\}\})$$

$$= maxim\{minim(\tfrac{-1}{2}, maxim\{\chi^-_A(\kappa * \mu), \chi^-_B(\kappa * \mu)\}),$$

$$minim(\tfrac{-1}{2}, maxim\{\chi^-_A(\kappa * \lambda), \chi^-_B(\kappa * \lambda)\})\}$$

$$= maxim\{maxim\{\chi^-_{!A}(\kappa * \mu), \chi^-_{!B}(\kappa * \mu)\},$$

$$maxim\{\chi^-_{!A}(\kappa * \lambda), \chi^-_{!B}(\kappa * \lambda)\}\}$$

$$= maxim\{\chi^-_{!A\cap !B}(\kappa * \mu), \chi^-_{!A\cap !B}(\kappa * \lambda)\}$$

Therefore $\chi^-_{!(A\cap B)}(\lambda * \mu) \leq maxim\{\chi^-_{!A\cap !B}(\kappa * \mu), \chi^-_{!A\cap !B}(\kappa * \lambda)\}$

(iv) Now, $\psi^+_{!(A\cap B)}(0) = minim(\frac{1}{2}, \psi^+_{A\cap B}(0))$

$$\leq minim(\tfrac{1}{2}, maxim\{\psi^+_A(\kappa), \psi^+_B(\kappa)\})$$

$$= maxim\{\psi^+_{!A}(\kappa), \psi^+_{!B}(\kappa)\} = \psi^+_{!A\cap !B}(\kappa)$$

Therefore $\psi^+_{!(A\cap B)}(0) \leq \psi^+_{!A\cap !B}(\kappa)$

Now, $\psi^-_{!(A\cap B)}(0) = maxim(\frac{-1}{2}, \psi^-_{A\cap B}(0))$

$$\geq maxim(\frac{-1}{2}, minim\{\psi^-_A(\kappa), \psi^-_B(\kappa)\})$$

$$= minim\{\psi^-_{!A}(\kappa), \psi^-_{!B}(\kappa)\} = \psi^-_{!A\cap !B}(\kappa)$$

Therefore $\psi^-_{!(A\cap B)}(0) \geq \psi^-_{!A\cap !B}(\kappa)$

(v) Now, $\psi^+_{!(A\cap B)}(\lambda * \mu) = minim(\frac{1}{2}, \psi^+_{A\cap B}(\lambda * \mu))$

$$\leq minim(\frac{1}{2}, maxim\{maxim\{\psi^+_A(\kappa * \mu), \psi^+_A(\kappa * \lambda)\}, maxim\{\psi^+_B(\kappa * \mu), \psi^+_B(\kappa * \lambda)\}\})$$

$$= maxim\{minim(\frac{1}{2}, maxim\{\psi^+_A(\kappa * \mu), \psi^+_B(\kappa * \mu)\}), minim(\frac{1}{2}, maxim\{\psi^+_A(\kappa * \lambda), \psi^+_B(\kappa * \lambda)\})\}$$

$$= maxim\{maxim\{\psi^+_{!A}(\kappa * \mu), \psi^+_{!B}(\kappa * \mu)\}, maxim\{\psi^+_{!A}(\kappa * \lambda), \psi^+_{!B}(\kappa * \lambda)\}\}$$

$$= maxim\{\psi^+_{!A\cap !B}(\kappa * \mu), \psi^+_{!A\cap !B}(\kappa * \lambda)\}$$

Therefore $\psi^+_{!(A\cap B)}(\lambda * \mu) \leq maxim\{\psi^+_{!A\cap !B}(\kappa * \mu), \psi^+_{!A\cap !B}(\kappa * \lambda)\}$

(vi) Now, $\psi^-_{!(A\cap B)}(\lambda * \mu) = maxim(\frac{-1}{2}, \psi^-_{A\cap B}(\lambda * \mu))$

$$\geq maxim(\frac{-1}{2}, minim\{minim\{\psi^-_A(\kappa * \mu), \psi^-_A(\kappa * \lambda)\}, minim\{\psi^-_B(\kappa * \mu), \psi^-_B(\kappa * \lambda)\}\})$$

$$= minim\{maxim(\frac{-1}{2}, minim\{\psi^-_A(\kappa * \mu), \psi^-_B(\kappa * \mu)\}), maxim(\frac{-1}{2}, minim\{\psi^-_A(\kappa * \lambda), \psi^-_B(\kappa * \lambda)\})\}$$

$$= minim\{minim\{\psi^-_{!A}(\kappa * \mu), \psi^-_{!B}(\kappa * \mu)\}, minim\{\psi^-_{!A}(\kappa * \lambda), \psi^-_{!B}(\kappa * \lambda)\}\}$$

$$= minim\{\psi^-_{!A\cap !B}(\kappa * \mu), \psi^-_{!A\cap !B}(\kappa * \lambda)\}$$

Therefore $\psi^-_{!(A\cap B)}(\lambda * \mu) \geq minim\{\psi^-_{!A\cap !B}(\kappa * \mu), \psi^-_{!A\cap !B}(\kappa * \lambda)\}$

Therefore $!(A\cap B) = !A\cap !B$ bipolar intuitionistic fuzzy α-ideal of Γ □

Theorem 3.3. *If A is a bipolar intuitionistic fuzzy α-ideal of Γ then $?A$ is the bipolar intuitionistic fuzzy α-ideal of Γ.*

Proof. Given A is bipolar intuitionistic fuzzy α-ideal of Γ. Consider $0, \kappa, \lambda, \mu \in A$.

(i) Now, $\chi^+_{?A}(0) = minim(\frac{1}{2}, \chi^+_A(0)) \geq minim(\frac{1}{2}, \chi^+_A(\kappa)) = \chi^+_{?A}(\kappa)$

Therefore $\chi^+_{?A}(0) \geq \chi^+_{?A}(\kappa)$

Now, $\chi^-_{?A}(0) = maxim(\frac{-1}{2}, \chi^-_A(0)) \leq maxim(\frac{-1}{2}, \chi^-_A(\kappa)) = \chi^-_{?A}(\kappa)$

Therefore $\chi^-_{?A}(0) \leq \chi^-_{?A}(\kappa)$

(ii) Now, $\chi^+_{?A}(\lambda * \mu) = minim(\frac{1}{2}, \chi^+_A(\lambda * \mu))$

$$\geq minim(\tfrac{1}{2}, minim\{\chi^+_A(\kappa * \mu), \chi^+_A(\kappa * \lambda)\})$$

$$= minim\{minim(\tfrac{1}{2}, \chi^+_A(\kappa * \mu)), minim(\tfrac{1}{2}, \chi^+_A(\kappa * \lambda))\}$$

$$= minim\{\chi^+_{?A}(\kappa * \mu), \chi^+_{?A}(\kappa * \lambda)\}$$

Therefore $\chi^+_{?A}(\lambda * \mu) \geq minim\{\chi^+_{?A}(\kappa * \mu), \chi^+_{?A}(\kappa * \lambda)\}$

(iii) Now, $\chi^-_{?A}(\lambda * \mu) = maxim(\frac{-1}{2}, \chi^-_A(\lambda * \mu))$

$$\leq maxim(\tfrac{-1}{2}, maxim\{\chi^-_A(\kappa * \mu), \chi^-_A(\kappa * \lambda)\})$$

$$= maxim\{maxim(\tfrac{-1}{2}, \chi^-_A(\kappa * \mu)), maxim(\tfrac{-1}{2}, \chi^-_A(\kappa * \lambda))\}$$

$$= maxim\{\chi^-_{?A}(\kappa * \mu), \chi^-_{?A}(\kappa * \lambda)\}$$

Therefore $\chi^-_{?A}(\lambda * \mu) \leq maxim\{\chi^-_{?A}(\kappa * \mu), \chi^-_{!A}(\kappa * \lambda)\}$

(iv) Now, $\psi^+_{?A}(0) = maxim(\frac{1}{2}, \psi^+_A(0)) \leq maxim(\frac{1}{2}, \psi^+_A(\kappa)) = \psi^+_{?A}(\kappa)$

Therefore $\psi^+_{?A}(0) \leq \psi^+_{?A}(\kappa)$

Now, $\psi^-_{?A}(0) = minim(\frac{-1}{2}, \psi^-_A(0)) \geq minim(\frac{-1}{2}, \psi^-_A(\kappa)) = \psi^-_{?A}(\kappa)$

Therefore $\psi^-_{?A}(0) \geq \psi^-_{?A}(\kappa)$

(v) Now, $\psi^+_{?A}(\lambda * \mu) = maxim(\frac{1}{2}, \psi^+_A(\lambda * \mu))$

$$\leq maxim(\tfrac{1}{2}, maxim\{\psi^+_A(\kappa * \mu), \psi^+_A(\kappa * \lambda)\})$$

$$= maxim\{maxim(\tfrac{1}{2}, \psi^+_A(\kappa * \mu)), maxim(\tfrac{1}{2},^+_A(\kappa * \lambda))\}$$

$$= maxim\{\psi^+_{?A}(\kappa * \mu), \psi^+_{?A}(\kappa * \lambda)\}$$

Therefore $\psi^+_{?A}(\lambda * \mu) \leq maxim\{\psi^+_{?A}(\kappa * \mu),^+_{?A}(\kappa * \lambda)\}$

(vi) Now, $\psi^-_{?A}(\lambda * \mu) = minim(\frac{-1}{2}, \psi^-_A(\lambda * \mu))$

$$\geq minim(\tfrac{-1}{2}, minim\{\psi^-_A(\kappa * \mu), \psi^-_A(\kappa * \lambda)\})$$

$$= minim\{minim(\tfrac{-1}{2}, \psi^-_A(\kappa * \mu)), minim(\tfrac{-1}{2}, \psi^-_A(\kappa * \lambda))\}$$

$$= minim\{\psi^-_{?A}(\kappa * \mu), \psi^-_{?A}(\kappa * \lambda)\}$$

Therefore $\psi^-_{?A}(\lambda * \mu) \geq minim\{\psi^-_{?A}(\kappa * \mu), \psi^-_{?A}(\kappa * \lambda)\}$

Therefore $?A$ is the bipolar intuitionistic fuzzy α-ideal of Γ. □

Theorem 3.4. *If A and B are bipolar intuitionistic fuzzy α-ideal of Γ, then $?(A \cap B) = ?A \cap ?B$ is bipolar intuitionistic fuzzy α-ideal of Γ.*

Proof. Let A and B be bipolar intuitionistic fuzzy α-ideal of Γ. Consider $0, \kappa, \lambda, \mu \in A \cap B$, then $0, \kappa, \lambda, \mu \in A$ and $0, \kappa, \lambda, \mu \in B$

(i) Now, $\chi^+_{?(A\cap B)}(0) = minim(\frac{1}{2}, \chi^+_{A\cap B}(0))$

$$\geq minim(\tfrac{1}{2}, minim\{\chi^+_A(\kappa), \chi^+_B(\kappa)\})$$

$$= minim\{\chi^+_{!A}(\kappa), \chi^+_{!B}(\kappa)\} = \chi^+_{?A\cap ?B}(\kappa)$$

Therefore $\chi^+_{?(A\cap B)}(0) \geq \chi^+_{?A\cap ?B}(\kappa)$

Now, $\chi^-_{?(A\cap B)}(0) = maxim(\frac{-1}{2}, \chi^-_{A\cap B}(0))$

$$\leq maxim(\tfrac{-1}{2}, maxim\{\chi^-_A(\kappa), \chi^-_B(\kappa)\})$$

$$= maxim\{\chi^-_{?A}(\kappa), \chi^-_{?B}(\kappa)\} = \chi^-_{?A\cap ?B}(\kappa)$$

Therefore $\chi^-_{?(A\cap B)}(0) \leq \chi^-_{?A\cap ?B}(\kappa)$

(ii) Now, $\chi^+_{?(A\cap B)}(\lambda * \mu) = minim(\frac{1}{2}, \chi^+_{A\cap B}(\lambda * \mu))$

$$\geq minim(\tfrac{1}{2}, minim\{minim\{\chi^+_A(\kappa * \mu), \chi^+_A(\kappa * \lambda)\},$$

$$minim\{\chi^+_B(\kappa * \mu), \chi^+_B(\kappa * \lambda)\}\})$$

$$= minim\{minim(\tfrac{1}{2}, minim\{\chi^+_A(\kappa * \mu), \chi^+_B(\kappa * \mu)\}),$$

$$minim(\tfrac{1}{2}, minim\{\chi^+_A(\kappa * \lambda), \chi^+_B(\kappa * \lambda)\})\}$$

$$= minim\{minim\{\chi^+_{?A}(\kappa * \mu), \chi^+_{?B}(\kappa * \mu)\},$$

$$minim\{\chi^+_{?A}(\kappa * \lambda), \chi^+_{?B}(\kappa * \lambda)\}\}$$

$$= minim\{\chi^+_{?A\cap ?B}(\kappa * \mu), \chi^+_{?A\cap ?B}(\kappa * \lambda)\}$$

Therefore $\chi^+_{?(A\cap B)}(\lambda * \mu) \geq minim\{\chi^+_{?A\cap ?B}(\kappa * \mu), \chi^+_{?A\cap ?B}(\kappa * \lambda)\}$

(iii) Now, $\chi^-_{?(A\cap B)}(\lambda * \mu) = maxim(\frac{-1}{2}, \chi^-_{A\cap B}(\lambda * \mu))$

$$\leq maxim(\tfrac{-1}{2}, maxim\{maxim\{\chi^-_A(\kappa * \mu), \chi^-_A(\kappa * \lambda)\},$$

$$maxim\{\chi^-_B(\kappa * \mu), \chi^-_B(\kappa * \lambda)\}\})$$

$$= maxim\{maxim(\tfrac{-1}{2}, maxim\{\chi^-_A(\kappa * \mu), \chi^-_B(\kappa * \mu)\}),$$

$$maxim(\tfrac{-1}{2}, maxim\{\chi^-_A(\kappa * \lambda), \chi^-_B(\kappa * \lambda)\})\}$$

$$= maxim\{maxim\{\chi^-_{?A}(\kappa * \mu), \chi^-_{?B}(\kappa * \mu)\},$$

$$maxim\{\chi^-_{?A}(\kappa * \lambda), \chi^-_{?B}(\kappa * \lambda)\}\}$$

$$= maxim\{\chi^-_{?A\cap ?B}(\kappa * \mu), \chi^-_{?A\cap ?B}(\kappa * \lambda)\}$$

Therefore $\chi^-_{?(A\cap B)}(\lambda * \mu) \leq maxim\{\chi^-_{?A\cap ?B}(\kappa * \mu), \chi^-_{!A\cap !B}(\kappa * \lambda)\}$

(iv) Now, $\psi^+_{?(A\cap B)}(0) = maxim(\frac{1}{2}, \psi^+_{A\cap B}(0))$

$$\leq maxim(\tfrac{1}{2}, maxim\{\psi^+_A(\kappa), \psi^+_B(\kappa)\})$$

$$= maxim\{\psi^+_{?A}(\kappa), \psi^+_{?B}(\kappa)\} = \psi^+_{?A\cap ?B}(\kappa)$$

Therefore $\psi^{+}_{?(A\cap B)}(0) \leq \psi^{+}_{?A\cap ?B}(\kappa)$

Now, $\psi^{-}_{?(A\cap B)}(0) = minim(\frac{-1}{2}, \psi^{-}_{A\cap B}(0))$

$\geq minim(\frac{-1}{2}, minim\{\psi^{-}_{A}(\kappa), \psi^{-}_{B}(\kappa)\})$

$= minim\{\psi^{-}_{?A}(\kappa), \psi^{-}_{?B}(\kappa)\} = \psi^{-}_{?A\cap ?B}(\kappa)$

Therefore $\psi^{-}_{?(A\cap B)}(0) \geq \psi^{-}_{?A\cap ?B}(\kappa)$

(v) Now, $\psi^{+}_{?(A\cap B)}(\lambda * \mu) = maxim(\frac{1}{2}, \psi^{+}_{A\cap B}(\lambda * \mu))$

$\leq maxim(\frac{1}{2}, maxim\{maxim\{\psi^{+}_{A}(\kappa * \mu), \psi^{+}_{A}(\kappa * \lambda)\},$

$maxim\{\psi^{+}_{B}(\kappa * \mu), \psi^{+}_{B}(\kappa * \lambda)\}\})$

$= maxim\{maxim(\frac{1}{2}, maxim\{\psi^{+}_{A}(\kappa * \mu), \psi^{+}_{B}(\kappa * \mu)\}),$

$maxim(\frac{1}{2}, maxim\{\psi^{+}_{A}(\kappa * \lambda), \psi^{+}_{B}(\kappa * \lambda)\})\}$

$= maxim\{maxim\{\psi^{+}_{?A}(\kappa * \mu), \psi^{+}_{?B}(\kappa * \mu)\},$

$maxim\{\psi^{+}_{?A}(\kappa * \lambda), \psi^{+}_{?B}(\kappa * \lambda)\}\}$

$= maxim\{\psi^{+}_{?A\cap ?B}(\kappa * \mu), \psi^{+}_{?A\cap ?B}(\kappa * \lambda)\}$

Therefore $\psi^{+}_{?(A\cap B)}(\lambda * \mu) \leq maxim\{\psi^{+}_{?A\cap ?B}(\kappa * \mu), \psi^{+}_{?A\cap ?B}(\kappa * \lambda)\}$

(vi) Now, $\psi^{-}_{?(A\cap B)}(\lambda * \mu) = minim(\frac{-1}{2}, \psi^{-}_{A\cap B}(\lambda * \mu))$

$\geq minim(\frac{-1}{2}, minim\{minim\{\psi^{-}_{A}(\kappa * \mu), \psi^{-}_{A}(\kappa * \lambda)\},$

$minim\{\psi^{-}_{B}(\kappa * \mu), \psi^{-}_{B}(\kappa * \lambda)\}\})$

$= minim\{minim(\frac{-1}{2}, minim\{\psi^{-}_{A}(\kappa * \mu), \psi^{-}_{B}(\kappa * \mu)\}),$

$minim(\frac{-1}{2}, minim\{\psi^{-}_{A}(\kappa * \lambda), \psi^{-}_{B}(\kappa * \lambda)\})\}$

$= minim\{minim\{\psi^{-}_{?A}(\kappa * \mu), \psi^{-}_{?B}(\kappa * \mu)\},$

$minim\{\psi^{-}_{?A}(\kappa * \lambda), \psi^{-}_{?B}(\kappa * \lambda)\}\}$

$= minim\{\psi^{-}_{?A\cap ?B}(\kappa * \mu), \psi^{-}_{?A\cap ?B}(\kappa * \lambda)\}$

Therefore $\psi^{-}_{?(A\cap B)}(\lambda * \mu) \geq minim\{\psi^{-}_{?A\cap ?B}(\kappa * \mu), \psi^{-}_{?A\cap ?B}(\kappa * \lambda)\}$

Therefore $?(A \cap B) = ?A \cap ?B$ bipolar intuitionistic fuzzy α-ideal of Γ □

Theorem 3.5. *If A is a bipolar intuitionistic fuzzy α-ideal of Γ then $\overline{?\overline{A}} = !A$ is also bipolar intuitionistic fuzzy α-ideal of Γ.*

Proof. Given A is bipolar intuitionistic fuzzy α-ideal of Γ. Consider $0, \kappa, \lambda, \mu \in A$.

(i) Now, $\chi^{+}_{\overline{?\overline{A}}}(0) = \chi^{+}_{?\overline{A}}(0) = maxim(\frac{1}{2}, \chi^{+}_{\overline{A}}(0)\} = maxim\{\frac{1}{2}, \chi^{+}_{A}(0)\}$

$\geq maxim\{\frac{1}{2}, \chi^{+}_{A}(\kappa)\} = \chi^{+}_{!A}(\kappa)$

Therefore $\chi^{+}_{\overline{?A}}(0) \geq \chi^{+}_{!A}(\kappa)$

Now, $\chi^{-}_{\overline{?A}}(0) = \chi^{-}_{?\overline{A}}(0) = minim(\frac{-1}{2}, \chi^{-}_{\overline{A}}(0)\} = minim\{\frac{-1}{2}, \chi^{-}_{A}(0)\}$

$\leq minim\{\frac{-1}{2}, \chi^{-}_{A}(\kappa)\} = \chi^{-}_{!A}(\kappa)$

Therefore $\chi^{-}_{\overline{?A}}(0) \leq \chi^{-}_{!A}(\kappa)$

(ii) Now, $\chi^{+}_{\overline{?A}}(\lambda * \mu) = \chi^{+}_{?\overline{A}}(\lambda * \mu) = maxim(\frac{1}{2}, \chi^{+}_{\overline{A}}(\lambda * \mu)\} = maxim\{\frac{1}{2}, \chi^{+}_{A}(\lambda * \mu)\}$

$\geq maxim\{\frac{1}{2}, minim\{\chi^{+}_{A}(\kappa * \mu), \chi^{+}_{A}(\kappa * \lambda)\}\}$

$= minim\{maxim\{\frac{1}{2}, \chi^{+}_{A}(\kappa * \mu), maxim\{\frac{1}{2}, \chi^{+}_{A}(\kappa * \lambda)\}\}$

$= minim\{\chi^{+}_{!A}(\kappa * \mu), \chi^{+}_{!A}(\kappa * \lambda)\}$

Therefore $\chi^{+}_{\overline{?A}}(\lambda * \mu) \geq minim\{\chi^{+}_{!A}(\kappa * \mu), \chi^{+}_{!A}(\kappa * \lambda)\}$

(iii) Now, $\chi^{-}_{\overline{?A}}(\lambda * \mu) = \chi^{-}_{?\overline{A}}(\lambda * \mu) = minim(\frac{-1}{2}, \chi^{-}_{\overline{A}}(\lambda * \mu)\} = minim\{\frac{-1}{2}, \chi^{-}_{A}(\lambda * \mu)\}$

$\leq minim\{\frac{-1}{2}, maxim\{\chi^{-}_{A}(\kappa * \mu), \chi^{-}_{A}(\kappa * \lambda)\}\}$

$= maxim\{minim\{\frac{-1}{2}, \chi^{-}_{A}(\kappa * \mu), minim\{\frac{1}{2}, \chi^{-}_{A}(\kappa * \lambda)\}\}$

$= maxim\{\chi^{-}_{!A}(\kappa * \mu), \chi^{-}_{!A}(\kappa * \lambda)\}$

Therefore $\chi^{-}_{\overline{?A}}(\lambda * \mu) \leq maxim\{\chi^{-}_{!A}(\kappa * \mu), \chi^{-}_{!A}(\kappa * \lambda)\}$

(iv) Now, $\psi^{+}_{\overline{?A}}(0) = \psi^{+}_{?\overline{A}}(0) = minim(\frac{1}{2}, \psi^{+}_{\overline{A}}(0)\} = minim\{\frac{1}{2}, \psi^{+}_{A}(0)\}$

$\leq minim\{\frac{1}{2}, \psi^{+}_{A}(\kappa)\} = \psi^{+}_{!A}(\kappa)$

Therefore $\psi^{+}_{\overline{?A}}(0) \leq \psi^{+}_{!A}(\kappa)$

Now, $\psi^{-}_{\overline{?A}}(0) = \psi^{-}_{?\overline{A}}(0) = maxim(\frac{-1}{2}, \psi^{-}_{\overline{A}}(0)\} = maxim\{\frac{-1}{2}, \psi^{-}_{A}(0)\}$

$\geq maxim\{\frac{-1}{2}, \psi^{-}_{A}(\kappa)\} = \psi^{-}_{!A}(\kappa)$

Therefore $\psi^{-}_{\overline{?A}}(0) \geq \psi^{-}_{!A}(\kappa)$

(v) Now, $\psi^{+}_{\overline{?A}}(\lambda * \mu) = \psi^{+}_{?\overline{A}}(\lambda * \mu) = minim(\frac{1}{2}, \psi^{+}_{\overline{A}}(\lambda * \mu)\} = minim\{\frac{1}{2}, \psi^{+}_{A}(\lambda * \mu)\}$

$\leq minim\{\frac{1}{2}, maxim\{\psi^{+}_{A}(\kappa * \mu), \psi^{+}_{A}(\kappa * \lambda)\}\}$

$= maxim\{minim\{\frac{1}{2}, \psi^{+}_{A}(\kappa * \mu), minim\{\frac{1}{2}, \psi^{+}_{A}(\kappa * \lambda)\}\}$

$= minim\{\psi^{+}_{!A}(\kappa * \mu), \psi^{+}_{!A}(\kappa * \lambda)\}$

Therefore $\psi^{+}_{\overline{?A}}(\lambda * \mu) \leq maxim\{\psi^{+}_{!A}(\kappa * \mu), \psi^{+}_{!A}(\kappa * \lambda)\}$

(vi) Now, $\psi^{-}_{\overline{?A}}(\lambda * \mu) = \psi^{-}_{?\overline{A}}(\lambda * \mu) = maxim(\frac{-1}{2}, \psi^{-}_{\overline{A}}(\lambda * \mu)\} = maxim\{\frac{-1}{2}, \psi^{-}_{A}(\lambda * \mu)\}$

$\geq maxim\{\frac{-1}{2}, minim\{\psi^{-}_{A}(\kappa * \mu), \psi^{-}_{A}(\kappa * \lambda)\}\}$

$= minim\{maxim\{\frac{-1}{2}, \psi^{-}_{A}(\kappa * \mu), maxim\{\frac{1}{2}, \psi^{-}_{A}(\kappa * \lambda)\}\}$

$= minim\{\psi^{-}_{!A}(\kappa * \mu), \psi^{-}_{!A}(\kappa * \lambda)\}$

Therefore $\psi^{-}_{\overline{\overline{?A}}}(\lambda * \mu) \geq minim\{\psi^{-}_{!A}(\kappa * \mu), \psi^{-}_{!A}(\kappa * \lambda)\}$

Therefore $\overline{\overline{?A}} = !A$ is also bipolar intuitionistic fuzzy α-ideal of Γ. □

Theorem 3.6. *If A is a bipolar intuitionistic fuzzy α-ideal of Γ then $!(?A) = ?(!A)$ is also bipolar intuitionistic fuzzy α-ideal of Γ.*

Proof. Given A is bipolar intuitionistic fuzzy α-ideal of Γ. Consider $0, \kappa, \lambda, \mu \in A$.

(i) Now, $\chi^{+}_{!(?A)}(0) = maxim(\frac{1}{2}, \chi^{+}_{?A}(0)\} = minim\{\frac{1}{2}, maxim\{\frac{1}{2}, \chi^{+}_{A}(0)\}\}$

$\geq minim\{\frac{1}{2}, maxim\{\frac{1}{2}, \chi^{+}_{A}(\kappa)\}$

$= minim\{\frac{1}{2}, \chi^{+}_{!A}(\kappa)\} = \chi^{+}_{?(!A)}(\kappa)$

Therefore $\chi^{+}_{!(?A)}(0) \geq \chi^{+}_{?(!A)}(\kappa)$

Now, $\chi^{-}_{!(?A)}(0) = minim(\frac{-1}{2}, \chi^{-}_{?A}(0)\} = maxim\{\frac{-1}{2}, minim\{\frac{-1}{2}, \chi^{-}_{A}(0)\}\}$

$\leq maxim\{\frac{-1}{2}, minim\{\frac{-1}{2}, \chi^{-}_{A}(\kappa)\}$

$= maxim\{\frac{-1}{2}, \chi^{-}_{!A}(\kappa)\} = \chi^{-}_{?(!A)}(\kappa)$

Therefore $\chi^{-}_{!(?A)}(0) \leq \chi^{-}_{?(!A)}(\kappa)$

(ii) Now, $\chi^{+}_{!(?A)}(\lambda * \mu) = maxim(\frac{1}{2}, \chi^{+}_{?A}(\lambda * \mu)\} = minim\{\frac{1}{2}, maxim\{\frac{1}{2}, \chi^{+}_{A}(\lambda * \mu)\}\}$

$\geq minim\{\frac{1}{2}, maxim\{\frac{1}{2}, minim\{\chi^{+}_{A}(\kappa * \mu), \chi^{+}_{A}(\kappa * \lambda)\}\}\}$

$= minim\{\frac{1}{2}, minim\{\chi^{+}_{!A}(\kappa * \mu), \chi^{+}_{!A}(\kappa * \lambda)\}\}$

$= minim\{minim\{\frac{1}{2}, \chi^{+}_{!A}(\kappa * \mu)\}, minim\{\frac{1}{2}, \chi^{+}_{!A}(\kappa * \lambda)\}\}$

$= minim\{\chi^{+}_{?(!A)}(\kappa * \mu), \chi^{+}_{?(!A)}(\kappa * \lambda)\}$

Therefore $\chi^{+}_{!(?A)}(\lambda * \mu) \geq minim\{\chi^{+}_{?(!A)}(\kappa * \mu), \chi^{+}_{?(!A)}(\kappa * \lambda)\}$

(iii) Now, $\chi^{-}_{!(?A)}(\lambda * \mu) = minim(\frac{-1}{2}, \chi^{-}_{?A}(\lambda * \mu)\} = maxim\{\frac{-1}{2}, minim\{\frac{-1}{2}, \chi^{-}_{A}(\lambda * \mu)\}\}$

$\leq maxim\{\frac{-1}{2}, minim\{\frac{-1}{2}, maxim\{\chi^{-}_{A}(\kappa * \mu), \chi^{-}_{A}(\kappa * \lambda)\}\}\}$

$= maxim\{\frac{-1}{2}, maxim\{\chi^{-}_{!A}(\kappa * \mu), \chi^{-}_{!A}(\kappa * \lambda)\}\}$

$= maxim\{maxim\{\frac{-1}{2}, \chi^{-}_{!A}(\kappa * \mu)\}, maxim\{\frac{-1}{2}, \chi^{-}_{!A}(\kappa * \lambda)\}\}$

$= maxim\{\chi^{-}_{?(!A)}(\kappa * \mu), \chi^{-}_{?(!A)}(\kappa * \lambda)\}$

Therefore $\chi^{-}_{!(?A)}(\lambda * \mu) \leq maxim\{\chi^{-}_{?(!A)}(\kappa * \mu), \chi^{-}_{?(!A)}(\kappa * \lambda)\}$

(iv) Now, $\psi^{+}_{!(?A)}(0) = minim(\frac{1}{2}, \psi^{+}_{?A}(0)\} = maxim\{\frac{1}{2}, minim\{\frac{1}{2}, \psi^{+}_{A}(0)\}\}$

$\leq maxim\{\frac{1}{2}, minim\{\frac{1}{2}, \psi^{+}_{A}(\kappa)\}$

$= maxim\{\frac{1}{2}, \psi^{+}_{!A}(\kappa)\} = \psi^{+}_{?(!A)}(\kappa)$

Therefore $\psi^{+}_{!(?A)}(0) \leq \psi^{+}_{?(!A)}(\kappa)$

Now, $\psi^-_{!(?A)}(0) = maxim(\frac{-1}{2}, \psi^-_{?A}(0)\} = minim\{\frac{-1}{2}, maxim\{\frac{-1}{2}, \psi^-_A(0)\}\}$

$\geq minim\{\frac{-1}{2}, maxim\{\frac{-1}{2}, \psi^-_A(\kappa)\}$

$= minim\{\frac{-1}{2}, \psi^-_{!A}(\kappa)\} = \psi^-_{?(!A)}(\kappa)$

Therefore $\psi^-_{!(?A)}(0) \geq \psi^-_{?(!A)}(\kappa)$

(v) Now, $\psi^+_{!(?A)}(\lambda * \mu) = minim(\frac{1}{2}, \psi^+_{?A}(\lambda * \mu)\} = maxim\{\frac{1}{2}, minim\{\frac{1}{2}, \psi^+_A(\lambda * \mu)\}\}$

$\leq maxim\{\frac{1}{2}, minim\{\frac{1}{2}, maxim\{\psi^+_A(\kappa * \mu), \psi^+_A(\kappa * \lambda)\}\}\}$

$= maxim\{\frac{1}{2}, maxim\{\psi^+_{!A}(\kappa * \mu), \psi^+_{!A}(\kappa * \lambda)\}\}$

$= maxim\{maxim\{\frac{1}{2}, \psi^+_{!A}(\kappa * \mu)\}, maxim\{\frac{1}{2}, \psi^+_{!A}(\kappa * \lambda)\}\}$

$= maxim\{\psi^+_{?(!A)}(\kappa * \mu), \psi^+_{?(!A)}(\kappa * \lambda)\}$

Therefore $\psi^+_{!(?A)}(\lambda * \mu) \leq maxim\{\psi^+_{?(!A)}(\kappa * \mu), \psi^+_{?(!A)}(\kappa * \lambda)\}$

(vi) Now, $\psi^-_{!(?A)}(\lambda * \mu) = maxim(\frac{-1}{2}, \psi^-_{?A}(\lambda * \mu)\} = minim\{\frac{-1}{2}, maxim\{\frac{-1}{2}, \psi^-_A(\lambda * \mu)\}\}$

$\geq minim\{\frac{-1}{2}, maxim\{\frac{-1}{2}, minim\{\psi^-_A(\kappa * \mu), \psi^-_A(\kappa * \lambda)\}\}\}$

$= minim\{\frac{-1}{2}, minim\{\psi^-_{!A}(\kappa * \mu), \psi^-_{!A}(\kappa * \lambda)\}\}$

$= minim\{minim\{\frac{-1}{2}, \psi^-_{!A}(\kappa * \mu)\}, minim\{\frac{-1}{2}, \psi^-_{!A}(\kappa * \lambda)\}\}$

$= minim\{\psi^-_{?(!A)}(\kappa * \mu), \psi^-_{?(!A)}(\kappa * \lambda)\}$

Therefore $\psi^-_{!(?A)}(\lambda * \mu) \geq minim\{\psi^-_{?(!A)}(\kappa * \mu), \psi^-_{?(!A)}(\kappa * \lambda)\}$

Therefore $!(?A) = ?(!A)$ is also bipolar intuitionistic fuzzy α-ideal of Γ. □

Theorem 3.7. *If A is a bipolar intuitionistic fuzzy α-ideal of Γ then $!(\Box A) = \Box(!A)$ is a bipolar intuitionistic fuzzy α-ideal of Γ.*

Theorem 3.8. *If A is a bipolar intuitionistic fuzzy α-ideal of Γ then $?(\Box A) = \Box(?A)$ is a bipolar intuitionistic fuzzy α-ideal of Γ.*

Theorem 3.9. *If A is a bipolar intuitionistic fuzzy α-ideal of Γ then $?(\Diamond A) = \Diamond(?A)$ is a bipolar intuitionistic fuzzy α-ideal of Γ.*

Theorem 3.10. *If A is a bipolar intuitionistic fuzzy α-ideal of Γ then $!(\Diamond A) = \Diamond(!A)$ is a bipolar intuitionistic fuzzy α-ideal of Γ.*

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