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SOME REVERSE SUPER EDGE MAGIC HARMONIOUS GRAPH STRUCTURES

M. Manju and Dr. M. Regees

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Abstract A reverse edge magic harmonious labeling of a (r, s) graph Γ is represented by a bijective mapping $\psi : V(\Gamma) \cup E(\Gamma) \rightarrow \{1, 2, 3, \ldots, r+s\}$ for every edge xy ∈ E(Γ), the value of $[\psi(xy) - (\psi(x) + \psi(y))(mod\ s)]$ is equal to a constant δ, called magic constant. This paper proves that the triangular ladder TL_n graph, the crown graph C_n^+, and the double crown graph C_n^{++} are reverse super edge magic harmonious graphs.

1 Introduction

Rosa introduced the concept of graph labeling in 1967 [4], which involved assigning integers to vertices, or edges, or both under particular conditions. Sedlacek introduced magic type labeling [9] in 1963. Kotzig and Rosa [6] introduced 'magic labeling' in 1970. Llado and Ringel introduced the term 'edge magic labeling' in 1996 [2], which was extended by Wallis in 2001 [11] to 'edge magic total labeling' referred to as 'super edge magic total labeling' when vertices were labeled with least positive integers. Kotte Amaranadha Reddy and S. Sherief Basha [5] introduced 'reverse edge magic labeling'. Graham and Sloane introduced 'harmonious labeling' [3]. They defined a graph as harmonious when an injection f from vertices to a group of integers modulo q results in distinct edge label $f(x) + f(y)(mod\ q)$. In 2013, Dushyant Tanna established some techniques in 'harmonious labeling' [1]. For further explanation, we refer to 'Gallian's dynamic survey of graph labeling' [4]. In 2020, Merrit Anisha. L, Regees .M and Nicholas. T [7] expanded this to 'edge magic and edge bimagic harmonious labeling'.

This paper presents the concept of reverse super edge magic harmonious labeling. Also, it proves that the triangular ladder graph TL_n for all $n \geq 2$, the crown graph C_n^+ and the double crown graph C_n^{++} for all odd $n \geq 3$ are reverse super edge magic harmonious graphs.

Definition 1.1. [10] The triangular ladder graph TL_n is obtained from the ladder graph L_n by adding additional edges $u_i v_{i+1}$, $1 \leq i \leq n-1$.

Definition 1.2. [8] The crown graph C_n^+ is obtained from a cycle C_n by adding one pendent edge to every vertex of C_n.

Definition 1.3. [8] The double crown graph C_n^{++} is obtained from a cycle C_n by adding two pendent edges to each vertex of C_n.

2 Reverse Edge Magic Harmonious Labeling of Graphs

This paper, we define a concept of reverse super edge magic harmonious labeling of graphs and establish that the triangular ladder TL_n for all $n \geq 2$, the crown graph C_n^+ and the double crown graph C_n^{++} for all odd $n \geq 3$ are reverse super edge magic harmonious graphs

Definition 2.1. A reverse edge magic harmonious (REMH) labeling of a (r, s) graph Γ is a bijective mapping $\psi : [V(\Gamma) \cup E(\Gamma)] \rightarrow \{1, 2, 3, \ldots, r+s\}$ for every edge xy ∈ E(Γ), the value $[\psi(xy) - (\psi(x) + \psi(y))(mod\ s)]$ is equal to a constant δ. A graph Γ is recognized as a REMH graph if it admits such a labeling.

Definition 2.2. A REMH labeling ψ is called a reverse super edge magic harmonious (RSEMH) labeling if $\psi(V(\Gamma)) = \{1, 2, 3, ..., r\}$ and $\psi(E(\Gamma)) = \{r+1, r+2, r+3, ..., r+s\}$. A graph that admits RSEMH labeling is known as an RSEMH graph.

Theorem 2.3. *The triangular ladder TL_n graph is an RSEMH graph for every positive integer $n \geq 2$.*

Proof. The triangular ladder graph TL_n is defined by a vertex set $V(TL_n) = \{a_i/i \in [1, n]\}$ and its edges are $E(TL_n) = \{a_i a_{i+1}, b_i b_{i+1}/i \in [1, n-1]\} \cup \{a_i b_i/i \in [1, n]\} \cup \{a_i b_{i+1}/i \in [1, n-1]\}$. And also, it consists of 2n vertices and 4n-3 edges.

For all $n \geq 2$, we define a bijection $\psi : [(V \cup E)(TL_n)] \to \{1, 2, 3, \ldots, 6n-3\}$ with the vertex labels denoted by $\psi(a_i) = 2i - 1, i \in [1, n]$ and $\psi(b_i) = 2i, i \in [1, n]$ and the edge labels are $\psi(a_i a_{i+1}) = 2n+4i+1, i \in [1, n-1]$; $\psi(b_i b_{i+1}) = 2n+4i+3, i \in [1, n-2]$; $\psi(b_{n-1} b_n) = 2n+2$; $\psi(a_i b_i) = 2n + 4i, i \in [1, n-1]$; $\psi(a_n b_n) = 2n + 3$; $\psi(a_i b_{i+1}) = 2n + 4i + 2, i \in [1, n-2]$ and $\psi(a_{n-1} b_n) = 2n + 1$. Applying these values to $[\psi(xy) - (\psi(x) + \psi(y))(mod\ s)]$, which is used for reverse edge magic harmonious labeling, we get the resulting constant $\delta = 2n + 1$.

Therefore, the triangular ladder graph TL_n with 2n vertices is distinctly labeled as $\{1, 2, \ldots, 2n\}$ for every positive integer $n \geq 2$ and thus the graph is an RSEMH graph. □

Example 2.4. An RSEMH labeling of the triangular ladder graph TL_9 and TL_6 are shown in Figures 1 and 2 respectively.

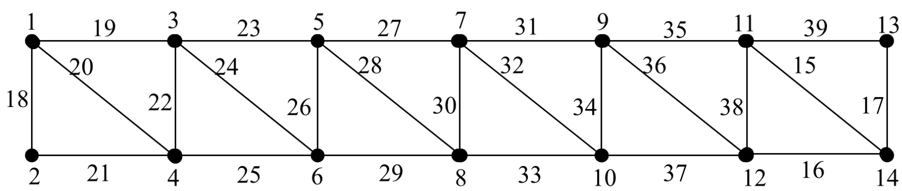

Figure 1. TL_9 with constant $\delta = 15$

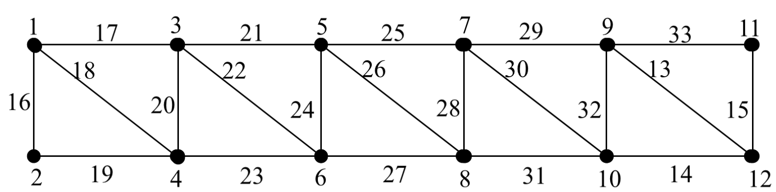

Figure 2. TL_6 with constant $\delta = 13$

Theorem 2.5. *The crown graph C_n^+ is an RSEMH graph for any odd positive integer $n \geq 3$.*

Proof. The crown graph C_n^+ is defined by a vertex set $V(C_n^+) = \{a_i, b_i/i \in [1, n]\}$ and its edges are $E(C_n^+) = \{a_i b_i/i \in [1, n]\} \cup \{a_i a_{i+1}/i \in [1, n-1]\} \cup \{a_1 a_n\}$. And also, it consists of 2n vertices and 2n edges.

For all odd $n \geq 3$, we define a bijection $\psi : [(V \cup E)(C_n^+)] \to \{1, 2, 3, \ldots, 4n\}$ with vertex labels denoted by $\psi(a_i) = i, i \in [1, n]$; $\psi(b_i) = n + i + 1, i \in [1, n-1]$ and $\psi(b_n) = n + 1$ and the edge labels are $\psi(a_i a_{i+1}) = 2n + 2i + 2, i \in [1, n-1]$; $\psi(a_n a_1) = 3n + 2$; $\psi(a_i b_i) = 3n + 2i + 2, i \in [1, \frac{n-1}{2} - 1]$; $\psi(a_i b_i) = n + 2i + 2, i \in [\frac{n-1}{2}, n-1]$ and $\psi(a_n b_n) = 2n + 2$. Applying these values in $[\psi(xy) - (\psi(x) + \psi(y))(mod\ s)]$, which is used for reverse edge magic harmonious labeling, we get the resulting constant $\delta = 2n + 1$.

Therefore, the crown graph C_n^+ with 2n vertices is distinctly labeled as $\{1, 2, \ldots, 2n\}$ for any positive odd integer $n \geq 3$, and thus the graph is an RSEMH graph. □

Example 2.6. An RSEMH labeling of the crown graph C_7^+ is shown in Figure 3.

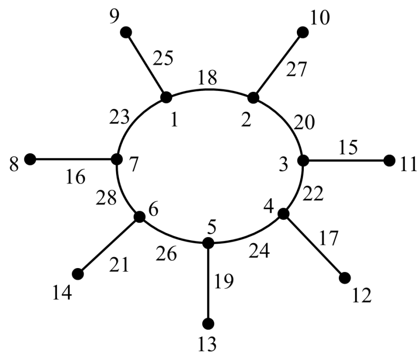

Figure 3. C_7^+ with constant $\delta = 15$

Theorem 2.7. *The double crown graph C_n^{++} is an RSEMH graph for any odd positive integer $n \geq 3$.*

Proof. The double crown graph C_n^{++} is defined by a vertex set $V(C_n^{++}) = \{a_i, b_i, c_i / i \in [1, n]\}$ and its edges are $E(C_n^{++}) = \{a_i b_i / i \in [1, n]\} \cup \{a_i c_i / i \in [1, n]\} \cup \{a_i a_{i+1}) / i \in [1, n-1]\} \cup \{a_1 a_n\}$. And also, it consists of 3n vertices and 3n edges.

For all odd $n \geq 3$, we define a bijection $\psi : [(V \cup E)(C_n^{++})] \to \{1, 2, 3, \ldots, 6n\}$ with vertex labels denoted by $\psi(a_i) = i, i \in [1, n]$; $\psi(b_i) = n + i + 1, i \in [1, n-1]$; $\psi(b_n) = n + 1$; $\psi(c_i) = 2n + i + 1, i \in [1, n-1]$; $\psi(c_n) = 2n + 1$ and the edge labels are $\psi(a_i a_{i+1}) = 3n + 2i + 2, i \in [1, n-1]$; $\psi(a_n a_1) = 4n + 2$; $\psi(a_i b_i) = 4n + 2i + 2, i \in [1, n-1]$; $\psi(a_n b_n) = 5n + 2$; $\psi(a_i c_i) = 5n + 2i + 2, i \in [1, \frac{n-1}{2} - 1]$; $\psi(a_i c_i) = 2n + 2i + 2, i \in [\frac{n-1}{2}, n-1]$ and $\psi(a_n c_n) = 3n + 2$. Applying these values to $[\psi(xy) - (\psi(x) + \psi(y))(mod\ s)]$, which is used for reverse edge magic harmonious labeling, we get the resulting constant $\delta = 3n + 1$

Therefore, the double crown graph C_n^{++} with 3n vertices is distinctly labeled as $\{1, 2, \ldots, 3n\}$ for any positive odd integer $n \geq 3$, and thus the graph is an RSEMH graph.

□

Example 2.8. An RSEMH labeling of the double crown graph C_7^{++} is shown in Figure 4.

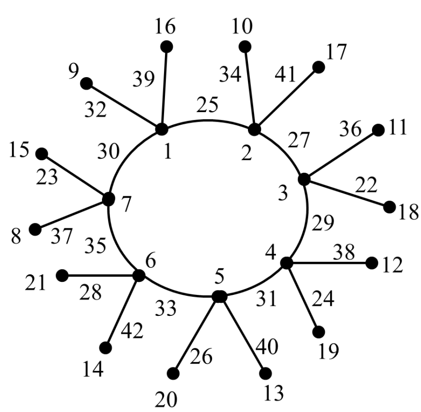

Figure 4. C_7^{++} with constant $\delta = 22$

3 Conclusion

Here, we prove that the triangular ladder graph TL_n for each positive integer $n \geq 2$, and similarly the crown graph C_n^+ and the double crown graph C_n^{++} for any positive odd integer $n \geq 3$ are RSEMH graphs. This finding helps us to understand these graphs simply and clearly.

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Author information

M. Manju, Department of Mathematics, Malankara Catholic College,
Affiliated to Manonmaniam Sundaranar University, Tirunelveli-627012, TamilNadu, India.
E-mail: mahimahi944200@gmail.com

Dr. M. Regees, Department of Mathematics, Malankara Catholic College, Tamil Nadu., India.
E-mail: regeesregees@gmail.com