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Dynamics of 1D Fuzzy Cellular Automata :Rule 22

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Abstract Cellular automata are discrete dynamical systems comprising fundamental, simple components that display various dynamical behaviours. This study examines the fuzzification of rule 22's disjunctive normal form and analyzes the behaviour of the resulting fuzzy rule. The diagonals on the left and right of that system converge to a single point. We also investigate how each local state, which converges to a fixed point, behaves as it converges.

1 Introduction

A fuzzy Cellular Automata (FCA) is obtained by fuzzification of the local function of a Boolean $\mathbf{CA}$: in the disjunctive normal form, $(a_1 \vee a_2)$ is replaced by $(a_1 + a_2), (a_1 \wedge a_2)$ by $(a_1 a_2)$, and $\neg a_1$ by $(1 - a_1)$. The resulting local rule is a real-valued function simulating the original function on $\{0,1\}^3$, with $l(a_1, 0) = 1 - a_1$ and $l(a_1, 1) = a_1$.
$g : \{0,1\}^3 \to \{0,1\}$, s.t $g(x,y,z) = \sum_{i=0}^{7} r_i \prod_{j=1}^{3} l(y_j, d_{ij})$

The usual fuzzification of the expression $a_1 \vee a_2$ is max$\{1, a_1 + a_2\}$ so as to ensure that the result is not larger than 1. However, keep in mind that using $(a_1 + a_2)$ for the $\mathbf{CA}$ fuzzification does not produce values larger than 1 since, according to rule 255, the total of all expressions is 1, meaning that every possible partial sum must be bounded by 1. [12]

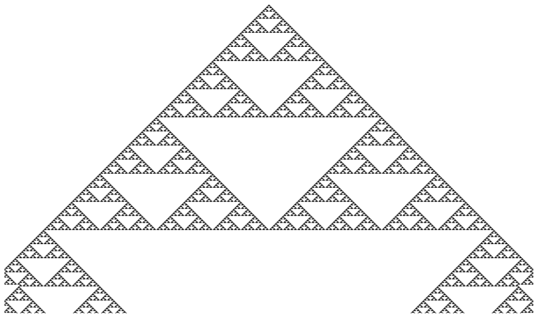
One dimensional cellular automata has 256 possible rules. This paper discusses the Boolean $rule$ 22 and the fuzzification of $rule$ 22.

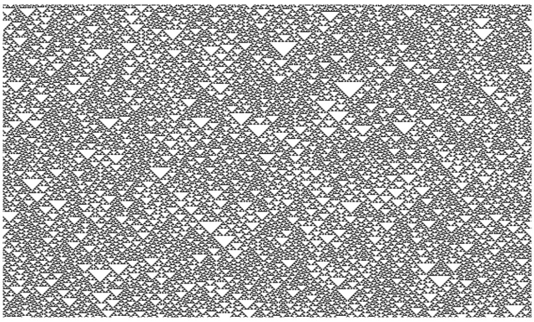
1.1 $Rule$ 22

2 Dynamics of fuzzy $Rule$ 22

One dimensional cellular automata has 256 possible rules. In this section we discuss the Boolean $rule$ 22 and the fuzzification of $rule$ 22.

$Rule$ 22 shows that random initial conditions can lead to a typical chaotic global behavior. Figure depicts the evolution from a single cell in state one as the initial condition. The pattern growth resembles a Sierpinski gasket and is fractal[7]. However, when applying the fuzzification of the disjunctive normal form [11] to $rule$ 22, a convergence pattern is required. Here we define a formula for the convergence of the left diagonal and right diagonal.





2.1 Fuzzification of $Rule$ 22

Rule 22 $= 2 + 2^2 + 2^4$
Its local rule $(000, 001, 010, 011, 100, 101, 110, 111)$ gives $(0, 1, 1, 0, 1, 0, 0, 0)$
The canonical expression of $rule$ 22 is

$$\begin{aligned} g_{22}(y_1, y_2, y_3) &= (y_1 \wedge \neg y_2 \wedge \neg y_3) \vee (\neg y_1 \wedge y_2 \wedge \neg y_3) \vee (\neg y_1 \wedge \neg y_2 \wedge y_3) \\ its\ fuzzification\ is & \\ g_{22}(y_1, y_2, y_3) &= y_1(1-y_2)(1-y_3) + (1-y_1)y_2(1-y_3) + (1-y_1)(1-y_2)y_3 \\ &= y_1 + y_2 + y_3 - 2y_1y_2 - 2y_2y_3 - 2y_1y_3 + 3y_1y_2y_3 \end{aligned}$$

Theorem 2.1. *Let $a \in (0, 1)$ be a single seed with fuzzy value in a zero background. The dynamics of fuzzy rule 22 are given below*
(i) $L_0^- = a$, $L_1^- = \frac{1}{2}$
(ii) The other limits are
$L_2^- = \frac{1}{2+a}$,
$L_3^- = \frac{2+a}{5+2a}$,
$L_4^- = \frac{a^2+4a+5}{2a^2+9a+12}$
In general for $k \geq 0$ $(k+1)^{th}$ left diagonal converges to

$$L_{k+1}^- = \frac{L_k^- - 2L_k^- L_{k-1}^- + L_{k-1}^-}{2L_k^- + 2L_{k-1}^- - 3L_k^- L_{k-1}^-}$$

(iii)All the remaining left diagonals converges to $1 - \frac{1}{\sqrt{3}}$
(iv) Vertical columns converge to $1 - \frac{1}{\sqrt{3}}$
Since the light cone is symmetric in nature the above results are true for right diagonals also

Proof. (i) If we consider the first diagonal, each term in the diagram is formed by 3 terms of the previous row . the left most and middle term are zeros
ie $g(0, 0, a) = 0 + 0 + a = a$
it is true for all elements in that diagonal. $\therefore L_0^-(a) = a$

First element y_0^1 in L_1^- diagonal is a. It is obtained from $g(0, a, 0) = a$
Second element y_{-1}^2 inL_1^- diagonal is obtained from
$g(0, a, a) = a(1-a) + (1-a)a$
ie $y_{-1}^2 = 2a - 2a^2$ Third element y_{-2}^3 inL_1^- diagonal is obtained from

$$\begin{aligned} g(0, a, 2a - 2a^2) &= a[1 - (2a - 2a^2)] + (1-a)(2a - 2a^2) \\ &= a + (2a - 2a^2) - 2a(2a - 2a^2) \\ y_{-2}^3 &= a + y_{-1}^2 - 2ay_{-1}^2 \end{aligned}$$

Similarly we get

$$y_{-4}^5 = a + y_{-3}^4 - 2ay_{-3}^4$$

general formula is

$$y^{m}_{-(m-1)} = a + y^{m-1}_{-(m-2)} - 2ay^{m-1}_{-(m-2)} \tag{2.1}$$

where $y^{m-1}_{-(m-2)} = a + y^{m-2}_{-(m-3)} - 2ay^{m-2}_{-(m-3)}$

substitute $y^{m-1}_{-(m-2)}$ in (1.1)

we get $y^{m}_{-(m-1)} = 2a - 2a^2 + (1-2a)^2 y^{m-2}_{-(m-3)}$

where $y^{m-2}_{-(m-3)} = 2a - 2a^2 + (1-2a)^2 y^{m-5}_{-(m-4)}$

$$\therefore y^{m}_{-(m-1)} = y^{2}_{-1} + \{1 + (1-2a)^2\} + (1-2a)^4 y^{5}_{-4}$$

where $y^{2}_{-1} = 2a - a^2$

again substitute $y^{m-5}_{-(m-4)}, y^{m-6}_{-(m-5)}, ...y^{2}_{-1}$

we get $y^{m}_{-(m-1)} = y^{2}_{-1}[1 + (1-2a)^2 + (1-2a)^4 + ...(1-2a)^{m-4}] + (1-2a)^{m-2} y^{2}_{-1}$

$= [1 + (1-2a)^2 + (1-2a)^4 + ...(1-2a)^{m-2}]x^{2}_{-1}$

$= \{1 + [1-2a]^2 + [(1-2a)^2]^2 + ...[(1-2a)^2]^{\frac{m-2}{2}}\}y^{2}_{-1}$

$= \{\frac{1-[(1-2a)^2]^{\frac{m-2}{2}}}{1-(1-2a)^2}\}x^{2}_{-1}$

$= \frac{1-(1-2a)^{m-2}}{4a-4a^2}(2a - 2a^2)$

$= \frac{1-(1-2a)^{m-2}}{2}$

as $m \to \infty$

$L^{-}_{1} = \frac{1-0}{2}$

$= \frac{1}{2}$

(ii) we already verified that

$L^{-}_{0} = a, L^{-}_{1} = \frac{1}{2}$

After a few iterations

For $\epsilon > 0$

$|y^{m}_{-(m-2)} - y^{m-1}_{-(m-3)}| \leq \epsilon$ for all $m \geq m_0$

ie as $m \to \infty$

$y^{m}_{-(m-2)} \to y^{m-1}_{-(m-3)} (= y)$

$y^{m}_{-(m-2)} = g(a, \frac{1}{2}, y^{m-1}_{-(m-3)})$

$\implies y = g(a, \frac{1}{2}, y)$

$y = a + \frac{1}{2} + y - \frac{2a}{2} - 2ay - y + \frac{3}{2}ay$

gives $y = \frac{1}{2+a}$

$\therefore L^{-}_{2} = \frac{1}{2+a}$

Next we have to prove that

$L^{-}_{3} = \frac{2+a}{5+2a}$

After a few iterations

For $\epsilon > 0$, $|y^{m}_{-(m-3)} - y^{m-1}_{-(m-4)}| \leq \epsilon$ for all $m \geq m_0$

ie as $m \to \infty$

$y^{m}_{-(m-3)} \to y^{m-1}_{-(m-4)} (= y)$

$y^{m}_{-(m-3)} = g(\frac{1}{2}, \frac{1}{2+a}, y^{m-1}_{-(m-4)})$

$\implies y = g(\frac{1}{2}, \frac{1}{2+a}, y)$

gives $y = \frac{2+a}{5+2a}$

$\therefore L^{-}_{3} = \frac{2+a}{5+2a}$

In the same way we have can prove

$L^{-}_{4} = \frac{a^2+4a+5}{2a^2+9a+12}$

We will show by induction that

$L^{-}_{k+1} = \frac{L^{-}_{k} - 2L^{-}_{k}L^{-}_{k-1} + L^{-}_{k-1}}{2L^{-}_{k} + 2L^{-}_{k-1} - 3L^{-}_{k}L^{-}_{k-1}}$

This fact has been varified for $k = 1$
Suppose that it is true for $k = n$
ie $L_n^- = \frac{L_{n-1}^- - 2L_{n-1}^- L_{n-2}^- + L_{n-2}^-}{2L_{n-1}^- + 2L_{n-2}^- - 3L_{n-1}^- L_{n-2}^-}$
Consider the element $y^m_{-(m-\overline{n+1})}$ in the diagonal L_{n+1}^-
$y^m_{-(m-\overline{n+1})} = g(y^{m-1}_{-(m-\overline{n-2})}, y^{m-1}_{-(m-\overline{n-1})}, y^{m-1}_{-(m-n)})$

For $m \geq m_0$
$y^m_{-(m-\overline{n+1})} \to y^{m-1}_{-(m-n)} (= y)$
This will give $L_{n+1}^- = \frac{L_n^- - 2L_n^- L_{k-1}^- + L_{n-1}^-}{2L_n^- + 2L_{n-1}^- - 3L_n^- L_{n-1}^-}$

$$\therefore L_{k+1}^- = \frac{L_k^- - 2L_k^- L_{k-1}^- + L_{k-1}^-}{2L_k^- + 2L_{k-1}^- - 3L_k^- L_{k-1}^-} \tag{2.2}$$

(iii) If L_k^- converges to L, then the same is true for L_{k-1}^- and L_{k+1}^-
$\therefore$ (1.2) can be written as $L = \dfrac{2L - 2L^2}{4L - 3L^2}$
The only solution of this equation that lies in (0,1) is $L = 1 - \frac{1}{\sqrt{3}} (= \mu)$
$\therefore$ Even though the limits of first few left diagonals are not $1 - \frac{1}{\sqrt{3}}$, the limits of the limits of the left diagonals is actually $1 - \frac{1}{\sqrt{3}}$

(iv) Since $L_k \to \mu$ as $k \to \infty$ choose M_1 large, so that $|L_n - L_m| < \epsilon/3$ for all $n, m \geq M_1$.
$|y^{p-1+n}_{-p} - L_n| < \epsilon/3$ for all $p \in Z$. Similarly choose M_2 so that $|y^{q-1+m}_{-q} - L_m| < \epsilon/3$
Combining these we get, if $m, n \geq M$ then
$|y^{p-1+n}_{-p} - y^{q-1+m}_{-q}| \leq |y^{p-1+n}_{-p} - L_n| + |y^{q-1+m}_{-q} - L_m| + |L_n - L_m| < \epsilon$
provided $M \geq max\{M_1, M_2\}$.
When M increases further we can take $p = q$ so that $|y^{p-1+n}_{-p} - y^{p-1+m}_{-p}| < \epsilon$ for all $m, n \geq M$
This proves that the vertical column converge to the limit μ
.

□

3 Representation of spatio-temporal evolution of fuzzy $rule$ 22 in tables

In this section, we give the representation of spatio- temporal evolution of fuzzy $rule$ 22**.**

Evolution from 0.25 in a zero background

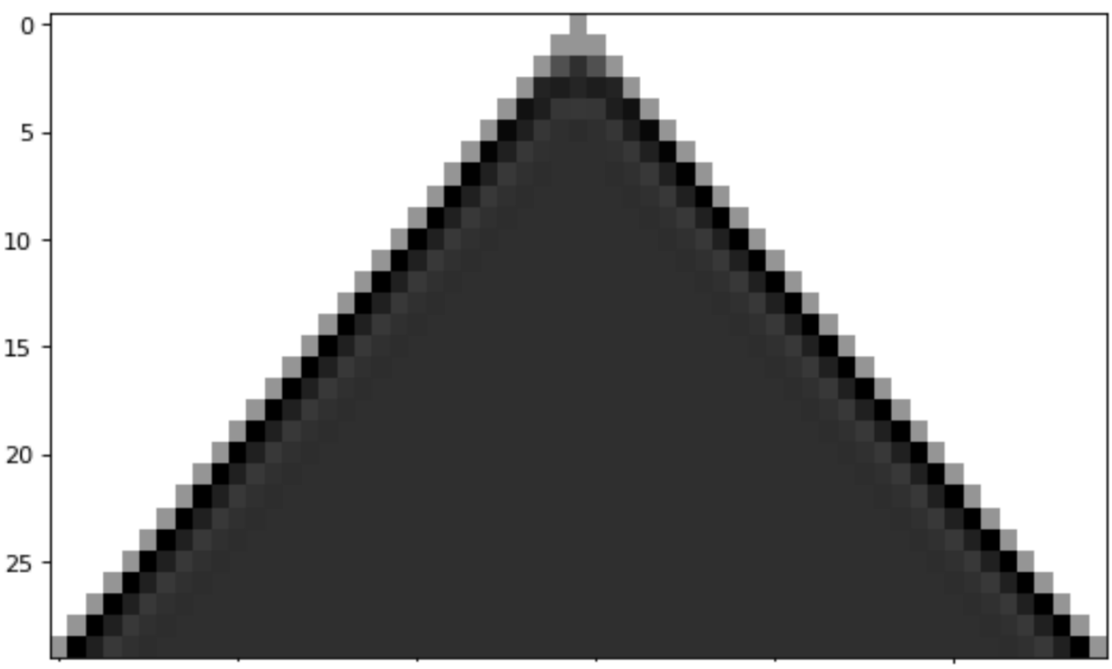
	Local states												
Time	...	-5	-4	-3	-2	-1	0	1	2	3	4	5	...
0	...	0	0	0	0	0	0.25	0	0	0	0	0	...
1	...	0	0	0	0	0.25	0.25	0.25	0	0	0	0	...
2	...	0	0	0	0.25	0.375	0.4219	0.375	0.25	0	0	0	...
3	...	0	0	0.25	0.4375	0.4419	0.4301	0.4419	0.4375	0.25	0	0	...
4	...	0	0.25	.468	0.4514	0.4121	0.4112	0.4121	0.4514	0.468	0.25	0	...
...	...	...	...	...	......	...	...	...	...	...	...	...	...

Evolution from 0.25 in zero background in 30 time steps

	-29	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.2500
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.3750	0.4219
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4375	0.4507	0.4358
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4688	0.4476	0.4133	0.4109
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4844	0.4459	0.4127	0.4224	0.4271
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4922	0.4452	0.4106	0.4208	0.4236	0.4220
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4961	0.4448	0.4099	0.4215	0.4247	0.4229	0.4225
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4980	0.4446	0.4095	0.4216	0.4245	0.4225	0.4223	0.4226
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4990	0.4445	0.4093	0.4217	0.4246	0.4226	0.4224	0.4227	0.4228
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4995	0.4445	0.4092	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4998	0.4445	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4227	0.4227
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4999	0.4445	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4226
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.4999	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4227
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226
18	0	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226
19	0	0	0	0	0	0	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
20	0	0	0	0	0	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
21	0	0	0	0	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
22	0	0	0	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
23	0	0	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
24	0	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
25	0	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
26	0	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
27	0	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
28	0	0	0.2500	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
29	0	0.25	0.5000	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
30	0.25	0.50	0.4444	0.4091	0.4217	0.4246	0.4225	0.4224	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0.3750	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0.4507	0.4375	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0.4133	0.4476	0.4688	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0.4224	0.4127	0.4459	0.4844	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0.4236	0.4208	0.4106	0.4452	0.4922	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0.4229	0.4247	0.4215	0.4099	0.4448	0.4961	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0.4223	0.4225	0.4245	0.4216	0.4095	0.4446	0.4980	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0.4227	0.4224	0.4226	0.4246	0.4217	0.4093	0.4445	0.4990	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4092	0.4445	0.4995	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0.4227	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4445	0.4998	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4445	0.4999	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.4999	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0	0	0	0	0	0	0	0
19	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0	0	0	0	0	0	0
20	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0	0	0	0	0	0
21	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0	0	0	0	0
22	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0	0	0	0
23	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0	0	0
24	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0	0
25	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0	0
26	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0	0
27	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.2500	0	0	0
28	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.5000	0.25	0	0
29	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.4444	0.50	0.25	0
30	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4224	0.4225	0.4246	0.4217	0.4091	0.44	0.50	0.25

Plotted as grey levels for evolution from 0.25 in a zero background in 30 time steps



Evolution from 0.15 in a zero background

Time	Local states												
	. . .	-5	-4	-3	-2	-1	0	1	2	3	4	5	. . .
0	. . .	0	0	0	0	0	0.15	0	0	0	0	0	. . .
1	. . .	0	0	0	0	0.15	0.15	0.15	0	0	0	0	. . .
2	. . .	0	0	0	0.15	0.255	0.3251	0.255	0.15	0	0	0	. . .
3	. . .	0	0	0.15	0.3285	0.4276	0.4369	0.4276	0.3285	0.15	0	0	. . .
4	. . .	0	0.15	0.37995	0.46154	0.4355	0.41880	0.4355	0.46154	0.37995	0.15	0	. . .
5	. . .	0.15	0.41596	0.46723	0.42244	0.415014	0.41922	0.415014	0.42244	0.46723	0.41596	0.15	. . .
. . .	. . .	. . .	. . .	. . .	. . . . . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

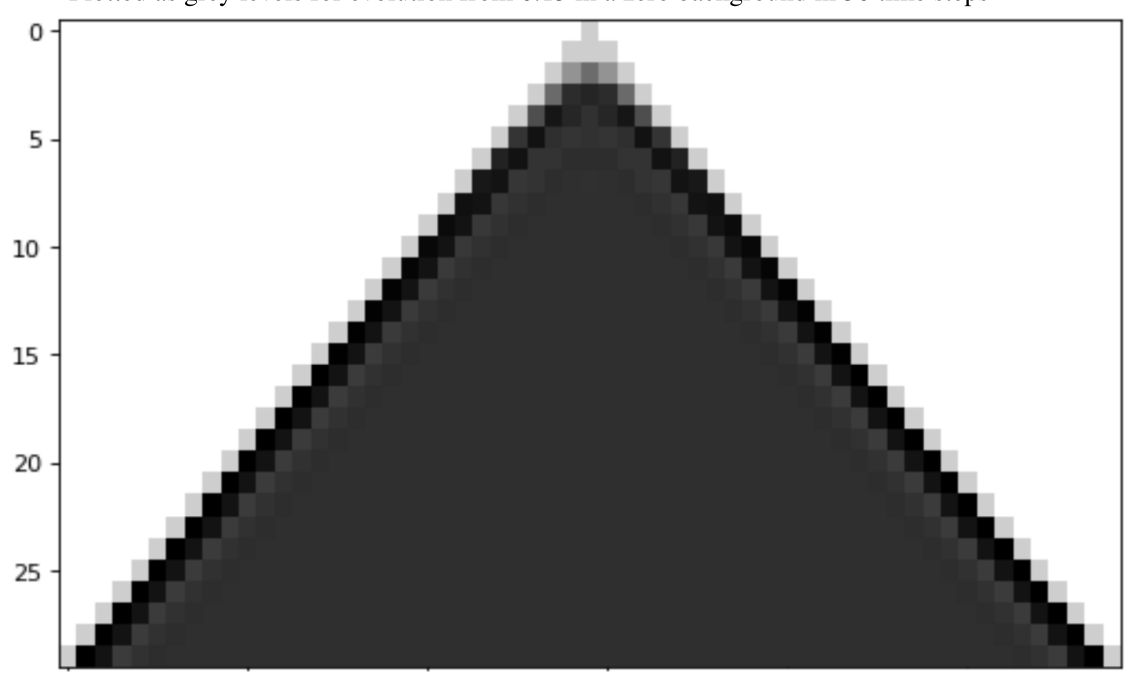
Evolution from 0.15 in zero background in 30 time steps

	-29	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1500
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1500	0.4901
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1500	0.4931	0.4653
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1500	0.4952	0.4652	0.4065
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1500	0.4966	0.4652	0.4063	0.4196
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1500	0.4976	0.4652	0.4061	0.4196	0.4253
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1500	0.4983	0.4652	0.4060	0.4197	0.4253	0.4227
18	0	0	0	0	0	0	0	0	0	0	0	0	0.1500	0.4988	0.4651	0.4059	0.4197	0.4253	0.4227	0.4223
19	0	0	0	0	0	0	0	0	0	0	0	0.1500	0.4992	0.4651	0.4058	0.4197	0.4253	0.4227	0.4223	0.4227
20	0	0	0	0	0	0	0	0	0	0	0.1500	0.4994	0.4651	0.4058	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227
21	0	0	0	0	0	0	0	0	0	0.1500	0.4996	0.4651	0.4057	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227	0.4226
22	0	0	0	0	0	0	0	0	0.1500	0.4997	0.4651	0.4057	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226
23	0	0	0	0	0	0	0	0.1500	0.4998	0.4651	0.4057	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227
24	0	0	0	0	0	0	0.1500	0.4999	0.4651	0.4057	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227
25	0	0	0	0	0	0.1500	0.4999	0.4651	0.4057	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226
26	0	0	0	0	0.1500	0.4999	0.4651	0.4057	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226
27	0	0	0	0.1500	0.5000	0.4651	0.4057	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4226
28	0	0	0.1500	0.5000	0.4651	0.4057	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226
29	0	0.15	0.5000	0.4651	0.4057	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226
30	0.15	0.50	0.4651	0.4067	0.4197	0.4253	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226

	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0.1500	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0.1500	0.1500	0.1500	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0.1500	0.2550	0.3251	0.2550	0.1500	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0.1500	0.3285	0.4276	0.4369	0.4276	0.3285	0.1500	0	0	0	0	0	0	0
5	0	0	0	0	0	0.1500	0.3800	0.4615	0.4355	0.4188	0.4355	0.4615	0.3800	0.1500	0	0	0	0	0	0
6	0	0	0	0	0.1500	0.4160	0.4672	0.4224	0.4150	0.4192	0.4150	0.4224	0.4672	0.4160	0.1500	0	0	0	0	0
7	0	0	0	0.1500	0.4412	0.4670	0.4170	0.4172	0.4244	0.4255	0.4244	0.4172	0.4170	0.4670	0.4412	0.1500	0	0	0	0
8	0	0	0.1500	0.4588	0.4664	0.4134	0.4178	0.4241	0.4228	0.4217	0.4228	0.4241	0.4178	0.4134	0.4664	0.4588	0.1500	0	0	0
9	0	0.1500	0.4712	0.4660	0.4111	0.4185	0.4246	0.4232	0.4226	0.4228	0.4226	0.4232	0.4246	0.4185	0.4111	0.4660	0.4712	0.1500	0	0
10	0.1500	0.4798	0.4657	0.4094	0.4189	0.4248	0.4229	0.4223	0.4226	0.4227	0.4226	0.4223	0.4229	0.4248	0.4189	0.4094	0.4657	0.4798	0.1500	0
11	0.4859	0.4655	0.4083	0.4191	0.4249	0.4229	0.4223	0.4227	0.4227	0.4227	0.4227	0.4227	0.4223	0.4229	0.4249	0.4191	0.4083	0.4655	0.4859	0.1500
12	0.4664	0.4075	0.4193	0.4251	0.4228	0.4223	0.4227	0.4227	0.4226	0.4226	0.4226	0.4227	0.4227	0.4223	0.4228	0.4251	0.4193	0.4075	0.4654	0.4901
13	0.4069	0.4194	0.4251	0.4228	0.4223	0.4227	0.4227	0.4226	0.4227	0.4227	0.4227	0.4226	0.4227	0.4227	0.4223	0.4228	0.4251	0.4194	0.4069	0.4653
14	0.4195	0.4252	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4252	0.4195	0.4065
15	0.4252	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4227	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4252	0.4196
16	0.4227	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4253
17	0.4223	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227
18	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223
19	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227
20	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227
21	0.4226	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226
22	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4226
23	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227
24	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227
25	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
26	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
27	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
28	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
29	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226
30	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226

	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0.1500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0.4931	0.1500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0.4652	0.4952	0.1500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0.4063	0.4652	0.4966	0.1500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0.4196	0.4061	0.4652	0.4976	0.1500	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0.4253	0.4197	0.4060	0.4652	0.4983	0.1500	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0.4227	0.4253	0.4197	0.4059	0.4651	0.4988	0.1500	0	0	0	0	0	0	0	0	0	0	0	0
19	0.4223	0.4227	0.4253	0.4197	0.4058	0.4651	0.4992	0.1500	0	0	0	0	0	0	0	0	0	0	0
20	0.4227	0.4223	0.4227	0.4253	0.4197	0.4058	0.4651	0.4994	0.1500	0	0	0	0	0	0	0	0	0	0
21	0.4227	0.4227	0.4223	0.4227	0.4253	0.4197	0.4067	0.4651	0.4996	0.1500	0	0	0	0	0	0	0	0	0
22	0.4226	0.4227	0.4227	0.4223	0.4227	0.4253	0.4197	0.4057	0.4651	0.4997	0.1500	0	0	0	0	0	0	0	0
23	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4253	0.4197	0.4057	0.4651	0.4998	0.1500	0	0	0	0	0	0	0
24	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4253	0.4197	0.4057	0.4651	0.4999	0.1500	0	0	0	0	0	0
25	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4253	0.4197	0.4057	0.4651	0.4999	0.1500	0	0	0	0	0
26	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4253	0.4197	0.4057	0.4651	0.4999	0.1500	0	0	0	0
27	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4253	0.4197	0.4057	0.4651	0.5000	0.1500	0	0	0
28	0.4226	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4253	0.4197	0.4057	0.4651	0.5000	0.1500	0	0
29	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4253	0.4197	0.4057	0.4651	0.5000	0.15	0
30	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4227	0.4223	0.4227	0.4253	0.4197	0.4057	0.4651	0.50	0.15

Plotted as grey levels for evolution from 0.15 in a zero background in 30 time steps



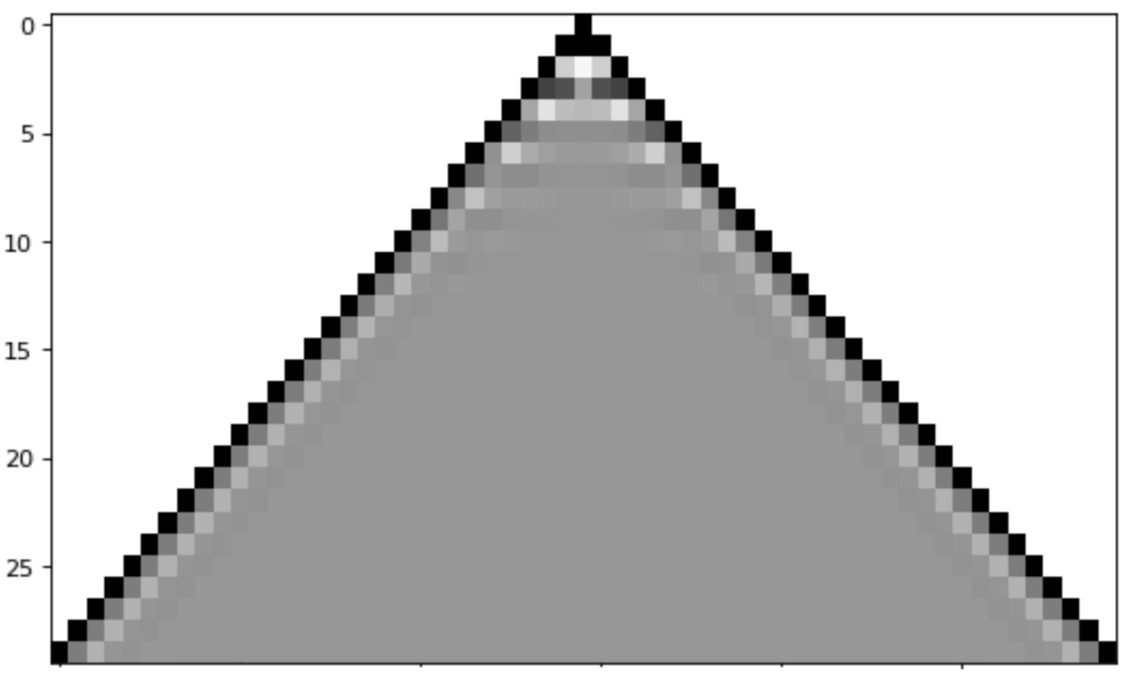
Evolution from 0.85 in a zero background

	Local states												
Time	...	-5	-4	-3	-2	-1	0	1	2	3	4	5	...
0	...	0	0	0	0	0	0.85	0	0	0	0	0	...
1	...	0	0	0	0	0.85	0.85	0.85	0	0	0	0	...
2	...	0	0	0	0.85	0.255	0.57375	0.255	0.85	0	0	0	...
3	...	0	0	0.85	0.6715	0.350343	0.4804	0.350343	0.6715	0.85	0	0	...
4	...	0	0.85	0.37995	0.2641	0.389	0.4393	0.389	0.2641	0.37995	0.85	0	...
5	...	0.85	0.584035	0.45436	0.44839	0.44851	0.430533	0.44851	0.44839	0.45436	0.584035	0.85	...
...	...	...	...	...	......	...	...	...	...	...	...	...	...

Evolution from 0.85 in zero background in 30 time steps

	-29	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.8500	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.2550	0.0574	0.2550	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.6715	0.6394	0.3900	0.6394	0.6715	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.3800	0.1685	0.3220	0.3320	0.3220	0.1685	0.3800	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.5840	0.5013	0.4511	0.4423	0.4443	0.4423	0.4511	0.5013	0.5840	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.4412	0.2513	0.3679	0.4000	0.4107	0.4123	0.4107	0.4000	0.3679	0.2513	0.4412	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.5412	0.4262	0.4515	0.4499	0.4348	0.4291	0.4276	0.4291	0.4348	0.4499	0.4515	0.4262	0.5412	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.4712	0.2997	0.3964	0.4127	0.4110	0.4151	0.4189	0.4198	0.4189	0.4151	0.4110	0.4127	0.3964	0.2997	0.4712	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.5202	0.3881	0.4416	0.4437	0.4295	0.4269	0.4261	0.4248	0.4242	0.4248	0.4261	0.4269	0.4295	0.4437	0.4416	0.3881	0.5202	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.4859	0.3253	0.4114	0.4225	0.4149	0.4175	0.4203	0.4211	0.4215	0.4217	0.4215	0.4211	0.4203	0.4175	0.4149	0.4225	0.4114	0.3253	0.4859	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.5099	0.3691	0.4341	0.4387	0.4256	0.4246	0.4250	0.4240	0.4234	0.4232	0.4231	0.4232	0.4234	0.4240	0.4250	0.4246	0.4256	0.4387	0.4341	0.3691	0.5099	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.4931	0.3382	0.4186	0.4276	0.4178	0.4194	0.4215	0.4218	0.4220	0.4222	0.4224	0.4224	0.4224	0.4222	0.4220	0.4218	0.4215	0.4194	0.4178	0.4276	0.4186	0.3382	0.4931	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.5048	0.3598	0.4298	0.4356	0.4233	0.4232	0.4241	0.4235	0.4231	0.4230	0.4229	0.4228	0.4228	0.4228	0.4229	0.4230	0.4231	0.4235	0.4241	0.4232	0.4233	0.4356	0.4298	0.3598	0.5048	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.4966	0.3446	0.4221	0.4301	0.4194	0.4204	0.4223	0.4222	0.4222	0.4224	0.4225	0.4225	0.4226	0.4226	0.4226	0.4225	0.4225	0.4224	0.4222	0.4222	0.4223	0.4204	0.4194	0.4301	0.4221	0.3446	0.4966	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.5024	0.3553	0.4276	0.4340	0.4221	0.4224	0.4236	0.4231	0.4228	0.4228	0.4228	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4228	0.4228	0.4228	0.4231	0.4236	0.4224	0.4221	0.4340	0.4276	0.3553	0.5024	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.4983	0.3478	0.4238	0.4313	0.4202	0.4210	0.4226	0.4225	0.4224	0.4225	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4225	0.4224	0.4225	0.4226	0.4210	0.4202	0.4313	0.4238	0.3478	0.4983	0.8500	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.5012	0.3530	0.4265	0.4332	0.4215	0.4220	0.4233	0.4229	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4229	0.4233	0.4220	0.4215	0.4332	0.4265	0.3530	0.5012	0.8500	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0.8500	0.4992	0.3494	0.4246	0.4319	0.4206	0.4213	0.4228	0.4226	0.4225	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4225	0.4226	0.4228	0.4213	0.4206	0.4319	0.4246	0.3494	0.4992	0.8500	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0.8500	0.5006	0.3519	0.4259	0.4328	0.4212	0.4218	0.4231	0.4228	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4228	0.4231	0.4218	0.4212	0.4328	0.4259	0.3519	0.5006	0.8500	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0.8500	0.4996	0.3501	0.4250	0.4322	0.4208	0.4214	0.4229	0.4227	0.4225	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4225	0.4227	0.4229	0.4214	0.4208	0.4322	0.4250	0.3501	0.4996	0.8500	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0.8500	0.5003	0.3514	0.4256	0.4326	0.4211	0.4217	0.4231	0.4228	0.4226	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4226	0.4228	0.4231	0.4217	0.4211	0.4326	0.4256	0.3514	0.5003	0.8500	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0.8500	0.4998	0.3505	0.4252	0.4323	0.4209	0.4215	0.4230	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4230	0.4215	0.4209	0.4323	0.4252	0.3505	0.4998	0.8500	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0.8500	0.5001	0.3511	0.4255	0.4325	0.4210	0.4216	0.4230	0.4228	0.4226	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4226	0.4228	0.4230	0.4216	0.4210	0.4325	0.4255	0.3511	0.5001	0.8500	0	0	0	0	0	0
25	0	0	0	0	0	0.8500	0.4999	0.3507	0.4253	0.4324	0.4209	0.4215	0.4230	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4230	0.4215	0.4209	0.4324	0.4253	0.3507	0.4999	0.8500	0	0	0	0	0
26	0	0	0	0	0.8500	0.5001	0.3510	0.4254	0.4325	0.4210	0.4216	0.4230	0.4228	0.4226	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4227	0.4226	0.4228	0.4230	0.4216	0.4210	0.4325	0.4254	0.3510	0.5001	0.8500	0	0	0	0
27	0	0	0	0.8500	0.5000	0.3508	0.4253	0.4324	0.4209	0.4216	0.4230	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4230	0.4216	0.4209	0.4324	0.4253	0.3508	0.5000	0.8500	0	0	0
28	0	0	0.85	0.5000	0.3509	0.4254	0.4325	0.4210	0.4216	0.4230	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4230	0.4216	0.4210	0.4325	0.4254	0.3509	0.5000	0.85	0	0
29	0	0.85	0.50	0.3508	0.4254	0.4324	0.4210	0.4216	0.4230	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4230	0.4216	0.4210	0.4324	0.4254	0.3508	0.50	0.85	0
30	0.85	0.50	0.351	0.4254	0.4325	0.4210	0.4216	0.4230	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4226	0.4227	0.4227	0.4226	0.4226	0.4227	0.4226	0.4226	0.4227	0.4230	0.4216	0.4210	0.4325	0.4254	0.35	0.50	0.85

Plotted as grey levels for evolution from 0.85 in a zero background in 30 time steps



An illustration of the convergence of the fuzzy rule 22 using the seed $a = 0.25$, 0.15,0.85 are shown in the tables.

These patterns indicate the convergence of the fuzzy rule 22 to the fixed point $1 - \frac{1}{\sqrt{3}}$.

4 Conclusion remarks

Boolean rule 22 makes a fractal pattern, but fuzzy rule 22 shows a converging nature. The evolution after fuzzification of rule 22 shows a convergence pattern in all left and right diagonal which is symmetric in nature and it converges to a fixed point $1 - \frac{1}{\sqrt{3}}$.

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