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EFFECTS OF SLIP CONDITIONS ON THE SYNOVIAL FLUID IN HUMAN JOINTS

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Abstract: *Within the ongoing inquiry, the consequences of slip stipulations regarding the motion of synovial liquid in Human joints have been explored. The overseeing equations of motion are simplified by employing fitting presumptions for the stream of synovial liquid over cartilage periphery. Here, it's presumed that the upper limit of the cartilage is shrouded by a porous plate, while the underlying limit is delineated by a rigid plate. Analytical expressions for axial pace, slip pace, and shear tension have been derived by utilizing Beavers-Joseph slip condition. The fundamental parameters, namely Darcy pace, permeable parameter and slip parameter are discussed for further examination. The effects of these parameters on axial pace, slip pace, and shear tension have been computed with employing the Beavers-Joseph slip condition and compared with Beavers-Joseph-Rudraiah slip condition. Noteworthy alterations in the stream of synovial fluid on the cartilage surface are perceived. These findings may possibly assist in regulating fluid dynamics within the cartilage in human joints.*

Nomenclature

h	*width of the channel*	$\alpha\sigma_0$	*porous parameter*
k	*flow consistency index*	η_1	*pressure gradient*
n	*flow behaviour index*	u_b	*slip velocity*
Q	*Darcy Velocity*	SS_U	*skin friction of upper porous plate*
α	*slip parameter*	SS_L	*skin friction of lower rigid plate*

1 Introduction

The human structure contains myriad bone intersections. At all connection points, the bone terminus is encased with pliable, substance-like composition labeled as articular cartilage. The space amid these cartilages is engulfed by a liquid recognized as synovial fluid. This liquid enables the motion of the bones. To grasp this motion, numerous inquiries have been undertaken - both experiential and abstract (Maroudas et al. [1], Peeyush Chandra [2], Levick & McDonald [3], Krzysztof Wierzcholski [4], Schmidt and Sah [5], Jaroslav Hron et al. [6] and Neelam Singh [7]). For theoretical scrutiny, the void in the middle of the cartilages is roughly calculated by various patterns. The stream of synovial fluid amid the parallel planes has been inspected with diverse edge situations. Rudraiah et al. [8] applied a dispersion pattern for the transport of nutrients from the synovial fluid to the cartilage. They neglected the permeableness of the cartilage surface. Blewis et al. [9] established a structure of synovial fluid lubricant composition in typical and impaired joints. Pustejovska [10] analyzed the flow of synovial fluid under precise conditions where it can be classified as an incompressible viscous non-Newtonian fluid. Albert

E Yousif and Ali Amer Al-allaq [11] investigated diverse character of synovial fluid throughout various intervals of joint movements and the ramifications in articular cartilage. Krzysztof Wierzcholski and Andrzej Miszczak [12] explored the curative application of magnetic induction fields on human joint cartilage. Alshehri and Sharma [13] introduced the theoretical examination of the convective diffusion mechanism occurring in the knee joint. Venna [14] explored mass migration problem of a power-law substance with slip of wall. Venna has observed that both the power-law exponent 'n' and the slip coefficient enhance the concentration dispersion and reduce the dispersal. Kapil Shekhar and Tyagi [15] developed an analytical solution for the normal human knee joint by treating the cartilage surfaces as porous and incorporating the changes in synovial fluid viscosity with hyaluronate concentration. Vijaya Kumar and Ratchagar [16] investigate the effects of various parameters on the convective diffusion of synovial fluid within a rectangular channel. Jai Chhimwal et al. [17] explored the use of computational dynamics in joint therapy and examined the flow patterns of synovial fluid within the joints. Noureddine Ouerfelli et al. [18] analyzed the rheological behavior of synovial fluid, highlighting it as a mathematical challenge. Recently, there has been significant attention given to studies on the mathematical modeling of the movement of synovial fluid in human joints (Rushi Kumar et al. [19], Al-Atawi et al. [20], Shahid Hasnain [21], Sushil Kumar and Yadav [22], Nawal Odah Ali-Atawi and Daoud Suleiman Mashat [23], Sadique and Sapna Ratan Shah [24]). In 1967, Beavers and Joseph's experiments [25] sparked interest in coupled parallel flow between a porous layer and free-space, as well as between composite porous layers. They observed that the mass flux through a free-space channel exceeded the mass flux predicted by Poiseuille flow under a no-slip condition. They attributed this discrepancy to a slip flow hypothesis at the interface and proposed a slip-flow condition (referred as BJ slip condition) that aligned closely with their experimental results. The BJ slip condition has attracted considerable attention in the literature. Rudraiah [26] determined that Brinkman's equation is better suited for porous layers with finite depth, while the BJ slip condition is more appropriate for semi-infinite porous layers. He took into consideration the depth of the layer and has adapted the Beavers-Joseph slip condition, also known as Beavers-Joseph-Rudraiah slip condition. Recently, Ramakrishnan and Swetha [27] scrutinized the effect of slip coefficients and depth of the permeable layer on the flow of synovial fluid within the Human joints. Dale Roach and Hamdan [28] reviewed recent advancements concerning the BJ slip condition for flow over porous layers and reported on its extensions to cover flow through free-space channels over non-Darcy porous layers and through composite porous layers. Therefore, this ongoing investigation, has analyzed the impacts of BJ slip conditions compared to BJR slip conditions on axial velocity, slip velocity, and shear stress in the movement of synovial fluid.

2 Formulation and Solution of the Problem

This depicted practical model is illustrated in Figure. 1 and 2. The issue of examination can be perceived as concurrent layers, comprising synovial fluid in the end joining bones enveloped by cartilage. Let's suppose, the top boundary on this cartilage is enveloped by a porous plate accordingly the underlying areas are enveloped a rigid boundary. A Cartesian coordinate system, where x-axis represents the central direction of the fluid pathway, while the y-axis is vertical to the surface of the cartilage, as shown in Figure 2 have been utilized. The boundaries enclosing each ends of the walls are denoted by the lines $y = \pm h$. Additionally, symmetry is presumed around the x-axis for the fluid pathway. The fluid flow is considered to be smooth, constant, and non-compressible.

The equation dictating the behavior of power-law fluid (Rudraiah et al. [8]) is outlined as,

$$\frac{d}{dy}\left[k\left(\frac{du}{dy}\right)^{n}\right] = \frac{dp}{dx} \tag{2.1}$$

where k representing the index of flow consistency whereas n denoting the index of flow behavior. The corresponding conditions are,

$$u = 0 \quad at \quad y = -1 \tag{2.2}$$

$$\frac{du}{dy} = \alpha\sigma_0 \left(u_b - Q\right) at \quad y = 1 \tag{2.3}$$

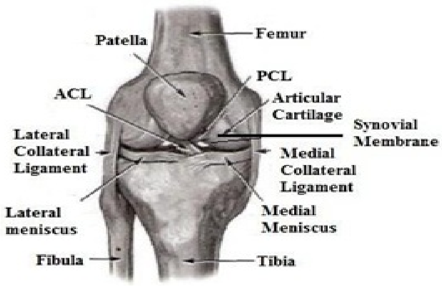

Figure 1. Joint of Human Knee

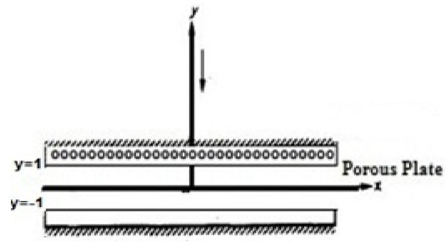

Figure 2. Human Knee Joint's Geometrical counterpart

Here, u_b stands for the slip velocity; Q represents the Darcy Velociity and $\alpha\sigma_0$ indicates the porous parameter. When $n = 1$ in Eqn.(2.1), the behaviour is observed as Newtonian fluid, with shear rate is directly proportional to shear stress. Take $n = 1$ in Eqn.(2.1), it reduces to

$$\frac{d}{dy}\left[k\left(\frac{du}{dy}\right)\right] = \frac{dp}{dx} \tag{2.4}$$

Solving Eqn.(2.4) using the boundary conditions (2.2) and (2.3), we have

$$u(y) = \frac{1}{2\eta_1}\left(\eta_1 y + d1\right)^2 + C \tag{2.5}$$

where

$$d_1 = \alpha\sigma_0\left(Q - u_B\right) - \eta_1 \tag{2.6}$$

$$\eta_1 = \frac{1}{k}\frac{dp}{dx} \tag{2.7}$$

$$C = -\frac{1}{2\eta_1}\left[\alpha\sigma_0\left(Q - u_B\right) - 2\eta_1\right]^2 \tag{2.8}$$

Using (2.5) in (2.3), the slip velocity is given by

$$U_b = \frac{2(\alpha\sigma_0 Q - \eta_1)}{1 + 2\alpha\sigma_0} \tag{2.9}$$

The shear stress in the lower rigid wall and the upper porous wall are given by

$$SS_U = \frac{k}{\eta_1}\left(\eta_1 + d_1\right) \tag{2.10}$$

$$SS_L = \frac{k}{\eta_1}\left(-\eta_1 + d_1\right) \tag{2.11}$$

3 Results and Discussion

For examining the impacts of Porous parameter, Darcy Velocity and Pressure gradient on the flow of synovial fluid, an exhaustive numerical calculation has been conducted for diverse values of porous parameter ($\alpha\sigma_0$), Darcy velocity (Q), and Pressure gradient(η). The impacts of these factors on axial velocity are examined through graphical representations, whereas their influence on slip velocity and skin friction is assessed via tabular analyses.

Figures 3 and 4 illustrate the effects of the porous parameter and Darcy velocity on the axial velocity profile at $\eta = 0.84$ and $\eta = 12.5$, respectively. In these figures, the solid line represents the axial velocity distribution without Darcy velocity, while the dotted lines show the presence of Darcy velocity. Both figures clearly show that the axial velocity distribution takes on a parabolic shape, reaching its maximum near the upper porous wall. The data confirm that the axial velocity increases when Darcy velocity is present. Furthermore, Figure 4 indicates that at $\eta = 12.5$, the

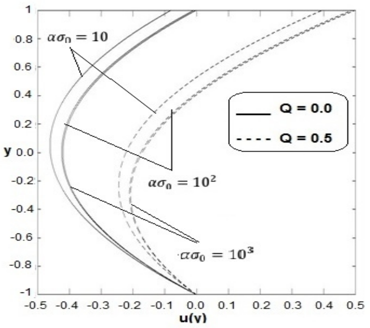

Figure 3. Variation of Axial velocity for different values of $\alpha\sigma_0$ and $\eta = 0.84$

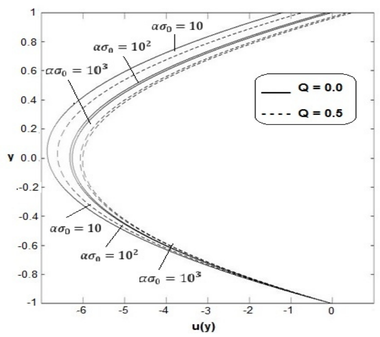

Figure 4. Variation of Axial velocity for different values of $\alpha\sigma_0$ and $\eta = 12.5$

axial velocity pattern is nearly symmetrical for lower values of $\alpha\sigma_0$. When the porous parameter exceeds 10^3, only minimal changes are observed, suggesting that in this scenario, the upper porous boundary behaves similarly to a rigid boundary.

The impact of the porous parameter and Darcy velocity on slip velocity is outlined in Tables 1 and 2 for $\eta = 0.84$ and $\eta = 12.5$, respectively. From Table 1, it can be seen that an increase in the porous parameter results in a numerical decrease in slip velocity, while a rise in Darcy velocity leads to an increase in slip velocity. These trends are also observed in Table 2. However, a comparison of the two tables highlights not only the quantitative differences, but also the fact that at $\eta = 12.5$, slip velocity remains significantly high even without Darcy velocity. Furthermore, it is noted that slip velocity almost same when the porous parameter exceeds 10^3 in the absence or present of Darcy velocity, suggesting that in this scenario, the plate acts as if it is impermeable, as previously mentioned.

Skin friction, which results from momentum transfer to a surface along the flow streamlines, leads to bodies submerged in moving fluids experiencing a force in the direction of the flow as a result of the fluid's deceleration. In the case of blood flow, skin friction primarily arises from the interaction between blood and the endothelial lining of the artery wall. Tables 3 and 4 provide specific numerical values of skin friction concerning a bottom rigid plate for $\eta = 0.84$ and $\eta = 12.5$, respectively. From these tables, it is clear that as the porous parameter increases, the skin friction at the lower rigid plate rises in both scenarios, whether Darcy velocity is present or absent.

Tabulations 5 and 6 revealed that, in the upper permeable plate, the skin friction numerically decreases as the porous parameter increases, which contrasts with the behavior observed in the lower plate. From a physical standpoint, a negative skin friction value signifies that the surface applies a dragging force on the fluid, whereas a positive value indicates the opposite effect.

Table 1: Variaton in Slip velocity when $\eta = 0.84$

$Q \downarrow \alpha\sigma_0 \rightarrow$	10^1	10^2	10^3
0.00001	*-0.0800*	*-0.0083*	*-0.00083*
0.1	*0.3962*	*0.4892*	*0.4989*

Table 2: Variaton in Slip velocity when $\eta = 12.5$

$Q \downarrow \alpha\sigma_0 \rightarrow$	10^1	10^2	10^3
0.00001	*-1.1905*	*-0.1244*	*-0.0125*
0.1	*-0.7143*	*0.3731*	*0.4873*

Table 3: Variation in Skin friction on bottom rigid boundary when $\eta = 0.84$

$Q \downarrow \alpha\sigma_0 \rightarrow$	10^1	10^2	10^3
0.00001	*0.9524*	*0.9950*	*0.9995*
0.1	*1.2358*	*1.2912*	*1.2970*

Table 4: Variation in Skin friction on bottom rigid boundary when $\eta = 12.5$

$Q \downarrow \alpha\sigma_0 \rightarrow$	10^1	10^2	10^3
0.00001	*0.9524*	*0.9950*	*0.9995*
0.1	*0.9714*	*1.0149*	*1.0195*

Table 5: Variation in Skin friction on the Upper Porous plate when $\eta = 0.84$

$Q \downarrow \alpha\sigma_0 \rightarrow$	10^1	10^2	10^3
0.00001	*-1.0476*	*-1.0050*	*-1.0005*
0.1	*-0.7642*	*-0.7068*	*-0.7030*

Table 6: Variation in Skin friction on the Upper Porous plate when $\eta = 12.5$

$Q \downarrow \alpha\sigma_0 \rightarrow$	10^1	10^2	10^3
0.00001	*-1.0476*	*-1.0050*	*-1.0005*
0.1	*-1.0286*	*-0.9851*	*-0.9805*

When comparing these results with the findings from BJR slip condition (Ramakrishnan and Swetha [27]), it is noted that the distribution of axial velocity shows no qualitative changes, although significant quantitative differences are observed. Regarding slip velocity variation under BJ and BJR slip conditions, slip velocity increases only when Darcy velocity is present along with a smaller pressure gradient in the case of BJ slip condition. In contrast, under BJR slip conditions, slip velocity increases with the thickness of the porous plate. Additionally, it is observed that slip velocity is greater in the case of BJ slip conditions compared to BJR slip condition.

When comparing skin friction under BJ and BJR slip conditions, significant qualitative changes are noted. For BJ slip condition, skin friction increases numerically for the lower rigid plate while it decreases for the upper porous plate. In contrast, the behavior is reversed under BJR slip condition. Furthermore, skin friction remains consistently positive near the upper porous plate in the case of BJR slip condition, whereas negative skin friction is observed with BJ slip condition.

Generally, the characteristics of fluid flow are influenced by parameters such as Darcy velocity, porous parameter, and pressure gradient under BJ slip conditions. When BJR slip conditions are applied, these changes are influenced by the width of the plate.

4 Conclusion Remarks

The investigation examines the effects of porous parameters, Darcy velocity, and slip parameters on synovial fluid flow in human joints. Each side of the cartilage is surrounded by a permeable plate above and a rigid plate below.

- *The axial velocity distribution exhibits a parabolic shape, peaking near the upper porous wall.*
- *An increase in the porous parameter of the upper plate results in a numerical decrease in axial velocity in the upper half of the flow channel.*
- *Slip velocity increases with higher Darcy velocity but decreases with increasing porous parameter values.*
- *For the lower rigid plate, skin friction rises with increased Darcy velocity and porous parameter, while skin friction on the upper porous plate decreases with rising parameter values.*
- *When comparing BJ and BJR slip conditions, it is observed that in BJ slip condition, Darcy velocity and porous parameter significantly influence flow alterations. In contrast, in BJR slip condition, the thickness of the porous plate is the primary factor.*

Therefore, by carefully selecting these parameters, it is possible to control synovial fluid flow on cartilage surfaces.

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