Palestine Journal of Mathematics

Vol 14 (Special Issue IV), (2025) , 177–186

Thermal Distribution Model to Study the Impact of Temperature on Building Construction Material

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MSC 2020 Classifications: 80A05, 74S40, 44A10, 80A23.

Keywords and phrases: Mittage Leffler Function, Wright Function, Laplace Transform, Natural Transform, Heat Flux.

We sincerely thank the editor and reviewers for their insightful feedback and thoughtful suggestions, which greatly enhanced the clarity and quality of our research work.

Abstract *Cracks are the most common problem and challenge that seem to persist in any building construction industry. The object of present paper is, first to derive a general expression for thermal flux with diffusivity and conductivity of solid materials in term of fractional order of one dimensional heat equation. We then addressing the impact of fractional order on heat flux during conduction process. The main object of present work is to extend a variability of formula for building construction materials and study the thermal grid points distribution on concrete in order to reducing the thermal cracks during construction process. This research is significant as it predicts thermal expression for various building construction mixtures of concrete (cement, sand and aggregates). The work also capturing the influence of thermal action on 2D and 3D behaviour on piece of concrete slab.*

1 Introduction

One of the frequent issues on any construction site is concrete cracking. The two basic forms of damage in any concrete structure are the cracks and cavities. These cracks and cavities, in turn, reduce the load endurance and hardness of the structure. Corrosion and weakness are the by-products of excessive and uncontrolled cracking of the concrete structure. Cracking not only spoils the ravishing appearance of the structure but it may also cause discomfort to the people staying in such buildings with heavy cracks and cavities. [1]. The causes of these cracks may vary, including external factors force, pressure etc. However, they are primarily related to concrete specifications and construction techniques. Cracks occur when the stress in a building component exceeds its strength, allowing water, air, and chloride to penetrate the concrete. Other factors like wind or seismic loads do generate stress in the building component and considered external forces. In contrast, dead load, moisture changes, thermal movements and chemical reactions are interior forces [2].
Cracks are generally classified into two types: Structural and Non –Structural Cracks. Non –Structural Cracks are typically caused by moisture or heat changes, chemical reactions, creep, elastic deformation, or problems with the foundation of soil (such as moving, settling or uncontrolled vegetation). These cracks do not result in structural weakness but are caused by internally generated stress in building materials. Researchers have identified various types of concrete cracks along with their causes preventing measures and treatments. Thermal cracking is caused by excessive temperature differences within a concrete structure or its surroundings due to environmental fluctuations. Concrete elements with large dimensions (min dimension > 1.5 m) may have a huge temperature difference between the center and outside at the time of curing. Among the prescribed type of concrete cracking and its cause, the one of unavoidable crack is thermal cracking shown in figure 1 and figure 2.

Description of the concrete crack problem: *Many researchers have investigated the effects*

(a) Figure 1: Vertical cracks in concrete

(b) Figure 2: Cracks due to change in temperature

of temperature, the angle of the interface crack and the strength of the concrete on the fracture characteristics. Numerous studies have focused on the high temperatures generated by cement hydration in concrete structures, as these conditions can lead to significant temperature-generated stresses and cracking. This makes it crucial to monitor the temperature development and implement effective cracking control measures. The case study of a concrete mass tunnel in Shantou City, China was investigated [3]. Thermal behavior and cracking risks were analyzed, with temperature monitored during curing. Gradient concrete and a plastic mesh were suggested to prevent cracks. The side wall had the highest risk. Cooling pipes weren't used, so their effect wasn't assessed.

2 Structure of mathematical model

A thorough assessment of damage in concrete exposed to elevated temperatures requires a clear understanding of how cracks develop and how localized failures occur over time. Present work introduces a theoretical model to monitor the temperature fluctuations during concrete casting process. The results are obtained by incorporating the concept of fractional variable operators with one, two and three dimensional heat equations. The work is divided into two segments. The first segment focused on the relationship between heat flux and fractional order of one dimensional heat equation. The second segment addresses the distribution of temperature grid points on a concrete slab in both cases two and three dimension.
To derive general expression of heat flux in solid material, we consider one dimensional heat equation of fractional order given by [4]:

$$D_t^{(\alpha,\beta)}V(x,t) = \lambda \frac{\partial^2 V(x,t)}{\partial x^2} \quad ; \quad 0 \le \alpha \le 1,\ \beta \le 1,\ t > 0,\ x \in \mathbb{R}, \tag{2.1}$$

here λ is the thermal diffusivity of solids. Initially at time $t = 0$ temperature of solid material is taken as $T_0 (T_0 > 0)$. The raising of temperature through external cause, is considered as $T_s > 0$, defining by $V(x,t) = \frac{T-T_0}{T_s-T_0}$ [5].Throughout the theoretical investigation the thermal conductivity of solid is considered as constant parameter. The boundary and initial conditions are given below:

$$\begin{aligned} V(x,0) &= 0, && x \ge 0 \\ V(0,t) &= 1, && t > 0 \\ \lim_{x\to\infty} V(x,t) &= 0, && t > 0 \end{aligned} \tag{2.2}$$

By applying Fourier sine transform, on both sides of the equation (2.1), we obtained

$$D_t^{(\alpha,\beta)}V_s(n,t) = \lambda\sqrt{\frac{2}{\pi}}\left[\sin(nx)\frac{\partial V}{\partial x}\right]_0^\infty - n\lambda\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\partial V}{\partial x}\cos(nx)\,dx,$$

On integrating, we get

$$D_t^{(\alpha,\beta)}V_s(n,t) = n\lambda\sqrt{\frac{2}{\pi}} - n^2\lambda V_s(n,t),$$

On taking Laplace transform of above equation and inverse Laplace transform, we obtained

$$V_s(n,t) = n\lambda\sqrt{\frac{2}{\pi}}\, t^{\alpha}\, E_{\alpha,\alpha+1}(-n^2\lambda t^{\alpha}),$$

On taking inverse Fourier sine transform of above equation, the solution of equation (2.1) in the form of wright function $w(\alpha,\beta;z)$ as given below,

$$V(x,t) = W\left(-\frac{\alpha}{2}, 1; \frac{-x}{\sqrt{\lambda t^{\alpha}}}\right). \tag{2.3}$$

The expression (2.3) is the temperature for solid material due to temperature fluctuations. Thereby a proposed theorem to discuss the rate of flow of thermal energy through heat transfer due to difference in temperature on solid material can be given as:

Theorem 2.1. *Let $D_t^{(\alpha,\beta)}V(x,t) = \lambda V''(x)$, where $\alpha > 0$, $\beta > 0$, $\lambda > 0$, and $x \in \mathbb{R}$, with the following boundary conditions:*

$$V(x,t=0) = 0, \quad V(x=0,t) = 1, \quad x \in \mathbb{R}, \quad \lim_{x\to\infty} V(x,t) = 0.$$

The temperature is given by:

$$V(x,t) = 1 - \frac{x}{\sqrt{\lambda t^{\alpha}}}\cdot\frac{1}{\Gamma\left(\frac{-\alpha}{2}+1\right)1!} + \left(\frac{x}{\sqrt{\lambda t^{\alpha}}}\right)^2\cdot\frac{1}{\Gamma(-\alpha+1)2!} - \cdots = W\left(-\frac{\alpha}{2}, 1; \frac{-x}{\sqrt{\lambda t^{\alpha}}}\right),$$

where $W(\cdot)$ is the Wright function.

Then, the heat flux at a certain point $x = 0$ on the surface of the solid is expressed as:

$$\Phi_q = \frac{C(T_s - T_0)}{\sqrt{\lambda t^{\alpha}}\Gamma\left(1-\frac{\alpha}{2}\right)}.$$

where $C > 0$, $\alpha > 0$, $\lambda > 0$, T_s is the surface temperature, and T_0 is the ambient temperature. Here, C is the conductivity of the solid and λ is the diffusivity of the solid.

Proof: *To prove this theorem, Fourier's law for the flow of thermal energy through conduction in solids is used. The surface heat flux of the solid is given by:*

$$\Phi_q = -C\left[\frac{\partial T}{\partial x}\right], \tag{2.4}$$

By applying the initial conditions and using the obtained result (2.3) in equation (2.4), at the point $x = 0$, we get:

$$\Phi_q = -C\,(T_s - T_0)\left(\frac{-1}{\sqrt{\lambda t^{\alpha}}}\right)\cdot\frac{1}{\Gamma\left(\frac{-\alpha}{2}+1\right)\cdot 1!}. \tag{2.5}$$

This is the desired proof of the theorem.

Theorem 2.2. *For the fractional order heat equation*

$$D_t^{(\alpha,\beta)}V(x,t) = \lambda V''(x), \quad (\alpha > 0, \beta > 0, \lambda > 0, x \in \mathbb{R}),$$

if the temperature on the surface of the solid is given by

$$V(x,t) = 1 - \frac{x}{\sqrt{\lambda t^{\alpha}}}\cdot\frac{1}{\Gamma\left(\frac{-\alpha}{2}+1\right)1!} + \left(\frac{x}{\sqrt{\lambda t^{\alpha}}}\right)^2\cdot\frac{1}{\Gamma(-\alpha+1)2!} - \cdots = W\left(-\frac{\alpha}{2}, 1; \frac{-x}{\sqrt{\lambda t^{\alpha}}}\right),$$

with boundary conditions

$$V(x,t=0) = 0, \quad V(x=0,t) = 1, \quad x \in \mathbb{R}, \quad \lim_{x\to\infty} V(x,t) = 0.$$

At a certain point $x = 0$, the heat flux on the surface of the solid is defined by:

$$\Phi_q = \frac{C(T_s - T_0)}{\sqrt{\lambda t^{\alpha}}\Gamma\left(1 - \frac{\alpha}{2}\right)},$$

where $C > 0$, $\alpha > 0$, $\lambda > 0$, $T_s > 0$, and $T_0 > 0$. Here, C is the conductivity of the solid and λ is the diffusivity of the solid.

Then, the accelerating heat flux of the n-th order is given by:

$$\left.\frac{d^n \Phi_q}{dt^n}\right|_t = \frac{(-1)^n C(T_s - T_0)\left(\frac{\alpha}{2}\right)_n}{\sqrt{\lambda} t^{\left(\frac{\alpha}{2}+n\right)}\Gamma\left(1 - \frac{\alpha}{2}\right)}.$$

where $\left(\frac{\alpha}{2}\right)_n$ is the Pochhammer symbol.

Proof: *From the obtained result of Theorem 1, and by performing successive differentiation with respect to time, the fractional order α appears in multiplicative series form of the respective ordered terms:*

$$\left(\frac{\alpha}{2}\right)\left(\frac{\alpha}{2} + 1\right)\left(\frac{\alpha}{2} + 2\right)\ldots, \quad \alpha > 0.$$

By adopting the principle of mathematical induction, we can derive the required proof of Theorem 2.

2.1 Numerical model of obtained result on concrete slab

To examine the pattern of heat flux on concrete, we consider a 15 cm thick concrete slab which is coated with black silicone paint for the study of flowing thermal energy. Since black silicone paint transfers negligible heat, we consider the coating as a heat insulator [6].

When one end of the concrete slab is heated through an external heat source, it is observed that at 1000 K, the concrete slab approximately behaves like a black body. Initially, at time $t = 0$, the temperature of the concrete slab is taken as $T_{0(Concrete)} = 300\,K$. After 2 minutes of heating ($t = 2\,min$), the temperature of the concrete surface reaches $T_{s(Concrete)} = 500\,K$.

We investigate the entire process by considering a varying range of fractional orders $\alpha = 0.5, 1, 1.5,$ and 2, and obtain the corresponding heat flux values as follows:

$$\phi_q = \begin{cases} 76709\,W/m^2, & \text{for } \alpha = 0.5 \\ 16021\,W/m^2, & \text{for } \alpha = 1 \\ 2366\,W/m^2, & \text{for } \alpha = 1.5 \\ 0\,W/m^2, & \text{for } \alpha = 2, \end{cases}$$

The geometrical representation of heat flux with respect to the fractional order is shown in Figure 3.

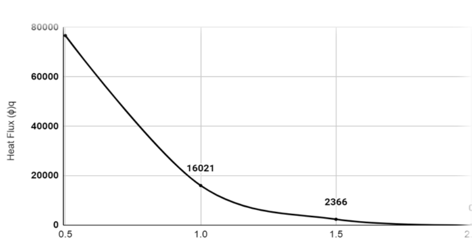

Figure 3: Linear curve of heat flux for concrete

2.2 Bivariate correspondence between heat flux and fractional order for concrete:

We now extend the present study related to heat transfer through a concrete slab. A magnified view of the rate of heat energy flow with respect to the fractional order is considered by reducing the range to $0.09 \leq \alpha \leq 0.5$.

To establish a tangible relationship between fractional order derivatives and the heat flux generated on the concrete slab, we define two variables: the first variable is the fractional order α*, denoted as* $X = variable_1$*; the second variable is the calculated heat flux* Φ_q*, denoted as* $Y = variable_2$.

We found that the correlation between heat flux and fractional order lies within the range $-0.75 \leq r \leq -1$. *A scatter plot with error bars for the concrete slab is presented in Figure 4.*

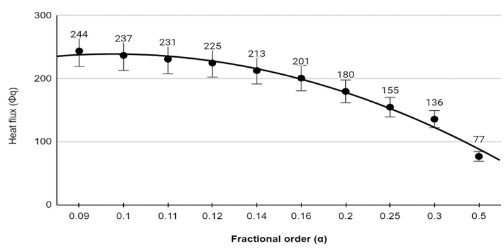

Figure 4: Scattered heat flux plots with fitted curve for concrete

The appropriate equation of the fitted curve is also investigated, given below

$$\Phi_q(\textit{Heat Flux}) = 2.22\alpha^2 + 36.5\alpha + 89, \quad \alpha > 0. \tag{2.6}$$

3 Thermal modelling for solid

According to thermodynamics, heat in the form of thermal energy can be transferred through three processes: conduction, convection, and radiation. Our investigation focuses on the conduction process of heat transfer in solid materials. We consider the two-dimensional (2D) and three-dimensional (3D) heat equations in the unsteady state.

In this study, we adopt a technique called the 'Adomian-Natural Transform Decomposition Technique' to obtain solutions of the 2D and 3D heat equations for solid materials. This technique [7], developed by combining two powerful tools—the Adomian Decomposition Method, introduced by Georg Adomian in 1980, and the Natural Integral Transform Method—is used to obtain the solution in the form of a decomposition series.

In this context, we discuss two cases: ***Case-1*** *and* ***Case-2****. In Case-1, the solution of the two-dimensional heat equation is obtained using a suitable initial condition, denoted as I.C.-1. In Case-2, an alternative series solution of the two-dimensional heat equation is obtained using a second initial condition, denoted as I.C.-2, for* $t > 0$.

Additionally, the two results obtained for both cases are numerically analyzed for a concrete slab. We also examine the behavior of temperature variation during the conduction process numerically. Furthermore, the variation in temperature is illustrated using surface graphs and contour wireframe curves.

3.1 Solution of fractional order two-dimensional heat equation:

We consider a rectangular shaped solid material, whose two-dimensional fractional order heat equation is given by [8],

$$\frac{\partial^\alpha V(x,y,t)}{\partial t^\alpha} = \lambda\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right), \quad x > 0,\ y > 0,\ t > 0. \tag{3.1}$$

Here λ *is the thermal diffusivity of solid material, which is considered as constant parameter throughout the investigation. Here* $V(x,y,t)$ *shows the temperature on the surface of solid at* *time $t > 0$.*

Case-1:*In this case to obtain the solution of equation (3.1), we consider the initial condition at time $t = 0$, such that*

$$\textit{I.C.-1:} \quad V_0 = V(x, y, 0) = (1 - x)e^y, \quad x > 0,\ y > 0. \tag{3.2}$$

By taking Natural transform [9] on equation (3.1), we get

$$\frac{s^\alpha}{u^\alpha} N^+ [V(x, y, t)] - \frac{s^{\alpha-1}}{u^\alpha} V(x, y, 0) = N^+ \left[\lambda \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right)\right],$$

By the Adomian Natural transform decomposition technique, we get the first initial term, given below:

$$V_0(x, y, t) = N^- \left[\frac{(1 - x)e^y}{s}\right],$$

And the recurrence relation in the following form

$$N^+ [V_{j+1}(x, y, t)] = \frac{u^\alpha}{s^\alpha} N^+ \left\{\lambda \left(\frac{\partial^2 V_j}{\partial x^2} + \frac{\partial^2 V_j}{\partial y^2}\right)\right\}, \quad j = 0, 1, 2, \ldots$$

By taking inverse Natural transform and by plugging the successive values of $j = 0, 1, 2, \ldots,$ we get the solution in series form, given below:

$$V(x, y, t) = (1 - x)e^y \left[1 + \frac{\lambda t^\alpha}{\Gamma(\alpha + 1)} + \frac{\lambda^2 t^{2\alpha}}{\Gamma(2\alpha + 1)} + \cdots\right], \tag{3.3}$$

Above expression show the solution of 2D heat equation (3.1) in the form of decomposition series. Now we convert the obtained solution (3.3) into closed form by putting $\alpha = 1$, steps are as follows:

$$V(x, y, t) = (1 - x)e^y \left[1 + \frac{\lambda t}{\Gamma(2)} + \frac{(\lambda t)^2}{\Gamma(3)} + \cdots\right],$$

$$V(x, y, t) = (1 - x)e^y \left[1 + \frac{\lambda t}{1!} + \frac{(\lambda t)^2}{2!} + \cdots\right],$$

$$V(x, y, t) = (1 - x)e^y e^{\lambda t},$$

or

$$V(x, y, t) = (1 - x)e^{(y+\lambda t)}. \tag{3.4}$$

The above expression shows the solution of considered equation (3.1) in compact form. The result (3.4) provide the exact value of temperature on the surface of solid at any time $t > 0$.

Numerical simulation for two dimensional concrete slab:*To observe the diversity of temperature for concrete and the flow of energy in the form of heat at $t = 1$ minute, we make certain assumptions in our model. We consider the length and breadth of the concrete slab to lie within the ranges $0 \leq y \leq 2$ and $0 \leq x \leq 1$ unit, respectively.*

By plugging in the value of the thermal diffusivity of concrete $\lambda_C = 0.75 \times 10^{-6} m^2/s$ [10], we obtain different temperature values $V(x, y, t)$ (in degrees Celsius) on the surface of the concrete slab. We present the surface graph of temperature variation in the xy-plane for the concrete slab in Figure 5(a). Additionally, we provide the V-contour wireframe graph for temperature to enhance the pattern of temperature variation on the concrete slab, as shown in Figure 5(b).

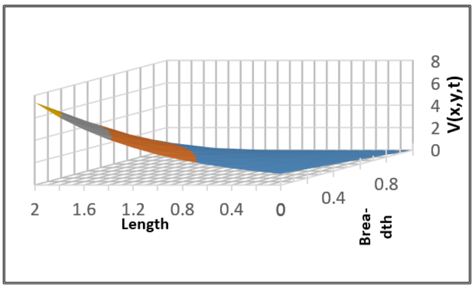

Figure 5(a)

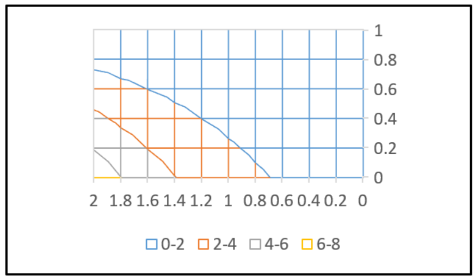

Figure 5(b)

3.2 Alternative solution of fractional order two-dimensional heat equation:

In this section, we expand our investigation with suitable second initial condition. We provide alternative solution of equation (3.1) through case -2.

Case-2: *We consider the second initial at* $t = 0$*, given by*

$$I.C.\text{-}2: \quad V_0 = V(x, y, 0) = e^{(2x+y)}, \quad x > 0,\ y > 0. \tag{3.5}$$

By adopting the technique of case 1 for the particular value of $\alpha = 1$*, we get the second solution of equation (3.1) in series form, given by (3.6),*

$$V(x, y, t) = e^{(2x+y)} \left[1 + \frac{5t}{1!} + \frac{(5\lambda t)^2}{2!} + \cdots \right], \tag{3.6}$$

or

$$V(x, y, t) = e^{(2x+y+5\lambda t)}, \quad x > 0,\ y > 0. \tag{3.7}$$

The above solution is compact form of solution of equation (3.1).
Numerical simulation of two dimensional concrete slab: *On considering the same assumption as taken in case-1, we generate the temperature values for concrete slab for case-2. The corresponding surface graph of temperature is shown by figure 6(a) and the V-contour wired curves for concrete are given in figure 6(b).*

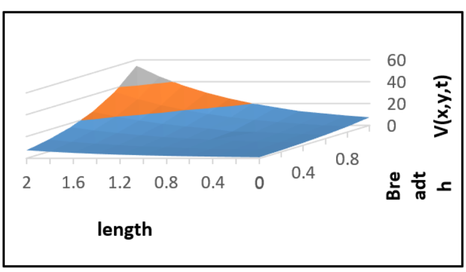

Figure 6(a)

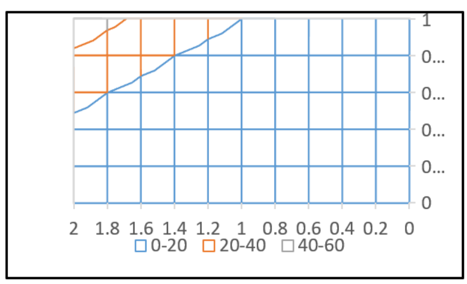

Figure 6(b)

4 Solution of three-dimensional fractional order heat equation:

Here we consider three-dimensional heat equation for solid material with independent three variables $x > 0, y > 0$ *and* $z > 0$ *is given by,*

$$\frac{\partial^\alpha V(x,y,z,t)}{\partial t^\alpha} = \lambda \left(\frac{\partial^2 V(x,y,z,t)}{\partial x^2} + \frac{\partial^2 V(x,y,z,t)}{\partial y^2} + \frac{\partial^2 V(x,y,z,t)}{\partial z^2} \right), \quad x > 0,\ y > 0,\ z > 0 \tag{4.1}$$

Here $V(x, y, z, t)$ *is the temperature on the surface of solid material and* λ *is the diffusivity of solid. We discuss two cases to get the solution of three-dimensional heat equation with two initial conditions at time* $t = 0$.

Case-1:*In this case, the initial condition at time* $t = 0$*, in form of product of exponential function and sine function has been taken, which is given as*

$$V_0(x, y, z, 0) = e^{(x+y)} \sin(z), \quad x > 0,\ y > 0,\ z > 0. \tag{4.2}$$

By taking Natural transform of the equation (4.1), we get

$$\frac{s^\alpha}{u^\alpha} N^+ [V(x, y, z, t)] - \frac{s^{\alpha-1}}{u^\alpha} V(x, y, z, 0) = N^+ \left[\lambda \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right)\right],$$

By taking the inverse Natural transform of above equation and adopting the Natural transform decomposition technique, we find the first initial term

$$V_0(x, y, z, t) = N^- \left[\frac{V_0(x, y, z, 0)}{s}\right].$$

The recurrence relation is given by

$$V_{j+1}(x, y, z, t) = N^- \left(\frac{u^\alpha}{s^\alpha} N^+ \left\{\lambda \left(\frac{\partial^2 V_j}{\partial x^2} + \frac{\partial^2 V_j}{\partial y^2} + \frac{\partial^2 V_j}{\partial z^2}\right)\right\}\right), \quad j = 0, 1, 2, 3, \ldots$$

By taking inverse Natural transform and by plugging the successive values of $j = 0, 1, 2, \ldots,$ *we get the solution in series form, given below:*

$$V(x, y, z, t) = e^{(x+y)} \sin(z) \left[1 + \frac{\lambda t^\alpha}{\Gamma(\alpha + 1)} + \frac{\lambda^2 t^{2\alpha}}{\Gamma(2\alpha + 1)} + \cdots\right],$$

$$V(x, y, z, t) = e^{(x+y+\lambda t)} \sin(z), \quad x > 0,\ y > 0,\ z > 0. \tag{4.3}$$

The above expression is the solution in compact form of equation (4.1), for $\alpha = 1$.

Numerical simulation of three dimensional concrete slab:*For the three-dimensional graphical representation of the obtained temperature given in equation (4.3), we consider a piece of concrete slab whose length lies along the* x*-axis, breadth along the* y*-axis, and thickness along the* z*-axis. The time duration for applying heat to the concrete slab from an external source (as a theoretical assumption) is one minute. We assume that the independent spatial variables vary along the coordinate axes within the range* $0 \le x, y, z \le 1$ *units. The resulting temperature distribution across the concrete slab is illustrated by the grids in Figure 7.*

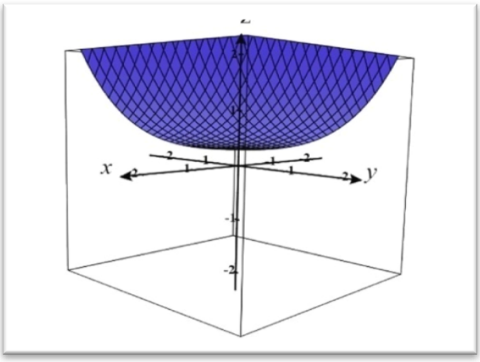

Figure 7: 3D Temperature grids on 3D concrete slab

To get Alternative solution of three-dimensional heat equation (4.1) here we discuss second case.

Case-2:*In this, we consider the second initial condition at* $t = 0$ *for* $x, y, z \ge 0$*, given by:*

$$V(x, y, z, t) = (1 - x)\, e^{(y+z)}. \tag{4.4}$$

By taking the Natural transform of the equation (4.1) and using the Adomian Natural transform decomposition technique we get solution in series form, given as

$$V(x,y,z,t) = (1-x)\,e^{(y+z)}\left[1+\frac{2\lambda t}{1!}+\frac{(2\lambda t)^2}{2!}+\cdots\right],$$

For $\alpha = 1$, solution has another face in compact form, given by

$$V(x,y,z,t) = (1-x)\,e^{(y+z+2\lambda t)}. \tag{4.5}$$

For the graphical representation of the variation of temperature given in equation (4.5) for the concrete slab, we maintain the previous assumptions as in Case 1. For the particular value of $\alpha = 1$, and using the thermal diffusivity of concrete $\lambda_C = 0.75 \times 10^{-6} m^2/s$, we obtain the temperature grids shown in Figure 8 below:

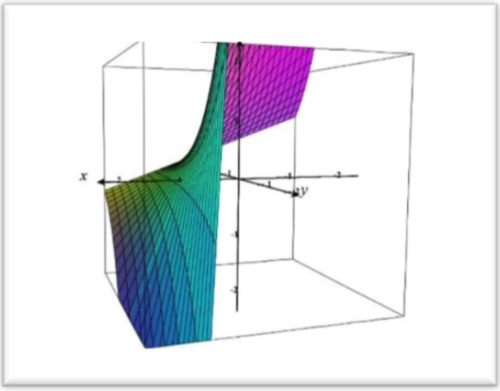

Figure 8: 3D Temperature grids on 3D concrete slab

5 Results and Discussion:

In this work, we present a thermal modelling approach to study the relationship between the fractional order of the one-dimensional heat equation and the calculated heat flux and temperature behaviour in building construction materials. The goal is to help reduce thermal cracks during the construction process. We derive a general expression for heat flux in terms of thermal conductivity and diffusivity of solid materials. A numerical simulation is also carried out, analysing the influence of fractional order on heat flux.

We also extend the analysis to the two-dimensional and three-dimensional heat equations. Compared to previous work on thermal modelling, the results of our model are more applicable to various kinds of concrete with differing thermal properties. This study is strictly limited to the conduction process in solid materials caused by temperature fluctuations and applies only to rectangular-shaped objects; it does not provide results for cylindrical or square-shaped geometries, which is considered a limitation of the current thermal model.

As a result of the numerical model for the one-dimensional heat equation, we observed a correlation of -0.9857 between the fractional order and heat flux, with the fitted curve for scattered heat flux dots given by:

$$\phi_q\,(\textit{Heat Flux}) = 2.22\alpha^2 + 36.5\alpha + 89, \quad \alpha > 0.$$

For the two-dimensional and three-dimensional heat equations, we found that the growth of temperature on the surface of the concrete slab is $7.389°C$ in Case 1, whereas for Case 2, the maximum growth of temperature on concrete is $54.598°C$, and the minimum temperature is noted as $0°C$. Additionally, a 3D surface plot of the temperature distribution in the concrete slab was generated.

6 Conclusion:

Thermal cracks in concrete occur when there is a significant temperature difference between the outer layer and the inner layers of the concrete. The temperature gradient causes energy to flow from regions of high temperature to regions of low temperature, resulting in a heat transfer rate described in terms of heat flux. Our model investigates this heat flux in terms of the fractional order of the one-dimensional heat equation. A strong correlation was identified, and within a small range of fractional orders, a scattered heat flux distribution was observed with a best-fit quadratic curve.

Since temperature difference is one of the causes of thermal cracking, we present a model for specific thermal conductivity and thermal diffusivity to understand the fluctuations of temperature in two-dimensional and three-dimensional concrete slabs. Under the considered initial conditions, we found consistent patterns of temperature distribution. Additionally, the obtained results are applicable for studying the temperature variation in different concrete mixtures such as NC1 and NC2, by incorporating their respective thermal conductivity and thermal diffusivity values.

References

[1] *Golewski, G. L. (2023), The Phenomenon of Cracking in Cement Concretes and Reinforced Concrete Structures: The Mechanism of Cracks Formation, Causes of Their Initiation, Types and Places of Occurrence, and Methods of Detection—A Review Buildings, 13(3), 765. https://doi.org/10.3390/buildings13030765.*

[2] *Askar, Mand & Al-Kamaki, Yaman & Ferhadi, Razaq & Majeed, Hisham. (2023), Cracks in Concrete Structures Causes And Treatments: A Review. The Journal of Duhok University. 26. 148-165. 10.26682/csjuod.2023.26.2.16.*

[3] *Li, X., Yu, Z., Chen, K., Deng, C., & Yu, F. (2023), Investigation of temperature development and cracking control strategies of mass concrete: A field monitoring case study. Case Studies in Construction 1 Materials, 18, e02144.*

[4] *Choudhary, A., Kumar, D., & Singh, J. (2016), Numerical simulation of a fractional model of temperature distribution and heat flux in the semi infinite solid. Alexandria Engineering Journal, 55, 87-91.*

[5] *Mills, A. F., & Ganesan, V. (2000), Heat transfer (pp. 97-105). Dorling Kindersley (India).*

[6] *Gu, X., Li, S., & Bao, H. (2020), Thermal conductivity of silicon at elevated temperature: Role of four-phonon scattering and electronic heat conduction. International Journal of Heat and Mass Transfer, 160, 120165. https://doi.org/10.1016/j.ijheatmasstransfer.2020.120165.*

[7] *Johansyah, D., Supriatna, A., Rusyaman, E., & Saputra, J. (2022), Solving Differential Equations of Fractional Order Using Combined Adomian Decomposition Method with Kamal Integral Transformation. Mathematics and Statistics, 10, 187-194. https://doi.org/10.13189/ms.2022.100117.*

[8] *Lin, J. (2021), Recovering temperature-dependent heat conductivity in 2D and 3D domains with homogenization functions as the bases. Engineering with Computers, 38, 1-15. https://doi.org/10.1007/s00366-021-01384-w.*

[9] *Khan, H., Shah, R., Kumam, P., & Arif, M. (2019), Analytical Solutions of Fractional-Order Heat and Wave Equations by the Natural Transform Decomposition Method. Entropy, 21(6), 597. https://doi.org/10.3390/e21060597.*

[10] *Malik, M., Bhattacharyya, S. K., & Barai, S. V. (2021), Thermal and mechanical properties of concrete and its constituents at elevated temperatures: A review. Construction and Building Materials, 270, 121398. https://doi.org/10.1016/j.conbuildmat.2020.121398.*

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