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Fuzzified Bisection Method for Improved Time Estimation in Therapeutic Drug Monitoring, Forensic Science and Groundwater Contamination.

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Abstract This paper explores innovative application of fuzzy numerical technique in environmental, medical and forensic sciences, focusing on three critical areas: groundwater contaminant depth analysis, therapeutic drug monitoring (TDM) and estimating time of death through body temperature analysis. The bisection method is a powerful root-finding method for solving complex problems in such fields. In medical science, Therapeutic Drug Monitoring (TDM) ensures that drug concentrations remain within therapeutic ranges to maximize efficacy and minimize toxicity. The bisection method is used to solve pharmacokinetic equations and determine the precise timing of drug administration. This numerical approach, along with fuzzification techniques, takes into consideration patient-specific variability and uncertainty in drug metabolism. In forensic science, numerical methods improve the accuracy of time estimations, which is important for reconstructing crime scenes and determining time of death. In environmental science, fuzzy numerical methods improve prediction and assessment of groundwater contaminant depth, which deals with uncertainties in environmental modelling.

1 Introduction

Accurate time estimation plays an important role in many fields, including therapeutic drug monitoring, forensic science, and groundwater contamination analysis. These areas require precise timing to ensure correct decision-making. For example, in therapeutic drug monitoring, determining the optimal time for drug administration can help optimize treatment effectiveness and prevent side effects. Similarly, in forensic science, estimating the time of an event, such as the occurrence of a crime, is important for investigations. In environmental studies, like groundwater contamination, understanding the time of contaminant movement is essential for planning clean-up strategies.

Classical methods for time estimation have limitations due to uncertainty or inadequate data. This may result in mistakes, particularly in complicated systems where data is complex to understand. The bisection method [2] is a widely used numerical approach for determining unknown values. It works by repeatedly dividing an interval in half until it converges on a solution. However, when applied to uncertain or inaccurate data, its effectiveness can be limited.

In this paper, we explore the Fuzzified Bisection Method as a way to enhance the accuracy of time estimation under uncertain conditions. In some cases, like therapeutic drug monitoring, forensic science, and groundwater pollution, we hope to increase the accuracy of time predictions by using fuzzy logic, a mathematical technique that handles uncertainty.

Our goal is to show that this modified method can make estimating time more accurate and reliable, which will lead to better results in all of these important areas.

2 Preliminaries

Here we shall state some basic concepts and results associated with trapezoidal fuzzy numbers (TrFNs) in fuzzy theory.

Definition 2.1. [1], [9], [10] Let X be a universal set. Then, the fuzzy subset $\tilde{P}$ of X is defined by its membership function $\mu_{\tilde{P}}$: $X \to [0, 1]$ which assign to each element $x \in X$ a real number $\mu_{\tilde{P}}(x)$ in the interval [0, 1], where the function value of $\mu_{\tilde{P}}(x)$ represents the grade of membership of x in $\tilde{P}$. A fuzzy set $\tilde{P}$ is written as
$\tilde{P} = \{(x, \mu_{\tilde{P}}(x)), x \in X, \quad \mu_{\tilde{P}}(x) \in [0, 1]\}$.

Definition 2.2. [1], [9], [10] A fuzzy set $\tilde{P}$, defined on the universal set of real number R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{P}$ is convex i.e.,
$\mu_{\tilde{P}}\,(\beta x_1 + (1-\beta)x_2) \geq \min(\mu_{\tilde{P}}(x_1), \mu_{\tilde{P}}(x_2)), \forall x_1, x_2 \in R, \ \forall \beta \in [0, 1]$.

(ii) $\tilde{P}$ is normal, i.e., $\exists\, x_0 \in R$ such that $\mu_{\tilde{P}}(x_0) = 1$.

(iii) $\mu_{\tilde{P}}$ is piecewise continuous.

Definition 2.3. [1], [8], [9] A fuzzy number $\tilde{P} = (p_1, p_2, p_3, p_4)$ is a TrFN in the general form if its membership function is as follows :

$$\mu_{\tilde{P}}(x) = \begin{cases} 0, & x \leq p_1 \\ \frac{x-p_1}{p_2-p_1}, & p_1 \leq x \leq p_2 \\ 1, & p_2 \leq x \leq p_3 \\ \frac{p_4-x}{p_4-p_3}, & p_3 \leq x \leq p_4 \\ 0, & x > p_4 \end{cases}$$

Definition 2.4. [1],[9] The sign of the TrFN $\tilde{P} = (p_1, p_2, p_3, p_4)$ can be classified as follows:

(i) $\tilde{P}$ is positive (negative) iff $p_1 \geq 0, (p_4 \leq 0)$

(ii) $\tilde{P}$ is zero iff $(p_1, p_2, p_3 \ \&\ p_4 = 0)$

(iii) $\tilde{P}$ is near zero iff $p_1 \leq 0 \leq p_4$

Definition 2.5. [1],[9] Operations on TrFNs.
The arithmetic operations on TrFNs are presented as follows:
Let $\tilde{P} = (p_1, p_2, p_3, p_4)$ and $\tilde{Q} = (q_1, q_2, q_3, q_4)$ be two TrFNs, then

(i) Addition :
$$\begin{aligned}\tilde{P} + \tilde{Q} &= (p_1, p_2, p_3, p_4) + (q_1, q_2, q_3, q_4)\\ &= (p_1 + q_1, p_2 + q_2, p_3 + q_3, p_4 + q_4)\end{aligned}$$

(ii) Subtraction :
$$\begin{aligned}\tilde{P} - \tilde{Q} &= (p_1, p_2, p_3, p_4) - (q_1, q_2, q_3, q_4)\\ &= (p_1 - q_4, p_2 - q_3, p_3 - q_2, p_4 - q_1)\end{aligned}$$

(iii) Symmetric image:
$$-\tilde{P} = (p_4, p_3, p_2, p_1)$$

(iv) Scalar multiplication : let $\beta \in R$, then

$$\beta \otimes (p_1, p_2, p_3, p_4) = \begin{cases} (\beta p_1, \beta p_2, \beta p_3, \beta p_4), & \beta \geq 0 \\ (\beta p_4, \beta p_3, \beta p_2, \beta p_1), & \beta < 0 \end{cases}$$

(v) Multiplication: The multiplication of fuzzy numbers is neither commutative nor associative. Thus, trapezoidal fuzzy numbers multiplication operations can be classified as follows :

Case I : If $\tilde{P} = (p_1, p_2, p_3, p_4)$ and $\tilde{Q} = (q_1, q_2, q_3, q_4)$ be two TrFNs, then

$$\tilde{P}\tilde{Q} = (r, s, t, u),$$

where

$$r = min(p_1q_1, p_1q_4, p_4q_1, p_4q_4)$$
$$s = min(p_2q_2, p_2q_3, p_3q_2, p_3q_3)$$
$$t = max(p_2q_2, p_2q_3, p_3q_2, p_3q_3)$$
$$u = max(p_1q_1, p_1q_4, p_4q_1, p_4q_4)$$

Case II : If $\tilde{P}, \tilde{Q} > 0$, then

$$\tilde{P}\tilde{Q} = (p_1q_1, p_2q_2, p_3q_3, p_4q_4)$$

Case III : If $\tilde{P}, \tilde{Q} < 0$, then

$$\tilde{P}\tilde{Q} = (p_4q_4, p_3q_3, p_2q_2, p_1q_1)$$

Case IV : If $\tilde{P} > 0, \tilde{Q} < 0$, then

$$\tilde{P}\tilde{Q} = (p_4q_1, p_3q_2, p_2q_3, p_1q_4)$$

Case V : If $\tilde{P} < 0, \tilde{Q} > 0$, then

$$\tilde{P}\tilde{Q} = (p_1q_4, p_2q_3, p_3q_2, p_4q_1)$$

(vi) Equality : The fuzzy numbers $\tilde{P} = (p_1, p_2, p_3, p_4)$ and $\tilde{Q} = (q_1, q_2, q_3, q_4)$ are equal iff

$$p_1 = q_1, p_2 = q_2, p_3 = q_3, p_4 = q_4.$$

Definition 2.6. [5] α - cut for TrFN : Let $\tilde{P} = (p_1, p_2, p_3, p_4)$ be a TrFN .

$$[\tilde{P}]^\alpha = [(p_2 - p_1)\alpha + p_1, p_4 - (p_4 - p_3)\alpha]$$

is the α - cut set for TrFN.

Definition 2.7. Defuzzification :

[6] The indirect comparison of fuzzy numbers is the fundamental reason for defuzzification. The comparison over fuzzy sets has no universal consensus. The fuzzy numbers has to be mapped initially to real values that can be compared for computing the magnitude values. Defuzzification process is mapping fuzzy numbers to real values that can be compared for computing the magnitudes. The computation may take several forms; the standard form which is the usage of centroid rule. The computation of the defuzzification process requires integrating the membership function over the fuzzy sets. This improves the effect over direct computation of centroid rule with centre of gravity that describes the fuzzy quantity. The defuzzification over trapezoid fuzzy numbers using median is evaluated at the risk rate:

For the trapezoid fuzzy number $\tilde{P} = (p_1, p_2, p_3, p_4)$, by the bisection of area , the median

$M_{\tilde{P}} = \frac{(p_1+p_2+p_3+p_4)}{4}$, only if , $p_2 \leq M_{\tilde{P}} \leq p_3$

3 Methodology

[3], [9] Let us consider an equation $G(x) = 0$. Let $G(x)$ be a continuous function and $G(x)$ can be algebraic. Let the function $G(x)$ changes sign over an interval $x = \tilde{P}$ and $x = \tilde{Q}$, where $\tilde{P} = (p_1, p_2, p_3, p_4)$ and $\tilde{Q} = (q_1, q_2, q_3, q_4)$. Then the root of $G(x) = 0$ lying between $\tilde{P}$ and $\tilde{Q}$. Now fuzzy membership function of $\tilde{P}$ and $\tilde{Q}$ are respectively,

$$\mu_{\tilde{P}}(x) = \begin{cases} \frac{x-p_1}{p_2-p_1}, & p_1 \leq x \leq p_2 \\ 1, & p_2 \leq x \leq p_3 \\ \frac{p_4-x}{p_4-p_3}, & p_3 \leq x \leq p_4 \\ 0, & otherwise \end{cases}$$

$$\mu_{\tilde{Q}}(x) = \begin{cases} \frac{x-q_1}{q_2-q_1}, & q_1 \leq x \leq q_2 \\ 1, & q_2 \leq x \leq q_3 \\ \frac{q_4-x}{q_4-q_3}, & q_3 \leq x \leq q_4 \\ 0, & otherwise \end{cases}$$

with respect to α - cuts as

$[\tilde{P}]^\alpha = [(p_2 - p_1)\alpha + p_1, p_4 - (p_4 - p_3)\alpha]$

$[\tilde{Q}]^\alpha = [(q_2 - q_1)\alpha + q_1, q_4 - (q_4 - q_3)\alpha]$

As a first approximation , the root $G(x) = 0$ is

$$\tilde{x_0} = \frac{\tilde{P} + \tilde{Q}}{2} = \frac{(p_1, p_2, p_3, p_4) + (q_1, q_2, q_3, q_4)}{2}$$

Let us consider $\tilde{x_0} = (x_0', x_0'', x_0''', x_0'''')$.

The fuzzy membership function (f.m.f) of $\tilde{x_0}$ is

$$\mu_{\tilde{x_0}}(x) = \begin{cases} \frac{x-x_0'}{x_0''-x_0'}, & x_0' \leq x \leq x_0'' \\ 1, & x_0'' \leq x \leq x_0''' \\ \frac{x_0''''-x}{x_0''''-x_0'''}, & x_0''' \leq x \leq x_0'''' \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_0}]^\alpha = [(x_0'' - x_0')\alpha + x_0',\ x_0'''' - (x_0'''' - x_0''')\alpha]$$

Suppose $G(\tilde{P})$ & $G(\tilde{x_0})$ are of opposite signs then the root is in between $\tilde{P}$ & $\tilde{x_0}$ and if $G(\tilde{x_0})$ & $G(\tilde{Q})$ are of opposite signs then the root is in between $\tilde{x_0}$ & $\tilde{Q}$.

If $G(\tilde{P}) < 0$ & $G(\tilde{Q}) > 0$ then the first approximation be

$$\tilde{x_0} = \frac{\tilde{P} + \tilde{Q}}{2} = \frac{(p_1, p_2, p_3, p_4) + (q_1, q_2, q_3, q_4)}{2}$$

Suppose $G(\tilde{x_0}) < 0$ then the root is located between $\tilde{x_0}$ & $\tilde{Q}$.

Then the 2^{nd} approximation is,

$$\tilde{x_1} = \frac{\tilde{x_0} + \tilde{Q}}{2} = \frac{(x_0', x_0'', x_0''', x_0'''') + (q_1, q_2, q_3, q_4)}{2}$$

let us consider , $\tilde{x_1} = (x_1', x_1'', x_1''', x_1'''')$

The f.m.f of $\tilde{x_1}$ is ,

$$\mu_{\tilde{X}_1}(x) = \begin{cases} \frac{x-x_1'}{x_1''-x_1'}, & x_1' \leq x \leq x_1'' \\ 1, & x_1'' \leq x \leq x_1''' \\ \frac{x_1''''-x}{x_1''''-x_1'''}, & x_1''' \leq x \leq x_1'''' \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_1}]^{\alpha} = [(x_1'' - x_1')\alpha + x_1',\ x_1'''' - (x_1'''' - x_1''')\alpha]$$

Suppose $G(\tilde{x_1})$ is positive then the roots lies between $\tilde{x_0}$ & $\tilde{x_1}$ and the third approximation is ,

$$\tilde{x_2} = \frac{\tilde{x_0} + \tilde{x_1}}{2} = \frac{(x_0', x_0'', x_0''', x_0'''') + (x_1', x_1'', x_1''', x_1'''')}{2}$$

let us consider , $\tilde{x_2} = (x_2', x_2'', x_2''', x_2'''')$
The f.m.f of $\tilde{x_2}$ is ,

$$\mu_{\tilde{x_2}}(x) = \begin{cases} \frac{x-x_2'}{x_2''-x_2'}, & x_2' \leq x \leq x_2'' \\ 1, & x_2'' \leq x \leq x_2''' \\ \frac{x_2''''-x}{x_2''''-x_2'''}, & x_2''' \leq x \leq x_2'''' \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_2}]^{\alpha} = [(x_2'' - x_2')\alpha + x_2',\ x_2'''' - (x_2'''' - x_2''')\alpha]$$

and so on.

4 Numerical Example

Example 1: Therapeutic Drug Monitoring
A patient is administered a drug intravenously and the drug concentration in the bloodstream is modeled by the function [7] $C(t) = C_0.e^{-kt}$ where,
$C(t)$: the drug concentration at time t.
C_0: is the initial drug concentration i.e.(100 mg/L).
k: is the elimination rate constant i.e.$(0.2\ hr^{-1})$.
we need to determine the time t when the drug concentration reaches a therapeutic level of 30 mg/L.

Let $f(t) = 100.e^{-0.2t} - 30$
we apply fuzzified bisection method using TrFNs to find t.
Choose an appropriate fuzzy interval where $f(t)$ changes sign.
Consider a = (0.1025, 0.1045, 0.1065, 0.1085) and b = (10.1025, 10.1045, 10.1065, 10.1085) such that $f(a)$ and $f(b)$ are of opposite signs.
$\therefore$ The root of $f(t) = 0$ lying between a and b.
Then
$x_0 = \frac{a+b}{2} = \frac{1}{2}\{(0.1025, 0.1045, 0.1065, 0.1085) + (10.1025, 10.1045, 10.1065, 10.1085)\}$

= (5.1025, 5.1045, 5.1065, 5.1085)
Now f.m.f. of x_0 is

$$\mu_{\tilde{x_0}}(x) = \begin{cases} \frac{x-5.1025}{5.1045-5.1025}, & 5.1025 \leq x \leq 5.1045 \\ 1, & 5.1045 \leq x \leq 5.1065 \\ \frac{5.1085-x}{5.1085-5.1065}, & 5.1065 \leq x \leq 5.1085 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[x_0]^\alpha = [(5.1045 - 5.1025)\alpha + 5.1025,\ 5.1085 - (5.1085 - 5.1065)\alpha]$$

$f(x_0) = f(5.1025, 5.1045, 5.1065, 5.1085) > 0$, so roots lies between x_0 and b.

$$x_1 = \frac{x_0 + b}{2} = \frac{1}{2}\{(5.1025, 5.1045, 5.1065, 5.1085) + (10.1025, 10.1045, 10.1065, 10.1085)\}$$

= (7.6025,7.6045,7.6065,7.6085)

$$\mu_{\tilde{x_1}}(x) = \begin{cases} \frac{x-7.6025}{7.6045-7.6025}, & 7.6025 \leq x \leq 7.6045 \\ 1, & 7.6045 \leq x \leq 7.6065 \\ \frac{7.6085-x}{7.6085-7.6065}, & 7.6065 \leq x \leq 7.6085 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[x_1]^\alpha = [(7.6045 - 7.6025)\alpha + 7.6025,\ 7.6085 - (7.6085 - 7.6065)\alpha]$$

$f(x_1) = f(7.6025, 7.6045, 7.6065, 7.6085) < 0$, so roots lies between x_0 and x_1. Continuing in this way, at 10^{th} iteration value of $f(x_{10})$ is near zero so we get fuzzy root at 10^{th} iteration.
$\therefore$ fuzzy root is (6.015585938, 6.017585938, 6.019585938, 6.021585938).
Now by applying defuzzification on the fuzzy root we get crisp root.
$\therefore$ crisp root is 6.018585938. Hence, the time t when the drug concentration reaches the therapeutic level of 30 mg/L is approximately 6.018585938 hours.

Example 2: Groundwater contaminant depth

Let us assume that, the contaminant concentration $C(z)$ is given by the function $C(z) = 10.e^{-0.5z} + 2$, where z : the depth in meters.$C(z)$ is the contaminant as a function of depth z. Our aim is to find depth z where $C(z)$ equals a threshold value $C_{threshold}$. Let set $C_{threshold} = 4$.
Define the function $f(z) = C(z) - C_{threshold} = 10.e^{-0.5z} - 2$ we apply fuzzified bisection method using TrFNs to find depth,z.
Choose an appropriate fuzzy interval where $f(z)$ changes sign.
Consider a = (0.2025, 0.2035, 0.2045, 0.2055) and b = (5.1015, 5.1025, 5.1035, 5.1045) such that $f(a)$ and $f(b)$ are of opposite signs.
$\therefore$ The root of $f(z) = 0$ lying between a and b.
Then
$x_0 = \frac{a+b}{2} = \frac{1}{2}\{(0.2025, 0.2035, 0.2045, 0.2055) + (5.1015, 5.1025, 5.1035, 5.1045)\}$

= (2.652, 2.653, 2.654, 2.655)

Now f.m.f. of x_0 is

$$\mu_{\tilde{x_0}}(x) = \begin{cases} \frac{x-2.652}{2.653-2.652}, & 2.652 \le x \le 2.653 \\ 1, & 2.653 \le x \le 2.654 \\ \frac{2.655-x}{2.655-2.654}, & 2.654 \le x \le 2.655 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[x_0]^\alpha = [(2.653 - 2.652)\alpha + 2.652,\ 2.655 - (2.655 - 2.654)\alpha]$$

$f(x_0) = f(2.652, 2.653, 2.654, 2.655) > 0$, so roots lies between x_0 and b.

$$x_1 = \frac{x_0 + b}{2} = \frac{1}{2}\{(2.652, 2.653, 2.654, 2.655) + (5.1025, 5.1045, 5.1065, 5.1085)\}$$
$$= (3.87675, 3.87775, 3.87875, 3.87975)$$

$$\mu_{\tilde{x_1}}(x) = \begin{cases} \frac{x-3.87675}{3.87775-3.87675}, & 3.87675 \le x \le 3.87775 \\ 1, & 3.87775 \le x \le 3.87875 \\ \frac{3.87975-x}{3.87975-3.87875}, & 3.87875 \le x \le 3.87975 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[x_1]^\alpha = [(3.87775 - 3.87675)\alpha + 3.87675,\ 3.87975 - (3.87975 - 3.87875)\alpha]$$

$f(x_1) = f(3.87675, 3.87775, 3.87875, 3.87975) < 0$, so roots lies between x_0 and x_1. Continuing in this way, at 9^{th} iteration value of $f(x_9)$ is near zero so we get fuzzy root at 9^{th} iteration.
$\therefore$ fuzzy root is (3.216533204, 3.217533204, 3.218533204, 3.219533204).
Now by applying defuzzification on the fuzzy root we get crisp root.
$\therefore$ crisp root is 3.218033204. Hence, the depth z where the contaminant concentration reaches the threshold $C_{threshold}$ is approximately 3.218033204 meters.

Example 3: Forensic Science: Estimating time of death.

[4] An officer was called to the home of a person who had died during afternoon. In order to estimate the time of death the coroner took the person's body temperature twice. At 3:30 pm the temperature was 84.7$^\circ F$ and after 25 mins the temperature was 81.5$^\circ F$. The room temperature stayed constant at 71$^\circ F$. Find the approximate time of death assuming the body temperature was normal at 98.6$^\circ F$ at the time of death. Let $f(t) = T(t) - T_{ambient} + (T_{initial} - T_{ambient})e^{-kt}$
Where,
$T(t)$ is the body temperature at time t.
$T_{ambient}$ is the surrounding temperature.
$T_{initial}$ is the initial body temperature.
k is the rate of constant.
t is the time since death.
Consider $k = 0.0106$
Now $f(t) = 81.5 - 71 + (84.7 - 71)e^{-0.0106t} = 27.6e^{-0.0106t} - 13.7$ we apply fuzzified bisection method using TrFNs to find time,t.
Choose an appropriate fuzzy interval where $f(t)$ changes sign.
Consider a = (60.1, 60.2, 60.3, 60.4) and b = (70.5, 70.6, 70.7, 70.8) such that $f(a)$ is positive and $f(b)$ is negative.
$\therefore$ The root of $f(t) = 0$ lying between a and b.
Then
$x_0 = \frac{a+b}{2} = \frac{1}{2}\{(60.1, 60.2, 60.3, 60.4) + (70.5, 70.6, 70.7, 70.8)\}$

= (65.3, 65.4, 65.5, 65.6)

Now f.m.f. of x_0 is

$$\mu_{\tilde{x_0}}(x) = \begin{cases} \frac{x-65.3}{65.4-65.3}, & 65.3 \leq x \leq 65.4 \\ 1, & 65.4 \leq x \leq 65.5 \\ \frac{65.6-x}{65.6-65.5}, & 65.5 \leq x \leq 65.6 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[x_0]^\alpha = [(65.4 - 65.3)\alpha + 65.3,\ 65.6 - (65.6 - 65.5)\alpha]$$

$f(x_0) = f(65.3, 65.4, 65.5, 65.6) > 0$, so roots lies between x_0 and b.

$$x_1 = \frac{x_0+b}{2} = \frac{1}{2}\{(65.3, 65.4, 65.5, 65.6) + (70.5, 70.6, 70.7, 70.8)\}$$

= (67.9, 68, 68.1, 68.2)

$$\mu_{\tilde{x_1}}(x) = \begin{cases} \frac{x-67.9}{68-67.9}, & 67.9 \leq x \leq 68 \\ 1, & 68 \leq x \leq 68.1 \\ \frac{68.2-x}{68.2-68.1}, & 68.1 \leq x \leq 68.2 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

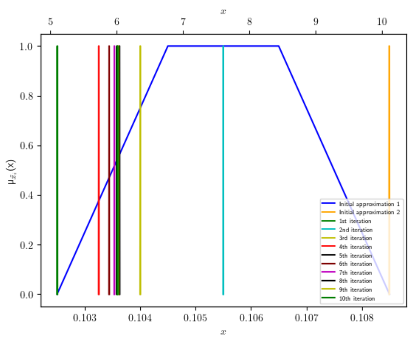
$$[x_1]^\alpha = [(68 - 67.9)\alpha + 67.9,\ 68.2 - (68.2 - 68.1)\alpha]$$

$f(x_1) = f(67.9, 68, 68.1, 68.2) < 0$, so roots lies between x_0 and x_1. Continuing in this way, at 4^{th} iteration value of $f(x_4)$ is near zero so we get fuzzy root at 4^{th} iteration.
$\therefore$ fuzzy root is (65.95, 66.05, 66.15, 66.25).
Now by applying defuzzification on the fuzzy root we get crisp root.
$\therefore$ crisp root is 66.1 min. Hence, the time t is approximately 66.1 minuts.
Which means a person died at 2.24 pm.

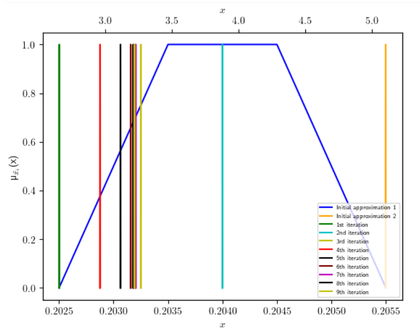
5 Result

Table 1: Comparison between classical bisection and fuzzified bisection method for the given examples.

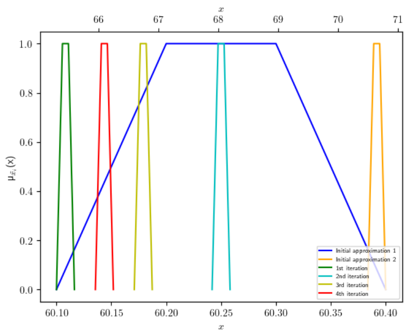
	Fuzzy bisection using TrFNs	***No. of Iterations***	***Classical bisection***	***No. of Iterations***
Therapeutic Drug Monitoring	6.018585938	11	6.019859	18
Groundwater contaminant depth	3.218033204	9	3.218876	24
Estimating time of death	66.1	4	66.077353	23



(a) Therapeutic Drug Monitoring



(b) Groundwater contaminant depth



(c) Estimating time of death

Figure 1: Graphical representation of a solution

From the table, results indicate that the approach suggested is more flexible and effective than the classical approach.

6 Conclusion

This study introduced a fuzzified bisection method to demonstrate a significant improvement in time estimation in areas like therapeutic drug monitoring, forensic science, and groundwater pollution analysis. This approach allows for more accurate time estimation, critical in therapeutic drug monitoring for optimal dosing intervals, in forensic science for establishing timelines, and in environmental studies to predict contaminant spread. This flexible method can be used in many different areas, making it helpful for real-life problems.

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