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An efficient algorithm to solve multi-objective transportation problem under uncertainty

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Abstract *Multi-objective transportation problems are useful in real-world situations where decision-makers need to balance multiple conflicting objectives while optimizing the transportation of goods or resources. These problems are mostly useful in various applications where decision-makers need to handle multiple conflicting goals in the presence of uncertain parameters. In this paper, we present an efficient algorithm for solving multi-objective transportation problem in which all parameters are uncertain, including supply and demand quantities, cost coefficients, and decision variables. The proposed algorithm is designed to directly solve multi-objective transportation problems without converting them into single-objective transportation problems. An efficient solution obtained through the proposed approach results in a compromise solution. Numerical examples are provided to compare results achieved by the proposed method with those obtained through existing approaches.*

1 | Introduction

The transportation problem (TP) involves finding the most cost-effective way to transport goods from a set of suppliers to a set of consumers. Multi-objective transportation problems (MOTP) are an extension of the classical TP, a type of linear programming problem commonly used in operations research and optimization. In MOTP, multiple conflicting objectives are considered simultaneously. These objectives can be related to different criteria, such as cost, time, or environmental impact. The objective is to obtain a set containing solutions that represent a compromise or trade-off among the various conflicting objectives. In everyday situations, variables associated with multi-objective transportation often present imprecision due to factors like insufficient information and vagueness in judgment. To manage this ambiguity, Zadeh [12] introduced the concept of fuzzy sets in 1965. In 1982, Oheigeartaigh [8] extended this concept by introducing the transportation problem in the context of uncertainty. In 1997, Verma et al. [10] presented a MOTP under uncertainty in which non-linear exponential and hyperbolic membership functions have been used. In 2000, Li and Lai [5] presented a model of MOTP under uncertainty. Zimmermann's fuzzy programming approach has been used in the model. Different methods to obtain the compromise optimal solution of MOTP under uncertainty have been studied by many authors such as Li, Ammar, Lee, Zangiabadi, Lohgaonkar and Wahed [5, 11, 1, 4, 13, 6]. In 2020, Ekanayake et al. [3] presented the Geometric mean (GM) method and ant colony optimization algorithm as solutions to MOTP in fuzzy environments. In 2023, Niluminda and Ekanayake [7] worked to create an alternative approach to solve MOTP by integrating GM with the penalty method. In 2023, Tadesse et al. [9] were given an application example that is solved using an optimization solver, namely LINGO Schrage from LINDO Systems (1997), as part of the proposed method.

In this paper, an algorithm has been formulated that centers on allocating resources to the lowest-cost cell for the s^{th} objective, corresponding to all cost values where the cell has the highest cost for the s^{th} objective. The suggested methodology surpasses established methods like the

Penalty method with GM [7] and Vogel's approximation method (VAM), offering a notably superior and unique efficient solution. Furthermore, the method is suited for MOTP involving different kinds of numerical representations, such as triangular, trapezoidal, and Pentagonal fuzzy numbers (PFN). The key highlights of this research include:

- *This paper introduces an efficient algorithm designed for solving MOTP under uncertainty.*
- *In this paper, all parameters including cost, supply, demand, and decision variables are considered fuzzy in the context of MOTP under uncertainty.*
- *The practical applicability of the proposed algorithm is highlighted by illustrative numerical examples.*
- *A comparative analysis with existing approaches demonstrates the superiority and reliability of the developed technique.*

Abbreviations and their full forms in Table 1

Table 1: Abbreviations and their full forms

MOTP	Multi-objective transportation problems
TP	Transportation problem
GM	Geometric mean
VAM	Vogel's approximation method
TFN	Trapezoidal fuzzy number
PFN	Pentagonal fuzzy number
LINDO	Linear, Interactive, and Discrete Optimizer

The paper continues with the following structure: Section 2 covers basic definitions, arithmetic operations, and ranking functions. Section 3 delineates the mathematical representation of fully fuzzy MOTP. Section 4 introduces an algorithm for determining an efficient solution of MOTP. Section 5 offers numerical examples pertaining to real-world problems. Section 6 presents the advantages of the proposed algorithm. Section 8 presents the disadvantages of the proposed algorithm. Section 9 provides comparative analyses using graphical representations and Section 7 contains the paper's concluding remarks.

2 | Preliminaries

Within this section, we discuss the essential concepts related to pentagonal and TFN, fundamental definitions, arithmetic rules, and a ranking function as elucidated in the work by Bisht [2].

2.1 Fuzzy Number

A fuzzy set $\tilde{A}$ on $\mathbb{R}$ is considered a fuzzy number when it complies with the following conditions: (i) Convex fuzzy set (ii) Normalized fuzzy set (iii) The fuzzy set's support must be bounded.

2.2 Pentagonal Fuzzy Number (PFN)

A fuzzy number $\tilde{A}^P = (p, q, r, s, t; \alpha_1, \alpha_2)$ is called a PFN if it complies with the following properties:
(a) $\mu_{\tilde{A}^P}(x)$ is a function which is continuous in $[0, 1]$.
(b) $\mu_{\tilde{A}^P}(x)$ is continuous and strictly increasing function in intervals $[\alpha_1, \alpha_2]$ and $[\alpha_2, \alpha_3]$.
(c) $\mu_{\tilde{A}^P}(x)$ is continuous and strictly decreasing function in intervals $[\alpha_3, \alpha_4]$ and $[\alpha_4, \alpha_5]$.
The membership function of a PFN is defined as:

$$\mu_{\tilde{A}^P}(x;\alpha_1,\alpha_2) = \begin{cases} \alpha_1(\frac{x-p}{q-p}), & if p \le x \le q \\ 1-(1-\alpha_1)(\frac{x-r}{q-r}), & if q \le x \le r \\ 1, & if x = r \\ 1-(1-\alpha_2)(\frac{x-r}{s-r}), & if r \le x \le s \\ \alpha_2(\frac{x-t}{s-t}), & if s \le x \le t \\ 0, & otherwise \end{cases}$$

Here p, q, r, s, t are real numbers such that $p \le q \le r \le s \le t$, α_1 and α_2 are the grades of q and s respectively.

2.3 Arithmetic Operations for PFN

Let $\tilde{A}^P = (p, q, r, s, t)$ and $\tilde{B}^P = (u, v, w, x, y)$ be two PFN. The operations as addition ($\oplus$), subtraction ($\ominus$) and multiplication ($*$) are presented as follows:
(a) Addition: $\tilde{A}^P \oplus \tilde{B}^P = (p+u, q+v, r+w, s+x, t+y)$
(b) Subtraction: $\tilde{A}^P \ominus \tilde{B}^P = (p-y, q-x, r-w, s-v, t-u)$
(c) Multiplication: $\tilde{A}^P * \tilde{B}^P = (p*u, q*v, r*w, s*x, t*y)$.

2.4 Ranking Function for PFN

The ranking function plays a crucial role in numerous mathematical models involving fuzzy numbers. A function denoted as $\Re : F(\mathbb{R}) \to \mathbb{R}$ is identified as a ranking function, where $F(\mathbb{R})$ serves to map any fuzzy number onto the real number line, representing the collection of fuzzy numbers firmly based on real numbers. The ranking function of a PFN is presented as $\Re(\tilde{A}^P) = \frac{p+t+z'}{3}$, where $z' = \frac{a'+r}{2}$, and $a' = \frac{qt-ps}{t-s-p+q}$.

2.5 Trapezoidal Fuzzy Number (TFN)

A TFN is described by a trapezoidal-shaped membership function. If $\alpha_1 = \alpha_2 = 1$, then the PFN simplifies to a TFN, $\tilde{A}^T$ =(p,q,s,t) and the membership function of a TFN is defined as:

$$\mu_{\tilde{A}^T}(x) = \begin{cases} (\frac{x-p}{q-p}), & if p \le x \le q \\ 1, & if q \le x \le s \\ (\frac{x-t}{s-t}), & if s \le x \le t \\ 0, & otherwise \end{cases}$$

2.6 Arithmetic Operations for TFN

Let $\tilde{A} = (p, q, s, t)$ and $\tilde{B} = (u, v, x, y)$ be two TFN. The operations $\oplus$ $\ominus$ and $\otimes$ are formulated as follows:
(a) Addition: $\tilde{A} \oplus \tilde{B} = (p+u, q+v, s+x, t+y)$
(b) Subtraction: $\tilde{A} \ominus \tilde{B} = (p-y, q-x, s-v, t-u)$
(c) Multiplication: $\tilde{A} \otimes \tilde{B} = (p^*, q^*, s^*, t^*)$
where $p^* = \min(pu, py, ut, ty)$, $q^* = \min(qv, qx, sv, sx)$, $s^* = \max(qv, qx, sv, sx)$, $t^* = \max(pu, py, ut, ty)$.

2.7 Ranking function for TFN

Consider a TFN, denoted as $\tilde{A}^T = (p, q, s, t)$. Then the ranking function of TFN, $\Re(\tilde{A}^T)$ can be formulated in the form of $\frac{1}{4}[(p+q+s+t)]$. This formulation covers a unique computation derived from the characteristics of the PFN.

3 | Mathematical Representation of the Fully Fuzzy MOTP under Uncertainty

In the day-to-day operations of logistics, organizers work towards accomplishing multiple goals simultaneously in the transportation of goods. The MOTP transports a multi-homogeneous product from m sources to n destinations with the lowest cost, minimum time, etc. in the desired posi-

tion. Mathematically, a MOTP in which cost coefficients, decision variables, and supply/demand quantities are considered fuzzy numbers can be written as:

$$
\begin{aligned}
Min\ \tilde{z}_1 &\approx \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^{1}\tilde{x}_{ij} \\
&\vdots \\
Min\ \tilde{z}_s &\approx \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}^{s}\tilde{x}_{ij}, \\
subject\ to & \\
& \sum_{j=1}^{n}\tilde{x}_{ij} \approx \tilde{a}_i, i = 1, 2, \ldots, m \\
& \sum_{i=1}^{m}\tilde{x}_{ij} \approx \tilde{b}_j, j = 1, 2, \ldots, n \\
& \tilde{x}_{ij} \succcurlyeq 0
\end{aligned}
$$

where
$\tilde{c}_{ij}^{s}$ *the multi-objective fuzzy transportation cost of the s-th objective,*
$\tilde{x}_{ij}$ *the fuzzy value representing the quantity of the product moved from the ith source to the jth destination,*
$\tilde{b}_j$ *fuzzy demand quantity at destination j,*
$\tilde{a}_i$ *fuzzy supply quantity by origin i,*
The above MOTP under uncertainty is said to be balanced if $\sum_{i=1}^{m}\tilde{a}_i = \sum_{j=1}^{n}\tilde{b}_j$, *otherwise unbalanced.*

4 | Proposed Method for Solving MOTP under Uncertainty:

The proposed algorithm is less time-consuming for solving MOTP under uncertainty. This algorithm helps in obtaining an efficient solution that closely approximates the ideal solution. The procedure of the proposed algorithm is as follows:

Step 1: Problem Representation: *Construct the provided transportation problem in the form of Table 2*

Step 2: Balancing the Unbalanced Problem: *In case of inequality between demand and supply (demand ≠ supply), the problem will be modified to form a balanced transportation problem (demand = supply) through the inclusion of dummy source or destination before proceeding to step 3.*

Step 3: Maximum Cost Calculation: *For all* s, *calculate* $K = \max_{1\leq i\leq m}\{\Re(\tilde{c}_{ij}^{s})\}$, *for fixed* j.

Step 4: Cell Selection and Allocation: *Select the cell that includes* K *among its objective values. If more than one such cell exists, select the one that maximizes the cost for an alternative objective. Subsequently, choose the cell that contains* $\min\{\Re(\sum_{i=1}^{m}\tilde{c}_{ij}^{s})$, *for fixed j} corresponding to the row and column of the previously chosen cell in Step 3. In case of a tie, select the one that allows the maximum allocation.*

Step 5: Procedure Repetitions and Stopping Criteria: *Proceeding with this process until all allocations are finished.*

Step 6: Efficient Solution: *Obtain an efficient solution by analyzing* $\tilde{z}^s = \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij}^{s}\otimes\tilde{x}_{ij})$ *for all* s.

Table 2: MOTP under Uncertainty.

Destination $\rightarrow$ Source $\downarrow$	D_1	D_2	$\dots$	D_n	Supply
S_1	$\tilde{c}^1_{11}$	$\tilde{c}^1_{12}$	$\dots$	$\tilde{c}^1_{1n}$	$\tilde{a}_1$
	$\dots$	$\dots$	$\dots$	$\dots$	
	$\tilde{c}^z_{11}$	$\tilde{c}^z_{12}$	$\dots$	$\tilde{c}^z_{1n}$	
S_2	$\tilde{c}^1_{21}$	$\tilde{c}^1_{22}$	$\dots$	$\tilde{c}^1_{2n}$	$\tilde{a}_2$
	$\dots$	$\dots$	$\dots$	$\dots$	
	$\tilde{c}^z_{21}$	$\tilde{c}^z_{22}$	$\dots$	$\tilde{c}^z_{2n}$	
S_3	$\tilde{c}^1_{31}$	$\tilde{c}^1_{32}$	$\dots$	$\tilde{c}^1_{3n}$	$\tilde{a}_3$
	$\dots$	$\dots$	$\dots$	$\dots$	
	$\tilde{c}^z_{31}$	$\tilde{c}^z_{32}$	$\dots$	$\tilde{c}^z_{3n}$	
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
S_m	$\tilde{c}^1_{m1}$	$\tilde{c}^1_{m2}$		$\tilde{c}^1_{mn}$	$\tilde{a}_m$
	$\dots$	$\dots$	$\dots$	$\dots$	
	$\tilde{c}^z_{m1}$	$\tilde{c}^z_{m2}$	$\dots$	$\tilde{c}^z_{mn}$	
Demand	$\tilde{b}_1$	$\tilde{b}_2$	$\dots$	$\tilde{b}_n$	

Figure A displays the flowchart corresponding to the proposed method for the balanced MOTP.

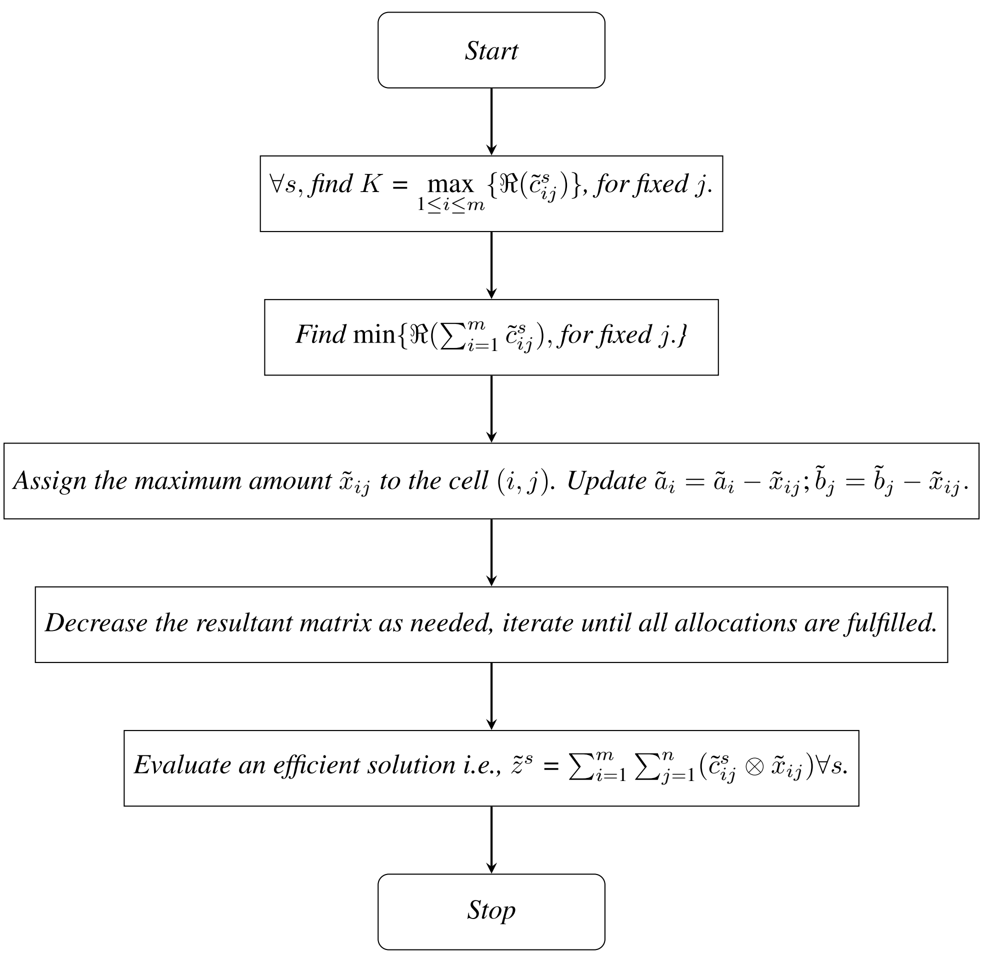

Figure A. Flow chart representation

5 | Numerical Examples

Numerical Example 1: Suppose there are three warehouses (W_1, W_2, W_3) *and four customers* (C_1, C_2, C_3, C_4)*. The logistics company focuses on minimizing transportation costs and delivery time simultaneously, considering pentagonal fuzzy nature of both factors.*
Warehouse W_1 *to Customer* C_1*: Cost* = $(6, 7, 8, 9, 13)$\$, *Time* = $(7, 8, 10, 11, 13)$ *days Warehouse* W_1 *to Customer* C_2*: Cost* = $(8, 9, 11, 13, 14)$\$, *Time* = $(2, 3, 4, 6, 8)$ *days Warehouse* W_1 *to Customer* C_3*: Cost* = $(7, 8, 10, 11, 13)$\$, *Time* = $(1, 2, 3, 4, 5)$ *days Warehouse* W_1 *to Customer* C_4*: Cost* = $(2, 4, 6, 8, 9)$\$, *Time* = $(3, 5, 7, 8, 9)$ *days*
Warehouse W_2 *to Customer* C_1*: Cost* = $(4, 5, 7, 9, 10)$\$, *Time* = $(6, 7, 8, 9, 13)$ *days*
Warehouse W_2 *to Customer* C_2*: Cost* = $(7, 8, 10, 11, 13)$\$, *Time* = $(8, 9, 11, 13, 14)$ *days*
Warehouse W_2 *to Customer* C_3*: Cost* = $(2, 3, 4, 6, 8)$\$, *Time* = $(6, 7, 8, 9, 13)$ *days*
Warehouse W_2 *to Customer* C_4*: Cost* = $(8, 9, 11, 13, 14)$\$, *Time* = $(1, 2, 3, 4, 5)$ *days*
Warehouse W_3 *to Customer* C_1*: Cost* = $(7, 8, 10, 11, 13)$\$, *Time* = $(2, 3, 4, 6, 8)$ *days*
Warehouse W_3 *to Customer* C_2*: Cost* = $(1, 2, 3, 4, 5)$\$, *Time* = $(6, 7, 8, 9, 13)$ *days*
Warehouse W_3 *to Customer* C_3*: Cost* = $(2, 3, 4, 6, 8)$\$, *Time* = $(1, 2, 3, 4, 5)$ *days*
Warehouse W_3 *to Customer* C_4*: Cost* = $(2, 3, 4, 6, 8)$\$, *Time* = $(2, 4, 6, 8, 9)$ *days.*

The given MOTP with supply of warehouses and demand of customers is written in Table 3

Table 3: MOTP with PFN.

Source → Destination ↓	C_1	C_2	C_3	C_4	Supply
W_1	(6,7,8,9, 13) (7,8,10,11, 13)	(8,9,11,13, 14) (2,3,4,6, 8)	(7,8,10,11, 13) (1,2,3,4, 5)	(2,4,6,8, 9) (3,5,7,8, 9)	(127,129,130, 132, 133)
W_2	(4,5,7,9, 10) (6,7,8,9, 13)	(7,8,10,11, 13) (8,9,11,13, 14)	(2,3,4,6, 8) (6,7,8,9, 13)	(8,9,11,13, 14) (1,2,3,4, 5)	(147,148,150, 151, 153)
W_3	(7,8,10,11, 13) (2,3,4,6, 8)	(1,2,3,4, 5) (6,7,8,9, 13)	(2,3,4,6, 8) (1,2,3,4, 5)	(2,3,4,6, 8) (2,4,6,8, 9)	(167,169,170, 172, 173)
Demand	(87,88,90, 92,93)	(97,99,100, 102,103)	(137,139,140, 142,143)	(117,118,120, 121,123)	

Table 3 represents the balanced MOTP i.e., $\sum_{i=1}^{3} a_i = \sum_{j=1}^{4} bj$ = 450.15. *To handle the problem using the proposed method, we get Minimum cost* = $(1, 2, 3, 4, 5) \otimes (97, 99, 100, 102, 103) \oplus (4, 5, 7, 9, 10) \otimes (87, 88, 90, 92, 93) \oplus (2, 3, 4, 6, 8) \otimes (54, 56, 60, 63, 66) \oplus (2, 3, 4, 6, 8) \otimes (64, 67, 70, 73, 76) \oplus (2, 4, 6, 8, 9) \otimes (117, 118, 120, 121, 123) \oplus (7, 8, 10, 11, 13) \otimes (4, 8, 10, 14, 16) = (943, 1543, 2270, 3174, 3896)$ = 2371.874
Minimum Time = $(6, 7, 8, 9, 13) \otimes (97, 99, 100, 102, 103) \oplus (6, 7, 8, 9, 13) \otimes (87, 88, 90, 92, 93) \oplus (6, 7, 8, 9, 13) \otimes (54, 56, 60, 63, 66) \oplus (1, 2, 3, 4, 5) \otimes (64, 67, 70, 73, 76) \oplus (3, 5, 7, 8, 9) \otimes (117, 118, 120, 121, 123) \oplus (1, 2, 3, 4, 5) \otimes (4, 8, 10, 14, 16) = (1847, 2441, 3080, 3629, 4973)$ = 3254.187. *Hence we obtained an efficient solution (ES) is* $(2371.874, 3254.187)$.

Numerical Example 2: The matrix form of MOTP under uncertainty in Table 4

Table 4: MOTP with TFN.

Source → Destination ↓	D_1	D_2	D_3	D_4	Supply
S_1	(0,1,1,2) (1,3,4,8)	(0,1,3,4) (1,3,4,8)	(2,5,7,14) (1,2,3,6)	(2,5,7,14) (1,3,4,8)	(3,5,8,16)
S_2	(0,1,1,2) (2,3,5,10)	(4,6,11,15) (3,5,8,16)	(1,2,3,6) (4,6,11,15)	(1,3,4,8) (7,8,11,14)	(15,18,19,24)
S_3	(3,5,8,16) (2,4,6,12)	(4,6,11,15) (0,1,3,4)	(1,3,4,8) (2,3,5,10)	(2,4,6,12) (0,1,1,2)	(13,16,17,22)
Demand	(8,9,12,15)	(1,2,3,6)	(10,13,14,19)	(12,15,16,21)	

6 | Comparative studies

In this section, we have shown an efficient solution of Example 1 in Table 5 using VAM, Penalty methods with GM [7] and the proposed algorithm of MOTP under uncertainty. Also, the graphical representation is shown by using VAM, Penalty methods with GM [7] and the proposed algorithm.

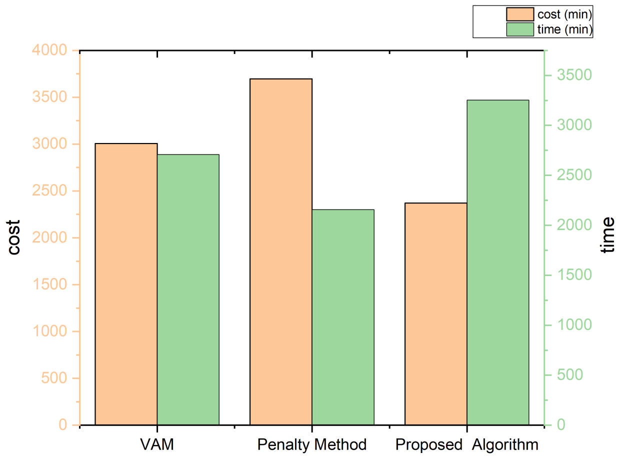

Figure 1: Solutions of Example 1 using different methods

The results of Example 1 are visually presented in Figure 1 using various established approaches and the proposed algorithm. In the proposed algorithm, we have observed that the cost is 2371.874 in MOTP, which is less than the cost by VAM and Penalty methods with GM [7]. In this example, the proposed algorithm outperforms in reducing the cost of MOTP. The graphical results suggest that the proposed method achieves better performance than both the VAM and Penalty methods using the GM [7].

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The outcomes of Example 2 are displayed in Table 6 and depicted graphically in Figure 2, comparing various established methods with proposed algorithm. In the proposed algorithm, we have observed that proposed algorithm is equivalent to the VAM yet it is best from the Penalty methods with GM [7] in MOTP. In this example, the proposed algorithm outperforms in reducing

Table 5: Example 1 results (ES)

Method	Obtained allocations	Obtained Cost	Obtained Time
Penalty Method with GM [7]	$\tilde{x}_{13} = (127, 129, 130, 132, 133)$, $\tilde{x}_{21} = (24, 27, 30, 33, 36)$, $\tilde{x}_{24} = (117, 118, 120, 121, 123)$, $\tilde{x}_{31} = (38, 43, 50, 56, 62)$, $\tilde{x}_{32} = (97, 99, 100, 102, 103)$, $\tilde{x}_{33} = (4, 7, 10, 13, 16)$	3696.120	2157.534
VAM	$\tilde{x}_{13} = (61, 66, 70, 75, 79)$, $\tilde{x}_{14} = (48, 53, 60, 65, 69)$, $\tilde{x}_{21} = (87, 88, 90, 92, 93)$, $\tilde{x}_{24} = (54, 56, 60, 63, 66)$, $\tilde{x}_{32} = (97, 99, 100, 102, 103)$, $\tilde{x}_{33} = (64, 67, 70, 73, 76)$	3007.467	2708.991
Proposed Method	$\tilde{x}_{32} = (97, 99, 100, 102, 103)$, $\tilde{x}_{21} = (87, 88, 90, 92, 93)$, $\tilde{x}_{23} = (54, 56, 60, 63, 66)$, $\tilde{x}_{33} = (64, 67, 70, 73, 76)$, $\tilde{x}_{14} = (117, 118, 120, 121, 123)$, $\tilde{x}_{13} = (4, 8, 10, 14, 16)$	2371.874	3254.187

MOTP. The graphical results suggest that proposed method achieves better performance than both the VAM and Penalty methods using the GM [7]. The proposed algorithm offers a unique efficient solution.

7 | Advantages of the proposed algorithm

(i) A new algorithm is proposed to solve MOTP under uncertainty, providing a direct compromise solution.

(ii) The proposed algorithm is suitable for MOTP involving different types of numerical representations, including triangular, trapezoidal, and PFN.

(iii) We compare our proposed algorithm with VAM and Penalty methods with GM [7], and conclude that our algorithm produces more accurate results.

(iv) The proposed algorithm is simple to apply and helps save time.

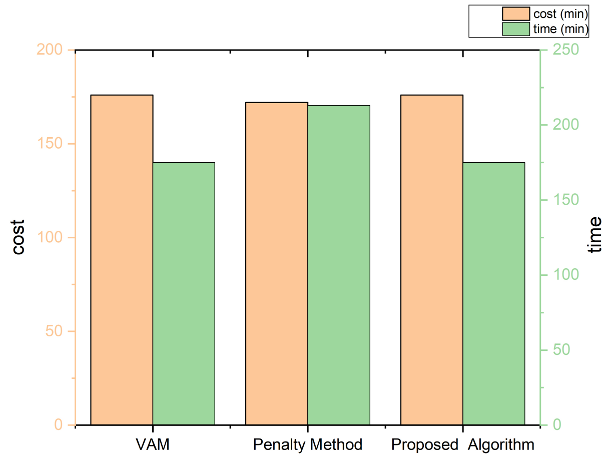

Figure 2: Solutions of Example 2 using different methods

Table 6: Example 2 results (ES)

Method	Obtained allocations	Obtained Cost	Obtained Time
Penalty Method with GM [7]	$\tilde{x}_{11}=(3,5,8,16)$, $\tilde{x}_{21}=(-8,1,7,12)$, $\tilde{x}_{32}=(1,2,3,6)$, $\tilde{x}_{34}=(7,13,15,21)$, $\tilde{x}_{23}=(10,13,14,19)$, $\tilde{x}_{24}=(-9,0,3,14)$	172	213
VAM	$\tilde{x}_{21}=(8,9,12,15)$, $\tilde{x}_{23}=(0,6,10,16)$, $\tilde{x}_{34}=(12,15,16,21)$, $\tilde{x}_{13}=(-3,2,6,15)$, $\tilde{x}_{33}=(-8,0,2,10)$, $\tilde{x}_{12}=(1,2,3,6)$	176	175
Proposed Method	$\tilde{x}_{12}=(1,2,3,6)$, $\tilde{x}_{21}=(8,9,12,15)$, $\tilde{x}_{13}=(-3,2,6,15)$, $\tilde{x}_{23}=(0,6,10,16)$, $\tilde{x}_{33}=(-8,0,2,10)$, $\tilde{x}_{34}=(12,15,16,21)$	176	175

8 | Disadvantages of the proposed algorithm

(i) The proposed algorithm is designed for linear MOTP under uncertainty. If the problem involves non-linear costs or other non-linearities, the approach might not be directly applicable without modification.

(ii) The approach may be sensitive to the quality of the data. Small changes in input data could potentially lead to significantly different solutions, especially when determining the "ideal" or "compromise" solution.

9 | Conclusion remarks

The proposed algorithm is designed to directly solve MOTP without converting them into single-objective transportation problems. The proposed algorithm is valuable for identifying a unique efficient solution. It is not required to generate more than one efficient solution. We have con-

cluded that the proposed method provides a better efficient solution than existing algorithms like VAM and Penalty methods with GM [7]. Also, this approach is straightforward to employ and consumes minimal time. The method is useful for solving MOTP with various types of numerical values like triangular, trapezoidal, PFN, etc. This approach will be extended in future work to tackle solid MOTP under uncertainty.

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