Palestine Journal of Mathematics

Vol 14(1)(2025) , 67–77

Fuzzified Bisection Method To Find The Root of An Algebraic Equation Using TrFNs

Kalyani Vable, Pravin Jadhav, Shrikishan Gaikwad and Dinkar Patil

Communicated by Thodoros Katsaounis

MSC 2010 Classifications: Primary 65D99; Secondary 03E72.

Keywords and phrases: Trapezoidal fuzzy number, Bisection method, Fuzzy membership function, Arithmetic operations on TrFNs, α - cut for TrFN.

The authors would like to thank the reviewers and editor for their constructive comments and valuable suggestions that improved the quality of our paper..

Corresponding Author: Kalyani Vable

Abstract Numerical methods are very useful techniques to obtain an approximate solution to hard problems. In 1965, fuzzy logic was introduced by Lotfi A. Zadeh. in conjunction with the proposal of fuzzy set theory. Fuzzy logic is applied by many researchers in various fields. A crisp set is transformed to a fuzzy set by the process of fuzzification, and by using defuzzification, a fuzzy set is reduced to a crisp set, or a fuzzy member can be converted into a crisp member. In this paper, we obtain the root of an algebraic equation by fuzzification of the bisection method using trapezoidal fuzzy numbers (TrFNs).

1 Introduction

In the real world, complexity arises from uncertainty in the form of ambiguity. The theory of probability is an effective tool to handle such uncertainty, but it can be used only in cases whose characteristics are based on random processes. Uncertainty may arise due to incomplete information or not fully reliable information, due to inherent imprecision in the language, or due to receiving information from more sources. Fuzzy set theory is an important mathematical tool for handling uncertainty due to vagueness.

In 1965, fuzzy logic was introduced by Lotfi A. Zadeh. in conjunction with the proposal of fuzzy set theory. Fuzzy logic is applied by many researchers in various fields. A value that lies in an expected range defined by the quantitative limits and described with non-specific categories with the help of fuzzy theory. Many researchers have investigated lots of methods of numerical analysis.

An approach for computing the product of various fuzzy numbers using α cut was proposed by Hassanzadeh et al.[4] in 2012. Ahmed Abdel Aziz Elsayed et al. developed numerical solutions for coupled trapezoidal fully fuzzy Sylvester matrix equations [1] in 2022. In that study, the author observed the effectiveness of the methods for solving the CTrFFSME and compared the results using various parameters like number of iterations, convergence factor, error, etc. The fuzzification of the bisection method was introduced by Hazarika and Bora [3] in 2015. In that study, the author introduced the fuzzy bisection method using triangular fuzzy numbers and solved numerical examples. In 2022, Seema Sharma and Sushma [6] developed a fuzzy methodology to analyze the behavior of the coal handling system in a thermal power plant. In that study, the author used trapezoidal fuzzy numbers to present a more realistic and flexible behavior analysis of the coal handling system.

George Cantor propounded the crisp set theory, which is nothing but the classical set theory. It is fundamental to the study of the fuzzy set. The fuzzy set survives on multistate membership [0-1]. Crisp logic is built on a 2-state truth value (True / False) and fuzzy logic is built on a multistate truth value. (True / False/ Very true/ partly true/ partly false and so on).

We recognize that many of the crisp values carry some considerable uncertainty. The variable is probably fuzzy, and it can be represented by a membership function because uncertainty arises due to vagueness.

For a system whose output is fuzzy, a crisp decision is taken easily if the output is a single scalar quantity. This process of converting a fuzzy set to a single crisp value is known as defuzzification.

In this paper, we apply the fuzzification of the bisection method to obtain the root of an algebraic equation using TrFNs. We present a new approach for finding the root of an algebraic equation with the trapezoidal fuzzy number. To solve the given algebraic equation, we consider a fuzzy interval, and with the help of arithmetic operations on TrFNs, we bisect the interval. If the value of the function at the midpoint of the fuzzy interval satisfies the required condition, then we stop the process and get the fuzzy root of the given algebraic equation. To obtain a crisp root, We apply defuzzification to a fuzzy root. We get a crisp root very close to the exact value of the function in the minimum iterations. Thus, the proposed method converges rapidly in comparison to the standard bisection method.

This paper is organized as Section 1 is from the introduction. Section 2 is devoted to some preliminary ideas and concepts. Section 3 includes methodology. Section 4 is for some numerical examples. The conclusion is in the fifth section.

Novelty: There are various methods to find the root of an algebraic polynomial, such as the classical bisection method, false position method, Newton-Raphson method, secant method, etc., but in the proposed method we use fuzzy interval, and we get the required result in fewer iterations in comparison to the classical bisection method. Thus, the proposed method is more effective than the classical bisection method.

2 Preliminaries

Here we shall state some basic definitions and results related to trapezoidal fuzzy numbers (TrFNs) in fuzzy theory [1], [4], [5], [6] .

Definition 2.1. Let X be a universal set. Then, the fuzzy subset $\tilde{P}$ of X is defined by its membership function $\mu_{\tilde{P}}$: $X \to [0,1]$ which assign to each element $x \in X$ a real number $\mu_{\tilde{P}}(x)$ in the interval [0, 1], where the function value of $\mu_{\tilde{P}}(x)$ represents the grade of membership of x in $\tilde{P}$. A fuzzy set $\tilde{P}$ is written as
$\tilde{P} = \{(x, \mu_{\tilde{P}}(x)), x \in X, \ \ \mu_{\tilde{P}}(x) \in [0,1]\}$.

Definition 2.2. A fuzzy set $\tilde{P}$, defined on the universal set of real number R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{P}$ is convex i.e.,
$\mu_{\tilde{P}}\,(\beta x_1 + (1-\beta)x_2) \geq \min(\mu_{\tilde{P}}(x_1), \mu_{\tilde{P}}(x_2)), \forall x_1, x_2 \in R, \ \forall \beta \in [0,1]$.

(ii) $\tilde{P}$ is normal, i.e., $\exists\, x_0 \in R$ such that $\mu_{\tilde{P}}(x_0) = 1$.

(iii) $\mu_{\tilde{P}}$ is piecewise continuous.

Definition 2.3. A fuzzy number $\tilde{P} = (p_1, p_2, p_3, p_4)$ is a TrFN in the general form if its membership function is as follows :

$$\mu_{\tilde{P}}(x) = \begin{cases} 0, & x \leq p_1 \\ \frac{x-p_1}{p_2-p_1}, & p_1 \leq x \leq p_2 \\ 1, & p_2 \leq x \leq p_3 \\ \frac{p_4-x}{p_4-p_3}, & p_3 \leq x \leq p_4 \\ 0, & x > p_4 \end{cases}$$

Definition 2.4. The sign of the TrFN $\tilde{P} = (p_1, p_2, p_3, p_4)$ can be classified as follows:

(i) $\tilde{P}$ is positive (negative) iff $p_1 \geq 0, (p_4 \leq 0)$

(ii) $\tilde{P}$ is zero iff $(p_1, p_2, p_3$ & $p_4 = 0)$

(iii) $\tilde{P}$ is near zero iff $p_1 \leq 0 \leq p_4$

Definition 2.5. Operations on TrFNs.
The arithmetic operations on TrFNs are presented as follows:
Let $\tilde{P} = (p_1, p_2, p_3, p_4)$ and $\tilde{Q} = (q_1, q_2, q_3, q_4)$ be two TrFNs, then

(i) Addition :

$$\begin{aligned}\tilde{P} + \tilde{Q} &= (p_1, p_2, p_3, p_4) + (q_1, q_2, q_3, q_4) \\ &= (p_1 + q_1, p_2 + q_2, p_3 + q_3, p_4 + q_4)\end{aligned}$$

(ii) Subtraction :

$$\begin{aligned}\tilde{P} - \tilde{Q} &= (p_1, p_2, p_3, p_4) - (q_1, q_2, q_3, q_4) \\ &= (p_1 - q_4, p_2 - q_3, p_3 - q_2, p_4 - q_1)\end{aligned}$$

(iii) Symmetric image:

$$-\tilde{P} = (p_4, p_3, p_2, p_1)$$

(iv) Scalar multiplication : let $\beta \in R$, then

$$\beta \otimes (p_1, p_2, p_3, p_4) = \begin{cases} (\beta p_1, \beta p_2, \beta p_3, \beta p_4), & \beta \geq 0 \\ (\beta p_4, \beta p_3, \beta p_2, \beta p_1), & \beta < 0 \end{cases}$$

(v) Multiplication: The multiplication between fuzzy numbers is neither commutative nor associative. Thus, TrFNs multiplication operations can be classified as follows :

Case I : If $\tilde{P} = (p_1, p_2, p_3, p_4)$ and $\tilde{Q} = (q_1, q_2, q_3, q_4)$ be two TrFNs, then

$$\tilde{P}\tilde{Q} = (r, s, t, u),$$

where

$$\begin{aligned}r &= min(p_1q_1, p_1q_4, p_4q_1, p_4q_4) \\ s &= min(p_2q_2, p_2q_3, p_3q_2, p_3q_3) \\ t &= max(p_2q_2, p_2q_3, p_3q_2, p_3q_3) \\ u &= max(p_1q_1, p_1q_4, p_4q_1, p_4q_4)\end{aligned}$$

Case II : If $\tilde{P}, \tilde{Q} > 0$, then

$$\tilde{P}\tilde{Q} = (p_1q_1, p_2q_2, p_3q_3, p_4q_4)$$

Case III : If $\tilde{P}, \tilde{Q} < 0$, then

$$\tilde{P}\tilde{Q} = (p_4q_4, p_3q_3, p_2q_2, p_1q_1)$$

Case IV : If $\tilde{P} > 0, \tilde{Q} < 0$, then

$$\tilde{P}\tilde{Q} = (p_4q_1, p_3q_2, p_2q_3, p_1q_4)$$

Case V : If $\tilde{P} < 0, \tilde{Q} > 0$, then

$$\tilde{P}\tilde{Q} = (p_1q_4, p_2q_3, p_3q_2, p_4q_1)$$

(vi) Equality : The fuzzy numbers $\tilde{P} = (p_1, p_2, p_3, p_4)$ and $\tilde{Q} = (q_1, q_2, q_3, q_4)$ are equal iff

$$p_1 = q_1, p_2 = q_2, p_3 = q_3, p_4 = q_4.$$

Definition 2.6. α - cut for TrFN : Let $\tilde{P} = (p_1, p_2, p_3, p_4)$ be a TrFN .

$$[\tilde{P}]^{\alpha} = [(p_2 - p_1)\alpha + p_1, p_4 - (p_4 - p_3)\alpha]$$

is the α - cut set for TrFN.

Definition 2.7. Defuzzification :

The indirect comparison of fuzzy numbers is the fundamental reason for defuzzification. The comparison over fuzzy sets has no universal consensus. The fuzzy numbers has to be mapped initially to real values that can be compared for computing the magnitude values. Defuzzification process is mapping fuzzy numbers to real values that can be compared for computing the magnitudes. The computation may take several forms; the standard form which is the usage of centroid rule. The computation of the defuzzification process requires integrating the membership function over the fuzzy sets. This improves the effect over direct computation of centroid rule with centre of gravity that describes the fuzzy quantity. The defuzzification over trapezoid fuzzy numbers using median is evaluated at the risk rate:

For the trapezoid fuzzy number $\tilde{P} = (p_1, p_2, p_3, p_4)$, by the bisection of area , the median

$M_{\tilde{P}} = \frac{(p_1+p_2+p_3+p_4)}{4}$, only if , $p_2 \leq M_{\tilde{P}} \leq p_3$

3 Methodology

Let us consider an equation $G(x)$ = 0. Let $G(x)$ be a continuous function and $G(x)$ can be algebraic. Let the function $G(x)$ changes sign over an interval $x = \tilde{P}$ and $x = \tilde{Q}$, where $\tilde{P} = (p_1, p_2, p_3, p_4)$ and $\tilde{Q} = (q_1, q_2, q_3, q_4)$. Then the root of $G(x)$ = 0 lying between $\tilde{P}$ and $\tilde{Q}$.

Now fuzzy membership function of $\tilde{P}$ and $\tilde{Q}$ are respectively,

$$\mu_{\tilde{P}}(x) = \begin{cases} \frac{x-p_1}{p_2-p_1}, & p_1 \leq x \leq p_2 \\ 1, & p_2 \leq x \leq p_3 \\ \frac{p_4-x}{p_4-p_3}, & p_3 \leq x \leq p_4 \\ 0, & otherwise \end{cases}$$

$$\mu_{\tilde{Q}}(x) = \begin{cases} \frac{x-q_1}{q_2-q_1}, & q_1 \leq x \leq q_2 \\ 1, & q_2 \leq x \leq q_3 \\ \frac{q_4-x}{q_4-q_3}, & q_3 \leq x \leq q_4 \\ 0, & otherwise \end{cases}$$

with respect to α - cuts as

$[\tilde{P}]^{\alpha} = [(p_2 - p_1)\alpha + p_1, p_4 - (p_4 - p_3)\alpha]$

$[\tilde{Q}]^{\alpha} = [(q_2 - q_1)\alpha + q_1, q_4 - (q_4 - q_3)\alpha]$

As a first approximation , the root $G(x) = 0$ is

$$\tilde{x_0} = \frac{\tilde{P} + \tilde{Q}}{2} = \frac{(p_1, p_2, p_3, p_4) + (q_1, q_2, q_3, q_4)}{2}$$

Let us consider $\tilde{x_0} = (x'_0, x''_0, x'''_0, x''''_0)$.

The fuzzy membership function (f.m.f) of $\tilde{x_0}$ is

$$\mu_{\tilde{x_0}}(x) = \begin{cases} \frac{x-x_0'}{x_0''-x_0'}, & x_0' \leq x \leq x_0'' \\ 1, & x_0'' \leq x \leq x_0''' \\ \frac{x_0''''-x}{x_0''''-x_0'''}, & x_0''' \leq x \leq x_0'''' \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_0}]^\alpha = [(x_0'' - x_0')\alpha + x_0',\ x_0'''' - (x_0'''' - x_0''')\alpha]$$

Suppose $G(\tilde{P})$ & $G(\tilde{x_0})$ are of opposite signs then the root lies between $\tilde{P}$ & $\tilde{x_0}$ and if $G(\tilde{x_0})$ & $G(\tilde{Q})$ are of opposite signs then the root lies between $\tilde{x_0}$ & $\tilde{Q}$.
If $G(\tilde{P}) < 0$ & $G(\tilde{Q}) > 0$ then the first approximation be

$$\tilde{x_0} = \frac{\tilde{P} + \tilde{Q}}{2} = \frac{(p_1, p_2, p_3, p_4) + (q_1, q_2, q_3, q_4)}{2}$$

Suppose $G(\tilde{x_0}) < 0$ then the root lies between $\tilde{x_0}$ & $\tilde{Q}$.
Then the second approximation is,

$$\tilde{x_1} = \frac{\tilde{x_0} + \tilde{Q}}{2} = \frac{(x_0', x_0'', x_0''', x_0'''') + (q_1, q_2, q_3, q_4)}{2}$$

let us consider , $\tilde{x_1} = (x_1', x_1'', x_1''', x_1'''')$
The f.m.f of $\tilde{x_1}$ is ,

$$\mu_{\tilde{x_1}}(x) = \begin{cases} \frac{x-x_1'}{x_1''-x_1'}, & x_1' \leq x \leq x_1'' \\ 1, & x_1'' \leq x \leq x_1''' \\ \frac{x_1''''-x}{x_1''''-x_1'''}, & x_1''' \leq x \leq x_1'''' \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_1}]^\alpha = [(x_1'' - x_1')\alpha + x_1',\ x_1'''' - (x_1'''' - x_1''')\alpha]$$

Suppose $G(\tilde{x_1})$ is positive then the roots lies between $\tilde{x_0}$ & $\tilde{x_1}$ and the third approximation is ,

$$\tilde{x_2} = \frac{\tilde{x_0} + \tilde{x_1}}{2} = \frac{(x_0', x_0'', x_0''', x_0'''') + (x_1', x_1'', x_1''', x_1'''')}{2}$$

let us consider , $\tilde{x_2} = (x_2', x_2'', x_2''', x_2'''')$
The f.m.f of $\tilde{x_2}$ is ,

$$\mu_{\tilde{x_2}}(x) = \begin{cases} \frac{x-x_2'}{x_2''-x_2'}, & x_2' \leq x \leq x_2'' \\ 1, & x_2'' \leq x \leq x_2''' \\ \frac{x_2''''-x}{x_2''''-x_2'''}, & x_2''' \leq x \leq x_2'''' \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_2}]^\alpha = [(x_2'' - x_2')\alpha + x_2',\ x_2'''' - (x_2'''' - x_2''')\alpha]$$

and so on.

4 Convergence Analysis

Theorem 4.1. *[2] Let f be a continuous on the closed interval [a,b] and suppose that $f(a).f(b) < 0$. The bisection method generates a sequence of approximation $\{p_n\}$ which converges to a root $p \in (a, b)$ with the property*

$$|p_n - p| \leq \frac{b-a}{2^n}$$

5 Stopping Condition

[2] Let ϵ be a specified convergence tolerance for any root finding technique, there are three primary measures of convergence with which to construct the stopping condition. These are

(i) **The absolute error in the location of the root.**

Terminate the iteration when $|p_n - p| < \epsilon$.

(ii) **The relative error in the location of root.**

Terminate the iteration when $|p_n - p| < \epsilon|p_n|$.

(iii) **The test for a root.**

Terminate the iteration when $|f(p_n)| < \epsilon$.

6 Numerical Example

In this section we solve some examples.
Example 1.
Let us consider an algebraic equation $G(x) = x^2 - 3x - 8$.
Let $\tilde{P} = (3.99, 4, 4.01, 4.02)$ and $\tilde{Q} = (4.99, 5, 5.01, 5.02)$. The function $G(x)$ changes sign over an interval $x = \tilde{P}$ and $x = \tilde{Q}$. Here, $G(\tilde{P}) < 0$ and $G(\tilde{Q}) > 0$.
$\therefore$ The root of $G(x) = 0$ lying between $\tilde{P}$ and $\tilde{Q}$.
Then
$\tilde{x_0} = \frac{\tilde{P}+\tilde{Q}}{2} = \frac{1}{2}\{(3.99, 4, 4.01, 4.02) + (4.99, 5, 5.01, 5.02)\} = (4.49, 4.5, 4.51, 4.52)$

Now f.m.f. of $\tilde{x_0}$ is

$$\mu_{\tilde{x_0}}(x) = \begin{cases} \frac{x-4.49}{4.5-4.49}, & 4.49 \leq x \leq 4.5 \\ 1, & 4.5 \leq x \leq 4.51 \\ \frac{4.52-x}{4.52-4.51}, & 4.51 \leq x \leq 4.52 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_0}]^\alpha = [(4.5 - 4.49)\alpha + 4.49,\ 4.52 - (4.52 - 4.51)\alpha]$$

$G(\tilde{x_0}) = G(4.49, 4.5, 4.51, 4.52) < 0$, so roots lies between $\tilde{x_0}$ and $\tilde{Q}$.

$\tilde{x_1} = \frac{\tilde{x_0}+\tilde{Q}}{2} = \frac{1}{2}\{(4.49, 4.5, 4.51, 4.52) + (4.99, 5, 5.01, 5.02)\} = (4.74, 4.75, 4.76, 4.77)$

Now f.m.f. of $\tilde{x_1}$ is

$$\mu_{\tilde{x_1}}(x) = \begin{cases} \frac{x-4.74}{4.75-4.74}, & 4.74 \leq x \leq 4.75 \\ 1, & 4.75 \leq x \leq 4.76 \\ \frac{4.77-x}{4.77-4.76}, & 4.76 \leq x \leq 4.77 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_1}]^{\alpha} = [(4.75 - 4.74)\alpha + 4.74,\ 4.77 - (4.77 - 4.76)\alpha]$$

$G(\tilde{x_1}) = G(4.74, 4.75, 4.76, 4.77) > 0$, so roots lies between $\tilde{x_0}$ and $\tilde{x_1}$.

$\tilde{x_2} = \frac{\tilde{x_0}+\tilde{x_1}}{2} = \frac{1}{2}\{(4.49, 4.5, 4.51, 4.52)+(4.74, 4.75, 4.76, 4.77)\} = (4.615, 4.625, 4.635, 4.645)$

Now f.m.f. of $\tilde{x_2}$ is

$$\mu_{\tilde{x_2}}(x) = \begin{cases} \frac{x-4.615}{4.625-4.615}, & 4.615 \leq x \leq 4.625 \\ 1, & 4.625 \leq x \leq 4.635 \\ \frac{4.645-x}{4.645-4.635}, & 4.635 \leq x \leq 4.645 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_2}]^{\alpha} = [(4.625 - 4.615)\alpha + 4.615,\ 4.645 - (4.645 - 4.635)\alpha]$$

$G(\tilde{x_2}) = G(4.615, 4.625, 4.635, 4.645) < 0$, so roots lies between $\tilde{x_1}$ and $\tilde{x_2}$.

$\tilde{x_3} = \frac{\tilde{x_1}+\tilde{x_2}}{2} = \frac{1}{2}\{(4.74, 4.75, 4.76, 4.77) + (4.615, 4.625, 4.635, 4.645)\}$

=(4.6775, 4.6875, 4.6975, 4.7075)

Now f.m.f. of $\tilde{x_3}$ is

$$\mu_{\tilde{x_3}}(x) = \begin{cases} \frac{x-4.6775}{4.6875-4.6775}, & 4.6775 \leq x \leq 4.6875 \\ 1, & 4.6875 \leq x \leq 4.6975 \\ \frac{4.7075-x}{4.7075-4.6975}, & 4.6975 \leq x \leq 4.7075 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_3}]^{\alpha} = [(4.6875 - 4.6775)\alpha + 4.6775,\ 4.7075 - (4.7075 - 4.6975)\alpha]$$

$G(\tilde{x_3}) = G(4.6775, 4.6875, 4.6975, 4.7075)$ is near zero, so root of $G(x) = 0$ is (4.6775, 4.6875, 4.6975, 4.7075).
Now by Defuzzification method, the defuzzified value is 4.6925.
$\therefore$ The crisp root is 4.6925.

Table 1. Comparison between classical bisection and fuzzified bisection method of example 1.

Method	Fuzzified bisection using TrFNs	Classical bisection
No. of Iterations	4	15
Root of $G(x)$	4.6925	4.7015

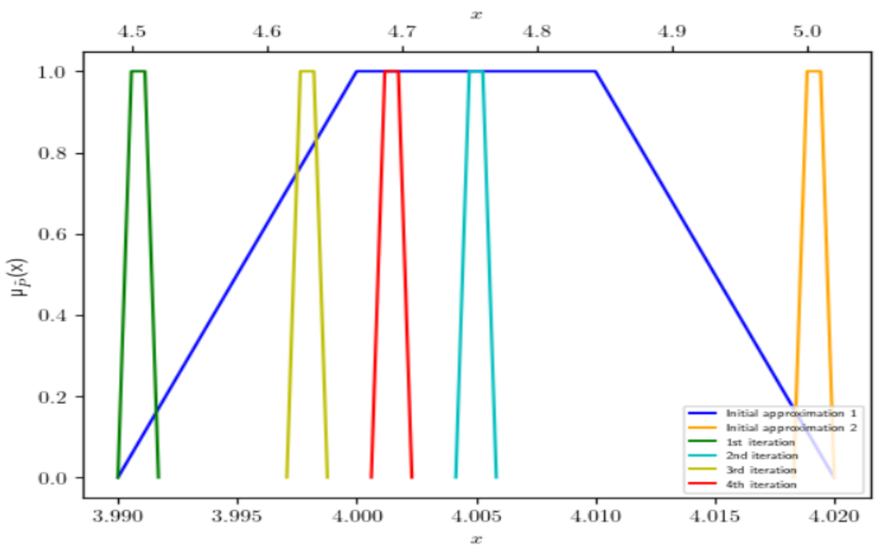

Figure 1. Graphical representation of a solution of Example 1.

In this example, we observed that the fuzzified bisection method using TrFNs gives the root in 4^{th} iterations and the classical bisection method gives the root in 15^{th} iterations. So the fuzzified bisection method using TrFNs is more effective than the classical bisection method.

Example 2.

Let us consider an algebraic equation $G(x) = x^3 - 18$.
Let $\tilde{P} = (1.89, 1.90, 1.91, 1.92)$ and $\tilde{Q} = (2.89, 2.90, 2.91, 2.92)$ .The function $G(x)$ changes sign over an interval $x = \tilde{P}$ and $x = \tilde{Q}$. Here, $G(\tilde{P}) < 0$ and $G(\tilde{Q}) > 0$.
$\therefore$ the root of $G(x) = 0$ lying between $\tilde{P}$ and $\tilde{Q}$.
Then

$\tilde{x_0} = \frac{\tilde{P}+\tilde{Q}}{2} = \frac{1}{2}\{(1.89, 1.90, 1.91, 1.92) + (2.89, 2.90, 2.91, 2.92)\} = (2.39, 2.4, 2.41, 2.42)$

Now f.m.f. of $\tilde{x_0}$ is

$$\mu_{\tilde{x_0}}(x) = \begin{cases} \frac{x-2.39}{2.4-2.39}, & 2.39 \leq x \leq 2.4 \\ 1, & 2.4 \leq x \leq 2.41 \\ \frac{2.42-x}{2.42-2.41}, & 2.41 \leq x \leq 2.42 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_0}]^{\alpha} = [(2.4 - 2.39)\alpha + 2.39,\ 2.42 - (2.42 - 2.41)\alpha]$$

$G(\tilde{x_0}) = G(2.39, 2.4, 2.41, 2.42) < 0.$ so roots lies between $\tilde{x_0}$ and $\tilde{Q}$.

$\tilde{x_1} = \frac{\tilde{x_0}+\tilde{Q}}{2} = \frac{1}{2}\{(2.39, 2.4, 2.41, 2.42) + (2.89, 2.90, 2.91, 2.92)\} = (2.64, 2.65, 2.66, 2.67)$

Now f.m.f. of $\tilde{x_1}$ is

$$\mu_{\tilde{x_1}}(x) = \begin{cases} \frac{x-2.64}{2.65-2.64}, & 2.64 \leq x \leq 2.65 \\ 1, & 2.65 \leq x \leq 2.66 \\ \frac{2.67-x}{2.67-2.66}, & 2.66 \leq x \leq 2.67 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_1}]^\alpha = [(2.65 - 2.64)\alpha + 2.64,\ 2.67 - (2.67 - 2.66)\alpha]$$

$G(\tilde{x_1}) = G(2.64, 2.65, 2.66, 2.67) > 0$, so roots lies between $\tilde{x_0}$ and $\tilde{x_1}$.

$\tilde{x_2} = \frac{\tilde{x_0}+\tilde{x_1}}{2} = \frac{1}{2}\{(2.39, 2.4, 2.41, 2.42)+(2.64, 2.65, 2.66, 2.67)\} = (2.515, 2.525, 2.535, 2.545)$

Now f.m.f. of $\tilde{x_2}$ is

$$\mu_{\tilde{x_2}}(x) = \begin{cases} \frac{x-2.515}{2.525-2.515}, & 2.515 \leq x \leq 2.525 \\ 1, & 2.525 \leq x \leq 2.535 \\ \frac{2.545-x}{2.545-2.535}, & 2.535 \leq x \leq 2.545 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_2}]^\alpha = [(2.525 - 2.515)\alpha + 2.515,\ 2.545 - (2.545 - 2.535)\alpha]$$

$G(\tilde{x_2}) = G(2.515, 2.525, 2.535, 2.545) < 0$, so roots lies between $\tilde{x_1}$ and $\tilde{x_2}$.

$\tilde{x_3} = \frac{\tilde{x_1}+\tilde{x_2}}{2} = \frac{1}{2}\{(2.64, 2.65, 2.66, 2.67) + (2.515, 2.525, 2.535, 2.545)\}$

= (2.5775, 2.5875, 2.5975, 2.6075)

Now f.m.f. of $\tilde{x_3}$ is

$$\mu_{\tilde{x_3}}(x) = \begin{cases} \frac{x-2.5775}{2.5875-2.5775}, & 2.5775 \leq x \leq 2.5875 \\ 1, & 2.5875 \leq x \leq 2.5975 \\ \frac{2.6075-x}{2.6075-2.5975}, & 2.5975 \leq x \leq 2.6075 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_3}]^\alpha = [(2.5875 - 2.5775)\alpha + 2.5775,\ 2.6075 - (2.6075 - 2.5975)\alpha]$$

$G(\tilde{x_3}) = G(2.5775, 2.5875, 2.5975, 2.6075) < 0$, so roots lies between $\tilde{x_1}$ and $\tilde{x_3}$.

$\tilde{x_4} = \frac{\tilde{x_1}+\tilde{x_3}}{2} = \frac{1}{2}\{(2.64, 2.65, 2.66, 2.67) + (2.5775, 2.5875, 2.5975, 2.6075)\}$

= (2.60875, 2.61875, 2.62875, 2.63875)

Now f.m.f. of $\tilde{x_4}$ is

$$\mu_{\tilde{x_4}}(x) = \begin{cases} \frac{x-2.60875}{2.61875-2.60875}, & 2.60875 \leq x \leq 2.61875 \\ 1, & 2.61875 \leq x \leq 2.62875 \\ \frac{2.63875-x}{2.63875-2.62875}, & 2.62875 \leq x \leq 2.63875 \\ 0, & otherwise \end{cases}$$

with respect to α - cut

$$[\tilde{x_4}]^\alpha = [(2.61875 - 2.60875)\alpha + 2.60875,\ 2.63875 - (2.63875 - 2.62875)\alpha]$$

$G(\tilde{x_4}) = G(2.60875, 2.61875, 2.62875, 2.63875)$ is near zero, so root of $G(x) = 0$ is (2.60875, 2.61875, 2.62875, 2.63875).
Now by Defuzzification method, the defuzzified value is 2.62375.
$\therefore$ The crisp root is 2.62375.

Table 2. Comparison between classical bisection and fuzzified bisection method of example 2.

Method	***Fuzzified bisection using TrFNs***	***Classical bisection***
No. of Iterations	5	17
Root of $G(x)$	2.62375	2.6207

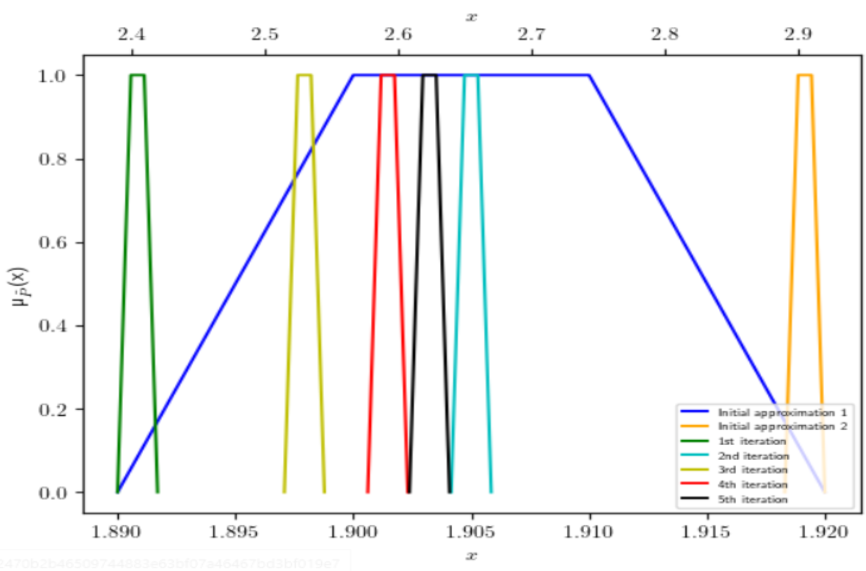

Figure 2. Graphical representation of a solution of Example 2.

In this example, we observed that the fuzzified bisection method using TrFNs gives the root in 5^{th} iterations and the classical bisection method gives the root in 17^{th} iterations. So the fuzzified bisection method using TrFNs is more effective than the classical bisection method.

7 Conclusion

In this paper, the fuzzification of the bisection method using trapezoidal fuzzy numbers is given. To find the root of an algebraic equation, we use the fuzzification of the bisection method using TrFNs. We observed that the root obtained by using both the methods—the fuzzified bisection method using TrFNs and the classical bisection method are almost the same. But by comparing the root obtained from both methods, the conclusion has been drawn that the fuzzified bisection method using TrFNs gives the root rapidly in comparison to the classical bisection method.

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Author information

Kalyani Vable, New Arts, Commerce and Science College, Ahmednagar, India.
E-mail: kvable24@gmail.com

Pravin Jadhav, Hon. Balasaheb Jadhav Arts, Commerce and Science College, Ale, India.
E-mail: pgjmaths@gmail.com

Shrikishan Gaikwad, New Arts, Commerce and Science College, Ahmednagar, India.
E-mail: sbgmathsnagar@gmail.com

Dinkar Patil, Adivasi Seva Samittee Nashik's Arts and Commerce College, Wadala, Nashik, India.
E-mail: sdinkarpatil95@gmail.com

Received: 2024-04-12
Accepted: 2024-11-10