

MATHEMATICAL REASONING ACROSS DISCIPLINES: A JOURNEY THROUGH LOGIC, LANGUAGE, AND LIMITS

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Abstract This reflection explores the pivotal yet often unexamined role of mathematical reasoning across diverse disciplines. While mathematics is commonly referred to as a “universal language,” this article questions whether such reasoning is truly indispensable, or perhaps merely an intellectual construct we have come to overvalue. We examine its contributions to both the exact sciences and the arts, investigate whether mathematics qualifies as a language from a cognitive neuroscience perspective, and clarify the distinctions between mathematical reasoning, logic, and other cognitive processes. Along the way, we highlight potential limitations and even unintended victims of the dominant narrative surrounding mathematical rigor. This cross-disciplinary inquiry, enriched with visual illustrations, argues that mathematical reasoning is not only foundational but also nuanced, sometimes fallible, and always intertwined with the broader human quest for understanding.

1 Introduction

Since its earliest entanglement with philosophy and the arts, mathematics has played a central role in satisfying humanity’s drive to question, classify, and make sense of the world. Long before the emergence of modern sciences such as physics and chemistry, mathematics served as a foundational tool, both practical and conceptual, for solving problems and exploring the structures underlying reality, [7, 13].

This article is structured in four sections. The first examines the contributions of mathematics across a range of disciplines, including both the exact sciences and the arts. The second addresses the question of whether mathematics qualifies as a language, drawing on perspectives from cognitive neuroscience. The third investigates various forms of reasoning, highlighting the distinctions between mathematical reasoning and other logical frameworks. The final section reflects on the limitations and challenges of mathematical reasoning, while reaffirming its indispensable, albeit imperfect, role in shaping human understanding. All figures included in this work are sourced from [1].

All academic disciplines ultimately strive to understand the phenomenon of life, particularly the complex relationship between human beings and the universe. Broadly speaking, these fields can be grouped into three major categories

- 1- Exact (or natural) sciences : disciplines that study the material universe, such as physics, chemistry, astronomy, and biology.
- 2- Humanities and social sciences: fields concerned with human existence and behavior, including history, philosophy, theology, and psychology.
- 3- The arts: although not traditionally classified as sciences due to their reliance on subjective judgment, the arts nonetheless express the human-universe relationship through literature, music, and visual forms.

It is important to note that the boundaries between these categories are often both fluid and

evolving. For example, economics includes highly technical subfields that rely on mathematics, equations, graphs, and statistical models, as well as human-centered domains that deal with marketing, behavioral studies, and sociology. This dual character spans both social sciences and applied mathematics.

Moreover, interdisciplinary fields are continually emerging, such as biomathematics, computational linguistics, and quantitative archaeology, reflecting a growing integration of digital tools and mathematical frameworks across domains. Indeed, information technology and digital analysis are increasingly embedded in nearly every specialty, giving rise to modern subdisciplines that reshape and renew their parent fields, [12].

2 Mathematics and other disciplines

To appreciate the importance of mathematical reasoning in other domains, one must first consider the role of mathematics across the full spectrum of human knowledge. The very ubiquity of mathematics in various disciplines attests to the foundational status of mathematical reasoning itself.

Discussing the role of mathematics in other fields is, in many ways, analogous to discussing the importance of a mother to her children, it is self-evident. This comparison is especially fitting, given that mathematics, philosophy, and the arts are among the earliest disciplines to have emerged alongside human consciousness. These fields developed naturally, as they respond to essential and innate human needs. Every human, at some point, has pondered existential questions, the domain of philosophy. The act of counting, categorizing, and distinguishing, such as separating sheep from cows, lies at the origin of mathematics. Likewise, the arts fulfill a deep-seated human desire to express and interpret the universe, as evidenced by prehistoric artifacts and geometric cave drawings created by early humans.

In contrast, the exact sciences, such as physics and chemistry, are relatively recent in the arc of intellectual history, with fields like biology and computer science being even more modern developments. This timeline underscores the enduring, foundational role that mathematics has played in shaping human inquiry since its earliest stages.

Mathematics in the Exact Sciences

Without attempting to be exhaustive, we may consider several illustrative examples of how mathematical tools underpin the major disciplines of the exact sciences. The role of ordinary differential equations (ODEs) in modeling dynamic phenomena, such as motion, heat flow, or electrical current, is well known and forms the basis of Newtonian mechanics. Similarly, the Schrödinger equation plays a fundamental role in quantum mechanics, modeling wave functions, diffusion, and oscillatory behavior over time. It provides the mathematical framework required to describe and predict the evolution of physical systems.

Linear algebra is equally indispensable. It underpins the analysis of multi-dimensional systems in areas as diverse as quantum physics, molecular vibrations, image processing, and data science. Probability theory forms the backbone of stochastic modeling and is crucial to rational decision-making in disciplines such as economics, finance, and actuarial science. The intersection of numerical analysis and computer science enables the simulation of complex processes, from planetary trajectories in astronomy to weather forecasting and biomedical modeling.

The list of examples is virtually endless. As knowledge expands, it becomes increasingly difficult to isolate a single mathematical tool as dominant within any given field. Rather, mathematical methods overlap, complement, and interweave, performing a kind of intellectual choreography that responds to the ever-growing needs of the sciences they support.

A particularly striking example of this interplay is artificial intelligence. The development and analysis of neural networks, for instance, rely on a vast array of mathematical disciplines, including linear algebra, statistics, probability, optimization theory, differential geometry, topology, and calculus. These mathematical components are not isolated; they form a tightly integrated architecture that makes AI systems possible.

Instead of reducing mathematics to a collection of calculations, algorithms, or abstract formalisms, though all of these are undeniably part of it, we should reflect on what truly weaves everything together: ideas. It is the interplay of ideas that makes mathematics so fertile. Think

of other disciplines as castles built with LEGO bricks: mathematics is not the bricks themselves, but the reasoning, the cement, that holds everything together. It is the foundation upon which these magnificent structures rise.

2.1 Mathematics in the Arts

Does mathematics play a role in the arts? Not in every artistic expression perhaps, but its presence in visual arts and music is both profound and undeniable.

In architecture, mathematics emerges through geometric forms, proportions, symmetry, and the use of perspective. It plays an equally fundamental role in music: the intervals of a musical scale are based on precise frequency ratios, while concepts such as time signatures and rhythmic subdivisions rely on arithmetic and fractions. More elaborate compositions, those involving counterpoint or polyphony, often reveal mathematical structures and patterns of symmetry. Even at a technical level, tools like Fourier analysis allow us to decompose and study sound waves, deepening our understanding of musical tones and timbres.

In the sections that follow, we will focus on two particularly striking examples: the golden ratio, often regarded as a mathematical ideal of beauty, and fractal art, which demonstrates how recursive structures can generate aesthetic complexity, [15].

The divine proportion

A proportion is said to be divine (or golden) when an object is divided into two unequal parts such that the ratio of the larger part, denoted (a), to the smaller part (b), is the same as the ratio of the whole ($a + b$) and the larger part (a). That is, both ratios are equal to what is known as the golden ratio

$$\frac{a}{b} = \frac{a+b}{a} = \Phi \approx 1.618$$

The golden ratio is not a measure of symmetry, but of harmony. Consider, for example, a finger that is disproportionately large relative to the hand: such a hand would appear visually unbalanced. This lack of harmony suggests that the ratio between the finger and the entire hand diverges from the golden ratio, denoted Φ . The name Φ (phi) is traditionally attributed to the ancient Greek mathematician Euclid, who referenced it in relation to aesthetic proportions. The symbol honors the sculptor Phidias, known for incorporating this ratio in his works. Centuries later, the Italian mathematician Leonardo Fibonacci introduced a now-famous numerical sequence in which each term is the sum of the two preceding ones. The ratio between successive terms in this sequence converges to the golden ratio.

The rectangle shown in the following image is known as a golden rectangle, characterized by the property that the ratio of its length to its width is approximately ($\simeq \Phi$). Remarkably, when a perfect square is removed from such a rectangle, the remaining shape is itself a smaller golden rectangle, revealing a recursive, self-similar structure. It turns out that this rectangle can be found

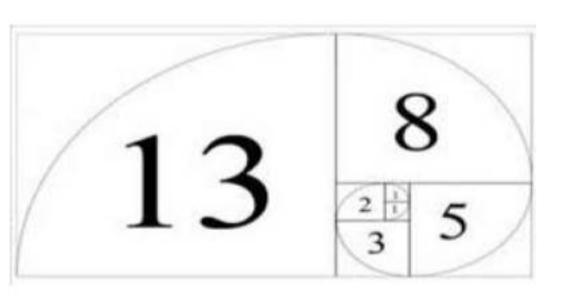


Figure 1. Golden rectangle.

in everything beautiful and harmonious. Many people enjoy searching for the golden rectangle everywhere, and when they do, they discover it in faces, cats, and butterflies! We know very well that humans have always imitated nature, which is why many artists draw inspiration from nature to achieve works with harmonious proportions and this is still the case in contemporary



Figure 2. Harmony everywhere.

times, [16]. For example, Leonardo De Vinci uses the golden ratio in the Mona Lisa and in the Vitruvian Man. Instagrammers have understood the interest of an aesthetically balanced and neat photograph thanks to the golden ratio, [4]. Logo designers also use the golden ratio frequently. The logo of national geographic for example is a simple golden rectangle.

Fractal art

Here are some computer-generated images from mathematical formulas, called fractal images. How do fractals work? The term fractal is derived from the idea of a non-integer (or fractional)



Figure 3. Some fractal art images.

dimension. While we are accustomed to whole-number dimensions, a point has dimension 0, a line has dimension 1, a plane has dimension 2, and so on, fractals occupy the space between these dimensions.

Consider, for instance, a sheet of paper representing a two-dimensional surface. When crumpled, it begins to occupy a three-dimensional volume. Yet it does not fully fill that volume like a liquid or a solid would. Its geometry is somewhere between dimensions 2 and 3, this is what we call a fractal dimension.

The conceptual foundations of fractals can be traced back to the work of Leibniz, Cantor, and Hausdorff, who explored the nature of continuity, dimension, and measure. However, the modern visual explosion of fractals owes much to the work of computer scientist Benoît Mandelbrot, who popularized fractal geometry through iconic images such as the Mandelbrot set, the Mandelbox, and the Mandelbulb. His work not only gave fractals a visual identity, but also highlighted their relevance across mathematics, nature, and digital art. Fractal objects have intricate, irregular

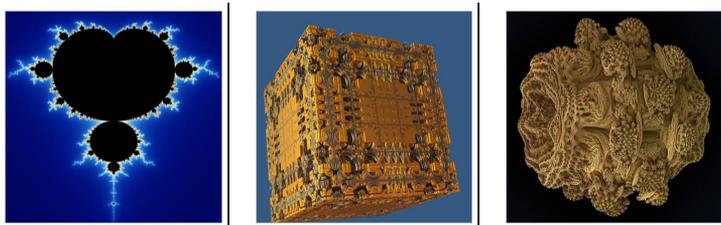


Figure 4. Left: The Mandelbrot set. Center: The Mandelbox. Right: The Mandelbulb

shapes with contours that fold and overlap endlessly.

The creation of fractals is guided by the principle that "simplicity breeds complexity", [6]. So, to generate a fractal image, start by defining a simple mathematical rule or formula that repeats iteratively. For example, the Mandelbrot set is created by iterating the relation $z_{n+1} = z_n^2 + c$,

where z and c are complex numbers. Each point on the plane is assigned a value for c , and the iteration determines whether the sequence remains bounded, [14]. The result is then visualized by mapping the number of iterations (or the speed of divergence) to colors, creating intricate, self-similar patterns. Specialized software like "Chaotica", "Fractint" or "Fractal Explorer" makes it easy to apply these formulas, adjust parameters, and add artistic effects such as gradient coloring. The beauty of fractals lies in their combination of mathematical precision and infinite complexity!

Fractals are widely used today in art galleries and multimedia projects. Computer scientists often add colors to enhance their aesthetic appeal, and fractal art has become a recognized art form.

3 Is Mathematics a Language?

The artistic manifestations of mathematics, whether through the golden ratio or fractals, illustrate not only its aesthetic power, but also its capacity to communicate patterns, structures, and balance. This communicative power raises a deeper philosophical question: could mathematics itself be a language?

The pedagogy of mathematics is clearly rooted in language. Numerous studies have shown that students who learn mathematics in a second language consistently underperform compared to those taught in their native tongue [18]. This suggests that language plays a critical role in how we understand diagrams, calculations, and abstract reasoning.

This intimate link between language and mathematics raises a deeper question: is mathematics itself a language? Galileo Galilei famously asserted, "The universe is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures; without these, one is wandering in a dark labyrinth" [19]. Later, Noam Chomsky reinforced this view by suggesting that mathematical abilities may emerge from an abstraction of linguistic operations, notably in his foundational work *Aspects of the Theory of Syntax*, [20].

The poetic dimension of mathematical thinking is also acknowledged by Sofia Kovalevskaya, who wrote: "It is impossible to be a mathematician without being a poet in soul" [21]. Indeed, mathematics teachers often insist that students "respect the syntax," because "math is a language."

However, this consensus is not without opposition. Albert Einstein offered a contrasting perspective rooted in his own experience as a thinker: "The words of language, as they are written or spoken, do not seem to play any role in my mechanism of thought... Conventional words or other signs have to be sought for laboriously only in a secondary stage" (as quoted in Hadamard, *The Psychology of Invention*, [22]).

So, how can we reconcile these opposing views? The debate over the relationship between mathematical language and natural language has recently been explored in cognitive neuroscience. Researchers from Sorbonne University, in collaboration with Stanford University, have begun to address this question empirically, using brain imaging to compare neural circuits involved in linguistic versus mathematical reasoning. Their findings suggest that the brain processes these two domains through distinct networks, reinforcing the idea that mathematics, while it borrows elements of language, also transcends it, [2, 3, 5].

The neural network responsible for processing natural language in the human brain, often referred to as the "semantic reflection network", has been well identified, even if its full functionality remains only partially understood. Remarkably, scientists have observed that the same brain areas are activated when individuals process syntactic structures, regardless of the spoken language, even in sign language.

But are these the same areas engaged during mathematical reasoning?

To investigate this question, a team of cognitive neuroscientists from the University of Sorbonne and Stanford University designed a novel experiment involving fifteen professional mathematicians. Each participant underwent MRI scans while responding to both mathematical and non-mathematical statements. They had only four seconds to decide whether each proposition was true, false, or nonsensical.

Some sample mathematical questions included:

- 1- Is any complex square matrix similar to an upper triangular matrix? **True**

- 2- Does there exist a continuous function from the unit sphere to itself that has no fixed point?
False

- 3- Does the stereographic projection admit $\sqrt{2}$ as its Euler characteristic?

For the less mathematically inclined, an equivalent sentence might read: “The potato flag was printed by the current king of the USA!”, yes, **Absurd**.

In addition to mathematical statements, participants were presented with questions from other domains: history, literature, and the arts, to serve as a control. The outcome was striking: mathematical reasoning systematically activated a specific brain network, involving regions such as the intraparietal sulcus and inferior temporal cortex. This network was clearly distinct from the language-processing areas traditionally engaged in reading and understanding natural language. Whether the problem was rooted in algebra, analysis, or geometry, the same mathematical network was at play.

These findings provide strong evidence that mathematical reasoning relies on its own dedicated neural architecture, separate from that of natural language. Mathematics, it seems, is not simply a form of language, it is something more, [25].

In other words, when we contemplate topics like what to make for dinner, the impact of climate change, or the role of women in society, our brains activate networks associated with semantic and social reasoning. Yet, when we demonstrate that any closed and bounded set is compact in finite-dimensional space, an entirely different brain system takes over.

One particularly insightful finding of the study was that sentences such as “Not all green plants are climbers”, which involve both quantifiers and negation, fail to activate the brain’s mathematical processing network. Instead, they are handled by the traditional language areas. Conversely, even a formula recalled by rote, such as $a^2 - b^2 = (a - b)(a + b)$, which seemingly requires little cognitive effort, still activates the brain’s mathematical reasoning areas. This indicates that such expressions, though familiar and possibly automated, are not interpreted as “language” by the brain, but as mathematics.

These observations led researchers to a striking conclusion: from the brain’s perspective, mathematics is not treated as a language. Its processing is neurologically distinct, relying on specific cortical regions unrelated to linguistic comprehension or production, [23, 24].

4 Mathematical Reasoning or Not?

If mathematics is not a language, as some cognitive studies suggest, perhaps it is something even more fundamental: a way of reasoning. Let us now explore the nature and structure of this reasoning, and how it differs (or not) from reasoning in other fields.

4.1 Basic concepts

Terms such as logical reasoning, mathematical logic, and mathematical reasoning are often used interchangeably by mathematicians, and not without justification. However, a closer look reveals important conceptual distinctions between logic, reasoning, and mathematical reasoning. Let’s begin with a few foundational definitions to clarify these differences

1- Logic

Logic is the formal study of valid reasoning and the structure of rational thought. It underpins our ability to construct, analyze, and communicate truths in a coherent manner. Logical thinking is essential not only in mathematics but also in everyday decision-making, critical thinking, and philosophical reflection. It is, in fact, what makes us rational beings. Nevertheless, engaging in specific mathematical tasks such as solving equations or drawing geometric constructions, does not, by itself, constitute existential reasoning.

2- Reasoning (in general)

Reasoning, whether mathematical or not, refers to the cognitive process by which we analyze situations, draw inferences, and reach conclusions. It is a tool for understanding reality and navigating complexity. The mechanisms of reasoning are what enable us to establish truths, refute errors, and make sound decisions.

Reasoning serves various purposes: it helps us justify claims, interpret facts, evaluate hypotheses, and even reflect ethically. Whether applied in mathematics, law, science, pedagogy, or philosophy, the overarching goal of reasoning is to minimize error, avoid fallacies, and maintain intellectual clarity. To reason effectively, we draw upon laws, principles, observations, and even intuitions.

3- Mathematical Reasoning

Mathematical reasoning involves demonstrating assertions within a formal system defined by axioms, statements accepted without proof, and governed by the principles of classical logic, such as the law of identity, the law of non-contradiction, and the law of the excluded middle.

Unlike informal reasoning, mathematical reasoning proceeds through abstraction, generalization, and strict deductive steps. It aims to establish truth in a way that is internally consistent and logically unavoidable.

Consider the following simple problem: *A pair of pants and a sweater are on sale. The pants cost €100 more than the sweater, and the total cost is €110. What is the price of the sweater?*

This puzzle belongs to a well-known family of cognitive reasoning tasks similar to the classic bat-and-ball problem, which has been widely used to demonstrate intuitive reasoning errors, [27]. This question, though elementary, illustrates the need for correct reasoning over intuitive guesswork, a theme we'll explore in the next section.

Many people instinctively believe that the price of the sweater is €10. However, someone employing more deliberate reasoning will quickly write the equation $x + (x + 100) = 110$ yielding $x = 5$ €. Therefore, the sweater costs €5. This simple example illustrates what we mean by mathematical reasoning, the application of structured logic within a quantitative framework to resolve a problem accurately.

Between logic and mathematical reasoning, there exists a form of entanglement¹: the two are so intimately connected that it becomes nearly impossible to draw a strict boundary between them. This interdependence mirrors the deep relationship observed in quantum physics, where entangled particles influence one another even across great distances. For this reason, logic and reasoning are often used synonymously in mathematics. In fact, acquiring any genuine mathematical understanding requires a firm grasp of basic logic. It is what allows learners to avoid classic pitfalls, such as confusing necessary conditions with sufficient ones, and to develop clarity and precision in their interpretation of mathematical statements.

Mathematics is often regarded as the discipline of rigor. To better understand what sets mathematical reasoning apart, let us explore several common types of reasoning used in various fields, and assess to what extent they meet the standards of precision required for mathematical demonstration.

1- Deductive Reasoning

Deduction is the process of deriving specific conclusions from general principles. It typically follows the structure of a syllogism, as in:

"All humans can understand mathematics".

"I am a human".

"Therefore, I can understand mathematics."

Deductive reasoning is a priori; it operates independently of experience and relies exclusively on logical structure. It is the gold standard in mathematical reasoning, underpinning demonstrations and formal proofs. Its reliability and universality make it a cornerstone of mathematical rigor.

2- Inductive Reasoning

Inductive reasoning proceeds from particular observations to broader generalizations. It involves identifying patterns or regularities and inferring a governing principle. For example:

After repeatedly observing that objects expand when heated, one may conclude that heat causes expansion.

¹A concept fundamental to quantum physics, where two or more particles become so interconnected that the state of one depends on the state of the other, even when separated by vast distances.

Unlike deduction, induction is a posteriori, grounded in empirical experience. While it is a powerful tool in the experimental sciences, such as pharmacology, where results from a sample population are generalized to the wider public, it is considered unacceptable in formal mathematics. The relationships it establishes may reflect coincidences rather than universal truths, and thus cannot serve as the basis for mathematical proof.

3- Analogical Reasoning

Analogical reasoning draws parallels between two concepts or situations based on their similarities. For example, when a student asks why the identity $x \cdot 0 = 0$ holds for all numbers x , a teacher may respond not with reference to the Peano axioms, but with an analogy:

“Imagine boxes that hold objects. If all your boxes are empty, no matter how many you have, you still have nothing.”

This type of reasoning can be pedagogically valuable, particularly in introducing abstract ideas. However, analogies are not reliable foundations for mathematical argumentation, as similarities may be superficial or context-dependent. Overextending analogy leads to faulty conclusions.

Nonetheless, analogical reasoning has played a notable role in scientific creativity. For instance, Schrödinger’s formulation of his famous wave equation was inspired by a formal analogy between optics and mechanics.

4- Dialectical Reasoning

Rooted in Socratic dialogue, dialectical reasoning unfolds through a dynamic exchange of ideas, progressing from a thesis, through antithesis, to a synthesis. It operates through interrogation, contradiction, and refinement. Consider the debate around personal freedom:

Thesis: “Freedom means doing whatever I want.”

Antithesis: “But if everyone does that, it leads to chaos.”

Question: “Then where should freedom be limited?”

Synthesis: “Your freedom ends where someone else’s begins.”

This form of reasoning fosters deeper understanding and is widely used in education, debate, and psychotherapy. Teachers use it to stimulate critical thinking; psychologists use it to guide patients toward insight.

However, dialectics lacks the precision and conclusiveness required in mathematics. No amount of dialogue about closed and bounded sets can demonstrate that they are compact, only a rigorous proof can.

In summary, mathematical reasoning is distinct in its capacity to eliminate coincidence, error, and vagueness. It stands as a model of intellectual rigor and reliability, offering a level of precision unmatched by other forms of reasoning, [11].

This naturally raises an important philosophical question: Has the privileged status of mathematical reasoning, often upheld as the epitome of intellectual rigor, somehow hindered progress in other domains of knowledge? One might wonder whether breakthroughs in the humanities, arts, or experimental sciences have instead been driven by alternative forms of reasoning, suggesting that mathematical reasoning may not be as universally essential as commonly assumed.

The answer, however, is no. Mathematical reasoning has not stifled scientific progress; on the contrary, as will be discussed in the concluding section, it has played a foundational and supportive role even in domains where it may not be explicitly visible.

4.2 Is mathematical reasoning perfect?

After reviewing the various types of reasoning used across disciplines: deduction, induction, analogy, and dialectics, it becomes clear that mathematical reasoning stands out for its rigor and precision. But does this make it a perfect system? This question leads us to a deeper reflection on the internal structure and limitations of mathematical reasoning itself. Is mathematical reasoning a flawless and self-sufficient system? Not quite! despite what some enthusiastic students or practitioners from other disciplines might think. To remain intellectually honest, it is essential to acknowledge that mathematical reasoning is not infallible.

Indeed, mathematics does not rely on a single universal system. Its conclusions always stem from axiomatic frameworks, and there exist multiple such systems, each tailored to specific domains of inquiry. For example: Euclid’s axioms form the basis of classical plane geometry, Peano’s axioms govern the arithmetic of natural numbers, Zermelo–Fraenkel set theory with the Axiom of Choice (ZFC) underpins most of modern pure mathematics.

Each of these systems assumes certain foundational truths without proof called axioms, from which all subsequent theorems are derived. But because different domains may adopt different axioms, truth in mathematics is always relative to the chosen framework. In this sense, mathematical reasoning is not an absolute system of universal truths, but rather a rigorous method of deduction grounded in agreed-upon assumptions, [8].

Among the most ambitious visions in the history of mathematics was that of David Hilbert [9], who aspired to construct a complete and consistent axiomatic system for all of mathematics. In such a system, every mathematical truth would be provable, and no contradictions would ever arise, a dream of absolute formal rigor.

However, this vision was profoundly disrupted in 1931 by a young Austrian logician named Kurt Gödel, who, at only 25 years old, published what are now known as the incompleteness theorems, [17]. Gödel’s first theorem states:

“Any sufficiently powerful axiomatic system is either incomplete or inconsistent.”

In simpler terms, no formal system that is capable of expressing arithmetic can be both complete and free of contradictions. There will always exist propositions within the system that are undecidable, statements that can neither be proven nor disproven using the system’s own rules.

Gödel’s result marked a turning point in the philosophy of mathematics, demonstrating that even the most rigorous systems are inherently limited. Far from weakening mathematics, however, this insight invites a deeper appreciation of the subtlety and humility required when navigating its foundations. A well-known example of undecidable propositions is the Goldbach’s



Figure 5. : Kurt Gödel (1906–1978)

Conjecture, a famous unsolved problem in number theory proposed by the German mathematician Christian Goldbach in 1742. It states: "Every even integer greater than 2 can be expressed as the sum of two prime numbers." The conjecture has been verified for even numbers up to very large values (trillions), but no general proof exists. It remains an open problem in mathematics. This example also underscores the contrast in mindset between mathematicians and scientists



Figure 6. The Goldbach’s Conjecture

from other disciplines. For instance, many computer scientists are content with empirical confirmation: after testing trillions of cases, they often feel confident enough to treat Goldbach's conjecture as a theorem. In their eyes, if no exception has been found up to 4×10^{18} , then the statement must be true. Goldbach's Conjecture (1742): "Every even integer greater than 2 is the sum of two prime numbers."

Verified computationally up to 4×10^{18} , [26]. However, for mathematicians, this kind of verification is never sufficient. Even trillions of confirmations do not constitute a rigorous proof. Their discipline demands absolute certainty: a single counterexample, no matter how far it lies beyond our current computational reach, would be enough to invalidate the entire conjecture. Therefore, until a formal proof is established, Goldbach's statement remains a conjecture, not a theorem.

This discussion naturally leads us to revisit Gödel's incompleteness theorems, which forever changed our understanding of what can be proven. While the first theorem showed that no axiomatic system can be both complete and consistent, Gödel's second theorem went further: "*No sufficiently powerful system can prove its own consistency*", [17].

In other words, even if a system appears free of contradictions, it cannot internally prove that it will never lead to one. This profound result reminds us that mathematical reasoning, though unparalleled in rigor, is not flawless. It is bounded by the very logic it seeks to master.

4.3 Collateral Victims of Mathematical Rigor

The prestige and authority of mathematical reasoning, with its emphasis on rigor and formal proof, have brought immense progress. However, this elevation of mathematics as the gold standard of reasoning has also produced unintended casualties.

First among them are individuals suffering from conditions such as dyscalculia² or innumeracy³. Dyscalculia is a persistent, often neurologically based disorder that affects a person's ability to understand and manipulate numbers, while innumeracy refers to acquired or temporary difficulties with mathematics often due to inadequate educational exposure. The latter is especially widespread in developing countries, where systemic challenges prevent early detection and intervention.

Unfortunately, primary school teachers are rarely trained to recognize or diagnose these learning differences. In systems where mathematics is viewed as a gateway to academic and professional success, children with undiagnosed dyscalculia are often stigmatized, misunderstood, or left behind. The effects can be long-lasting, leading to low self-esteem, disengagement from school, and the false belief that they lack intelligence, [10].

Beyond individual struggles, the over-glorification of mathematical reasoning can also delegitimize other valid intellectual approaches such as intuitive, analogical, or artistic reasoning. When only formal logic is considered respectable, disciplines that rely on subjectivity, creativity, or interpretation may be unfairly marginalized. Even in academia, some researchers internalize the idea that work not based on quantitative models is inherently less "serious."

Thus, while mathematical reasoning remains a pillar of rational inquiry, it is essential to remember that not all minds think mathematically and that's not a failure, it's a diversity. Valuing plurality in reasoning is just as important as striving for precision.

5 Conclusion

Mathematical reasoning occupies a unique and irreplaceable position in the intellectual landscape. It is a model of rigor, clarity, and internal coherence that has transformed not only the exact sciences, but has also deeply influenced the way we think across disciplines. Its capacity to eliminate ambiguity, to formalize truth, and to construct logically solid structures makes it a cornerstone of modern knowledge.

However, as this reflection has shown, mathematical reasoning is not infallible, nor is it universally applicable. Gödel's incompleteness theorems remind us that even the most refined

²Dyscalculia: A cognitive disorder, often neurological in origin, affecting mathematical abilities, unfortunately persistent at the adult age.

³Innumeracy: A temporary or acquired difficulty with mathematics, typically resulting from a lack of education or exposure to the subject, widespread in developing countries.

logical systems have boundaries. The over-reliance on mathematics as the ultimate mode of thought can unintentionally marginalize other reasoning styles, diminish human-centered disciplines, and exclude individuals whose cognitive profiles diverge from the norm.

Moreover, mathematical reasoning does not—and should not—claim monopoly over truth or meaning. Other forms of reasoning, such as induction, analogy, and dialectics, though imperfect in a mathematical sense, are essential to scientific discovery, human understanding, and educational practice. They allow us to navigate uncertainty, ask ethical questions, interpret the world's complexity, and connect with others beyond the realm of abstraction.

In the end, the value of mathematical reasoning lies not in its exclusivity, but in its complementarity. It teaches us how to reason well, but not how to feel, create, or understand the human condition. Embracing the full spectrum of reasoning is not a rejection of mathematics, it is on the contrary, a tribute to its influence and an invitation to dialogue with other forms of knowledge.

6 Future Perspectives

This reflection invites not only a reevaluation of mathematical reasoning but also a broader rethinking of how we teach, apply, and value different cognitive approaches in education and research. Several directions merit further exploration:

Interdisciplinary Education: Developing curricula that integrate mathematical rigor with philosophical inquiry, linguistic analysis, and artistic intuition could foster more holistic thinkers. Bridging the gap between abstract reasoning and human-centered reflection is a challenge worth pursuing.

Teacher Training and Inclusion: Greater awareness and training are needed to detect and support students with learning differences such as dyscalculia and mathematical anxiety. Promoting differentiated instruction and inclusive pedagogy can ensure that reasoning is not a barrier but a bridge to learning.

Epistemological Inquiry: The debate around whether mathematics is a language, a tool, or a way of thinking is far from closed. Future studies in cognitive neuroscience and the philosophy of mathematics could help refine our understanding of how the human brain processes abstract reasoning.

AI and Automated Reasoning: The rise of artificial intelligence raises fascinating questions about the nature of reasoning. Can machines "understand" mathematics the way humans do? Do they reason, or merely compute? These questions echo deeper philosophical concerns about meaning, consciousness, and truth.

Global Access to Mathematical Culture: In a world marked by educational inequality, promoting access to mathematical reasoning as a universal skill, not just an elite one, could contribute to intellectual emancipation and collective progress.

Ultimately, reflecting on mathematical reasoning opens doors far beyond mathematics itself. It allows us to better understand the nature of thought, the limits of logic, and the infinite richness of the human mind.

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