

Balancing numbers that are concatenations of three repdigits in base b

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Abstract. In this note, we show that the balancing numbers that can be represented as concatenations of three repdigits in base- b with $2 \leq b \leq 10$ are

$$\begin{aligned}
 B_2 &= 6 = (\overline{110})_2, \\
 B_3 &= 35 = (\overline{100011})_2 = (\overline{1022})_3 = (\overline{203})_4 = (\overline{120})_5, \\
 B_4 &= 204 = (\overline{540})_6 = (\overline{411})_7 = (\overline{314})_8 = (\overline{246})_9 = (\overline{204})_{10}, \\
 B_5 &= 1189 = (\overline{3316})_7 = (\overline{2245})_8 = (\overline{1189})_{10}.
 \end{aligned}$$

1 Introduction

A sequence of balancing numbers denoted by (B_n) is defined by the recurrence relation $B_n = 6B_{n-1} - B_{n-2}$ for $n \geq 2$ with initial conditions $B_0 = 0, B_1 = 1$. The closed-form formula for B_n is given by

$$B_n = \frac{\alpha^n + \beta^n}{4\sqrt{2}},$$

where $\alpha = 3 + \sqrt{8}$ and $\beta = 3 - \sqrt{8}$. The inequality

$$\alpha^{n-1} \leq B_n < \alpha^n \tag{1.1}$$

can be proved by induction. Non-negative integers whose digits are all equal are called repdigits. For integers b, a and d satisfying $2 \leq b \leq 10, 1 \leq a \leq b - 1$ and $d \geq 1$, we have the following equality

$$\underbrace{(\overline{a \dots a})}_d = \frac{a \cdot (b^d - 1)}{b - 1}.$$

Concatenations of k repdigits can be written in the form

$$\overbrace{a_1 \dots a_1 a_2 \dots a_2 \dots a_k \dots a_k}^{d_1 \text{ times } d_2 \text{ times } d_k \text{ times}},$$

where $k \geq 2, d_1, d_2, \dots, d_k \geq 1, 0 \leq a_1, a_2, a_3, \dots, a_k \leq 9$ and $a_1 \neq 0$.

In [3] and [16] the authors found all the Fibonacci and balancing numbers that are concatenations of two repdigits. In [10], the author dealt with the problem of finding the tribonacci numbers that are concatenations of two repdigits. In [13] and [14], we found all Lucas numbers, which are concatenations of two and three repdigits, respectively. Several studies on different integer sequences can be found in the literature; see, for example, [1], [2], [4], [5], [11], [17],

[18] and [19]. Moreover, investigators of [12] determined Pell and Pell-Lucas numbers formed by the concatenation of three repdigits.

Independently of the present study, Erduvan and Keskin [15] investigated the part of this problem related to balancing numbers. Using similar tools, we extend these results from base 10 to base b with $2 \leq b \leq 10$. Namely, we tackle the equation

$$B_n = \underbrace{\overline{d_1 \dots d_1}}_{m_1 \text{ times}} \cdot b^{m_2+m_3} + \underbrace{\overline{d_2 \dots d_2}}_{m_2 \text{ times}} \cdot b^{m_3} + \underbrace{\overline{d_3 \dots d_3}}_{m_3 \text{ times}}, \tag{1.2}$$

where $m_1, m_2, m_3 \geq 1$, $2 \leq b \leq 10$, $0 \leq d_1, d_2, d_3 \leq b - 1$ and $d_1 \neq 0$. Our result is the following.

Theorem 1.1. *The only balancing numbers which are concatenations of three repdigits in base b with $2 \leq b \leq 10$ are 6, 35, 204, 1189.*

2 Tools

The proof of Theorem 1.1 depends on Baker’s method and some tools from Diophantine approximation. Now, let’s recall some information. Let ζ be an algebraic number of degree d over the rationals, with minimal polynomial

$$a_0 (x - \zeta^{(1)}) (x - \zeta^{(2)}) \dots (x - \zeta^{(d)}) \in \mathbb{Z}[x].$$

Then the absolute logarithmic height of ζ is defined by

$$h(\zeta) = \frac{1}{d} \left(\log a_0 + \sum_{i=1}^d \log \left(\max \left\{ |\zeta^{(i)}|, 1 \right\} \right) \right), \tag{2.1}$$

where $a_0 > 0$. The following properties

$$h(\eta \pm \gamma) \leq h(\eta) + h(\gamma) + \log 2, \tag{2.2}$$

$$h(\eta\gamma^{\pm 1}) \leq h(\eta) + h(\gamma), \tag{2.3}$$

$$h(\eta^m) = |m|h(\eta) \tag{2.4}$$

can be found in [8]. The following theorem is given in [7] as Theorem 9.4.

Lemma 2.1. *Assume that $\gamma_1, \gamma_2, \dots, \gamma_t$ are positive real algebraic numbers in a real algebraic number field \mathbb{K} of degree D , b_1, b_2, \dots, b_t are rational integers, and*

$$\Lambda := \gamma_1^{b_1} \dots \gamma_t^{b_t} - 1$$

is not zero. Then

$$|\Lambda| > \exp \left(-1.4 \cdot 30^{t+3} \cdot t^{4.5} \cdot D^2 (1 + \log D) (1 + \log B) A_1 A_2 \dots A_t \right),$$

where

$$B \geq \max \{ |b_1|, \dots, |b_t| \} \text{ and } A_i \geq \max \{ Dh(\gamma_i), |\log \gamma_i|, 0.16 \}$$

for all $i = 1, \dots, t$.

The following two lemmas are given in [6] and [9], respectively.

Lemma 2.2. *Let M be a positive integer, let p/q be a convergent of the continued fraction of the irrational number γ such that $q > 6M$, and let A, B, μ be some real numbers with $A > 0$ and $B > 1$. Let $\epsilon := \|\mu q\| - M\|\gamma q\|$, where $\|\cdot\|$ denotes the distance from the nearest integer. If $\epsilon > 0$, then there exists no solution to the inequality*

$$0 < |u\gamma - v + \mu| < AB^{-w},$$

in positive integers u, v , and w with

$$u \leq M \text{ and } w \geq \frac{\log(Aq/\epsilon)}{\log B}.$$

Lemma 2.3. *Let $s, x \in \mathbb{R}$. If $0 < s < 1$ and $|x| < s$, then*

$$|\log(1+x)| < \frac{-\log(1-s)}{s} \cdot |x|$$

and

$$|x| < \frac{s}{1-e^{-s}} \cdot |e^x - 1|.$$

3 Proof of the Theorem 1

Assume that equations (1.2) holds. We examined the first 54 balancing numbers and found the solutions of (1.2), which are

$$\begin{aligned} B_2 &= 6 = (\overline{110})_2, \\ B_3 &= 35 = (\overline{100011})_2 = (\overline{1022})_3 = (\overline{203})_4 = (\overline{120})_5, \\ B_4 &= 204 = (\overline{540})_6 = (\overline{411})_7 = (\overline{314})_8 = (\overline{246})_9 = (\overline{204})_{10}, \\ B_5 &= 1189 = (\overline{3316})_7 = (\overline{2245})_8 = (\overline{1189})_{10} \end{aligned}$$

for $d_1, m_1, m_2, m_3 \geq 1$, $2 \leq b \leq 10$ and $0 \leq d_1, d_2, d_3 \leq b-1$. From now on, we assume that $n \geq 55$. Considering (1.2), we can write

$$B_n = \frac{d_1(b^{m_1}-1)}{b-1}b^{m_2+m_3} + \frac{d_2(b^{m_2}-1)}{b-1}b^{m_3} + \frac{d_3(b^{m_3}-1)}{b-1}, \quad (3.1)$$

or

$$B_n = \frac{1}{b-1} (d_1b^{m_1+m_2+m_3} - (d_1-d_2)b^{m_2+m_3} - (d_2-d_3)b^{m_3} - d_3). \quad (3.2)$$

Combining (1.1) and (3.1), we can see that

$$2^{m_1+m_2+m_3-1} < B_n < \alpha^n < 2^{3n},$$

and so $m_1 + m_2 + m_3 < 3n + 1$. Now, together with the closed-form formula of B_n , we can rewrite equation (3.2) as

$$\begin{aligned} \left| \frac{(b-1)\alpha^n}{4\sqrt{2}} - d_1b^{m_1+m_2+m_3} \right| &\leq \frac{(b-1)\beta^n}{4\sqrt{2}} + |d_1-d_2| \cdot b^{m_2+m_3} + |d_2-d_3| \cdot b^{m_3} + d_3 \\ &\leq \frac{(b-1)\alpha^{-n}}{4\sqrt{2}} + (b-1) \cdot b^{m_2+m_3} + (b-1) \cdot b^{m_3} + (b-1) \\ &< 1.8 \cdot (b-1) \cdot b^{m_2+m_3}, \end{aligned}$$

where we have used the fact that $n \geq 55$. From this, we obtain

$$\left| \frac{(b-1)}{4\sqrt{2}d_1} \cdot \alpha^n \cdot b^{-m_1-m_2-m_3} - 1 \right| < \frac{1.8}{b^{m_1-1}}. \quad (3.3)$$

Here, we can apply Lemma 2.1 with

$$\gamma_1 := (b-1)/(4\sqrt{2}d_1), \gamma_2 := \alpha, \gamma_3 := b$$

and $b_1 := 1, b_2 := n, b_3 := -(m_1 + m_2 + m_3)$. It is obvious that $D = 2$. Let

$$\Lambda_1 := \frac{(b-1)}{4\sqrt{2}d_1} \alpha^n b^{-m_1-m_2-m_3} - 1.$$

If $\Lambda_1 = 0$, then we get

$$\alpha^n = \frac{4\sqrt{2}d_1}{b-1} \cdot b^{m_1+m_2+m_3}.$$

Since α^{2n} is irrational for $n \geq 1$, this is not possible. So, $\Lambda_1 \neq 0$. Since

$$h(\gamma_1) = h\left(\frac{b-1}{4\sqrt{2}d_1}\right) \leq h(b-1) + h(d_1) + h(4\sqrt{2}) < 3 \log b,$$

and

$$h(\gamma_2) = h(\alpha) = \frac{\log \alpha}{2} < 0.9, h(\gamma_3) = h(b) = \log b,$$

we can choose $(A_1, A_2, A_3) := (6 \log b, 1.8, 2 \log b)$. As $m_1 + m_2 + m_3 < 3n + 1$, we can take $B := 3n + 1$. Let

$$C := 1.4 \cdot 30^6 \cdot 3^{4.5} \cdot 2^2(1 + \log 2) \cdot 1.8 \cdot 2 \log b.$$

Using (3.3) and Lemma 2.1, we conclude that

$$1.8 \cdot b^{-(m_1-1)} > |\Lambda_1| > \exp(-C \cdot (1 + \log(3n + 1)) \cdot 6 \log b),$$

i.e.,

$$m_1 \log b < 4.2 \cdot 10^{13} (\log b)^2 \log n. \tag{3.4}$$

Here, we have used the fact that $1 + \log(3n + 1) < 2 \log n$ for $n \geq 55$. We can rearrange (3.2) as

$$\frac{(b-1)\alpha^n}{4\sqrt{2}} - (d_1 b^{m_1} - (d_1 - d_2)) b^{m_2+m_3} = \frac{(b-1)\beta^n}{4\sqrt{2}} - (d_2 - d_3) b^{m_3} - d_3.$$

The above inequality tell us that

$$\begin{aligned} \left| \frac{(b-1)\alpha^n}{4\sqrt{2}} - (d_1 b^{m_1} - (d_1 - d_2)) b^{m_2+m_3} \right| &\leq \frac{(b-1)\beta^n}{4\sqrt{2}} + |d_2 - d_3| \cdot b^{m_3} + d_3 \\ &\leq \frac{(b-1)\alpha^{-n}}{4\sqrt{2}} + (b-1) \cdot b^{m_3} + (b-1) \\ &< 1.6 \cdot b^{m_3+1}. \end{aligned}$$

Therefore, we obtain

$$\left| \left(\frac{b-1}{4\sqrt{2}(d_1 b^{m_1} - (d_1 - d_2))} \right) \alpha^n b^{-m_2-m_3} - 1 \right| < \frac{1.6}{b^{m_2-1}}. \tag{3.5}$$

Taking

$$\gamma_1 := \frac{b-1}{4\sqrt{2}(d_1 b^{m_1} - (d_1 - d_2))}, \gamma_2 := \alpha, \gamma_3 := b$$

and $(b_1, b_2, b_3) := (1, n, -m_2 - m_3)$, we can use Lemma 2.1 again. Here $D = 2$. Let

$$\Lambda_2 := \left(\frac{b-1}{4\sqrt{2}(d_1 b^{m_1} - (d_1 - d_2))} \right) \alpha^n b^{-m_2-m_3} - 1.$$

By the same arguments used before for Λ_1 , we conclude that $\Lambda_2 \neq 0$. By using (2.2), (2.3), and (2.4), we get

$$\begin{aligned} h(\gamma_1) &= h \left(\frac{b-1}{4\sqrt{2}(d_1 b^{m_1} - (d_1 - d_2))} \right) \\ &\leq h(b-1) + h(4\sqrt{2}) + h(d_1 b^{m_1}) + h(d_1 - d_2) + \log 2 \\ &< 3 \log(b-1) + \frac{1}{2} \log 32 + m_1 \log b + \log 2 \\ &< 6m_1 \log b. \end{aligned}$$

So, we can choose $A_1 := 12m_1 \log b$. We also know $A_2 := 1.8$ and $A_3 := 2 \log b$. On the other hand, as $m_2 + m_3 < 3n$, we can see that $B := 3n$. Comparing (3.5) and Lemma 2.1, we have

$$1.6 \cdot b^{-(m_2-1)} > |\Lambda_2| > \exp(-C \cdot (1 + \log 3n) \cdot 12m_1 \log b),$$

or

$$m_2 \log b < 8.4 \cdot 10^{13} \log n \cdot m_1 (\log b)^2. \tag{3.6}$$

With the necessary arrangement, equality (3.2) is transformed into

$$\left| 1 - \left(\frac{4\sqrt{2}(d_1 b^{m_1+m_2} - (d_1 - d_2)b^{m_2} - (d_2 - d_3))}{b-1} \right) \alpha^{-n} b^{m_3} \right| \leq 5.66 \cdot \alpha^{-n}. \quad (3.7)$$

Taking

$$(\gamma_1, \gamma_2, \gamma_3) := \left(\frac{4\sqrt{2}(d_1 b^{m_1+m_2} - (d_1 - d_2)b^{m_2} - (d_2 - d_3))}{b-1}, \alpha, b \right),$$

and $(b_1, b_2, b_3) = (1, -n, m_3)$, we can apply Lemma 2.1. Put

$$\Lambda_3 := 1 - \left(\frac{4\sqrt{2}(d_1 b^{m_1+m_2} - (d_1 - d_2)b^{m_2} - (d_2 - d_3))}{b-1} \right) \alpha^{-n} b^{m_3}.$$

We can ensure that $\Lambda_3 \neq 0$, just as for Λ_1 . By using (2.2), (2.3), and (2.4), we obtain

$$\begin{aligned} h(\gamma_1) &= h \left(\frac{4\sqrt{2}(d_1 b^{m_1+m_2} - (d_1 - d_2)b^{m_2} - (d_2 - d_3))}{b-1} \right) \\ &\leq 4 \log(b-1) + \frac{1}{2} \log 32 + (m_1 + m_2) \log b + m_2 \log b + 2 \log 2 \\ &< 6 \log b + m_1 \log b + 2m_2 \log b. \end{aligned}$$

So, we can take $A_1 := 12 \log b + 2m_1 \log b + 4m_2 \log b$, $A_2 := 1.8$, and $A_3 := 2 \log b$ for $D = 2$. As $B \geq \max\{|1|, |-n|, |m_3|\}$, we can choose $B := 3n$. Thanks to the inequality (3.7) and Lemma 2.1, we have

$$5.66 \cdot \alpha^{-n} > |\Lambda_3| > \exp(-C \cdot (1 + \log 3n) (12 \log b + 2m_1 \log b + 4m_2 \log b)),$$

or

$$n \log \alpha - \log 5.66 < 7 \cdot 10^{12} \log n (12 \log b + 2m_1 \log b + 4m_2 \log b). \quad (3.8)$$

The inequalities (3.4), (3.6), and (3.8), give us that $n < 9.22 \cdot 10^{47}$ for $2 \leq b \leq 10$. Let

$$z_1 := (m_1 + m_2 + m_3) \log b - n \log \alpha - \log((b-1)/(4\sqrt{2}d_1)).$$

From (3.3), we get

$$|\Lambda_1| = |e^{-z_1} - 1| < \frac{1.8}{b^{m_1-1}} < 0.9,$$

for $m_1 \geq 2$. By Lemma 2.3, we have

$$|z_1| = |\log(\Lambda_1 + 1)| < \frac{\log 10}{0.9} \cdot \frac{1.8}{b^{m_1-1}} < \frac{4.61}{b^{m_1-1}},$$

which yields

$$0 < \left| (m_1 + m_2 + m_3) \frac{\log b}{\log \alpha} - n - \frac{\log((b-1)/(4\sqrt{2}d_1))}{\log \alpha} \right| < 2.62 \cdot b^{-(m_1-1)}. \quad (3.9)$$

Put

$$\gamma := \frac{\log b}{\log \alpha} \notin \mathbb{Q}, \mu := -\frac{\log((b-1)/(4\sqrt{2}d_1))}{\log \alpha}, A := 2.62, B := b,$$

and $w := m_1 - 1$. One can take $M := 2.77 \cdot 10^{48}$, since $m_1 + m_2 + m_3 < 3n + 1$. For all $2 \leq b \leq 10$, we find that $q_{107} > 6M$ and

$$\epsilon := \|\mu q_{107}\| - M \|\gamma q_{107}\| > 0.002.$$

Hence,

$$m_1 - 1 < \frac{\log(Aq_{107}/\epsilon)}{\log B} < 179.38.$$

This inequality shows that $m_1 \leq 180$. Comparing (3.6) and (3.8), we obtain $n < 7.76 \cdot 10^{33}$. Applying Lemma 2.2 again with $M := 2.33 \cdot 10^{34}$, we have $q_{76} > 6M, \epsilon > 0.001, m_1 \leq 125$ and $n < 5.34 \cdot 10^{33}$. Now, put

$$z_2 := (m_2 + m_3) \log b - n \log \alpha - \log \left(\frac{b-1}{4\sqrt{2}(d_1 b^{m_1} - (d_1 - d_2))} \right).$$

From (3.5), it follows that

$$|\Lambda_2| = |e^{-z_2} - 1| < 1.6 \cdot b^{-(m_2-1)} < 0.8$$

for $m_2 \geq 2$. By Lemma 2.3, we obtain

$$|z_2| = |\log(\Lambda_2 + 1)| < \frac{\log 5}{0.8} \cdot \frac{1.6}{b^{m_2-1}} < \frac{3.22}{b^{m_2-1}}.$$

This shows that

$$\left| (m_2 + m_3) \log b - n \log \alpha - \log \left(\frac{b-1}{4\sqrt{2}(d_1 b^{m_1} - (d_1 - d_2))} \right) \right| < 3.22 \cdot b^{-(m_2-1)}$$

i.e.,

$$0 < \left| \frac{(m_2 + m_3) \log b}{\log \alpha} - n - \frac{\log \left((b-1)/(4\sqrt{2})(d_1 b^{m_1} - (d_1 - d_2)) \right)}{\log \alpha} \right| < 1.83 \cdot b^{-(m_2-1)}. \quad (3.10)$$

Taking $\gamma := \frac{\log b}{\log \alpha}$ and $m_2 + m_3 < M := 1.61 \cdot 10^{34}$, we find that $q_{89} > 6M$. Furthermore, we can choose

$$\mu := -\frac{\log \left((b-1)/(4\sqrt{2})(d_1 b^{m_1} - (d_1 - d_2)) \right)}{\log \alpha}, A := 1.83, B := b$$

and $w := m_2 - 1$ in Lemma 2.2. We compute that

$$\epsilon := \|\mu q_{89}\| - M \|\gamma q_{89}\| > 0.00001.$$

for $1 \leq m_1 \leq 125, 1 \leq d_1 \leq b-1$ and $0 \leq d_2 \leq b-1$. Thus, we conclude that

$$m_2 - 1 < \frac{\log(Aq_{89}/\epsilon)}{\log B} < 144.38.$$

Therefore, $m_2 \leq 145$. As $m_1 \leq 125$ and $m_2 \leq 145$, substituting this upper bounds for m_1 and m_2 into (3.8), we obtain $n < 3.11 \cdot 10^{17}$. Applying Lemma 2.2 again with $M := 9.33 \cdot 10^{17}$, we have $q_{47} > 6M, \epsilon > 0, m_2 \leq 91$ and $n < 2.29 \cdot 10^{17}$. Now, let

$$z_3 := m_3 \log b - n \log \alpha + \log \left(\frac{4\sqrt{2}(d_1 b^{m_1+m_2} - (d_1 - d_2) b^{m_2} - (d_2 - d_3))}{b-1} \right).$$

From (3.7), we can write

$$|\Lambda_3| = |e^{z_3} - 1| < 5.66 \cdot \alpha^{-n} < 0.01$$

for $n \geq 55$. By Lemma 2.3, we have

$$|z_3| = |\log(\Lambda_3 + 1)| < \frac{\log(100/99)}{0.01} \cdot \frac{5.66}{\alpha^n} < \frac{5.7}{\alpha^n},$$

which shows that

$$0 < \left| m_3 \frac{\log b}{\log \alpha} - n + \frac{\log \left(4\sqrt{2}(d_1 b^{m_1+m_2} - (d_1 - d_2) b^{m_2} - (d_2 - d_3)) / (b-1) \right)}{\log \alpha} \right| < 3.24 \cdot \alpha^{-n}. \quad (3.11)$$

Put $\gamma := \frac{\log b}{\log \alpha}$. As $m_3 < M := 6.87 \cdot 10^{17}$, we find that q_{76} , the denominator of the 76 th convergent of γ exceeds $6M$. Taking

$$\mu := \frac{\log \left(4\sqrt{2} (d_1 b^{m_1+m_2} - (d_1 - d_2) b^{m_2} - (d_2 - d_3)) / (b - 1) \right)}{\log \alpha}$$

and considering the fact that $1 \leq m_1 \leq 125$, $1 \leq m_2 \leq 91$, $1 \leq d_1 \leq b-1$ and $0 \leq d_2, d_3 \leq b-1$, we obtain

$$\epsilon := \left| \mu q_{76} \right| - M \left| \gamma q_{76} \right| > 0.00002.$$

Let $A := 3.24$, $B := \alpha$, and $w := n$ in Lemma 2.2. Hence,

$$n < \frac{\log(Aq_{76}/\epsilon)}{\log B} < 54.69,$$

and so $n \leq 54$. This contradicts our assumption that $n \geq 55$. This completes the proof.

4 Conclusion

The aim of this paper is to investigate balancing numbers that can be represented as concatenations of three repdigits in base b , where $2 \leq b \leq 10$. Our results show that only the balancing numbers B_2, B_3, B_4 and B_5 satisfy these conditions, and for each of them all possible bases and corresponding concatenations are explicitly determined. The proof relies on bounds derived from linear forms in logarithms, which allow us to effectively limit the size of potential solutions. Moreover, the resulting large upper bounds are further reduced by means of a reduction method, yielding sharp bounds suitable for a complete computational verification. Consequently, no other balancing numbers can be written as concatenations of three repdigits in the prescribed range of bases.

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